Money and Velocity During Financial Crises: From the Great Depression to the Great Recession

Richard G. Anderson
Senior Research Fellow, School of Business and Entrepreneurship
Lindenwood University, St Charles, Missouri, rganderson@alum.mit.edu

Michael Bordo
Rutgers University
National Bureau of Economic Research
Hoover Institution, Stanford University
bordo@econ.rutgers.edu

John V. Duca*
Associate Director of Research and Vice President, Research Department
Federal Reserve Bank of Dallas, P.O. Box 655906, Dallas, TX 75265
john.v.duca@dal.frb.org
and Adjunct Professor, Southern Methodist University, Dallas, TX

November 18, 2016

Abstract
This study models the demand for a broad monetary aggregate (M2) from the Great Depression through the Great Recession. Key to the model is the interaction between a measure of time-variation in economic agents’ perceived financial risk and an index of the cost of portfolio adjustment. The finding of a useful money demand relationship suggests that skepticism regarding the indicator role of a broad, liquid money aggregate as a policy guide may be exaggerated. Further, our model provides some guidance for policymakers who face the challenge of unwinding large balance sheets as risk premia return to normal and velocity adjusts.

JEL codes: E410, E500, G11

Key words: money demand, financial crises, monetary policy, liquidity, financial innovation

*Corresponding author. We thank Jens Christensen, Benjamin Doring, Samuel Reynard, and participants at the 2014 Paul Woolley Conference in Sydney, 2015 FMA European Conference, 2015 Bundesbank Workshop on Central Banks and Crises—Historical Perspectives, the 2015 Swiss Society for Financial Market Research Conference, and the 2015 conference, Large-Scale Crises: 1929 vs. 2008 for suggestions and comments. We thank J.B. Cooke and Elizabeth Organ for excellent research assistance. This paper reflects our intellectual debt to many monetary economists, especially Milton Friedman, Stephen Goldfeld, Richard Porter, Robert Rasche, Anna Schwartz, and James Tobin. The views expressed are those of the authors and are not necessarily those of the Federal Reserve Banks of Dallas and St. Louis, or the Federal Reserve System. Any errors are our own.
1. Introduction

The Great Depression and the Great Recession are seen as the defining American financial crises of the past century.\(^1\) Although Federal Reserve policy was better in the latter crisis, it did not fully meet its full employment and price stability objectives as reflected in high unemployment from 2008 to 2013 and inflation being below 2 percent since 2013.\(^2\) Despite Federal Reserve efforts, the Great Recession saw a shortfall in nominal demand despite solid M2 growth (Figures 1 and 2). Our study asks, when shifts in money demand are incorporated, whether M2 growth suggests that monetary policy provided adequate liquidity during the recent crisis.

The Great Depression and the Great Recession, like many financial crises, were marked by two mutually self-reinforcing factors: sharp increases in risk premia and flights to quality. As Bagehot (1873) famously wrote, during the crises he observed from 1825 to 1866, the public could be satisfied only with gold or Bank of England notes; no other asset would do—extreme risk aversion and flight-to-quality drove everyone. Similar behavior accompanied the onset of both the Great Depression and Great Recession. Many authors have noted that financial crises tend to develop after peaks in economic activity and slowing business conditions shake confidence. These downturns might have been ordinary cyclical fluctuations except for events that triggered sharp rises in risk premia (e.g., the spread between Baa corporate and 10-year Treasury yields, Figure 3) and flights-to-quality (lower M2’s velocity, Figure 4), for examples, were signatures of the evolution of slowdowns in economic activity into the Great Depression and the Great Recession.

Our results quantify the important role of the risk premia (and also central bank policy) for the behavior of velocity before and after the crises. Fundamental to our analysis, also, are changes

---

1 In a related paper, Bordo and Haubrich (2012) discuss other American financial crises.
2 In the Great Depression, the unemployment rate peaked near 25 percent (Lebergott, 1957) and the CPI fell for five straight years for a cumulative 25 percent decline, whereas in the Great Recession, the unemployment rate peaked at 9.8 percent and the CPI registered an annual decline for only one year, dipping 0.4 percent in 2009.
Figure 1: The Fed Better—But Imperfectly—Stabilized Nominal GDP Growth in the Great Recession than in the Great Depression

Nominal GDP annual growth rate, percent

Source: Bureau of Economic Analysis.

Figure 2: M2 Declined in the Great Depression, But, Except in 2010, Rose Solidly in the Great Recession

M2 annual growth rate, percent

Sources: Board of Governors of the Federal Reserve System and Friedman and Schwartz (1970).
Figure 3: Financial Market Risk Premium Circa Two Financial Crises
(Baa - Treasury bond yield spread)

Sources: Moodys, Board of Governors of the Federal Reserve System, NBER Macrohistory Database, and authors' calculations.

Figure 4: M2 Velocity Circa Two Financial Crises
(normalized to equal 1 in 1929 and in 2006)

Sources: Bureau of Economic Analysis, Board of Governors of the Federal Reserve System, Friedman and Schwartz (1970), and authors' calculations.
in transaction costs—lower costs decrease the threshold at which investors react to movements in risk premia. In the Depression, uncertainty ebbed quickly after the Bank Holiday and President Roosevelt’s March 12, 1933, “fireside chat.” Velocity rapidly regained its earlier level following stabilization of the banking system (Friedman and Schwartz, 1963); in the Great Recession, it has not. Our empirical results also address this unusual result.

Although excess reserves rose in both crises, the reasons differ. In the Great Depression, banks increased excess reserves owing to low opportunity costs (short-term rates were near zero) and bank fears of deposit runs. In contrast, high excess reserves in the recent crisis are an intended result of the Federal Reserve’s Large Scale Asset Purchase (LSAP) program, an initiative to lower long-term interest rates via large scale purchases of long-term Treasury and mortgage-backed securities. These purchases also artificially boosted M2. First, capital and stress test requirements dissuaded banks from lending the newly created reserves, plus new liquidity requirements (e.g., the liquidity coverage ratio) induced banks to increase their holdings of high quality liquid assets, including deposits at the Federal Reserve Banks (that is, excess reserves). Second, banks were offered interest on deposits at the Federal Reserve above Treasury bill rates and, at its overnight reverse repo facility, interest near T-bill rates. These two factors further discouraged banks from reducing excess reserves. On the liability side of banks’ balance sheets, new deposits at the Federal Reserve were matched by increases in insured retail deposits in M2 (new liquidity regulations encouraged banks to reduce their use of uninsured deposits and runnable debt). Because Federal Reserve purchases of securities are made by use of deposits at the Federal Reserve banks, such purchases increase both the deposits at banks at the Federal Reserve and the deposits of bank customers at the bank—in short, quantitative easing (QE) is indirectly funded, in part, by increases in M2. This regime shift is not tracked by standard opportunity cost spreads as bond yields reflect
not only private assessments of term premia, but also new policies and regulations that lower bond yields and induce banks to hold excess reserves funded by M2 deposits. This regime shift interpretation accords with Bordo and Jonung’s view (1987, 1990, 2004) that changes in financial institutions or practices can notably affect money demand.

Judging the relative impacts on velocity is thus complicated by sorting out the build-up and later unwinding of flight-to-quality effects, as well as the effects of new policies on V2. Our study addresses these issues. Illustrating some of our findings, Figure 4 shows M2 velocity (V2) and a counterfactual path in the Great Recession based on our estimates of QE effects on V2.

This is relevant to assessing current monetary policy with monetary aggregates. Reflecting both the insights of Bagehot and lessons learned from the Great Depression, in recent years it was well-understood that financial crises require aggressive central bank intervention. A central bank typically should respond to crises by expanding its balance sheet via asset purchases or lending. After the crisis, the central bank needs to remove excess liquidity by shrinking its balance sheet at an appropriate pace, a task complicated by portfolio shifts associated with reversion of risk premia to normal levels and any unusual effects on velocity from new monetary or regulatory policies.

This paper can provide guidance to monetary policy by contributing a dynamic empirical model of the demand for broad money that well-fits the two preeminent American financial crises of the past century, and also money demand in the years between the crises, including the early-2000s period of “missing money.” By treating carefully innovations in financial intermediation and time-variation in financial-asset risk premia, our study makes visible a dynamic, non-linear interaction in which lower transaction costs (particularly for bond and equity mutual funds)
encourage more rapid portfolio rebalancing in response to shifting risk premia. Our analysis extends, to earlier periods, the models of M2 demand of Anderson and Duca (2014) that permit velocity to respond to higher uncertainty during crises and later revert to normal levels, ceteris paribus. To include the Great Depression, we extend estimates of mutual fund costs back to the 1920s and develop pre-WWII measures of average interest paid on M2 balances. By controlling for shifts in risk premia, financial innovations, and unconventional monetary policy, our framework provides a statistically sound and internally consistent way of modeling money demand in the short- and long-runs, and also reconciles the low inflation, weak nominal income growth, and moderately robust M2 growth of the recent economic recovery. The quality of our model is shown in Figure 5 by its ability to track M2 velocity since the early 1930s.

To establish these and other results, this study is organized as follows. Section 2 reviews previous studies of money demand. Section 3 discusses our empirical specification, and Section 4, the variables used. Section 5 presents our estimated dynamic error correction model and some simulations of velocity and nominal GDP. Section 6 draws implications for interpreting M2.

2. Previous Studies of Long-Run Money and Financial Crises

Studies of money over long periods must confront shifts in money demand. Traditionally, studies of the demand for broad money in the U.S. asserted or assumed that the effects of increasing financial sophistication and innovation are well-captured either within the definition of a broad money aggregate (reconstructed for earlier periods using contemporary definitions) or the path of nominal (or real) income. Although the importance of financial innovation was acknowledged, its

---

3 Two assets are “extremely close” as media of exchange if the transaction cost of exchanging one for the other is de minimis. By explicitly measuring the economically meaningful “distance” between assets with out-of-pocket costs, we reject the view that this distance is well-measured by differences in assets’ market yields (e.g., Barnett, 1980).

4 By shifting the neutral Wicksellian real rate, financial innovations and frictions challenge gauging monetary policy with the Taylor-Rule. Barsky, et al. (2014) find that the neutral real rate has varied by 7 ½ percent since 1990. Real interest rate frameworks entail tracking inflation expectations since the 1930s (Jalil and Rua, 2015), which we avoid.

5 Figure 5 and our models use various controls to address money demand shifts associated with World War II.
effects often were addressed only via exogenous dummy variables. “Breakdowns” in empirical money demand relationships, most often, were traced to the inadequacy of such variables.

The literature on the role of financial innovation is vast. Well before the recent financial crisis, Ford and Mullineux (1996) summarized the issues for money demand and financial stability:

“The recent decades, and more particularly the last two, have seen the most substantial evolution, maybe we should say revolution, in the financial and monetary sectors of the developed nations of the world. In the financial sector in the broad sense, many new types of financial claims (both assets and liabilities) have emerged. Some of these claims have appeared in what we might strictly call the banking sector. [Others] have arisen because of attempts to insure against the uncertainty in financial markets, which has become an increasingly important feature of the global economic scene. The volatility that has occurred in those markets probably owes a significant part of its existence to the integration, and liberalization, of markets that have been dominant phenomena in many western economies.”

A decade later, Federal Reserve Chairman Ben Bernanke (2006) echoed this theme⁶:

---

⁶ Bernanke (2006) notes that the Federal Reserve Board’s P* model (Hallman, et al., 1991) was developed to predict long-run inflation using long-run potential output and velocity. The model performs better when improved to account for financial innovation, specifically, the decreasing transaction cost and increasing use of bond mutual funds; see
Why have monetary aggregates not been more influential in U.S. monetary policymaking, despite the strong theoretical presumption that money growth should be linked to growth in nominal aggregates and to inflation? In practice, the difficulty has been that, in the United States, deregulation, financial innovation, and other factors have led to recurrent instability in the relationships between various monetary aggregates and other nominal variables.

…the rapid pace of financial innovation in the United States has been an important reason for the instability of the relationships between monetary aggregates and other macroeconomic variables.

…the empirical relationship between money growth and variables such as inflation and nominal output growth has continued to be unstable at times.

Historically, the economic function of financial innovation has been to increase the liquidity of otherwise less-liquid assets, i.e., to reduce the transaction costs of converting assets that are not medium-of-exchange into medium-of-exchange (e.g., Hasbrouck, 2009). Some such innovation has occurred with the regulated, chartered banks. But innovation also has occurred elsewhere, most notably in the mutual fund industry, where the costs of transferring assets into mutual fund accounts have plunged, prompting greater stock ownership rates among middle-income families.

This study focuses on the broad monetary aggregate, M2. Previous long-run money demand studies, focused on the transactions motive for money demand, generally have used M1. Friedrich and Schwartz (1970), however, argue that data quality supports only a broad money aggregate for long-run studies because before the mid-1930s there was little economic difference between different types of bank deposits because banks often waived early withdrawal penalties on time deposits. (The difference between M1 and M2 became important during the 1930s when regulatory changes introduced statutory reserve requirements by type of deposit and prohibited the payment of interest on demand deposits.) They study the behavior of a broad definition of money (currency plus all deposits held by the public at commercial banks) from the mid-1870s to the mid-

---

Footnotes:
7 Although there are a number of previous studies of long-run U.S. money demand, most such studies have used M1, not a broad monetary aggregate, e.g., Wang (2011), Lucas (1988), Stock and Watson (1993), and Bull (2001).
Two forces, prominent in their study and ours are increasing financial sophistication and increasing per capita real income—the former tending to raise velocity and the latter to reduce it. Bordo and Jonung (1987, 1990, 2004) examine the period 1880 to 2000, and Bordo, Jonung, and Siklos (1997) examine the period from the late 1800s to the late 1900s. Our research echoes theirs by stressing the roles of increasing financial sophistication and decreasing transaction costs.

An additional recent study, complementary to ours, is Benati et al. (2016), which explores demand for a narrow monetary aggregate (M1) in a panel of 31 countries since 1851, and, similar to our results, locates a stable long-run money demand functions. The design of that study—to compare and contrast a large panel of countries over a long time span—limits its focus on the United States alone. We focus herein on demand in the United States for a broad aggregate (M2) rather than M1, following Friedman and Schwartz (1970) who argued that American banking practices prior to the mid-1930s did not allow separation with confidence between transaction deposits in M1 and the liquid non-transaction deposits included in the non-M1 portion of M2. Further, since 1994, measures of M1 have been contaminated by the effects of retail deposit sweeping (Anderson and Rasche, 2001). Although the Federal Reserve has never collected from banks data regarding sweeping activity, analysts (including one author on this paper) concluded that at the time of the onset of the global financial crisis as much as half of all transaction deposits in U.S. banks were being reported to the Federal Reserve as savings deposits, not checkable deposits. The wide sweep of Benati et al also limits their exploration of admissible econometric

---

8 Friedman and Schwartz (1982) use their deposit data for 1867 to 1946, their currency data through 1942, and Federal Reserve data after. Later changes to M2’s definition make untenable using exactly the same figures. We use Rasche’s data for 1946 to 1958 and earlier data from Friedman and Schwartz, see Anderson (2003). We avoid Friedman and Schwartz’s difficulties with income and prices by using annual Department of Commerce data since 1929. We also use standard time-series methods, not the “reference phase” framework of Friedman and Schwartz.

9 See Anderson (2003) for discussion. The data in that study were published in Carter, et al. (2006).

10 At one time, the Federal Reserve Bank of St. Louis posted estimates of the amount of deposits affected by retail deposit sweeping. These estimates, prepared by Federal Reserve Board staff, were crude approximations, not compilations of reported actual amounts swept, and should not be used in empirical work.
alternatives to exploring a single equation, the relationship between the velocity of M1 and a short-
term interest rate. Here, our narrow focus on the United States permits a richer econometric
analysis that includes measures of investors’ changing perceptions of risk and of the transaction
costs of portfolio switching between M2 and equity mutual funds.

Large shocks to the underlying data generating process are the bane of all long-term empirical analyses. Financial crises are the major focus of our analysis. Wars, however, are
nuisance parameters that cannot be ignored. In our sample period, only World War II appears to have affected money demand by distorting consumption, saving, and asset holding. Velocity rose rapidly early in the war, thereafter declining steadily but slowly during 1944-46.11 These movements reflected very low short-term rates and the Federal Reserve “pegging” long-term bond yields.12 13 How to model the effects of WWII? For example, including defense spending or a dummy for the onset and lifting of price controls is unlikely to fully reflect the interaction of these effects and the expectations impacts arising from them. Also, including direct measures of defense spending raises simultaneity issues. After considerable experimentation, we proxied the effects of mobilization in 1941-1942 via a two-step intervention variable and the effects of demobilization with a separate intervention variable in 1945. Combined, the variables permit a flexible temporary shift in velocity but not a long-run effect of the war. This is issue is addressed further below.

11 In contrast, velocity fell during most of WW I. See Friedman and Schwartz (1982), chapter 5.
12 Friedman and Schwartz (1982, pp. 101-2) argue wartime real output is overstated because “price control meant that price increases took indirect and concealed forms not recorded in the indexes” and that “the large rise in price indexes when price control was repealed in 1946 consisted largely of an unveiling of the earlier concealed increase.” They argue that true average prices during the war were unobservable as some transactions occurred above controlled prices or were black market transactions. Friedman and Schwartz (1982, chapter 4) adjust prices and output over 1943-46. Owing to the “different economic circumstances” during WW I and II, they often present separate results calculated with and without the war years. They also adjust data for the 1971-73 price controls.
13 One could argue that the measured prices are accurate but, due to rationing, were not market-clearing and the 1946 increase reflected only an adjustment to those prices. Friedman and Schwartz argue that measured nominal income was less distorted by illegal activity than prices, and use the method of interpolation-by-related-series (interpolating the price index by net national product) to construct an adjusted price index for 1943-46. Although elegant, we do not pursue the Friedman and Schwartz adjustment. Friedman and Schwartz (1982, pp. 104-105) make a similar adjustment for the 1971-74 price controls. For reasons above, we do not make such an adjustment.
3. The Empirical Model: Specification

The canonical model of the demand for broad, liquid money, developed during the 1980s (Small and Porter, 1989, and Moore, Porter, and Small, 1990), represented velocity as a function of its opportunity cost, often measured as the gap between money’s own rate of return and a short-term default risk-free market yield (e.g., 6-month Treasury bill yield). Among other things, these models hinged on assumptions that (i) the income elasticity of broad money demand was unity, (ii) the structure of risk premia (across risky assets) was (largely) temporally stable (i.e., not highly time-dependent), and (iii) the effects of financial innovations were internalized within broad money aggregates. Combined, these conditions implied that the opportunity cost of M2 and its velocity were mean-reverting. The long-run equilibrium for such a model might be written as

\[
\ln V2^* - \alpha_0 - \alpha_i OC^* = 0
\]  

(1)

where \( V2^* \) denotes the unobserved, latent equilibrium velocity and \( OC^* \), the latent equilibrium opportunity cost (i.e., a spread between the average rate on M2 and a Treasury bill rate). In long-run equilibrium, \( V2^* = V2_t \) and \( OC^* = OC \). An estimable model can be written as

\[
\ln V2_t = \alpha_0 + \alpha_i \ln V2^* + \alpha_3 (OC_t - OC^*) + \epsilon_t
\]  

(2)

with short-run deviations from the long-run equilibrium as:

\[
\Delta \ln V2_t = \beta_0 + \beta_1 (\ln V2_{t-1} - \ln V2^*_{t-1}) + \beta_2 OC_{t-1} + \beta_3 \Delta \ln V2_{t-1} + \epsilon_t
\]  

(3)

The omission from the model of assets other than short-term Treasury securities asserts that portfolio substitution, as it affects M2 holdings, is between liquid broad money (M2) and short-

---

14 These models build off earlier insights from Friedman (1956), Goldfeld (1976), and Tobin (1958).
15 This practice, at times, resulted in large numbers of “monetary aggregates,” M1, M2, M3, M4 M5, M6, etc. Some financial innovation is not subsumed in the aggregate. We include these explicitly in the model, below.
16 This argument builds off Moore, Porter and Small (1990) and Hallman, Porter and Small (1991).
17 For clarity, we write the model in single-equation error-correction form and in the econometric appendix, as a four-equation VAR and a reduced-rank VECM. No loss of generality occurs by use of the former versus the latter.
term government debt. Such a specification likely was defensible until the late 1980s; see Friedman and Schwartz (1982), Lucas (1988), Meltzer (1998), Moore, Porter and Small (1990), Hallman, Porter, and Small (1991), Small and Porter (1989), Rasche (1989, 1992), and more recently Judson, et al. (2014). The accompanying short-run dynamic model, aside from dummy variables for special events, then necessarily asserts that changes in V2 reflected changes in money’s opportunity cost.

Empirical difficulties with this framework became apparent in the early 1990s, when an episode of unexpectedly high velocity became known as a period of “missing money.”18 Almost immediately, suspicion fell on two omitted variables: (i) financial innovation, and (ii) sharp swings in risk premia. Although broad M2 had successfully incorporated within itself many innovations that affected depository institution liabilities, financial innovation now was reducing the cost of asset exchanges between M2 and liquid money market instruments, and also between M2 and bond and equity mutual funds.19 Initially, this effect appeared as time variation in the elasticity of substitution between M2 and alternative assets (Duca, 2000), and was most pronounced for small-denomination time deposits (Carlson, et al., 2000).20 Experiments to “repair” the problem included narrowing M2 to omit small-denomination time deposits, “M2 minus” (e.g., Carlson, et al., 2000) or expanding M2 to include new assets, such as bond funds, in “M2 plus” (Besci and Duca, 1994).

*Ad hoc* efforts to repair M2 demand models by redefining M2 were ultimately unsuccessful. A more satisfactory approach is to explicitly model the changing margin of substitution between M2 and alternatives. The increased household holdings of mutual funds is the most important—and most neglected—financial innovation of recent decades. Mutual funds

---

18 See for example Carlson, Hoffman, Keen, and Rasche (2000) for a discussion of the missing M2 and also Goldfeld (1976) for a discussion of another period of missing money, the mid-1970s, when M1 velocity rose.
19 We are not the first to mention financial innovation as a culprit. As noted above, Friedman and Schwartz (1982) mention it throughout their analysis, often combined with the hope that omitting measures of innovation from their equations does little harm. Bordo and Jonung (1987, 2004) stress financial sophistication, which is closely related.
20 The modelling challenge presented by such shifts is not be underestimated. Judson et al (2014), for example, simply omit 1990-1993 from their regressions because the regressions do not fit the data.
are the main vehicle for most households to feasibly own a diversified portfolio of stock and bonds. Moreover, mutual fund transaction costs are *proportional*, not fixed—the type of cost that Brunner and Meltzer (1967) argued was most likely to affect portfolio behavior and, hence, money velocity. Duca (2000) was among the first to include such costs in an empirical model, arguing that omitting transfer costs between M2 and mutual funds created omitted variable bias in M2 demand models,

\[
\ln V2^* - \alpha_0 - \alpha_1 OC^* - \alpha_2 \ln(\text{load}) = 0 \tag{4}
\]

where \( \text{load} \) is the average proportional fee for transferring assets into or out of mutual funds.

Such transaction costs were not the only omitted variable afflicting money demand: portfolio theory implies risk premia matter. In the decades before the 1990s, risk premia moved within a small range; thereafter, relative to earlier decades, the swings became increasingly sharp.  

By making asset prices less predictable, these swings have altered investors’ perceptions of the liquidity of nonmonetary assets, including stocks and bonds. The importance of time-variation in the risk premia’s variance has long been noted by monetary theorists, perhaps most prominently Tobin (1958), who emphasized the speculative demand for money, and Friedman and Schwartz (1963, chapters 11 and 12), who noted the link between higher corporate bond risk premia and the fall in money velocity during the Great Depression. Empirical attempts to model such effects by including variables such as changes in stock returns had mixed success (e.g., Carlson and Schwartz, 1999; Hamburger, 1966, 1977) likely because the empirical relationship between money demand and risk premia was neither sufficiently stable nor strong (e.g., Duca, 2000).

What has not been recognized so far, is that the omitted variable problem hinges on the interaction between financial innovation and time-varying risk premia. Financial innovations that

\[21 \text{ Most analysts seek to measure risk premia in equity markets. Unfortunately, the lack of an effective risk-free rate makes difficult measurement before 1925 (see Goetzmann and Ibbotson, 2008). In more recent data, survivorship bias complicates precise measurement (see Brown, Goetzmann, and Ross, 1995).} \]
reduce asset trading costs lower both the cost of diversification and of hedging risk. As predicted in Tobin’s (1958) general equilibrium model, combined, these have altered underlying portfolio behavior with respect to how risk premia affect money demand: lower transaction costs reduce investors’ costs of responding to shifts in risk. Moreover, smaller transaction costs narrow the “no-action zone” in inventory models and raise the optimal size of each portfolio rebalancing, for a given size change in relative returns or risk. Among such events are “flight-to-quality” dynamics.

Liu (2004) constructs a model in which fixed and proportional asset transfer costs affect the optimal consumption and portfolio behavior of households with constant relative risk aversion, and concludes that portfolio shares reflect differentials in pecuniary yields between safe and risky assets (e.g., the Treasury yield premium or a corporate-Treasury bond yield differential) scaled by proportional asset transfer costs in levels. More specifically, he finds that portfolio shares approximately reflect negative linear tradeoffs between expected return differentials and proportional asset transfer costs. In log specifications, this implies that the logs of a risk premium and an asset transfer cost series enter as separate factors determining long-run equilibrium velocity:

\[
\ln V_2^+ = \alpha_0 + \alpha_1 \text{OC}_t + \alpha_2 \ln(\text{load}) + \alpha_3 \ln(\text{Baa10TR}) + \varepsilon_t
\]

where \text{Baa10TR} is the difference between the Moody’s Baa corporate and 10-year Treasury yields.

In our empirical work, we cannot reject the hypothesis that the variables \(V2, \text{load}, \text{Baa10TR}, \text{and OC}\) are usefully modeled as I(1), that is, that the first-differences are covariance stationary. Further, we cannot reject the statistical and economic significance of both variables; hence, omitting either variable induces a composite error and inconsistent parameter estimates.

---

22 Transaction costs create a no-action zone in which it is optimal not to trade until portfolio misalignment is large enough to warrant incurring transfer costs. This zone’s width is inversely proportional to the transfer cost. As proportional transfer costs (e.g., loads) fall, the zone of portfolio inaction narrows (e.g., Davis and Norman, 1990; Liu and Loewenstein, 2002; Zakamouline, 2002). The models imply that lower transfer costs make it more likely that households realign portfolios in response to a change in the relative risk or return on money versus other assets.
Empirically, the issue is manifest in the correlation between the size of the Baa-Treasury yield spread and the levels of corporate equity and bond prices. In flights to quality, the spread widens as stock and bond prices decrease and Treasury prices rise. As a result, current-period corporate yields fall and Treasury yields rise. In the future, corporate yields will increase relative to the current period and Treasury yields will fall. In such models, increases in opportunity costs are asserted to reduce the quantity demanded and increase velocity—but this need not happen after flights to quality because default and liquidity risk premia change. Although imperfect, including Baa10TR helps prevent flight-to-quality effects from biasing opportunity cost coefficients.

We complete the model by augmenting eq. (5) with a dynamic short-run equation, in the form of an error-correction model:

\[
\ln V^*_t = \alpha_0 + \alpha_1 \ln(\text{load}_t) + \alpha_2 \ln(Baa10TR_t) + \alpha_3 \text{OC}_t + \epsilon_t \\
\Delta \ln V^*_t = \beta_0 + \beta_1 \left( V^*_t - V^*_{t-1} \right) + \beta_2 \Delta \ln(\text{load}_{t-1}) + \beta_3 \Delta \ln(Baa10TR_{t-1}) + \beta_4 \Delta \text{OC}_{t-1} + \beta_5 D_t + \nu_t
\]

where \( D_t \) is a vector of dummies used as short-run controls for events not otherwise captured.

4. The Empirical Model: Data

This section explores measures of two innovations in our study: the declining costs of exchanging M2-type assets for bond and stock mutual funds, and the increasing volatility of the U.S. financial risk premia. We also discuss the conventional measure of M2’s opportunity cost.

**Mutual Fund Loads**

Exchanges into bond and equity mutual funds are the most important substitution margin for the households that hold M2-type assets.\(^{23}\) A fall in mutual fund transaction costs both tends to increase bond and stock fund ownership rates among households and to ease portfolio substitution..

---

\(^{23}\) M2 includes currency but is dominated by deposits at depositories in minimum balances under $100,000. Firms also hold these but are more likely to have low transaction cost alternatives with near-market returns, including
between M2 and those funds, altering the measured interest elasticity of money demand. Heaton and Lucas (2000) demonstrated that high asset transfer costs for households that exhibit habit formation in consumption can lower stock ownership rates. Duca (2005, 2006), using data from the Federal Reserve’s Survey of Consumer Finances, found a significant negative correlation (about –1) between average equity fund transaction costs (loads) and stock ownership, for both direct and indirect (via mutual funds) stock ownership (Figure 6). He also found that higher equity participation arose mainly from greater mutual fund ownership among middle-income families; in turn, these families M2-type assets had grown more slowly relative to high-income families.

In this study, we update and extend data on mutual fund costs used in Anderson and Duca (2014) and Duca (2000). In those studies of post-war quarterly M2 demand, transaction costs sweeping deposits into Treasury bills or institutional money market mutual funds (e.g., Fleissig and Jones, 2015). Unfortunately, little data is available regarding household versus business ownership of deposits at depositories.

![Figure 6: Equity Fund Loads Fall and Stock Ownership Rates Rise](image-url)

Sources: CDA/Wiesneberger, IBC/Donaghue, Morningstar, Duca (2005), Surveys of Consumer Finances, and authors’ calculations.
measured by bond fund loads outperformed those based on stock fund loads. This study, however, measures transaction costs using stock fund, rather than bond fund, loads, for two reasons. First, as discussed below, we use a proxy for risk premia—the spread between yields on Baa-rate corporate and 10-year Treasury bonds—which reflects the riskiness of stocks and private bonds. Second, our purpose here is to examine a longer span of time: in contrast to stock funds, bond funds do not cover the 1920s and 1930s and provide limited evidence on the 1940s. Further, although we have been able to locate only limited information exists, we conjecture that the equity funds’ charges also, perhaps, are indicative of the brokerage fees incurred by any customer seeking to directly own equity as a substitute for M2.

As in Duca (2005), we measure stock fund loads as the fee incurred when a fund is purchased (front-end load) or when a fund is sold after less than one year of ownership (back-end fee), both expressed as a proportion of assets (Figure 7). While this series cannot capture all aspects of asset transfers, for two reasons it likely well-proxies time series movements in asset transfer costs. First, empirical evidence suggests that technological change (that is, falling information technology costs), rather than economies of scale, has been the primary driver of falling mutual fund costs. Specifically, the difference between the limited (1968-1998) available information technology costs, rather than economies of scale, has been the primary driver of falling mutual fund costs. The difference between the limited (1968-1998) available data on banking sector productivity (the closest time-series proxy for financial sector productivity) and data on mutual funds costs are stationary, and weak exogeneity tests suggest that bank productivity

24 Only one bond fund existed before 1950 and it started in 1940, whereas a few stock mutual funds existed in the 1930s, with two prominent ones starting in the 1920s.
25 Among other sources, we explored the Missouri History Museum in St. Louis, which holds an extensive collection of documents related to the rise of St. Louis after the Civil War as prominent financial center. Unfortunately, only a small sample of published “retail” price schedules exist, not records of actual charges to customers.
26 We also tested loads using a 5-year horizon and/or adjusted for expense ratios (see Duca, 2005). These were significant but were outperformed by the series we use which corresponds more closely with asset transfer costs.
Granger-causes stock mutual fund loads in a long-run sense (Duca, 2005). Second, empirical evidence suggests that mutual fund costs notably influence the composition of household portfolios. Specifically, stock mutual fund costs are cointegrated with—and are highly and negatively correlated with—stock ownership rates, and weak exogeneity tests indicate causality running from long-run trends in mutual fund loads to equity participation rates (Duca, 2013).

At the aggregate time series level, several empirical patterns imply that mutual fund loads are a driver of M2 velocity. Duca (2005) found that stock mutual fund loads mainly reflect evolving financial technology. In vector error-correction models, loads were weakly exogenous with respect to the use of mutual funds to own stock but not vice versa, while loads were not weakly exogenous to financial sector productivity, but the converse was true. Granger and Lin (1995) would view such results as evidence that mutual fund use is caused, in a long-run sense, by loads, which are caused by financial technology. Similarly, below we find that stock mutual fund
loads are weakly exogenous to M2 velocity, while velocity is not weakly exogenous to stock fund loads. These results imply that trends in loads lead those in velocity, consistent with the view that asset transfer costs Granger cause money demand in a long-run sense.

**Risk Premia**

Households’ perceived risk premia is an important variable in our analysis. We measure the risk premia as the spread between yields on Baa corporate bonds and the 10-year Treasury constant-maturity yield. This measure includes compensation to investors for the higher default and liquidity risk on the Baa bond (the lowest investment grade corporate bond). Evidence suggests that, via arbitrage, it also partly measures risk in stock prices: an equity risk premium measured as the gap between the earnings-price ratio for nonfinancial corporate stocks and a real ex post bond yield is more stable when measured using the Baa corporate yield, rather than the 10-year Treasury yield (chart available upon request), suggesting a common risk factor to stocks and private bonds.

We construct our measure using the Moody’s Baa yield from the 1920s to 2015. For Treasury yields, from April 1954-2015 we use the constant maturity 10-year Treasury yield. For 1941-54, we use Federal Reserve data on the average yield on long-term U.S. Government securities, and for 1926-41 we use a separate U.S. government bond yield series. The series are spliced by matching the levels during overlapping periods with small additive adjustments. The series are compared in Figure 1 (above) for the periods of the Great Depression and the Great Recession, while entire the time series is plotted in Figure 8 below.

ADF tests suggest that shocks to our risk premia measure are persistent, that is, the hypothesis of I(1) “permanent” shocks cannot be rejected. Persistence in the risk premia measure is even more pronounced when scaled by stock fund loads (Figure 8), which according to Liu’s (2004) model, should be correlated with portfolio behavior.
In addition to the measured risk premia, our empirical baseline model includes stock loads in the cointegrating vector and the short-run dynamic equations; in the latter, it seeks to capture the time-varying sensitivity of M2’s velocity to stock returns and risk premia.

**Conventional Opportunity Cost of Money**

Our baseline specification includes a conventional measure of the opportunity cost of M2: a short-term Treasury yield minus the weighted-average measured rate of return on M2 balances. The former is measured for 1934-2015 as the bond-equivalent yield on 3-month Treasury bills and for 1926-33 as the average of the 3-month and 6-month Treasury bill rate. The average pecuniary yield on M2 for 1958-2015 is annual averages of Federal Reserve Board estimates. Prior to 1958, we calculate the average rate on demand and time deposits from OCC reports on active banks, and weight these rates by demand and time deposit shares of M2, taking into account currency...
outstanding. Because the resulting annual opportunity cost series (OC, Figure 9) has a unit root and a few negative values, its level enters the model by being a determinant of long-run velocity, thus affecting the change in velocity via the error-correction term and lagged first-difference terms.

Other Data

Our model uses annual observations 1927-2015. Nominal GDP for 1927-1928 is from Balke and Gordon (1986), thereafter from the U.S. Department of Commerce. Money stock is broad money (M2), for 1927-1945 from Friedman and Schwartz (1970), for 1946-1958 from Rasche (1992), and thereafter from the Federal Reserve Board. Interest rates are annualized Treasury constant-maturity rates on a bond-equivalent basis. The Baa rate is the average yield to

Figure 9: M2 Opportunity Costs Relative to T-Bill Rates

Opportunity cost terms use authors' calculations of M2 own rates of return, a spliced 3 month Treasury Bill rate series.

---

27 Series are spliced where they overlap with a small additive adjustment (Appendix B is available upon request).
28 We use an M2 measure as consistent as possible over our sample, with M2 as currently defined by the Federal Reserve Board. For discussion of historical US monetary aggregates, see Anderson (2003) and Carter, et al. (2006).
maturity on corporate bonds rated Baa by Moody’s Investor Services. We measure stock fund loads as in Duca (2005), equal to the proportional fee (percent of assets) levied when a mutual fund is purchased or the fee levied for withdrawing funds within a year of purchase (Figure 9).

5. The Empirical Model: Estimates

Our empirical framework is Johansen’s workhorse multivariate, reduced-rank, “cointegration” model (Johansen, 1995; Hoffman and Rasche, 1996; Juselius, 2006). Letting \( x_t \) denote a \( p \times 1 \) vector of time series, the unrestricted \( k \)-th order VAR may be written as

\[
x_t = \mu_0 + \Pi_1 x_{t-1} + \cdots + \Pi_k x_{t-k} + \Phi D_t + \varepsilon_t, \quad t = 1, \ldots, T
\]

where \( \varepsilon_t \) is \( \mathcal{N}(0, \Sigma) \), \( x_0, \ldots, x_{t-k} \) are assumed fixed, and \( D_t \) is a vector of deterministic components (e.g., dummies). It is well-known that the dynamic properties of the model are summarized by the roots of its characteristic polynomial \( (1 - \Pi_1 L - \cdots - \Pi_k L^k) = \Pi(L) \): when the VAR process is non-stationary, \( \Pi(L) \) is non-invertible, i.e., \( \Pi(L) \) must be estimated via reduced-rank methods. Results with respect to “unit roots,” “cointegration,” and “common trends” may be summarized in the roots of the companion matrix: the dynamic process governing the evolution of \( x_t \) is stationary if all eigenvalues of the companion matrix are inside the unit circle, nonstationary at least one root lies on the unit circle, and explosive if any roots lie outside the unit circle.\(^{30}\) The \( k \)-th order VAR maybe re-written, without loss of generality or imposing any restrictions on the parameters or the the likelihood function, in “vector error-correction” form as

\[
\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \cdots + \Gamma_k \Delta x_{t-k+1} + \Pi x_{t-1} + \Phi D_t + \varepsilon_t, \quad t = 1, \ldots, T
\]

\(^{29}\) We tested loads at a longer 5 year horizon and/or adjusted for expense ratios (see Duca, 2005). These variants were significant but were outperformed by the series used here which corresponds more closely with asset transfer costs.\(^{30}\) Throughout our discussion, see Johansen (1995) or Juselius (2006) for details.
where $\Pi = -(I - \Pi_1 - \cdots - \Pi_k)$ and $\Gamma_1 = -\Pi_k$. Assuming the various $\Delta x_t$ terms and $\varepsilon_t$ are stationary requires, for a well-defined regression, that $\Pi x_{t-k+1}$ is stationary, i.e., there exists a linear combination of the stationary and nonstationary terms in $x_t$ that is stationary. If $\Pi$ is of full rank $(p)$, then estimation of the VAR or VECM form is equivalent, though one may be more easily interpreted. If $\text{rank}(\Pi) = r < p$, a reduced-rank estimator is required and the formulation is further specialized by assuming that $\Pi x_{t-1} = \alpha \beta x_{t-1}$ where $\beta$ is the “cointegrating vector” and $\alpha$, the “factor loadings.” The separation of $\alpha$ and $\beta$ follows from imposing identification conditions on $\beta$, including normalizing each row on a specific variable. For details, see Juselius (2006).

**Stationarity**

Recall our baseline empirical framework is $x_t = \{v2, OC, Sload, BaaTR\}$, where $v2$ is the (natural) log of M2 velocity; $OC$ is the opportunity cost of holding M2, measured as the difference between the yield on a short-term Treasury security and the weighted-average returns on the components of M2; $Sload$ is an index number for the cost of portfolio rebalancing, measured as the log of cost of one roundtrip into and out of an equity mutual fund during a calendar year, including frontend (investment), backend (redemption), and annual management fees; and $BaaTR$ tracks investor perceived risk, measured as the log of the difference between the yield on securities rated by Moody’s Investor Services as Baa and the weighted-average yield on Treasury securities with 10 years or more to maturity. A long-run upward trend is apparent in log velocity (Figure 31).

---

31 Here, we follow the usual convention in economics of showing $\Pi x_{t-1}$ with a one-period lag. But, more generally, this may be written as $\Pi x_{t-m}$ where the lag “m” may be varied so as to best fit the data.

32 It is sometimes asserted, incorrectly, that the vector $x_t$ must contain only nonstationary terms. But, in the multivariate model, each stationary term included in the cointegrating vector will suggest an added unit root.

33 We use the log of the difference of the two rates, rather than the difference between the logs of the two rates because we interpret the variable as a measure of investors’ perceived risk conditioned on their weighted-average risk aversion.
1), and a long-run downward trend is apparent in the transaction costs of using equity mutual funds (Figure 7), both perhaps affected by improvements in information technology that have altered payment practices (and hence the demand for M2). The first-differences suggest well-behaved series with perhaps modest outliers for World War II and, in some cases, modestly long duration periods away from the mean difference.

We assess the stationarity of \( \{ x_t \} \) in four ways by: ADF test statistics (Table 1); visually examining \( \{ \Delta x_t \} \); visually examining the stochastic trends in \( \{ x_t \} \); and tests of stationarity within the ML VAR/VECM multivariate framework. Tests imply the variables are well-regarded as I(1), i.e., the differences are I(0) stationary. Details available in the Econometric Appendix.

Stochastic trends are displayed in an appendix available upon request. (The stochastic trend is defined as the partial sums of the residuals resulting from regressions of \( \{ x_t \} \) on deterministic time trends. The presence or absence of cointegration is a feature of the relationships among the stochastic trends. All four stochastic trends suggest nonstationary behavior. Further, the first three series appear to have similar stochastic trends, with the fourth trend somewhat a distorted mirror image of the other three. Overall, we judge the stochastic trends as supporting our conclusion to model the series in \( \{ x_t \} \) as I(1) and as suggestive of potential cointegration.

**Unrestricted VAR and Reduced-Rank VECM**

Estimates of the unrestricted VAR:

\[ \{ x_t \} = \{ v2, OC, Sload, BaaTR, trend \} \]

\[ \Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \cdots + \Gamma_k \Delta x_{t-k+1} + \Pi x_{t-1} + \Phi D_t + \epsilon_t, \quad t = 1, \ldots, T \]

as revealed in market equilibrium yields, rather than as an opportunity cost, although we are not aware of any general equilibrium theory that recommends the concavity induced by the log function.
are shown in an available Econometric Appendix.\textsuperscript{34} It is important that deterministic variables be chosen carefully because the presence and form of $\Phi D_t$ generally affects inference regarding cointegration rank. Deterministic terms should not be chosen solely to “soak up” large residuals or pre-ordain desired statistical results, but rather to reflect exogenous shocks from outside the economic system.\textsuperscript{35} Wars perhaps are the most difficult.\textsuperscript{36} Here, we include the following: $D_{1941}$ ($= 0.5$ in 1941 and 1942, and 0, otherwise) to reflect the onset of World War II; $D_{1946}$ ($= -0.5$ in 1945 and 1946 and 0, otherwise) to reflect the end of World War II mobilization, rationing, and price controls; $D_{1950}$ ($= 0.5$ in 1950 and 1951, -1 in 1952, and 0 otherwise) to control for Korean War price control and other effects; and $D_{1985}$ ($= 1$ in 1985, otherwise) to reflect a sharp rise in interest rates and market risk premia during the mid-1980s oil bust. All are statistically significant.

Estimates of the reduced-rank VECM are shown in section 2 of the Econometric Appendix. We conclude that $\{x_t\}$ is well-represented with one co-integrating vector (unit root) and three common (stochastic) trends. Our conclusion regarding the co-integrating space is informed by Johansen’s rank test and by examining the roots of the companion matrix.\textsuperscript{37} Normalizing on log M2 velocity, we interpret the cointegrating vector as a long-run money demand relationship.

\textit{Model Estimates}

Several variants of equations (9a, b) are presented in Table 1. The models differ with respect to samples and definition of M2. All models include the four aforementioned dummies

\textsuperscript{34} We tested a deterministic trend in the VAR. A likelihood-ratio (chi-square) test for excluding the trend had a p-value of 0.019, so we omit it.
\textsuperscript{35} We are grateful to referee for arguing that we might have exceeded this boundary in a previous version of this work.
\textsuperscript{36} Many empirical models omit the World War II years. Friedman and Schwartz (1982 instead, elaborately adjust a putatively understated national price level, thereby reducing measured GDP. Because a large-scale war is a break from a normal economy, all solutions are arbitrary. In our dataset, we did not need regime switches or other powerful breaks in the cointegrating vector; for such a model in a related context, see Anderson, Chauvet and Jones (2015).
\textsuperscript{37} It is well-known that the rank test tends to have low power and that the presence of deterministic terms shifts its asymptotic distribution to the right; hence, when deterministic terms are present, critical values may be obtained via simulation. Juselius (2006) suggests that inference via the roots of the companion matrix is a valuable complement.
and are estimated with a lag length of 3, leaving a “full-sample” period of 1931-2015. The lag length was chosen judgmentally according to three criteria: a unique cointegrating vector, a rapid speed of adjustment, and clean residuals. A time trend was not included in the models’ cointegrating vectors, but trends are permitted in the variables.

The models shown in columns 1 and 2 are based on a long-run equilibrium that includes M2’s opportunity cost, the corporate-Treasury yield spread, and stock fund loads. The model in column 3 specifies a long-run equilibrium that omits stock fund loads; the models in column 4 omit stock fund loads and the corporate-Treasury yield spread. The models in columns 1, 3, and 4 are estimated using the full sample, the model in column 2 uses a shorter pre-2006 sample to illustrate the robustness of Model 1’s coefficients to events beginning 2006 that foreshadowed the recent financial crisis. Model 3 helps assess the robustness of Model 1 to omission of the stock fund load series, while model 4 omits both stock fund loads and the corporate bond spread.

A unique and statistically significant cointegrating vector was identified in the models (Models 1 and 2) that included stock loads and bond risk premia. The estimated coefficient on each of the long-run terms was highly significant with the expected sign, and recursive estimation (Econometric Appendix) reveals that these coefficient estimates are very stable. We conclude that our baseline specification is robust to both whether it is estimated over a pre-crisis sample and in the samples including the Great Recession. Higher stock fund loads and higher corporate risk spreads reduce velocity because higher asset transfer costs lower the liquidity of non-M2 assets and raise the demand for M2. In contrast, higher opportunity costs of M2 with respect to Treasury bill rates \((OC)\) reduce the incentive to hold M2 balances, increasing velocity. The VECM coefficients on the error-correction term in regressions of the other three long-run variables indicate that they are weakly exogenous to velocity, implying that long-run trends in risk premia,
mutual fund costs, and conventional M2 opportunity costs move temporally ahead of those in velocity (Granger and Lin, 1995).

The model of M2 velocity that includes stock fund loads and the bond spread and is estimated over the full sample (Model 1) easily outperforms models 3 and 4—which omit stock fund loads. First, the SIC and H-Q test statistics clearly favor model 1, as do the log likelihood statistics corrected R² statistics. This reflects information from stock fund loads (and compared to model 5, also from corporate bond spreads) coming through the error-correction term and lagged first differences. As illustrated earlier in Figure 5, the implied equilibrium level of velocity from model 1 tracks actual velocity well. Second, the error-correction coefficient for model 1 is much larger than that for models 3 and 4.

6. Unwinding QE: M2 Velocity and the Federal Reserve’s QE Programs

We argued above that the recent behavior of M2’s velocity has been affected by the Federal Reserve “unconventional” monetary policy, including the large-scale asset purchase program (LSAP) under which it has purchased approximately $2.5 trillion in securities since early 2009. Here we explore that policy relative to our model. Specifically, we argued that a purchase by the Federal Reserve of (say) $1 billion in securities generates two new “deposits” in the financial system: $1 billion of increased deposits held by banks at the Federal Reserve (in Fedspeak, “Reserve Balances”) and $1 billion of increased deposits held by the nonbank public at banks. In other words, we assert that the “reserves multiplier” for new M2 deposits during this period, driven by increases in reserve balances, is unity. Clearly, this is a rough approximation—but with

---

38 Large-scale balance sheet expansion began with the December 2007 introduction of the Term Auction Facility to lend reserve balances to banks. In March 2009, reserve balances provided by increased Federal Reserve outright purchases of securities began to displace TAF funds. The TAF volume decreased rapidly and was ended in early 2010.

39 Throughout, we use the generic term “bank” to refer to any depository financial institution.
the aggregate volume of bank lending relatively stagnant since the global financial crisis except for draw-downs on loan commitments, this seems a reasonable assumption as the textbook “multiple expansion of deposits” has been inactive.

Also as discussed in the introduction, the Federal Reserve has unusually altered the term premium in interest rates by buying term premium risk with these policies, thereby altering the substitutability between money and bonds held by the public in ways not tracked by standard opportunity cost terms. By inducing banks to hold excess reserves, the central bank is essentially engaging in a carry-trade funded ultimately by M2 deposits. Essentially, part of M2 is funding central bank portfolio holdings of assets not private sector holdings. These subset of M2 balances are imparting a persistent upward shift in M2 holdings (and a downward shift in velocity) by altering the architecture of financial intermediation. Our model, which embodies opportunity costs as well as indexes of investor’s perceived risk and the transaction costs of acting on that risk, is well-suited to assessing the challenge of unwinding these policy actions.

To fix ideas, Figure 10 displays deposits and loans plus leases since 2006 at all U.S.: robust sustained deposit growth has accompanied sluggish bank lending and weak income (GDP) growth. Increases in reserves balances are shown in Table 3.

Figure 11 displays measured (actual, published) and an “adjusted” counterfactual M2 velocity since 1980. The counterfactual is constructed by subtracting from M2, in each year, the amount of reserve balances held by banks. Relative to the size of M2, the amount of reserve balances prior to 2008 is not material—the two series coincide (especially after 1994, as large banks aggressively started using retail deposit sweeping to reduce the amounts of reserve balances that they must hold at the Federal Reserve to satisfy statutory reserve requirements). The impact of Federal Reserve QE actions after 2007 is dramatic.
Table 3, Depository Reserve Balances at the Federal Reserve

To more carefully assess the implications of QE-induced shifts for M2 velocity, we compare forecasts from our reduced-rank VECM to two counterfactual scenarios. In Scenario 1, we assume that the Federal Reserve finds it too costly (or disruptive in markets) to shrink its balance sheet and, as a result, decides to sustain reserve balances at the 2015 level through 2025, thereby inducing no increase or decrease in M2. We assume that GDP during the same period increases at a 3 percent annual rate, approximately the consensus in the November 2016 Blue Chip survey.
In Scenario 2, we assume the Federal Reserve shrinks its balance sheet such that reserve balances decrease linearly from $2,600 billion in 2015 to $600 billion in 2025. (Of course, that may not be the complete adjustment: banks held $9 billion in reserve balances prior to the financial crisis.) To begin, Figure 12 displays two dynamic forecasts from the VECM and observed (actual) velocity: One forecast (the upper line) uses parameters estimated 1931-2006, the other (lower line) parameters estimated over the full sample (1931-2015). In both cases, the model is then simulated through 2025 (the only exogenous inputs are the dummy variables). Actual values of velocity are shown in the third line. Obviously the QE-induced post-2006 decrease in velocity matters: the model that sees no actual data after 2006 projects a fairly flat trajectory for velocity, while the model estimated with data through 2015 projects further decreases even when it has no actual data after 2006. Remarkable, perhaps, is that the model, in both simulations, “resists” following the path of actual velocity, which reached 1.48 in 2015.

---

40 The forecasts are fully dynamic across the 4-equation model. The dummy variables are the only exogenous inputs.
Figure 13 continues our analysis by comparing Scenario 1 and Scenario 2 to the VECM model forecast that is anchored in 2006 (estimated with data through 2006). Scenario 1 permits the Federal Reserve to passively “outgrow” the QE expansion—by 2025, with 3 percent annual nominal GDP growth, Scenario 1’s velocity projection intersects that from the model. In Scenario 2, an aggressive shrinkage of reserve balances at $200 billion annually—combined with the same GDP growth—causes velocity to intersect our forecast five years earlier.

Of course, both scenarios (and our model forecast) are only experiments: the real world will evolve differently. But the scenarios and forecasts emphasize that the Federal Reserve’s quantitative easing policy since 2007 (beginning in December 2007 with the Term Auction Facility) has caused substantial distortion to M2 as well as bank reserves. In this matter, a well-behaved error-correction model of M2’s velocity perhaps furnishes guidance when forming policies to return toward normalcy.
7. Conclusion

This study presents a model of U.S. broad money demand since the onset of the Great Depression. The model is able to track more than 80 years of money demand, through both the Great Depression and the Great Recession by incorporating *interactions* among three variables: (i) the traditional opportunity cost of M2, (ii) long-run declines in the transaction costs of using M2 substitutes, and (iii) a measure of financial market participants’ perceived risk. All three variables are economically and statistically significant in our long-run model. Since all three are covariance non-stationary and mutually correlated, omitting any from the empirical model would cause an implicit nonstationary disturbance and inconsistent parameter estimates. We conjecture that past “velocity shifts” and cases of “missing M2” are statistical consequences of such miss-specification that arises from not fully accounting for these important determinants of money demand.

Because our estimated dynamic model tracks velocity well over the long time period spanning the two major U.S. financial crises of the past century, it can help extract information...
important to the conduct of monetary policy from movements in the path of a broad monetary aggregate. Models that accurately track M2 velocity are particularly valuable during periods when risk premia change quickly. Following crisis periods, economic activity is returning to normal but, at the same time, velocity is increasing as risk premia recede from crisis peaks. This is particularly relevant in comparing the Great Depression and the Great Recession. The starts of these crisis periods were marked by sharp increases in risk premia and decreases in velocity. One key difference is velocity recovered rapidly in the mid-1930s as risk premia retreated, whereas the recovery in velocity after the Great Recession was offset by the impact of the Dodd-Frank Act, which induced shifts into money from other assets by altering the structure of the U.S. banking and financial system, consistent with Bordo and Jonung (1987, 1990, 2004) and Duca (2016). As illustrated in Figure 2, our model suggests that, absent the impact of Dodd-Frank and related regulatory changes, velocity likely would have recovered much as it did during the Great Depression. More specifically, average growth in M2 at a 6.5 percent annual rate since onset of the global financial crisis in 2007-08 did not translate into moderately strong nominal GDP growth in the United States because financial reform and risk premia effects worked to increase money demand and lower velocity. The estimated speeds of adjustment in our preferred model strongly suggest that further velocity declines from the Dodd-Frank act are not likely to continue. Indeed, simulations based on our model estimates indicate that velocity is likely to rise toward a somewhat higher equilibrium level in coming years, with dynamic simulations showing a smaller and later uptick in velocity than in static simulations.
References


Baumol, W., 1952. The transactions demand for cash: an inventory theoretic approach. Q. J. Econ. 66, 545-56.


CDA/Wiesenberger (b) Mutual Funds Panorama. Various annual issues, CDA Investment Technologies: Rockville, Maryland.


Committee on Banking and Currency, 1935. Summary of statements by Marriner S. Eccles, governor of the Federal Reserve Board, in reply to questions posed by members of the
Committee on Banking and Currency of the House of Representatives, at hearings on The Banking Bill of 1935. March 4-20, 1935.


Edward Elgar, New York.


IBC/Donoghue. Mutual Funds Almanac, various annual issues, IBC/Donoghue: Ashland, Massachusetts, USA.


Morningstar, Morningstar Mutual Funds, various issues.


### Table 1: Vector Error Correction Models of Log M2’s Velocity

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Model</td>
<td>Base Model</td>
<td>Omit Sload</td>
<td>Omit Sload &amp; BaaTR</td>
<td></td>
</tr>
<tr>
<td>Long-Run Equilibrium: ( \ln V_2 = \beta_1 + \beta_2 OC_1 + \beta_3 \ln Sload_1 + \beta_4 \ln BaaTR + \mu )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M2OC )</td>
<td>0.054**</td>
<td>0.059**</td>
<td>0.151**</td>
<td>0.102**</td>
</tr>
<tr>
<td></td>
<td>(2.367)</td>
<td>(3.602)</td>
<td>(3.864)</td>
<td>(3.353)</td>
</tr>
<tr>
<td>( Sload )</td>
<td>-0.132**</td>
<td>-0.186**</td>
<td>-0.253*</td>
<td>(3.825)</td>
</tr>
<tr>
<td></td>
<td>(2.931)</td>
<td>(4.872)</td>
<td></td>
<td>(2.356)</td>
</tr>
<tr>
<td>( BaaTR )</td>
<td>-0.218**</td>
<td>-0.134**</td>
<td>-0.253*</td>
<td>(3.825)</td>
</tr>
<tr>
<td></td>
<td>(3.449)</td>
<td>(2.356)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-Run: ( \Delta V_2 = \alpha_0 + \alpha_1 EC_{t-1} ) + short-run dynamics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( EC_{t-1} )</td>
<td>-0.149**</td>
<td>-0.202**</td>
<td>-0.059**</td>
<td>-0.081*</td>
</tr>
<tr>
<td></td>
<td>(4.349)</td>
<td>(3.577)</td>
<td>(3.308)</td>
<td>(2.129)</td>
</tr>
<tr>
<td>( D1941 )</td>
<td>0.141**</td>
<td>0.136**</td>
<td>0.150**</td>
<td>0.102**</td>
</tr>
<tr>
<td></td>
<td>(4.839)</td>
<td>(4.604)</td>
<td>(4.816)</td>
<td>(2.973)</td>
</tr>
<tr>
<td>( D1946 )</td>
<td>0.117*</td>
<td>0.128*</td>
<td>0.096</td>
<td>0.112*</td>
</tr>
<tr>
<td></td>
<td>(2.408)</td>
<td>(2.55)</td>
<td>(1.876)</td>
<td>(2.005)</td>
</tr>
<tr>
<td>( D1950 )</td>
<td>0.070**</td>
<td>0.061*</td>
<td>0.075**</td>
<td>0.072*</td>
</tr>
<tr>
<td></td>
<td>(2.932)</td>
<td>(2.516)</td>
<td>(2.986)</td>
<td>(2.49)</td>
</tr>
<tr>
<td>( D1985 )</td>
<td>-0.049</td>
<td>-0.055</td>
<td>-0.056</td>
<td>-0.061</td>
</tr>
<tr>
<td></td>
<td>(1.671)</td>
<td>(1.837)</td>
<td>(1.771)</td>
<td>(1.709)</td>
</tr>
<tr>
<td>( \Delta \ln V_{2,t-1} )</td>
<td>(0.378**</td>
<td>0.417**</td>
<td>0.394**</td>
<td>0.379**</td>
</tr>
<tr>
<td></td>
<td>(3.803)</td>
<td>(3.745)</td>
<td>(3.846)</td>
<td>(3.314)</td>
</tr>
<tr>
<td>( \Delta \ln V_{2,t-2} )</td>
<td>-0.019</td>
<td>-0.066</td>
<td>-0.042</td>
<td>-0.112</td>
</tr>
<tr>
<td></td>
<td>(0.211)</td>
<td>(0.663)</td>
<td>(0.441)</td>
<td>(1.047)</td>
</tr>
<tr>
<td>( \Delta \ln V_{2,t-3} )</td>
<td>-0.096</td>
<td>-0.036</td>
<td>-0.105</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>(1.124)</td>
<td>(0.381)</td>
<td>(1.17)</td>
<td>(0.165)</td>
</tr>
<tr>
<td>( R^2 ) (corrected)</td>
<td>0.61</td>
<td>0.61</td>
<td>0.556</td>
<td>0.411</td>
</tr>
<tr>
<td>( H-Q )</td>
<td>-17.59</td>
<td>-17.36</td>
<td>-10.278</td>
<td>-7.03</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>867.423</td>
<td>769.602</td>
<td>506.916</td>
<td>336.059</td>
</tr>
</tbody>
</table>

Notes: (i) Absolute t-statistics are in parentheses. **(*) denotes significant at the 99% (95%) confidence level. (ii) Long-run: Maximum likelihood estimates of the long-run equilibrium relationship: \( \ln V_2 = \beta_0 + \beta_1 OC_1 + \beta_2 \ln Sload_1 + \beta_3 \ln BaaTR + \mu \) using a four equation system with (at most) one cointegrating vector. (iii) Short-run: OLS estimates of the speed of adjustment and short-run dynamics using the estimated equilibrium correction terms in (ii). \( EC_{t-1} = V_{2,t-1} - \beta_0 OC_{1,t-1} - \beta_2 \ln Sload_{t-1} - \beta_3 \ln BaaTR_{t-1} \) (iv) Lagged first difference terms of elements in the long-run cointegrating vector and the constant in the short-run model are omitted to conserve space. (v) Lag lengths chosen to obtain unique significant vectors with sensible coefficients and clean residuals. Equal to 4 in all models.
Appendix A: Mutual Fund Data
Not Intended for Publication, Available to Readers

Because data before the mid-1980s are sketchy and incomplete, mutual fund costs were based on a sample of large mutual funds. Funds were selected if their assets were at least $1 billion at year-end 1991 if the fund existed before the mid-1980s; were at least $2 billion at year-end 1994 if the fund's inception date occurred after 1983; were at least $5 billion at year-end 2003; or were at least $250 million at year-end 1975. The first criterion reflects whether a fund was sizable during early missing M2 period of the early 1990s. The second criterion reflects whether a growing but new fund was large near the end of the missing M2 period. The third criterion reflects whether a fund remained large following the stock market bust of the early 2000s. Given the stock and bond appreciation of the early 1990s, the hurdles for newer funds were higher for the 1994 and 2003 cutoff dates to keep data gathering costs from exploding. The fourth criterion avoids excluding funds that were relatively large in 1975 from distorting averages when fewer funds existed. Also excluded were funds that were closed-end, only open to employees of a specific firm, or institutional. Also omitted are funds with high minimum balances (100,000 or more) because such high hurdles make such funds poor substitutes for M2, which is predominantly held by middle income households. 46 non-municipal bond and 133 equity mutual funds are in the sample (a list is available from the author) using data from the funds and various issues of Morningstar, IBC/Donoghue, and CDA/Wiesenberger (a, b). The aggregate load series are based on using the size of net assets under management of a fund relative to the sum of all assets managed by funds in the sample. Year-end asset data are available since 1946.

For each year over 1926-45 we proxied asset weights on each fund by its 1946 asset weight divided by the sum of all 1946 asset weights for funds that in operation after the year of their
inception. For the years before and during a fund’s inception its 1946 weight is replaced by zero. These proxied asset weights are combined with front-end loads levied by the funds (there were no funds charging deferred or back-end loads until the 1980s). Since most funds operating before 1946 charged loads between 6.5 and 8.5 percent, with the largest fund (Massachusetts Investors Trust) levying 7.5 percent, the annual weighted average load series SLD1 fluctuated in a narrow range between 7.5 and 7.8 percent over 1926-45, much as it did over 1946-1959. This suggests that the use of proxied annual asset weights before 1946 had a minimal effect on the resulting annual aggregate series. Annual expense ratios before 1946 were not available and pre-1945 expense ratios were assumed to equal their 1946 levels. As with the load series, the annual average expense ratios for 1926-45 were similar to those seen over 1946-59. Moreover, the analysis focuses on using the load series without expense ratio adjustments—as it performed better than an expense-adjusted series, consistent with the non-adjusted series entering the money demand (velocity) specifications mainly as a proxy for asset transfer costs.
Appendix B: Historical Own Rates of Return on M2 and M2 Opportunity Costs
Not Intended for Publication, Available to Readers

Conventional measures of the opportunity cost of M2 equal an average “own rate” of return on M2 minus a risk-free short-term interest rate. For the latter we spliced 1927-33 data on the average short-term (three to six months) Treasury interest rate (NBER MacroHistory DataBase) with the three month Treasury interest rate converted from a discount basis (360 days per year) to a 365 day basis. Consistent measures of the own rate of return on M2 are available from 1958 to present (source: Federal Reserve Bank of St. Louis), necessitating the construction of earlier readings. Pre-1958 readings equal the non-currency share of M2 multiplied by the average interest rate paid on deposits at financial intermediaries (banks, S&Ls, credit unions and mutual savings banks), which in turn equals the deposit-weighted average interest rate paid on demand and time deposits.

Average Interest Rates on M2 Balances

Prior to the Federal Reserve’s September 1933 implementation of the Banking Act of 1933 banks were allowed to pay interest on the demand deposits and between 1933 and 1939, there were a small and declining number of grandfathered account balances which could offer the interest. Note that the distinction between demand and time deposits was more ambiguous in the 1930s than in more recent decades because it was not until the mid-1930s that the Federal Reserve started imposing different reserve requirement ratios on the two deposit types. Hence the distinction between M1 and nonM1 M2 deposits was less clear-cut and this measurement issue was among the reasons Friedman and Schwartz preferred M2 over M1. Using data from active (i.e., not suspended) national bank mid-year reports to the OCC, the average annual interest rate on demand deposits equaled the total interest paid over the prior 12 months on demand deposits divided by
average of mid-year demand deposit balances for years t and t-1. This average rate fell from a peak of 1.21 percent in 1929 to 0.01 in 1938 and 0 thereafter.

The average annual interest rate on time deposits equaled the total interest paid over the prior 12 months on time deposits divided by the mid-year total of time deposits using national bank mid-year reports to the OCC until 1939, and from 1940-58 the average time deposit rate equaled the December reported annual total of interest paid divided by the average deposit level for that year—approximated by the average of the year t and t-1 December deposit balances. Thrift institutions (mutual savings banks, savings and loans, and credit unions) typically offered either a common share or several time deposit accounts, which typically offered somewhat higher interest rates on what are typically classified as time or savings deposits. The interest rate on share deposits at mutual savings banks (MSBs), for example, typically exceeded the average time deposit rate paid at commercial banks using interest rate data at MSBs available before 1930 and after 1945. For this reason, our measure of time deposit rates and the average own rate on M2 (available using published data from the OCC up to 1964), while consistently measured over time for national (commercial) banks, likely understates what a more ideal and comprehensive series spanning commercial banks and thrift institutions, such as that from the Federal Reserve System.

Consistent with this view, overlapping data for the period 1959-61 indicate that our pre-1958 measure understated M2 own rates by between 0.27 and 0.29 percentage points. To splice the two series, we add the 28 basis point average gap between them for 1959-61 to the pre-1958 raw average M2 yields. The resulting series is plotted in Figure 3. As a check on the splicing, we recalculated the average own rate on M2 using the balance-weighted average yield on currency, commercial (national bank) and thrifts using annually data on weights and 1929-32
and 1945-61 published data on MSB average share interest rates. The two series are plotted in Appendix Table A1. The resulting difference between this series and official Federal Reserve estimates for the 1959-61 overlap years were between 0.01 and 0.03 percentage points, implying that the splice is reasonable. In addition, the difference between the spliced and MSB-based series was about 0 between 1927 and 1930 and 1956-61, with the MSB series understating the spliced series by between 0.01 and 0.15 percentage points. Because MSB interest rate data are unavailable for 1933-45 and do not fully reflect interest rates offered at other types of thrift institutions, we use the spliced series in Figure 1 as the own rate on M2.

**Figure B1: Weighted Average (Own) Interest Rate Paid on M2 Balances**
Appendix C: Projected Paths of Key Determinants of V2 for Simulations and In-Sample Model Residuals (Not Intended for Publication, available upon request)

Projected Path for Ln Baa-Treasury Spread

Projected Path for Conventional M2 Opportunity Cost

Projected Path of Ln Stock Fund Loads
Second Stage Residuals for the first difference of logs of several main V2 determinants, each of which are clean.
Levels and Differences

Log M2 velocity

M2 opportunity cost, percent annual rate

Log stock load, percent

Log (Baa - Long-Term Treasury Yield)

First difference, log M2 velocity

First difference, M2 Own Rate

First difference, log stock load

First difference, log (Baa - Long-Term Treasury Yield)
Econometric Appendix (Not intended for publication, but available to readers).

Section 1: Unrestricted 4-Equation VAR (in VECM form)

This section presents details on estimating our preferred model of V2 using an unrestricted four-equation VAR in VECM form. Our key finding is that a unique cointegrating relationship among V2, OC, Sload, and BaaTR can be found, with well-behaved residuals.

Data vector: \( x_t = \{ V2, OC, Sload, BaaTR \} \) (t-statistics in parentheses, constant term omitted)

Model: \( \Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \Gamma_3 \Delta x_{t-3} + \Pi x_{t-1} + \Phi D_t + \epsilon_t, \ t = 1932, \ldots, 2015 \)

\[
\begin{align*}
\Pi = & \begin{bmatrix} -0.148 & 0.004 & -0.015 & -0.032 \\ 1.23 & -0.287 & 0.251 & -0.499 \\ 0.059 & -0.008 & 0.005 & -0.004 \\ -0.375 & 0.080 & -0.108 & -0.021 \\ & (2.21) & (0.68) & (0.99) & (2.96) \\ & (0.83) & (2.17) & (0.74) & (2.12) \\ & (1.43) & (2.16) & (0.58) & (0.62) \\ & (0.81) & (1.94) & (1.02) & (0.287) \\ \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\Gamma_1 = & \begin{bmatrix} 0.338 & -0.012 & -0.316 & -0.029 \\ -0.833 & 0.168 & -5.061 & -0.066 \\ -0.095 & 0.011 & 0.611 & -0.004 \\ -0.034 & 0.100 & 0.987 & 0.252 \\ & (2.91) & (1.51) & (1.93) & (1.486) \\ & (0.33) & (0.96) & (1.40) & (0.15) \\ & (1.33) & (2.29) & (6.09) & (0.33) \\ & (0.042) & (1.83) & (0.88) & (1.87) \\ \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\Gamma_2 = & \begin{bmatrix} -0.038 & 0.007 & 0.136 & 0.078 \\ 1.30 & -0.195 & -1.104 & 0.507 \\ -0.063 & 0.009 & -0.389 & 0.016 \\ -1.17 & -0.029 & -0.714 & -0.497 \\ & (.389) & (0.98) & (0.76) & (4.17) \\ & (0.60) & (1.27) & (0.28) & (1.23) \\ & (1.06) & (2.20) & (3.56) & (1.39) \\ & (1.73) & (0.60) & (0.58) & (3.86) \\ \end{bmatrix}
\end{align*}
\]

\[\text{Maximum likelihood estimates. Rats 9.1 and Cats 2.07, Windows 10.}\]
\[
\begin{array}{cccc}
\Gamma_3 = & -0.100 & -0.026 & -0.367 & -0.052 \\
& (1.04) & (3.44) & (2.34) & (2.77) \\
& -2.132 & -0.193 & -3.28 & -0.244 \\
& (1.04) & (1.15) & (0.95) & (0.59) \\
& -0.051 & 0.003 & 0.473 & -0.001 \\
& (0.90) & (0.66) & (4.91) & (0.13) \\
& -0.05 & 0.113 & 1.57 & 0.111 \\
& (0.08) & (2.16) & (1.45) & (0.854) \\
\end{array}
\]

Dummy Variables

<table>
<thead>
<tr>
<th></th>
<th>Dt1941</th>
<th>Dt1946</th>
<th>Dt1950</th>
<th>Dt1985</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.129</td>
<td>0.136</td>
<td>0.066</td>
<td>-0.041</td>
</tr>
<tr>
<td></td>
<td>(4.22)</td>
<td>(2.64)</td>
<td>(2.79)</td>
<td>(1.32)</td>
</tr>
<tr>
<td></td>
<td>0.083</td>
<td>0.324</td>
<td>0.298</td>
<td>-1.30</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.29)</td>
<td>(0.57)</td>
<td>(1.89)</td>
</tr>
<tr>
<td></td>
<td>-0.008</td>
<td>0.001</td>
<td>-0.009</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(.45)</td>
<td>(0.29)</td>
<td>(0.59)</td>
<td>(0.426)</td>
</tr>
<tr>
<td></td>
<td>-0.149</td>
<td>0.436</td>
<td>-0.111</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>(0.71)</td>
<td>(1.23)</td>
<td>(0.67)</td>
<td>(0.56)</td>
</tr>
</tbody>
</table>

Residual Analysis

(p-values in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>LM(1)</th>
<th>LM(2)</th>
<th>LM(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Multivariate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>32.5 (0.00)</td>
<td>23.1 (0.111)</td>
<td>12.0 (0.75)</td>
</tr>
<tr>
<td>ARCH</td>
<td>165.6 (0.00)</td>
<td>267.6 (0.001)</td>
<td>366.0 (.005)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ARCH</th>
<th>LM(4)</th>
<th>Normality</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Univariate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ log V2</td>
<td>11.71 (0.02)</td>
<td>0.019 (0.99)</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>ΔM2 opportunity cost</td>
<td>7.30 (0.12)</td>
<td>5.29 (0.71)</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>Δ log(stock load)</td>
<td>16.26 (0.003)</td>
<td>13.73 (0.001)</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>Δ log(Baa – Long Rate)</td>
<td>1.20 (0.89)</td>
<td>5.54 (0.63)</td>
<td>0.46</td>
<td></td>
</tr>
</tbody>
</table>

Section 2: Residuals, Standardized Residuals, ACF and Normal Density

DLV2

DM2OC

57
Section 3: The Reduced-Rank VECM (Cointegrated VAR) Model

This section presents detailed results on estimating our preferred specification of V2 using reduced-rank VECM. A critical finding is that a unique cointegrating relationship among V2, OC, Sload, and BaaTR can be found, with reasonably well-behaved residuals. Another important finding is that the long-run coefficients in the velocity relationship are stable according to recursive estimates.

Data vector: \( x_t = \{v2, OC, Sload, BaaTR\} \) (t-statistics in parentheses).

\[
\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \Gamma_3 \Delta x_{t-3} + \Pi x_{t-1} + \Phi D_t + \varepsilon_t, \quad t = 1932, \ldots, 2015
\]

\[
= \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \Gamma_3 \Delta x_{t-3} + \alpha' \beta x_{t-1} + \Phi D_t + \varepsilon_t
\]

\[\alpha' = \begin{bmatrix} -0.149 \\ -1.143 \\ 0.019 \\ -0.138 \end{bmatrix} \begin{bmatrix} \text{4.35} \\ \text{1.46} \\ \text{0.89} \\ \text{-0.57} \end{bmatrix} \]

\[\beta = \begin{bmatrix} 1.00 \\ -0.054 \\ 0.132 \\ 0.218 \end{bmatrix} \begin{bmatrix} \text{2.37} \\ \text{2.93} \\ \text{3.83} \end{bmatrix} \]

\[\Gamma_1 = \begin{bmatrix} 0.378 \\ 2.55 \\ -0.037 \\ -0.765 \end{bmatrix} \begin{bmatrix} \text{3.80} \\ \text{1.12} \\ \text{0.60} \\ \text{1.09} \end{bmatrix} \begin{bmatrix} -0.017 \\ -0.118 \\ 0.006 \\ 0.174 \end{bmatrix} \begin{bmatrix} \text{2.65} \\ \text{0.82} \\ \text{1.50} \\ \text{3.95} \end{bmatrix} \begin{bmatrix} -0.249 \\ -1.211 \\ 0.685 \\ -0.053 \end{bmatrix} \begin{bmatrix} \text{1.66} \\ \text{0.35} \\ \text{7.30} \\ \text{0.05} \end{bmatrix} \begin{bmatrix} -0.034 \\ -0.456 \\ -0.014 \\ 0.338 \end{bmatrix} \begin{bmatrix} \text{0.46} \\ \text{1.12} \\ \text{1.23} \\ \text{2.67} \end{bmatrix} \]

\[\Gamma_2 = \begin{bmatrix} -0.019 \\ 3.24 \\ -0.032 \\ -1.55 \end{bmatrix} \begin{bmatrix} \text{0.21} \\ \text{1.57} \\ \text{0.58} \\ \text{2.43} \end{bmatrix} \begin{bmatrix} 0.004 \\ -0.403 \\ 0.005 \\ 0.019 \end{bmatrix} \begin{bmatrix} \text{0.67} \\ \text{2.91} \\ \text{1.42} \\ \text{0.45} \end{bmatrix} \begin{bmatrix} 0.151 \\ -0.874 \\ -0.388 \\ -0.889 \end{bmatrix} \begin{bmatrix} \text{0.84} \\ \text{0.21} \\ \text{3.46} \\ \text{0.70} \end{bmatrix} \begin{bmatrix} 0.079 \\ 0.344 \\ 0.011 \\ -0.493 \end{bmatrix} \begin{bmatrix} \text{4.33} \\ \text{0.83} \\ \text{0.94} \\ \text{3.84} \end{bmatrix}
\[ \Gamma_3 = \begin{array}{cccc}
-0.096 & -0.030 & -0.310 & -0.062 \\
(1.12) & (4.71) & (2.08) & (3.81) \\
-0.635 & -0.428 & -0.480 & -0.775 \\
(0.32) & (0.14) & (2.65) & (2.09) \\
-0.027 & -0.001 & 0.526 & -0.013 \\
(0.50) & (0.30) & (5.63) & (1.27) \\
-0.200 & 0.177 & 0.726 & 0.255 \\
(0.331) & (3.91) & (0.69) & (2.27) \\
\end{array} \]

Dummy Variables

<table>
<thead>
<tr>
<th></th>
<th>Dt1941</th>
<th>Dt1946</th>
<th>Dt1950</th>
<th>Dt1985</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.141</td>
<td>0.117</td>
<td>0.070</td>
<td>-0.049</td>
</tr>
<tr>
<td></td>
<td>(4.84)</td>
<td>(2.41)</td>
<td>(2.93)</td>
<td>(1.67)</td>
</tr>
<tr>
<td></td>
<td>0.529</td>
<td>-0.624</td>
<td>0.339</td>
<td>-1.91</td>
</tr>
<tr>
<td></td>
<td>(0.80)</td>
<td>(0.56)</td>
<td>(0.62)</td>
<td>(2.83)</td>
</tr>
<tr>
<td></td>
<td>-0.001</td>
<td>-0.015</td>
<td>-0.008</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.50)</td>
<td>(0.51)</td>
<td>(1.16)</td>
</tr>
<tr>
<td></td>
<td>-0.314</td>
<td>0.719</td>
<td>-0.148</td>
<td>0.261</td>
</tr>
<tr>
<td></td>
<td>(1.53)</td>
<td>(2.09)</td>
<td>(0.88)</td>
<td>(1.25)</td>
</tr>
</tbody>
</table>

Residual Analysis
(p-values in parentheses)

<table>
<thead>
<tr>
<th>Multivariate</th>
<th>LM(1)</th>
<th>LM(2)</th>
<th>LM(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation</td>
<td>34.7 (0.004)</td>
<td>18.6 (0.29)</td>
<td>18.7 (0.29)</td>
</tr>
<tr>
<td>ARCH</td>
<td>138.6 (0.006)</td>
<td>231.1 (0.07)</td>
<td>353.6 (.02)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Univariate</th>
<th>ARCH LM(4)</th>
<th>Normality</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ log V2</td>
<td>9.16 (0.06)</td>
<td>0.385 (0.83)</td>
<td>0.61</td>
</tr>
<tr>
<td>ΔM2 opportunity cost</td>
<td>7.01 (0.14)</td>
<td>5.697 (0.06)</td>
<td>0.33</td>
</tr>
<tr>
<td>Δ log(stock load)</td>
<td>12.0 (0.02)</td>
<td>13.17 (0.001)</td>
<td>0.64</td>
</tr>
<tr>
<td>Δ log(Baa – Long Rate)</td>
<td>1.82 (0.77)</td>
<td>5.31 (0.07)</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Tests for Reduced Rank of $\Pi = \alpha'\beta$

Conclusion: Single cointegrating vector (one unit root)

**Rank Test**

<table>
<thead>
<tr>
<th>p-r</th>
<th>r</th>
<th>Eigenvalue</th>
<th>Trace Stat</th>
<th>Trace Stat*</th>
<th>p-value</th>
<th>p-value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>0.245</td>
<td>39.9</td>
<td>29.9</td>
<td>0.016</td>
<td>0.167</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.127</td>
<td>16.0</td>
<td>11.3</td>
<td>0.180</td>
<td>0.496</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.048</td>
<td>4.4</td>
<td>3.0</td>
<td>0.356</td>
<td>0.553</td>
</tr>
</tbody>
</table>

* :: With calculated Bartlett small sample correction

Critical values are calculated via simulation to account for deterministic variables $\Phi D_t$. 
Recursive Stability Tests

“R” denotes the concentrated model obtained by concentrating-out estimates of dynamic and deterministic terms using coefficients estimated from the entire sample period (see Juselius, 2006). Estimation period for recursions begins 1932-1972.
Beta 1 (R1-model)
Residuals, Standardized Residuals, ACF and Normal Density
## Appendix Table 1: Augmented Dickey-Fuller statistics

<table>
<thead>
<tr>
<th>Specification</th>
<th>m</th>
<th>Δm</th>
<th>p</th>
<th>Δp</th>
<th>y</th>
<th>Δy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longest Lag</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ADF t-value</td>
<td>-2.52</td>
<td>-2.72</td>
<td>-3.31</td>
<td>-6.07</td>
<td>-2.48</td>
<td>-3.36</td>
</tr>
<tr>
<td>c.v. 5%</td>
<td>-3.46</td>
<td>-1.94</td>
<td>-3.46</td>
<td>-2.89</td>
<td>-3.46</td>
<td>-1.94</td>
</tr>
<tr>
<td>c.v. 1%</td>
<td>-4.06</td>
<td>-2.59</td>
<td>-4.06</td>
<td>-3.51</td>
<td>-4.06</td>
<td>-2.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>v</th>
<th>Δv</th>
<th>M2OC</th>
<th>ΔM2OC</th>
<th>ln(SLoad)</th>
<th>Δln(SLoad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longest Lag</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ADF t-value</td>
<td>-1.60</td>
<td>-6.62</td>
<td>-2.84</td>
<td>-8.74</td>
<td>-1.89</td>
<td>-2.71</td>
</tr>
<tr>
<td>c.v. 5%</td>
<td>-3.46</td>
<td>-1.94</td>
<td>-3.46</td>
<td>-1.94</td>
<td>-3.46</td>
<td>-1.94</td>
</tr>
<tr>
<td>c.v. 1%</td>
<td>-4.06</td>
<td>-2.59</td>
<td>-4.06</td>
<td>-2.59</td>
<td>-4.06</td>
<td>-2.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baa</th>
<th>ΔBaa</th>
<th>Treasury Yield Long (TR)</th>
<th>ΔTreasury yield Long</th>
<th>ln(Baa-TR Long)</th>
<th>Δln(Baa-TR long)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longest Lag</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ADF t-value</td>
<td>-1.10</td>
<td>-6.56</td>
<td>-1.23</td>
<td>-7.44</td>
<td>-2.65</td>
<td>-8.84</td>
</tr>
<tr>
<td>c.v. 5%</td>
<td>-3.46</td>
<td>-1.94</td>
<td>-3.46</td>
<td>-1.94</td>
<td>-3.46</td>
<td>-1.94</td>
</tr>
<tr>
<td>c.v. 1%</td>
<td>-4.06</td>
<td>-2.59</td>
<td>-4.06</td>
<td>-2.59</td>
<td>-4.06</td>
<td>-2.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>M2 Own Rate</th>
<th>ΔM2 Own Rate</th>
<th>Treasury Yield Short</th>
<th>ΔTreasury Yield Short</th>
<th>m-p</th>
<th>Δ(m-p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longest Lag</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ADF t-value</td>
<td>-0.87</td>
<td>-6.57</td>
<td>-1.60</td>
<td>-7.88</td>
<td>-1.79</td>
<td>-4.23</td>
</tr>
<tr>
<td>c.v. 5%</td>
<td>-3.46</td>
<td>-1.94</td>
<td>-3.46</td>
<td>-1.94</td>
<td>-3.46</td>
<td>-1.94</td>
</tr>
<tr>
<td>c.v. 1%</td>
<td>-4.06</td>
<td>-2.59</td>
<td>-4.06</td>
<td>-2.59</td>
<td>-4.06</td>
<td>-2.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>OCST</th>
<th>YC</th>
<th>ΔYC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longest Lag</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ADF t-value</td>
<td>-9.06</td>
<td>-4.07</td>
<td>-7.76</td>
</tr>
<tr>
<td>c.v. 5%</td>
<td>-3.46</td>
<td>-3.46</td>
<td>-1.94</td>
</tr>
<tr>
<td>c.v. 1%</td>
<td>-4.06</td>
<td>-4.06</td>
<td>-2.59</td>
</tr>
</tbody>
</table>

Notes:

Levels variables are in logs. Interest rate variables are in levels except SLoad. m=M2, p=GDP price deflator, y=GDP, v=velocity of M2, TR=average yield on long-term (10-year) Treasury securities, M2OC=M2 own rate minus short-term Treasury rate, Sload=equity mutual fund front-end load, Baa=Moody’s Baa bond yield, OCST=opportunity cost of M2 relative to stock returns, YC=yield curve slope equal to yield on long Treasury minus yield on short Treasury bills.