The Knowledge Trap: Human Capital and Development Reconsidered*

Benjamin F. Jones†

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Abstract

This paper presents a model where human capital differences can explain several central phenomena in the world economy. Human capital differences emerge in the quality and quantity of skilled labor. Low quality occurs when skilled workers fail, collectively, to embody advanced knowledge. Traditional human capital accounting is shown to underestimate resulting skill differences between rich and poor nations. The theory may explain price, wage and income differences across countries and further suggests novel interpretations of immigrant outcomes, poverty traps, and the brain drain, among several other applications.

Keywords: human capital, education, technology, TFP, relative prices, wages, cross-country income differences, international trade, multinationals, poverty traps, migration

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†Kellogg School of Management and NBER. Contact: 2001 Sheridan Road, Room 609, Evanston, IL 60208. Email: bjones@kellogg.northwestern.edu.
1 Introduction

To explain several central phenomena in economics, from the wealth and poverty of nations to patterns of world trade, economists often rely on large, residual productivity differences. That is, explanations rely on some critical factor of production that is distinct from the contributions of physical and human capital. This paper presents an alternative view: I show how to put “ideas” back into people, presenting a model where human capital differences can play an expanded role in the world economy and may help explain many stylized facts.

In the model, cross-country differences emerge through the quality and quantity of skilled labor. Quality depends on the capacity of skilled workers to obtain and aggregate advanced knowledge. Quantity depends on workers’ choices to obtain higher education. The model thus characterizes human capital on two dimensions; a choice over training content (implying quality) and a choice over training duration (implying quantity). The tandem of quality and quantity drives the macroeconomic implications.

In this paper, skilled workers are seen as vessels of ideas. The notion of worker quality extends beyond the productivity of the educational production function to embrace the division of labor, which is needed to aggregate advanced knowledge. I characterize an environment where, conditional on obtaining higher education, one might obtain general skills (e.g. general medicine, general engineering) with modest knowledge about multiple tasks, or obtain specialized skills (e.g. thoracic surgery, endocrine oncology, tomographic imaging, satellite material science) with deep knowledge at a particular task. Quality advantages emerge in the collective productivity of skilled workers, where specialists working in teams aggregate greater knowledge in production.

The theory thus builds on Adam Smith’s foundational observation that the division of labor can bring high productivity. In this paper, labor division is essential for advanced production, simply because the set of advanced knowledge is too large for one person to know. The acquisition of advanced knowledge is also challenging to achieve. Three challenges are emphasized. First, deep expertise may be hard to acquire locally (e.g. university quality is low). Second, coordination costs in production may be especially high. The idea that coordination costs of teamwork limit the gains from specialization follows Becker &
be strategic complementarities in decisions to specialize, creating persistent poverty when
the initial supply of specialists is low. For any (or all) of these reasons, a low-productivity
equilibrium may persist. I call such outcomes a "knowledge trap" because the unspecialized
equilibrium features shallower collective knowledge.\(^2\)

Such quality differences exist among skilled workers. Importantly, however, the quantity
of skilled labor also adjusts. This quantity adjustment pins down the model, where, in
equilibrium, real income effects are shared equally by skilled and unskilled workers alike.
This quantity effect follows naturally when labor supply is endogenous: as more workers
become skilled to take advantage of high wage returns, the relative price (and hence relative
wages) of skilled versus unskilled services falls. This adjustment decouples the wage returns
from the quality of skilled labor.

This quantity adjustment is crucial to the macroeconomic effects. Among other impli-
cations, it poses significant challenges to traditional human capital accounting methods.
The traditional approach infers cross-country skill differences from within-country returns
to schooling, but in this model the entire wage distribution shifts, so that within-country
wage equilibria on their own say nothing about cross-country skill differences. Estimation
approaches based on immigrant behavior face similar problems. The wage gains experienced
by unskilled workers who immigrate from poor to rich countries need not be explained by
technology residuals, as some infer; in this model, unskilled wage gains follow simply because
unskilled workers, working as farm hands or taxi drivers, gain from their relative scarcity
or their complementarities with skilled workers.

In sum, human capital is viewed as the embodiment of ideas into people. Rich countries
attain deeper collective knowledge among skilled workers. Resulting adjustments in the
quantity of skilled workers mean that the real wages of skilled and unskilled workers rise
in equal proportion, even though unskilled workers have no more skill in rich than poor

Murphy (1992). More broadly, the limits to specialization considered in this paper are based on local
frictions, rather than on the extent of the market as in Smith (1776).

\(^2\)This perspective has a somewhat different emphasis from classic descriptions of specialization that
emphasize the extent of the market (the demand side) or, in more modern literature, coordination costs
as limits on specialization (Smith 1776, Becker and Murphy 1992). In those perspectives, specialization is
good when it can be achieved. By contrast, in the above perspective, specialization is essential to accessing
the stock of advanced ideas in a modern economy. That the division of labor is necessary for employing
advanced ideas in production seems inevitable when the stock of productive knowledge is too great for one
individual to know. See also Jones (2009).
countries. One thus finds a skill-based interpretation of cross-country income differences that can also get wages right, while providing interpretations for many other stylized facts about the world economy.

This paper is organized as follows. Section 2 introduces the core ideas. Section 3 presents a formal model, examining mechanisms for the existence of knowledge traps and their general equilibrium effects. Section 4 discusses several applications and relates them to existing empirical evidence in addition to new evidence about the quality of skilled workers. I show that the model provides an integrated perspective on (i) cross-country income differences, (ii) immigrant labor market outcomes, and (iii) poverty traps, as well as price phenomena, including (iv) why some goods are especially cheap in poor countries and (v) why "Mincerian" wage structures appear in all countries. Section 4 also offers possible insights about (vi) the brain drain, (vii) the role of multinationals in development and closes by discussing generalizations to inform international trade patterns, skill-biased technical change, and income divergence across countries. Section 5 concludes.

Related Literature Many existing papers explore theoretical aspects of the division of labor (e.g. Kim 1989, Becker and Murphy 1992, Garicano 2000). Other papers explore multiple equilibria in human capital (e.g. Kremer 1993, Acemoglu 1996), and still others explore specialization in intermediate goods, i.e. at the firm level, as the source of development failures (e.g. Ciccone and Matsuyama 1996, Rodriguez-Clare 1996, Acemoglu et al. 2006). A key innovation in this paper is to imagine specialization in training as a basis for different organizational forms of labor supply. More precisely, this paper imagines a two-dimensional education decision where both the breadth and duration of education are endogenous choices. There is thus a division of labor among skilled workers (based on breadth), and a division of labor between skilled and unskilled workers (based on duration).

This theoretical approach allows a reinterpretation of several empirical literatures, including the "macro-Mincer" approach in the vast development accounting literature (surveyed in Caselli 2005), which attempts to assess the role of human capital in cross-country income differences. These empirical literatures will be discussed in detail below.
2 The Core Ideas

This section introduces the core ideas in this paper. First, the quality of skilled workers is considered. Second, the quantity of skilled workers is considered. As one application, the tandem of quality and quantity differences is shown to disrupt traditional macroeconomic accounting methods, leading to an understatement of cross-country skill differences. Section 3 integrates these ideas into a formal model.

2.1 The Quality of Skilled Labor

Modern production in rich countries appears to involve an enormous variety of expert knowledge, from microprocessors to jet propulsion, from polymer synthesis to optical switches, from radiation oncology to accounting consistent with the GAAP. As measures of this differentiation, consider that the United States Patent and Trademark Office recognizes 475 different technology classes, the ISI Web of Science organizes research journals into 252 different fields, and the American Board of Medical Specialties recognizes physician certifications in 145 different areas.\(^3\)

It seems infeasible for one individual to know more than a fraction of a modern economy’s advanced knowledge. A basic challenge is then how – and whether – economies load this advanced knowledge into people’s minds. If the set of productive knowledge is greater than what one person can acquire, then the acquisition of advanced knowledge becomes a collective enterprise - it depends on a division of labor.

To fix ideas, imagine there are two tasks, \(A\) and \(B\), which are complementary in the production of a good. For example, the ultimate output could be a microprocessor, a gas turbine, or heart surgery, each of which builds on knowledge across complementary tasks.\(^4\)


\(^4\)Heart surgery requires surgical expertise (surgery), pain control (anesthesiology), as well as various complementary skills around diagnosis, infection control, and post-operative care. Microprocessor production combines microprocessor photolithography (the etching of the processor onto silicon, which draws on material science and optics), microprocessor design (including the instruction set architecture, memory, control and data path design, thermal analysis, etc), and microprocessor software (the assembler, compiler, debugger, etc) all of which draw on very different kinds of knowledge. Turbine production involves the integrated design and manufacture of turbine blades, turbofans, compressors, combustors, control systems, fuel systems, nozzles, et cetera, which draw on disparate and highly specific engineering expertise, including thermodynamics, material science, fluid mechanics, rotational and vibrational dynamics and high-heat electronics. One broadly-trained engineer working alone may be able to produce a simple integrated circuit.
Now imagine individuals must train to acquire skill. One might train as a "generalist", developing skill at both tasks. Alternatively, one might focus training on one task, becoming especially adept at that task. For simplicity, let training as a generalist produce a skill level 1 at both tasks, while training as a specialist produces a skill level $m > 1$ at one task and 0 at the other.

As an example, let production be $Y = \sqrt{H^A H^B}$ when working alone and $cY$ when pairing with another worker. This Cobb-Douglas production function captures the complementarity between skills, and the term $c < 1$ represents a coordination penalty from working in a team. Output is per unit of clock-time, and the amount of skill applied to a particular task, e.g. $H^A$, is the summation of skill applied per unit of clock-time.

In this setting, a generalist working alone does best by dividing his time equally between tasks and earning $Y = \frac{1}{2}$. A pairing of complementary specialists optimally applies each worker to their specialty, producing $Y = mc$ for every unit of clock time, or $\frac{1}{2}mc$ per team member. The specialist organizational form is therefore more productive as long as $mc > 1$; that is, as long as coordination penalties do not outweigh the benefits of deeper expertise.

A "knowledge trap" occurs when the unspecialized state is a stable equilibrium, thus failing to access frontier knowledge. In a poor country, this may occur most simply because, locally, $m$ is small. To motivate this idea, Table 1 compares the available instruction in the mechanical engineering departments of a top-ranked engineering school in East Africa (the University of Khartoum) and a top-ranked engineering school in the United States (MIT).

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[Khartoum most directly draws students from Sudan, Ethiopia, and Eritrea, countries with a combined population of 142 million. Within this part of East Africa, the University of Khartoum appears to define the upper limit for technical education; it currently enrolls 16,800 undergraduate and 6,000 graduate students, while MIT enrolls 4,300 undergraduate and 6,300 graduate students.

Makerere University in Uganda tends to be the highest rated university in Sub-Saharan Africa (outside South Africa). Makerere’s mechanical engineering department looks similar to Khartoum’s in terms of undergraduate course offerings (43) and number of faculty (21, of which 7 have Ph.D.s). The comparison in the text examines Khartoum because more complete information is available about the Khartoum curriculum.]
within mechanical engineering. Overall, MIT offers 3.4 times as many subjects, which appear more specialized, as evident from the course titles in Table 1. This comparison is conservative, in the sense that MIT has two additional departments (Aeronautics and Astronautics, Nuclear Science and Engineering) that provide 141 additional courses in subject areas where Khartoum offers a total of 2 courses. This type of evidence suggests that the advanced knowledge underlying many high-value added industries may be difficult to access through higher education in poorer countries.  

While the ability to learn narrow, deep knowledge may be limited in poor countries ($m$ is low), specialization may also be inhibited by coordination penalties ex-post in production ($c$ is low). Such coordination penalties – downstream of skill acquisition – reduce the gains from narrow expertise and may thus dissuade workers from acquiring deep knowledge at complementary tasks. Hence, most simply, poor countries may feature $mc < 1$ while a rich country has $mc > 1$, leading to potentially very large differences in the quality of skilled labor.

More subtly, the unspecialized state may persist due to thin supply of complementary specialist types. To see this, imagine being born into an economy of generalists and consider the decision to become a specialist instead. The best you could do as a lone specialist would be to pair with an existing generalist. In such a pairing, the specialist focuses on the task where they have expertise, the generalist on the other, and the optimal output is $Y = \sqrt{mc}$. The generalist would have to be paid at least their outside option, $\frac{1}{2}$, to willingly join the specialist in such a team. The most income the specialist could earn is therefore $\sqrt{mc} - \frac{1}{2}$, which itself must exceed $\frac{1}{2}$ to prefer training as a specialist. Hence the unspecialized equilibrium is stable to individual deviations if $\sqrt{mc} < 1$. We thus have a potential trap: for any coordination penalty in the range $\frac{1}{m} < c < \frac{1}{\sqrt{m}}$, mutual specialization is more productive and yet the generalist equilibrium is stable.  

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6 Based on Table 1, and looking just at mechanical engineering, these industries would appear to include the production of modern airplanes, helicopters, satellites, and ships, as well as industries that rely on automated manufacturing (e.g. modern automobile production, modern chemical manufacturing), MEMS technologies (e.g. optical switches in telecommunications, gyroscopes in smart phones, accelerometers in air bags, piezoelectronics in inkjet printers), and many others.

7 Becker and Murphy (1992) discuss numerous types of coordination costs that can inhibit the division of labor.

8 This type of knowledge trap would be resolved by mutual specialization in complementary tasks, and
I call this set of specialization failures a "knowledge trap" because skilled workers in the generalist equilibrium have shallower knowledge. While generalists may still invest substantial time in training, specialists acquire deeper knowledge about individual tasks, collectively acquiring far greater knowledge and productivity. To see the implications of these quality differences, we must further consider the quantity of skilled labor, which we turn to next.

2.2 The Quantity of Skilled Labor

We now consider the division of labor between skilled and unskilled labor, where workers must choose whether or not to become skilled. This choice connects variation in the quality of skilled workers to their resulting quantity. This nexus of quality and quantity drives the model's macroeconomic implications.

To motivate the quantity dimension, and see how it helps pin down the macroeconomy in an interesting way, it is useful to start with an application: human capital accounting. A large accounting literature has concluded that schooling variation across countries is too small to explain cross-country income differences (see Caselli 2005 for a survey). This inference is primarily drawn when computing human capital from data on the wage-schooling relationship (e.g. Klenow and Rodriguez-Clare 1997, Hall and Jones 1999). If workers are paid their marginal products, the argument goes, then wage gains from schooling would seem to inform how schooling influences productivity. Wage-schooling relationships are usually taken to follow the log-linear, i.e. "Mincerian", form (Mincer 1974),

\[ w(s) = w(s')e^{r_m(s-s')} \]  

where \( s \) is schooling duration, \( w(s) \) is the wage, and \( r_m \) is the percentage increase in the one may ask why this coordination problem isn't resolved naturally in the market, especially by firms. The implicit assumption for this mechanism is that important educational decisions are primarily made prior to the interactions of individuals and firms, so that firms cannot coordinate major educational investments but rather make production decisions given the skill set of the labor force. This seems a reasonable characterization empirically, since skilled workers (engineers, lawyers, doctors, etc.) typically train for many years in educational institutions that are distinct from firms, before entering the workforce. In this sense, it then falls to other institutions to solve this type of coordination problem. These issues will be discussed further in Section 4.

\(^9\)For example, a generalist medical doctor would know something about anesthesiology, surgery, infectious disease, oncology, psychiatry, ophthalmology, etc. Learning something about all of these areas may require a lot of education.
wage for an additional year of schooling.\footnote{Such log-linear wage-schooling relationships have been estimated in many countries around the world (see Psacharopolous 1994).}

To see how such wage relationships can be misinterpreted in inferring skill, consider what happens when the quantity of skilled workers is endogenous. In particular, define a worker’s lifetime income as

\[ y(s) = \int_s^\infty w(s)e^{-rt}dt \]  

(2)

where individuals earn no wage income during their \( s \) years of training and face a discount rate \( r \). If in equilibrium workers cannot deviate to other schooling decisions and be better off, then for any two schooling levels

\[ y(s) = y(s') \]

and therefore (1) follows with \( r_m = r \).\footnote{This arbitrage argument follows in the spirit of Mincer (1958). Integrating (2) gives \( y(s) = \frac{1}{r}w(s)e^{-rs} \) so that \( y(s) = y(s') \) implies \( w(s) = w(s')e^{r(s-s')} \). Equivalently, (1) follows if workers choose schooling duration to maximize lifetime income. That is, with \( s^* = \arg\max y(s) \) we have \( w'(s^*) = rw(s^*) \) which is just the log-linear wage structure expressed as a marginal condition.}

In this simple setting, the log-linear wage structure in (1) becomes an inevitable equilibrium, where similar wage returns become consistent with very different mappings between skill and schooling. When individuals decide whether to invest time in training, they give up wages today in exchange for higher wages later, and the wage returns become pinned down by the expected return on investment - i.e. the discount rate.\footnote{Here the interest rate and the return to schooling are equivalent. A richer model would introduce other aspects, such as ability differences, progressive marginal income tax rates, out-of-pocket costs for education, and finite time horizons which could, for example, drive the return to schooling above the real interest rate. See Heckman et al. (2005) for a broader characterization of lifetime income.}

Quality differences in education don’t appear in the wage data, because quantity decisions react to ensure the same equilibrium rate of return.

Given this quantity effect, now consider how one can interpret skill. Imagine that there are two services, service 1 (e.g. haircuts) produced by unskilled workers with no education and service 2 (e.g. engineering) that requires \( S \) years of training to perform. Imagine as above that skill, \( h \), and time, \( L \), are the only inputs to production, so that \( x_1 = h_1l_1 \) and \( x_2 = h_2l_2 \). The marginal product for each service is then \( w_1 = p_1h_1 \) and \( w_2 = p_2h_2 \), and
we have
\[ h_2 = \frac{p_1}{p_2} h_1 e^{rS} \]  \hspace{1cm} (3)
where \( w_2/w_1 = e^{rS} \) follows from the quantity decision as above.

To compare skill across countries, traditional accounting methods typically estimate skilled labor quality, \( h_2 \), as
\[ h_2 = h_1 e^{rS} \]
an approach that now appears problematic. When the quantity of skilled labor is endogenous, equilibrium wage returns hide potentially enormous quality differences. As shown in (3), one must also confront the relative prices \( (p_1/p_2) \) of skilled and unskilled labor services – where the general equilibrium shifts in quantity will be felt.\(^{13}\) Moreover, under the innocuous assumptions that poor countries are relatively abundant in low skill and that demand is downward sloping, \( p_1/p_2 \) will be relatively small in poor countries. Hence the skill gains from education \( (h_2/h_1) \) must be adjusted upwards in rich countries relative to poor countries.\(^{14}\) These observations suggest not only that wage returns do not imply skill returns, but also that the traditional method may systematically understate skill differences across countries. Skill differences may therefore play a more important role in the world economy than a large literature has suggested.

The following section presents a general equilibrium model, integrating analysis of the quality and quantity of skilled labor supply and exploring mechanisms for endogenous differences across countries. Section 4 then details several applications and reconsiders established empirical evidence from the model’s perspective.

\(^{13}\) Relative price differences across countries are large and motivate purchasing power parity (PPP) price corrections when comparing real incomes. Note that the relative prices of interest here are not easy to observe directly, since generally they are prices of intermediate service outputs, rather than finished goods. In Section 4, we will consider calibrations where production function assumptions allow estimation without observing these intermediate prices.

\(^{14}\) The standard accounting method assumes that the output of different skilled workers are perfect substitutes. In this case \( p_1 = p_2 \) (effectively, there is one good only). Under this assumption, one could estimate \( h_2/h_1 \) based purely on \( w_2/w_1 \). However, this assumption is unrealistic if we believe that worker types are less than perfect substitutes. More realistically, any number of high school students are unlikely to successfully perform angioplasty, assemble a jet engine, or write a contract consistent with the UCC. Different types of workers produce different types of goods that face downward sloping demand. Hence, skill endowments will matter in making inferences about human capital. This will be discussed further in Section 4.
3 The Model

Imagine a world where workers are born, invest in skills, and then work, possibly in teams. They can work in one of two sectors. One sector requires only unskilled labor, and output is insensitive to the education level of the worker. Output in the other sector depends on formal education.

The key decision problem for the individual is what skills to learn. Skill type is chosen to maximize expected lifetime income. Once educated, the worker enters the labor force and produces output, which occurs efficiently conditional on the education decisions made and the ability to form appropriate teams. The educational decision is thus the key to the model.

3.1 Environment

There is a continuum of individuals of measure $L$. Individuals are born at rate $r > 0$ and die with hazard rate $r$, so that $L$ is constant. Individuals are identical at birth and may either start work immediately in the unskilled sector or invest $S$ years of time to undertake education. If they choose to educate themselves, they may develop skill at two tasks, A and B. We denote an individual’s skill level $h = \{h_A, h_B\}$. An individual may choose to become a "generalist" and learn both skills, developing skill level $h = \{h, h\}$. Alternatively, one may focus on a single skill and develop deeper but narrower expertise, attaining skill level $h = \{mh, 0\}$ or $h = \{0, mh\}$ where $m > 1$.

3.1.1 Timing

For the individual, the sequence of events is:

1. The individual is born.

2. The individual makes an educational decision, becoming one of four types of workers\textsuperscript{15}

\textsuperscript{15}For simplicity, the model is developed where skilled workers – generalists or specialists – choose the same duration of education. The model could be alternatively developed where specialists undertake longer education than generalists (e.g., a Ph.D. on top of an undergraduate degree). That potentially increased level of realism increases the complexity of the exposition but does not add substantial theoretical insights and is therefore left aside.
(a) Type U workers ("unskilled") undertake no education, $s^U = 0$, and have skill level $h^U = \{0, 0\}$.

(b) Type G workers ("generalists") undertake $s^G = S$ years of education and learn both tasks, developing skill level $h^G = \{h, h\}$.

(c) Type A workers ("A-specialists") focus $s^A = S$ years on task A, developing skill level $h^A = \{mh, 0\}$.

(d) Type B workers ("B-specialists") focus $s^B = S$ years on task B, developing skill level $h^B = \{0, mh\}$.

3. The individual enters the workforce.

(a) Unskilled workers (type U) go to work immediately in the unskilled sector.

(b) Skilled workers (types G, A, B) enter the skilled sector after $S$ years and may choose to work alone or pair with other skilled workers.

i. Unpaired skilled workers randomly meet other unpaired skilled workers with hazard rate $\lambda$.

ii. If paired and your partner dies (at rate $r$), then you become unpaired again.

3.1.2 Preferences

Expected utility is given by

$$U^k = \int_0^\infty u(C^k(t))e^{-rt}dt$$

where $u(C)$ is increasing and concave. The effective rate of time preference is given by $r$, the hazard rate of death, which is equivalent to the discount rate.\textsuperscript{16} This equivalence implies that an individual’s consumption does not change across periods, by the standard Euler equation.\textsuperscript{17}

\textsuperscript{16} There is no physical capital in this model, so there is no rental rate of capital. However, there are loans, since players are born with no wealth and therefore those in school must borrow to consume. We imagine a zero-profit competitive annuity market where individuals hand over rights to their future lifetime income, $W$, upon birth in exchange for a payment, $a$, every period. This payment must be $a = rW$ by the zero profit condition. Therefore, the rate of interest on loans is the same as the hazard rate of death.

\textsuperscript{17} The Euler equation is $\frac{du(C)/dt}{u(C)} = r - r = 0$, so that $u(C)$ and hence $C$ are constant with time.
Let preferences across goods be

\[ C^k(x_1, x_2) = (\gamma x_1^\rho + (1 - \gamma) x_2^\rho)^{1/\rho} \]  \hspace{1cm} (4)

where \( x_1 \) is the good produced by the unskilled sector, \( x_2 \) is the good produced by the skilled sector, and \( \varepsilon = \frac{1}{1-\rho} \) is the elasticity of substitution between goods, which we assume is finite.

### 3.1.3 Income

The expected present value of lifetime income for a worker of type \( k \) is

\[ W^k = \int_{s^k}^{\infty} rV^k e^{-r\tau} d\tau \]  \hspace{1cm} (5)

where \( s^k \in \{0, S\} \) is the duration of education. Time subscripts are suppressed because we will focus on steady-state equilibria. \( V^k \) is the value of being a type \( k \) worker at the moment your education is finished, which is the expected value of being an unpaired worker of type \( k \). For unskilled workers, \( rV^U = w_1 \), where \( w_1 \) is the wage earned from producing the unskilled good. For skilled workers we have

\[ rV^k = w_2^k + \lambda \sum_{j \in \Omega^k} \Pr(j) (V^{kj} - V^k) \]  \hspace{1cm} (6)

The flow value of being unpaired, \( rV^k \), equals the wage from working alone, \( w_2^k \), in the skilled sector plus the expected marginal gain from a possible pairing. You meet other unpaired skilled workers at rate \( \lambda \), and the unpaired skilled worker is type \( j \) with probability \( \Pr(j) \).

We assume a uniform chance of meeting any particular unpaired skilled worker, so that

\[ \Pr(j) = \frac{L_j^k}{L_p} \]  \hspace{1cm} (7)

where \( L_j^k \) is the measure of workers of type \( j \) who are unpaired and \( L_p = \sum_j L_j^k \).\(^\text{18}\) You accept the match if \( V^{kj} \geq V^k \) and reject otherwise, which defines the "acceptance set",

\[^{18}\text{Note that this specification guarantees that the aggregate rate at which type } k \text{ people bump into type } j \text{ people } (\lambda \Pr(j)L_p^k) \text{ is the same as the rate at which type } j \text{ people bump into type } k \text{ people } (\lambda \Pr(k)L_p^j). \text{ Specifically,}

\[ \lambda \Pr(j)L_p^k = \lambda \left( \frac{L_j^k}{L_p} \right) L_p^k = \lambda \left( \frac{L_j^k}{L_p} \right) \left( \frac{L_p^k}{L_p} \right) L_p^k = \lambda \Pr(k)L_p^j \]
\[ \Omega^k \subset \{G, A, B\}, \text{ the set of types that a player of type } k \text{ is willing to match with. If you reject, you remain in the matching pool. If you accept, you leave the matching pool and earn } V^{kj}, \text{ which is defined} \]

\[ rV^{kj} = w_{2}^{kj} - r(V^{kj} - V^k) \]  

(8)

The flow value of being paired, \( rV^{kj} \), is equal to the wage you receive in this pairing, \( w_{2}^{kj} \), less the expected loss from becoming a solo worker again, which occurs when your partner dies (with probability \( r \)).

Paired workers split the value of their joint output by Nash Bargaining, dividing the joint output such that

\[ w_{2}^{kj} = \arg \max_{w_{kj}} (V^{kj} - V^k)^{1/2} (V^{jk} - V^j)^{1/2} \]  

(9)

Meanwhile, a solo worker earns the total value of his output when working alone.

3.1.4 Output

Sector 1 produces a simple good, \( x_1 \), with unskilled labor and with no advantage to skill in tasks A or B. Each worker in sector 1 produces with the technology

\[ x_1 = z \]

per unit of clock time.

Sector 2 produces a good where skill at tasks A and B matters. Workers in sector 2 may work alone or with a partner, with the production function

\[ x_2 = c(n)(H_A^n + H_B^n)^{1/\alpha}, \quad H_k = \sum_i t_i^k h_i^k \]  

(10)

where \( \sigma = \frac{1}{1-\alpha} \) is the elasticity of substitution between the two skills and we assume \( \sigma \leq 1 \), so that both inputs are necessary for positive production.\(^{19}\) The term \( c(n) \in [0,1] \) captures the coordination penalty from working in a team of size \( n \in \{1,2\} \). Without loss of generality set \( c(1) = 1 \) and \( c(2) = c \). The time devoted by individual \( i \) to task \( k \) is \( t_i^k \), and members of a team split their time across tasks to produce maximum output.

\(^{19}\) The CES production function in (10) is used for simplicity. The theory can be developed from a more general production function, \( x_2 = c(n)f(H_A, H_B) \), where \( f(H_A, H_B) \) is a symmetric, constant returns to scale function. Gross complements (\( \sigma \leq 1 \)) provides substantial tractability but is not a necessary condition for the main results.
3.2 Equilibrium

An equilibrium is a decision by each worker that maximizes her utility given the decisions of other workers. The choice involves (a) maximizing lifetime income, and (b) maximizing utility of consumption given this lifetime income. We look at equilibria where all players of skilled type \( k \) have the same matching policy \( \Omega^k \) that is constant with time.

It is convenient to define the equilibrium in terms of aggregate variables. Let \( L^k \) be the measure of living individuals who have chosen to be type \( k \), and let \( L_q \) be the measure of workers actively producing the good of type \( q \). Let \( X^S_q, X^D_q, \) and \( p_q \) respectively be the total supply, total demand, and price of good \( q \).

**Definition 1** A steady-state equilibrium consists of \( W^k, V^k, C^k, L^k \) for all worker types \( k \in \{U, G, A, B\} \); \( V^{kj}, \Omega^k, L^k_j \) for all skilled worker types \( k, j \in \{G, A, B\} \); and \( L_q, X^S_q, X^D_q, p_q \) for each good \( q \in \{1, 2\} \) such that

1. (Income maximization: Choice of worker type) \( W^k \geq W^j \) \( \forall k \in \{U, G, A, B\} \) such that \( L^k > 0 \), \( \forall j \in \{U, G, A, B\} \)
2. (Income maximization: Matching policy) \( j \in \Omega^k \) for any \( j \in \{G, A, B\} \) such that \( V^{kj} \geq V^k \), \( \forall k \in \{G, A, B\} \)
3. (Consumer optimization) \( C^k(x_1, x_2) \geq C^k(x'_1, x'_2) \) \( \forall x_1, x_2, x'_1, x'_2 \) such that \( p_1 x_1 + p_2 x_2 \leq r W^k \) and \( p_1 x'_1 + p_2 x'_2 \leq r W^k \), \( \forall k \in \{U, G, A, B\} \)
4. (Market clearing) \( X^D_q = X^S_q \) \( \forall q \in \{1, 2\} \)
5. (Steady-state) \( L^k \) is constant \( \forall k \in \{U, G, A, B\} \) and \( L^k_j \) is constant \( \forall k \in \{G, A, B\} \)

We will further focus on equilibria in the "full employment" setting, where \( \lambda \to \infty \).

3.3 Analysis

We analyze the equilibria in this model in two stages. First, we focus on the skilled sector. We investigate two different equilibria that can emerge in the organization of skilled labor, a "generalist" equilibrium and a "specialist" equilibrium. Second, we introduce the unskilled sector and demand to close the economy.
3.3.1 Organizational Equilibria in the Skilled Sector

The value of being a skilled worker of type $k$ at the moment one’s education is complete is, from (6) and (8),

$$V^k = \frac{1}{r} \frac{w^k_2 + \frac{1}{2r} \sum_{j \in \Omega^k} \Pr(j) w^{kj}}{1 + \frac{1}{2r} \sum_{j \in \Omega^k} \Pr(j)}$$

so that the value of being a type $k$ worker depends on (a) the wage you earn if you work alone, $w^k_2$, (b) the wage you can earn in pairings you are willing to accept, $w^{kj}_2$, and (c) the rate such pairings occur, $\lambda \Pr(j)$. To solve this model, we consider the wages and pairings that can be supported in equilibrium.

The equilibrium definition requires that no individual be able to deviate and earn higher income. Hence we must have $W^k = W$ for all active worker types in any equilibrium and therefore, by (5),

$$V^k = V \text{ for all } k \in \{G, A, B\}$$

That is, each type of skilled worker must have the same expected income upon finishing school. If one type did better than the others, an individual would switch to become this type.

This common value, $V$, means that in any equilibrium individuals have the same outside option when wage bargaining. Defining $x^{kj}_2$ as the maximum output individuals of type $k$ and $j$ can produce when working together, it then follows from Nash Bargaining, (9), that in any accepted pairing $V^{kj} = V^{jk}$ and

$$w^{kj}_2 = \frac{1}{2} p_2 x^{kj}_2$$

so that in equilibrium a worker team splits its joint output equally. Meanwhile, if skilled workers work alone, then they earn the total product, so that

$$w^k_2 = p_2 x^k_2$$

where $x^k_2$ is the maximum output an individual of type $k$ can produce when working alone.

These results lead to a limited set of matching behaviors that can exist in equilibrium.

**Lemma 1 (Matching Rules)** In equilibrium, matching behavior is either $\{\Omega^A, \Omega^B, \Omega^G\} = \{\{B\}, \{A\}, \{\emptyset\}\}$ or $\{\Omega^A, \Omega^B, \Omega^G\} = \{\{B, G\}, \{A, G\}, \{A, B\}\}$
Proof. See appendix.

This result states in part that types never match with themselves. This is intuitive because matching with one own’s type provides no productivity advantage but incurs coordination costs. The lemma also states that a specialist is always willing to match with the other specialist type in equilibrium. This is intuitive because an AB pairing produces the highest wages. A second, intuitive equilibrium property follows from the symmetry between specialists and their desire not to be unemployed.

**Lemma 2 (Balanced Specialists)** In equilibrium, \( L^A = L^B \).

Proof. See appendix.

This lemma limits the class of possible equilibria. If \( L^s \) is the total mass of skilled workers, then we can distinguish three potential equilibria: (1) a "generalist" equilibrium where \( \{L^A, L^B, L^G\} = \{0, 0, L^s\} \); (2) a "specialist" equilibrium where \( \{L^A, L^B, L^G\} = \{\frac{1}{2}L^s, \frac{1}{2}L^s, 0\} \); and (3) a "mixed" equilibrium where \( \{L^A, L^B, L^G\} = \{L', L', L^s - 2L'\} \) for some \( L' \) such that \( 0 < L' < \frac{1}{2}L^s \).

**Proposition 1 (Knowledge Trap)** With full employment, where \( \lambda \to \infty \), a "generalist" equilibrium exists \( \iff \) \( x^{AG}_2 \leq 2x^G_2 \) and a "specialist" equilibrium exists \( \iff \) \( x^{AB}_2 \geq 2x^G_2 \). With full employment, any "mixed" equilibrium limits to the "generalist" equilibrium. For some parameter values, both a generalist and specialist equilibrium can exist. These equilibria are summarized in Figure 1.

Proof. See appendix.

The intuition for these results is straightforward. As \( \lambda \to \infty \), workers meet at such a high rate that they match instantaneously in equilibrium and are never unemployed. Hence skilled workers choose matches based simply on wages. In the "generalist" case, skilled workers earn \( w^G_2 = p_2x^G_2 \). If a player deviates to be a specialist, say type A, then the best he can do is pair with an existing generalist and earn \( p_2x^{AG}_2 - w^G_2 \). Therefore, a world of generalists is an equilibrium iff \( p_2x^{AG}_2 - w^G_2 \leq w^G_2 \), or

\[
x^{AG}_2 \leq 2x^G_2
\]

\( ^{20} \)With full employment, the deviating player captures the joint output net of the other player’s outside wage. With finite \( \lambda \), the possibility of unemployment further affects the wage bargain - see Appendix.
In the "specialist" case, skilled workers produce in teams and earn a wage $w_{2}^{AB} = \frac{1}{2}p_2x_2^{AB}$. If a player deviates to be a generalist, then he could either (a) work alone and earn $w_2^G$ or (b) pair with an existing specialist and earn $p_2x_2^{AG} - w_{2}^{AB}$. The latter option cannot be worthwhile. In particular, since $x_2^{AG} < x_2^{AB}$, deviating to be a generalist only to pair with a specialist is not better than remaining as a specialist in the first place. We therefore only need consider the first case, where the deviating generalist works alone. Hence, this world of specialists is an equilibrium iff $w_2^G \leq w_{2}^{AB}$, or

$$x_2^{AB} \geq 2x_2^{G}$$

These existence conditions can be rewritten in terms of the model's exogenous parameters, using the production functions, where the condition for specialist stability, $x_2^{AB} \geq 2x_2^{G}$, is simply $mc \geq 1$, and the condition for generalist stability, $x_2^{Ag} \leq 2x_2^{G}$, is $mc \leq \left(\frac{2}{1 + mc}\right)^{\frac{1}{mc - 1}}$. The equilibria are plotted in Figure 1.

Figure 1: The Knowledge Trap

A country where coordination costs are low (i.e. high $c$), or the skill gains from narrow training are large (i.e. high $m$) will tend towards the specialist equilibrium. A country where
coordination costs are high or gains from focused training are modest will tend towards the
generalist equilibrium. The failure to develop deep specialists could therefore be viewed
as institutional problems, where the important policy parameters are $m$ and $c$, as will be
discussed below. There are also, however, regions of the parameter space where different
equilibria may emerge even if $m$ and $c$ are the same, providing the possibility of multiple,
pareto-ranked equilibria. In general, a country with specialized skilled workers is $mc$ times
more productive than an economy with generalist skilled workers. Moreover, the ratio of
income between generalist and specialist equilibria is potentially unbounded even where
both are stable.

**Corollary 1 (Gains from Specialization)** Output in the skilled sector is $mc$ times larger in a
"specialist" equilibrium than in a "generalist" equilibrium. Moreover, the range of potential
combinations $mc$ where both a generalist and specialist equilibria exist is unbounded from
above.

**Proof.** See appendix. ■

Note the important roles of (1) coordination costs and (2) task complementarity in sup-
porting a sub-optimal generalist equilibrium. Deviating to become a specialist only to pair
with an existing generalist is less appealing when coordination costs are high (i.e. smaller $c$)
or complementarities of tasks are high (i.e. smaller $\sigma$). With sufficient coordination costs or
complementarity, $m$ (and hence $mc$) can become unboundedly large, so that the generalist
case is stable even though the specialist organization produces unboundedly higher income.
For example, with Leontief task aggregation ($\sigma = 0$), $mc$ can be unboundedly large for
arbitrarily small coordination costs.

Lastly, note the role of a "thick market" problem for supporting a robust generalist
equilibrium despite large $mc$. The generalist equilibrium is stable to the extent that find-
ing a complementary specialist type is challenging were you to deviate yourself. With
finite $\lambda$, the generalist equilibrium is stable to trembles where positive masses of specialists
appear, because the search friction impedes easy matching. The convenient case of "full
employment", where $\lambda \to \infty$, is the limit of trembling hand perfect equilibria.\footnote{In the limit, the model still features a "needle in a haystack" friction where, although search is extremely}
3.3.2 The Equilibrium Economy

Given the possible organizational equilibria in the skilled sector, we now consider the influence of this organizational equilibrium on the economy at large. Denote with the superscript \( n \) the organizational equilibrium in the skilled sector, where \( n = G \) defines the "generalist" outcome and \( n = AB \) defines the "specialist" outcome. The equilibrium in the skilled sector will influence the endogenous outcomes in both the skilled and unskilled sectors, including labor allocations, prices, and wages.

The first result concerns wages.

**Lemma 3 (Log-linear Wages).** In any full employment equilibrium

\[
     w_2^n = w_1^n e^{rS} \tag{14}
\]

**Proof.** See appendix.

This functional form follows from (a) exponential discounting and (b) the opportunity cost of time. Through endogenous decisions to become skilled or unskilled, an identical Mincerian wage structure emerges regardless of the organizational equilibrium in the skilled sector.

Given this wage relationship, we can now pin down prices. In equilibrium, workers in each sector are paid

\[
    w_1^n = p_1^n z
\]
\[
    w_2^n = p_2^n A^n
\]

where skilled workers’ productivity depends on their organizational equilibrium,

\[
    A^n = 2\frac{1}{2} h \times \left\{ \begin{array}{ll} 1, & n = G \\ mc, & n = AB \end{array} \right. \]

Therefore, using the wage ratio, the price ratio on the supply side is determined as a function of exogenous parameters\(^{22}\)

\[
    \frac{p_1^n}{p_2^n} = A^n e^{-rS} \tag{15}
\]

\(^{22}\)The price ratio is determined entirely by the supply side because both the skilled and unskilled sectors exhibit constant returns to scale.
Now consider the demand side to close the model. With CES preferences, aggregate demands are such that
\[ \frac{X^n_1}{X^n_2} = \left( \frac{\gamma}{1-\gamma} \right)^\varepsilon \left( \frac{p^n_1}{p^n_2} \right)^{-\varepsilon} \]
Market clearing implies \( p^n_1 X^n_1 = w^n_1 L^n_1 \) and \( p^n_2 X^n_2 = w^n_2 L^n_2 \) so that labor allocations are also pinned down given relative prices
\[ \frac{L^n_1}{L^n_2} = \left( \frac{\gamma}{1-\gamma} \right)^\varepsilon \left( A^n e^{-rS} \right)^{1-\varepsilon} e^{rS} \tag{16} \]
where \( L^n_q \) is the measure of people actively working in sector \( q \).

Real income per-capita, \( y^n = Y^n/L \), is also pinned down given relative prices
\[ y^n = z \left( \gamma^\varepsilon + (1-\gamma)^\varepsilon \left( A^n e^{-rS} \right)^{\varepsilon-1} \right)^\frac{1}{\varepsilon-1} \tag{17} \]
and we can define human capital’s contribution to output as \( H^n = y^n L/z \).

4 Applications and Discussion

This section examines several applications of the model. One primary application uses general equilibrium reasoning to show why human capital can play a much larger role in the world economy than traditional accounting estimates suggest. A series of further applications show that "knowledge traps" may provide a parsimonious interpretation of several stylized facts in the world economy.

\footnote{There are also a number of students who are training in sector 2 and not yet active workers. Given the hazard rate of death \( r \), we have \( e^{-rs} L^n_2 \) people currently training and working in sector 2, so that total labor supply is \( L = L^n_1 + e^{-rs} L^n_2 \).}

\footnote{Real national income (\( Y^n \)) is given by \( p^n Y^n = w^n_1 L_1 + w^n_2 L_2 \), where the aggregate price level is \( p^n = \left( \gamma^\varepsilon \left( p^n_1 \right)^{1-\varepsilon} + (1-\gamma)^\varepsilon \left( p^n_2 \right)^{1-\varepsilon} \right)^\frac{1}{\varepsilon-1} \). Real per-capita income (\( y^n = Y^n/L \)) is
\[ y^n = \frac{w^n}{p^n} \left( \frac{L^n_1}{L^n_2} + \frac{w^n L^n_1}{w^n L^n_2} \right) = \frac{w^n}{p^n} \]
Thus average per-capita income is equivalent to the real wage in the low-skilled sector. This follows in equilibrium because workers’ net present value of lifetime wage income is equivalent at birth. We can alternatively write this in terms of sector 2 wages, since \( w^n_1 = e^{-rs} w^n_2 \).

\footnote{Note that the model, which considers two final goods in consumption, is equivalent to a model that considers a single final good and treats \( x_1 \) and \( x_2 \) as two intermediates. In that interpretation, where (4) is now a production function instead of a preference aggregator, we would write the production function as \( y^n = z \left( \gamma \left( L^n_1 \right)^\phi + (1-\gamma) \left( A^n L^n_2 \right)^\phi \right) / L \), which can be shown to be equivalent to (17), where total factor productivity is interpreted as \( z \), and the contribution of human capital is \( H^n = \left( \gamma \left( L^n_1 \right)^\phi + (1-\gamma) \left( A^n L^n_2 \right)^\phi \right)^{1/\phi} \).}
4.1 Wages, Prices, and Labor Allocations

When people choose to be highly educated, any excessive wage gains to the highly-educated can be arbitraged away by an increase in the supply of such workers. In the model, this choice problem generates the log-linear "Mincerian" wage structure and pins the skilled wage premium to the interest rate, as in (14).\(^{26}\)

One key implication is that two countries can have vastly different mappings between schooling duration and skill, and yet have identical wage returns to schooling in equilibrium. In fact, skill differences are hidden by the wage structure. It is prices and labor supply that shift to ensure the equilibrium wage-schooling relationship.

In particular, from (15), prices adjust in the model such that

\[
\frac{p_{1}^{AB}/p_{2}^{AB}}{p_{1}^{G}/p_{2}^{G}} = mc
\]

This result says that low-skilled services will be cheaper in the poor country. This feature of the equilibrium may be appealing, as it provides a Balassa-Samuelson effect in relative prices (e.g. Harrod 1933, Balassa 1964, Samuelson 1964). The model may thus inform a central observation in development, which is that certain goods are relatively cheap in poor countries, an effect that motivates the need for PPP price corrections when comparing real income across countries.\(^{27}\) The knowledge trap model provides an endogenous basis for this phenomenon, where low-skilled goods (e.g. haircuts) are relatively cheap in a poor country because low skill is relatively abundant there.\(^{28}\)

Meanwhile, from (16) and (18), labor supply adjusts such that

\[
\frac{L_{1}^{AB}/L_{2}^{AB}}{L_{1}^{G}/L_{2}^{G}} = (mc)^{1-\varepsilon}
\]

\(^{26}\)Note that this simple perspective suggests a positive correlation between interest rates and returns to schooling across countries. In fact, the literature has suggested both (a) higher interest rates in poor countries (e.g. Banerjee and Duflo 2005) and (b) higher rates of return to schooling in poor countries (Psacharopoulos 1994). Also see footnote \(^{27}\).

\(^{27}\)Classic explanations for this price phenomenon imagine exogenous cross-country differences in technology (Balassa 1964, Samuelson 1964) or factor endowments (Bhagwati 1984).

\(^{28}\)Note that the model considers price differences between a final good completely produced through skilled labor and a final good completely produced through unskilled labor. In looking at microeconomic price data, one would consider input mixes of skilled and unskilled labor away from these extremes, which would attenuate the observed price differences in final goods. In a generalization of the model, the observable price effect would appear such that goods that use unskilled labor relatively intensively would be relatively expensive in rich countries.
Together, the equilibrium price and labor supply adjustments in (18) and (19) decouple the wage returns to schooling from the skill-gains from schooling. These results pose substantial challenges to the traditional macro-Mincer calibration method, which we turn to next.

4.2 Calibration Evidence

Many analyses have concluded that human capital plays a relatively modest role in explaining the wealth and poverty of nations, leaving residual variation in total factor productivity as a major explanation (see Caselli 2005 for a review). This conclusion is reached using the "macro-Mincer" method to account for human capital (Klenow and Rodriguez-Clare 1997, Hall and Jones 1999). In this method, each economy’s human capital is calculated as the labor supply at each level of education, weighted by the average wage at that education level; i.e., in this paper’s notation, \( H_M = L_1 + e^S L_2 \). The returns to education are taken as \( e^S \), and countries differ in their human capital to the extent that they have more or less educated workers. To see why this method is problematic, first consider a simple example.

**Example 1** With Cobb-Douglas aggregation (\( \varepsilon = 1 \)), it follows from (19) that \( L_1^{AB} / L_2^{AB} = L_1^G / L_2^G \), so that the labor allocation does not vary with the skill gains from education, \( mc \). Therefore, the macro-Mincer human capital stock calculation, \( H_M = L_1 + e^S L_2 \), would not vary with the skill gains from education. Mincerian accounting would therefore suggest no role for human capital, even should human capital explain unboundedly large income differences across countries.

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29 Some calibrations also allow \( e^S \) to vary across countries, based on observed educational returns. To focus on the core methodological issue, the following theoretical results will abstract from variation in \( r \). Calibration evidence discussed below will incorporate such variation as well. See also footnote 30.

30 Moreover, a regression of per-capita income on average schooling duration would also show no relationship. With Cobb-Douglas preferences (\( \varepsilon = 1 \)) the average schooling in a population is

\[ s^n = \frac{S L_2}{L} = (1 - \gamma) Se^{-rS} \]

a constant independent of which equilibrium is attained. For average schooling to be positively associated with income (which it is), we require the elasticity of substitution between skilled and unskilled labor to be greater than 1. Then countries with high quality skilled-labor (i.e. specialization) will see an endogenous increase in the supply of such skilled workers.
The general intuition can be stated as follows. With downward sloping demand for different labor classes, countries that are very good at producing high skill will find that goods and services produced by low-skill workers are scarce, which drives up low-skilled wages. In particular, with relative wages pinned down by the discount rate, as in (14), workers allocate themselves so that the percentage wage gains for skilled and unskilled workers rise or fall in equal proportion. Wages are Mincerian in each country, but this within-country equilibrium does not inform human capital differences across countries. Rather, the wage-schooling relationship shifts vertically depending on the skilled equilibrium. This is shown in Figure 2 for the Cobb-Douglas case, in which price adjustments fully offset productivity differences, requiring no labor adjustment.

Because Mincerian accounting rules out the scarcity effect on unskilled wages, it will in fact systematically understate human capital differences across countries given the observed allocations of labor. Define the ratio of actual human capital differences across countries to the Mincerian calculation of these differences as

\[ R_H = \frac{H^{AB}/H^G}{H^{AB}_{Mincer}/H^G_{Mincer}} \]

**Lemma 4** *(Mincer as Lower Bound)* \( R_H \geq 1 \) for all \( \varepsilon \in [0, \infty) \). Moreover, \( \lim_{\varepsilon \to 1} R_H = \infty \)
for a given labor allocation $L_G^1/L_A^1AB \neq 1$.

**Proof.** See appendix.

This lemma states that Mincerian human capital accounting is only a lower-bound on the actual human capital differences across countries. The lemma further says that the magnitude of the underestimate may be arbitrarily large, depending on the elasticity of substitution between skilled and unskilled labor. The reasoning follows from (19). For example, fixing the observed labor allocation, $(L_A^{1AB}/L_A^{2AB}) (L_G^1/L_G^2) < 1$, reducing $\varepsilon$ towards 1 calls for greater $mc$, which makes for a larger human capital difference between these countries.\(^{31}\) Put another way, once educational attainment is seen as a choice problem, it is natural to ask why so many more workers seek higher education in rich countries. The larger supply of such workers is reconciled in equilibrium by larger skill gains from schooling. As $\varepsilon$ falls, the human capital differences must increase to compensate if we are to explain the observed supply of skilled workers.

It is clear that the elasticity of substitution between skilled and unskilled labor becomes a key parameter in assessing the role of human capital. The literature suggests values of $\varepsilon \in [1, 2]$.\(^{32}\) Figure 3 provides estimates of $R_H$, fixing the observed labor allocation and exploring the effect of $\varepsilon$. I use Barro-Lee educational attainment data to describe the labor allocations, counting the unskilled, $L_n^1$, as those with no more than secondary school education and comparing labor allocations in the United States to the mean labor allocation in the poorest 10% of countries.\(^{33}\) Taking the Katz and Murphy (1992) estimate of $\varepsilon = 1.4$, we see that macro-Mincer calibration would understate human capital differences by a factor of 2.7. As $\varepsilon$ falls to 1.2, human capital differences would be understated by a factor of 7.5.

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\(^{31}\) In practice, we see $(L_A^{1AB}/L_A^{2AB}) (L_G^1/L_G^2) < 1$, which is consistent with $\varepsilon > 1$.

\(^{32}\) See, e.g., the review by Katz and Autor (1999).

\(^{33}\) The educational attainment data comes from Barro and Lee (2001). The poorest countries are determined using Penn World Tables v. 5.6 (Summers and Heston 1991). These are the same data sources used in Caselli and Coleman (2006), which is discussed below.
This simple calibration can be pushed further. In particular, one may ask whether residual TFP differences are still needed to explain cross-country income differences once these skilled productivity differences are accounted for. In fact, Caselli and Coleman (2006) provide such a calibration, using realistic values of $\varepsilon$, although their interpretation is different. Caselli and Coleman (2006) calibrated separate productivity terms for skilled ($s$) and unskilled ($u$) workers across countries using the production function
\[ y = k^n \left[ (A_u L_u)^\rho + (A_s L_s)^\rho \right]^{\frac{1}{\rho}} \]
which is the analogue of (4) in this paper with the addition of physical capital, $k$.\(^{34}\) They find an enormous productivity advantage of skilled workers ($A_s$) in rich countries while the productivity of unskilled workers ($A_u$) is no higher there, leaving little need for residual TFP differences.\(^{35}\) This calibration is closely consistent with the predictions of the knowledge

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\(^{34}\)They calculate $L_u$ and $L_s$ by aggregating workers within lower schooling ranges ($L_u$) and upper schooling ranges ($L_s$) with perfect substitutes assumed within each range.

\(^{35}\)In their preferred specification, Caselli and Coleman further argue that unskilled workers are actually \textit{less} productive in rich than poor countries. That is, with low enough $\varepsilon$, the productivity of the educated must rise so much across countries that it overexplains income differences and thus requires TFP to be negatively correlated with income. More generally, their result for unskilled workers is sensitive to the calibration parameters and how one defines "unskilled worker". If one classifies such workers as having less than high school or less than college-level education, then unskilled workers in their calibration become mildly more productive in rich countries. What appears highly robust about their specification is that skilled workers have enormous productivity advantages in rich countries, as is consistent with the knowledge trap model.
trap model: enormous productivity advantages in rich countries that are limited to skilled workers. By contrast, Caselli and Coleman argue, citing traditional Mincerian-based reasoning, that the skilled productivity differences are too large to be explained as human capital, and therefore view $A_s$ as a residual productivity advantage. However, the division of labor can support the massive productivity advantages that such a calibration suggests, and the Mincerian evidence, as seen through Lemma 4, does not in fact limit the human capital differences across countries. Since the productivity gains appear limited to the educated, it seems natural to imagine that education itself provides some critical advantage. I will next consider related evidence from immigration, to help assess whether these differences in skilled workers’ productivity really are independent of the individual worker’s human capital.

4.3 Immigrant Wages and Occupations

An alternative approach to assessing human capital’s role in cross-country income differences is to examine what happens when workers trained in poor countries are placed in rich countries. If human capital differences were critical, it is argued, then such workers should experience significant wage penalties in the rich country’s economy. Noting that immigrants from poor to rich countries earn wages broadly similar to workers in the rich country, authors have concluded that human capital plays at most a modest role in explaining productivity differences across countries (Hendricks 2002). However, this estimation approach as implemented faces the same issue as standard accounting approaches, by limiting the effect of scarce labor supply.

36 Another important calibration is Manuelli and Sheshadri (2005), who estimate human capital by considering it as an endogenous choice variable. They find large quality differences in human capital across countries that, once accounted for, require little or no TFP differences. Their estimation suggests large advantages in the quality of education in rich countries even at entrance to primary school. This skill advantage at very low-education levels differs from the "knowledge trap" approach, which emphasizes differences that are limited to the highly skilled and differs from the Caselli and Coleman (2006) calibration, where quality advantages are found to exist only among those with more education. Manuelli and Sheshadri’s imputed quality differences at all skill levels follow because their model does not allow for the relative price effects that occur when skilled and unskilled workers produce different intermediate or final goods.

37 The calibrations imply that skilled productivity differences between the richest and poorest countries are on the order of 100, depending on choices of $\varepsilon$. One reason to emphasize the division of labor - where skilled laborers are the vessels of advanced ideas, the stock of which is too large to be accessed without specialization - is because this explanation seems capable of producing such large productivity differences, whereas simpler conceptions of quality in the educational production function would have greater difficulty.

38 The main estimates in Hendricks (2002) assume workers output at different skill classes are perfect substitutes thus eliminating any effect of scarcity on the wages of the unskilled. To the extent calibrations
The knowledge trap model predicts that low-skilled immigrants, who are the majority of immigrants, will enjoy (a) much higher real wages than they left behind and (b) face no wage penalty in the rich economy vis-a-vis other unskilled workers. Indeed, why would education matter for the uneducated, working as taxi drivers, retail workers, and farm hands? Wage gains follow naturally when the low-skilled immigrant moves to a place where his labor type is relatively scarce. The over-riding role of scarcity, rather than productivity, for unskilled workers is corroborated by the calibration discussed above. The potentially more informative implications of the knowledge trap model lie among skilled immigrants.

**Corollary 2 (Immigrant Workers)** An unskilled worker who migrates from a poor to a rich country will earn a higher real wage. The skilled generalist who migrates from a poor to a rich country will work in the unskilled sector and earn the unskilled wage, which may provide more or less real income than staying at home.

**Proof.** See Appendix. ■

Skilled immigrants, as generalists, are unable to find local specialists willing to team with them. Moreover, they won’t work alone; the specialized equilibrium of the rich country raises the low-skilled wage enough to make unskilled work a more enticing alternative to the immigrant generalist than using his education. Hence, for example, we can see immigrant Ph.D.’s who drive taxis.

Friedberg (2000) demonstrates that the source of education does matter to immigrant wages, but the literature does not appear to have looked explicitly at higher education. Descriptive facts can be assembled however using census data. I divide individuals in the 2000 U.S. Census into three groups: (1) US born, (2) immigrants who arrive by age 17, and (3) immigrants who arrive after age 30. The idea is that those who immigrated by age 17 likely received any higher education in the United States, while those who immigrated after age 30 likely did not.

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39 I divide individuals in the 2000 U.S. Census into three groups: (1) US born, (2) immigrants who arrive by age 17, and (3) immigrants who arrive after age 30. The idea is that those who immigrated by age 17 likely received any higher education in the United States, while those who immigrated after age 30 likely did not.

39 The data and methods are detailed further in the Appendix.
Figure 4a shows two important facts. First, controlling for age and English language ability, the location of higher education appears to matter. Among highly educated workers, those who immigrate after age 30 experience significant wage penalties, of 50% or more. Meanwhile there is no wage penalty if the immigrant arrived early enough to receive higher education in the United States. Second – and conversely – the location of high-school education does not matter. Wages do not differ by birthplace or immigration age for workers with an approximately high-school level education. Hence, the location of education matters for high skill workers but not so much for low skill workers, as this paper’s model suggests.\footnote{Note also that immigrants with high school or less education have extremely similar wage outcomes regardless of immigration age. This further suggests that early-age immigrants are an adequate control group for late-age immigrants, highlighting that differing labor market outcomes only occur at higher education levels. Lastly, it is clear that very-low education immigrants (e.g. primary school) do significantly better than very-low educated US born workers. Such limited education is very rare among the US born and likely reflects individuals with developmental difficulties, which may explain that wage gap.}

Figure 4b considers related evidence based on occupation type. To construct this graph, each occupation in the census is first categorized by the modal level of educational attainment for workers in that occupation. For example, taxi drivers typically have high school degrees, physicians typically have professional degrees, and physicists typically have Ph.D.’s. The figure shows the propensity for workers with professional or doctoral degrees to work in different occupations. We see that US born workers and early immigrants have extremely similar occupational patterns. However, late immigrants with professional or doctoral degrees have a much smaller propensity to work in occupations that rely on such degrees. Instead, they tend to shift down the occupational ladder into jobs that require only college degrees and even, to a smaller extent, into occupations typically filled by those with high school or less education. This pattern is further reflected in Figure 4a, which shows that late immigrants with professional or Ph.D. degrees earn average wages no better than a locally educated college graduate.

This evidence is consistent with this paper’s model but inconsistent with a pure technology story, in which the location of education would not matter. More broadly, the evidence is consistent with the idea that human capital differences across countries exist primarily among the highly educated, as suggested independently by the calibration discussed above.
4.4 Poverty Traps and Education

The theory in this paper emphasizes that educational choices are not sufficiently described by schooling duration. Rather, the content of education is critical. If workers organize themselves to learn narrow, deep knowledge and aggregate it through work in teams, then they can collectively access the knowledge frontier. But workers may organize as generalists, avoiding reliance on teamwork and failing to embody frontier knowledge. This section further considers challenges to collective skill improvement from the perspective of the model.\textsuperscript{41}

4.4.1 The Quality of Higher Education

Income differences across countries may persist if countries are in different regions of Figure 1. Countries with $mc < 1$ will have shallow knowledge and remain in poverty. This may occur if acquiring deep skills is hard in poor countries ($m$ is small). One can think of $m$ as a policy parameter, where, for example, $m$ increases through public investment in higher education. Small $m$ also follows naturally if knowledge acquisition is limited by local access to others with deep skill - i.e. expert teachers. For example, becoming skilled at protein synthesis will be difficult without access to existing skilled protein synthesists: their lectures, advice, the ability to train in their laboratories, etc. In this setting, we can imagine a simple, further type of knowledge trap. If we write $m^n$, where $m^G < m^{AB}$, then countries that start in the generalist equilibrium will remain there if $m^G c < 1$.

Escaping such a trap involves importing skill from abroad to train local students or sending students abroad and hoping they will return. Both approaches face an incentive problem however, since those with deep skills will earn higher real wages by remaining in the rich country. The model thus suggests a "brain drain" phenomenon.

**Corollary 3** (Brain Drain) Once trained as a specialist in the rich country, one will prefer to stay.

\textsuperscript{41}The following discussion focuses on poverty-inducing mechanisms that follow from the supply of human capital. Coordination costs (the parameter $c$) may also be important, especially should coordination costs be more severe in poorer countries. Lastly, market size may be an important limiting factor in specialization. This last possibility may also bear consideration in confronting data but is not incorporated in the model for focus and brevity.
Proof. See appendix. ■

Specialists in rich countries prefer to stay because they can work with complementary specialists there and thus earn higher wages. Hence students who migrate to the U.S. for their Ph.D.‘s face real wage declines if they go home - even though they are scarce at home. Related, it is clear that students from rich countries do not migrate to developing countries for their education, even though university and living expenses are considerably lower. This may further suggest that the quality of education is low.42

This result suggests that wage subsidies or other incentives may be required to attract skilled experts to the poor country and improve local training. China, for example, has been actively engaging in such policies (see, e.g., Zweig 2006).

4.4.2 The Coordination of Higher Education

Even if poor countries can produce high-quality higher education, there is still an organizational challenge. Countries may be in the middle region of Figure 1, facing the same parameters m and c but sitting in different equilibria. Here a country cannot escape poverty without creating thick measures of specialists with complementary skills.43 This may be hard. Any intervention must convince initial cohorts of students to spend years in irreversible investments as specialists, which would be irrational if complementary specialists were not expected. Hence we need a "local push".44,45 Yet it is not obvious what institutions have the incentives or knowledge to coordinate such a push. A firm may have little

42I thank Kevin Murphy for pointing this out.
43One could alternatively construe the "trap" as being a deterministic function of the initial conditions, where a sufficient mass of specialists of each type creates a stable, high income state, while insufficient supply of specialists creates a stable, low income state.
44Note that the type of trap allowed in the model differs from poverty traps that envision aggregate demand externalities (e.g. Murphy et al. 1989). Rather, knowledge traps can be overcome locally, when workers achieve greater collective skill. A challenge for aggregate demand models is that many poor economies are quite open to trade or have large GDP on their own despite low per-capita GDP, so it is unclear that aggregate demand is a credible obstacle. Meanwhile, booms are often local, whether it is city-states like Hong Kong or Singapore, or cities within countries, like Bangalore, Hyderabad, and Shenzhen, which have led growth in India and China. Yet such booms are also rare, which suggests that local coordination problems are themselves not trivial to overcome.
45Some authors see such coordination failures as easily solved due to trembling hand type arguments (e.g. Acemoglu 1997). However, there are several reasons to think that small "trembles" are unlikely to undo a generalist equilibrium. First, we are considering many years of education for an individual, so that a "tremble" must be rather large. Second, while we consider two tasks for simplicity, there may be $N > 2$ tasks needed for positive output, which would then require simultaneous trembles over many specialties. Third, with greater search frictions in the market (smaller $\lambda$), trembles must occur over a large mass of workers. Fourth, in tradeable sectors, one must leap to the skill equilibrium of the rich countries to compete internationally - small skill trembles won’t suffice.
incentive to make these investments when students can decamp to other firms. Public institutions may not produce the right incentives either. Developing deep expertise requires time, so that the fruits of educational investments may not be felt for many years, depressing the interest of public leaders (or firms), who may have short time horizons. Even if local leaders wish to intervene, it may be challenging to envision the set of skills to develop, especially if there are many required skills and deep knowledge does not exist locally. These difficulties suggest a need for "visionary" public leaders. They also suggest an intriguing role for multinationals in triggering escapes from poverty.

4.4.3 Multinationals and Poverty Traps

Intra-firm trade can allow for production teams that span national borders, and I discuss here how a multinational can play a unique role in helping countries escape poverty.

**Corollary 4** *(Desirable Cheap Specialists)* A firm of specialists in a rich country would hire specialists in poor countries, if they could be found.

**Proof.** See appendix. ■

This result follows because the skilled wage in the poor country is held down by the Mincerian wage equilibrium, making a specialist there attractive. Hence, production would shift to incorporate a skilled specialist in the poor country if such a type existed. But now we have a cross-border coordination problem. A multinational will only be able to find these specialists if they exist in sufficient measure, and no one in the poor country will want to become such a specialist unless the multinational will be able to find them.

The interesting aspect is that a multinational allows the local educational institutions to avoid producing all required specialities locally. The multinational provides the complementary worker types from abroad. For example, in working to initiate the economic boom in Hyderabad, governor Naidu both subsidized a vast expansion in engineering education and personally convinced Bill Gates to employ these workers in Microsoft’s global

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46Contracts may help here, but labor contracts that prevent workers from departing a firm (i.e., in an extreme form, slavery) are typically illegal. Labor market frictions may allow firms to do some training if frictions give the firm some monopsony power (e.g., Acemoglu and Pischke 1998). Still, it is clear that foundational training and specialization, such as the Ph.D., occur in educational institutions, prior to engagement with firms.
production chain, so that computer programmers in Hyderabad now team with other skilled specialists in advanced economies. Here, the "visionary" leader need not recreate Microsoft but simply produce a sufficient quantity of one specialist type that Microsoft will hire. To the extent that a thick supply of this specialist type triggers complementary specialization locally, the local economy may escape from the trap broadly.

4.5 Generalizations: Trade, Skill Bias, and Divergence

The emphasis on the division of labor among skilled workers also suggests natural generalizations to international trade and growth contexts, with the possibility to inform (i) comparative advantage, (ii) skill-biased technical change and (iii) cross-country income divergence.

First, knowledge traps may provide a useful perspective on comparative advantage. The factor endowment model of trade, Heckscher-Ohlin, explains why Saudi Arabia exports oil but is famously poor at predicting trade flows based on capital and labor endowments – the so-called "Leontief Paradox" (Leontief 1953, Maskus 1985, Bowen et al. 1987, etc.). International trade analysis, much like cross-country income analysis, has therefore relied on substantial residual productivity terms to explain the empirical patterns (e.g. Trefler 1993, 1995, Harrigan 1997).

With knowledge traps, the rich country has a comparative advantage in the skilled good while the poor country has a comparative advantage in the low-skilled good. Yet these comparative advantages - based in the division of labor - won’t appear in standard calculations of labor endowments. The division of labor suggests a human-capital interpretation


48 With only two types of specialists, the emergence of one type in the poor country can trigger the emergence of the other, and the poor country will become rich. With more than two specialist types, or with an inability to train locally in the other skill, the emergence of one type may not inspire the local creation of the other types. Here, a multinational can continue to employ a narrow type of skilled specialists in one country without triggering a general escape from poverty. Here we will see both off-shoring and persistently "cheap engineers".

49 In terms of the model, we can consider two small open economies who can trade both goods 1 and 2. With world prices, $p_1/p_2$, such that

$$\frac{p^G_1}{p^G_2} < \frac{p_1}{p_2} < \frac{p^{AB}_1}{p^{AB}_2}$$

the country in the generalist equilibrium exports the low-skilled good (1) while the country in the specialist equilibrium exports the high skilled good (2).

50 For example, the degree of specialization won’t appear in designations like "professional" or "highly edu-
of residual productivity terms, where rich countries are net exporters of skilled goods not simply because they have more skilled workers (quantity), but because their skilled workers have much more collective skill (quality).

The emphasis on skilled workers as the vessels of advanced ideas also suggests a natural generalization to skill-biased technical change. Along the growth path in advanced economies, the empirical tendency for skilled wage premiums to hold steady, or even rise, despite large increases over time in skilled labor supply is consistent with the rising quality of skilled labor compared to unskilled labor (see, e.g. Katz and Autor 1999). This tendency would occur naturally in a generalization of the model. In particular, if growth is associated with the creation of new ideas and consequent expansion of frontier knowledge, which can be modeled as an increase in \( m \), then growth is intrinsically skill-biased.

Similar reasoning would also predict cross-country income divergence over time. To the extent that workers in poor countries, organized for general knowledge, do not access this deepening set of ideas, cross-country divergence in per-capita income becomes the natural outcome empirically, as is the usual case (Jones 1997, Pritchett 1997). As with skill-biased technical change, this dynamic result would follow from the same extension of the model, where advanced ideas drive growth and these advanced ideas are accessed in the workforce through education.\(^51\)

5 Conclusion

This paper offers a human-capital based interpretation of several phenomena in the world economy and therefore a possible guide to core obstacles in development. The model shows how endogenous differences in the quality and quantity of skilled labor may persist across economies, emphasizing the importance of skilled workers as vessels of ideas and high productivity. As one application, the theory shows how standard human capital accounting

\[ \text{cated}^n \] worker, which can explain why attempts to save Heckscher-Ohlin through finer-grained classifications of labor endowments have failed (e.g. Bowen et al. 1987).

\(^51\) In an endogenous growth framework, some fraction of skilled workers would produce productivity enhancing ideas that lead to growth in \( m \). Such accumulation of knowledge may require innovators to become more specialized along the growth path, so that the number of tasks at the frontier (2 in this model) becomes endogenous and increases with time. See Jones (2009) for such a growth model, as well as empirical evidence that knowledge workers in the U.S. become increasing specialized with time and work in larger teams.
may severely underestimate cross-country skill differences. More broadly, the model may provide an integrated perspective on cross-country income differences, labor allocations, wages, price differences, migrant behavior, poverty traps, and other phenomena in a way that appears broadly consistent with important facts.

By expanding the human capital decision to incorporate the content of education, rather than simply its duration, the theory may substantially amplify human capital’s role in understanding development. The division of labor provides a natural mechanism for individuals to solve the basic problem that frontier knowledge in an economy is too much for any one individual to know. By suggesting specific mechanisms, including institutional mechanisms, that disrupt learning deep knowledge, the theory further suggests tangible, micro-empirical avenues for future work.

This paper also speaks directly to a long-running debate over the roles of "human capital" and "technology" in explaining income differences across countries. I close by further considering this distinction. Much existing literature imagines human capital and ideas as distinct inputs into a production function and, using macro-Mincer accounting, suggests a modest role for human capital, pushing education toward the periphery in understanding key issues in development. What is called technology, the residual, has consequently occupied a central position and is often imagined as a set of techniques, methods, facts, models, et cetera that impact production. At root, this paper attempts to reconfigure this debate and, in some sense, sidestep it. This paper shows how human capital may play a central role. At the same time, this paper embraces the critical importance of ideas. People are born with empty minds, and education is seen as the process of acquiring knowledge. Rather than conceiving technology as a distinct, disembodied input to production, this paper imagines that ideas must first be embodied in people. It is thus the emphasis on embodiment, rather than the role of "ideas", that distinguishes this paper from other approaches. In this perspective, technological progress, the expansion of the set of ideas, may well drive economic development, but here too the effects of knowledge will likely be felt – and understood – not in contest with human (or physical) capital, but through its embodiment into the people and machines that actually produce things.
Proof. The lemma follows from five intermediate results.

(1) Workers are never willing to match with their own type ($k \notin \Omega^k \forall k$)

In equilibrium, all skilled types have some $V > 0$. A type $k$ never matches with type $k$ if $V^{kk} < V$. For As or Bs, the joint output when teaming with one’s own type is zero. Hence (8) implies $V^{AA} = V^{BB} = \frac{1}{2}V < V$. Therefore, neither As or Bs will match with their own type. For Gs, (8) implies $V^{GG} = \frac{1}{2}w_{22}^{GG}/r + \frac{1}{2}V$. Noting that $V \geq w_{22}^G$ (G’s income if he never matches, from (6)) and that $w_{22}^{GG} < w_{22}^G$ (GG matches provide no skill advantage but incur a coordination penalty), it follows that $V^{GG} < V$. Hence no type will match with her own type.

(2) Type $k$ is willing to match with type $j$ iff type $j$ is willing to match with type $k$ ($k \in \Omega^j \iff j \in \Omega^k$)

A type $k$ is willing to match with type $j$ if $V^{kj} \geq V$. With the Nash Bargaining Solution and common $V$ in equilibrium, it follows that $V^{kj} = V^{jk}$. Hence $k \in \Omega^j \iff j \in \Omega^k$.

(3) As are willing to match with Gs iff Bs are willing to match with Gs ($G \in \Omega^A \iff G \in \Omega^B$)

As are willing to match with Gs if $V^{AG} \geq V$. In equilibrium, $V^{AG} = V^{BG}$. This follows from (8) because with (a) common $V$ and (b) $x_2^{AG} = x_2^{BG}$, Nash Bargaining implies $w_2^{AG} = w_2^{BG}$. Hence, $V^{AG} \geq V \iff V^{BG} \geq V$, so that As are willing to match with Gs iff Bs are willing to match with Gs.

(4) If an A or B is willing to match with Gs, then the A or B is also willing to match with the complementary specialist type ($G \in \Omega^A \Rightarrow B \in \Omega^A$ and $G \in \Omega^B \Rightarrow A \in \Omega^B$)

If As are willing to match with Gs, then $V^{AG} \geq V$ and $w_2^{AG} = \frac{1}{2}x_2^{AG}/2$. But $w_2^{AB} = \frac{1}{2}p_2x_2^{AB} \geq \frac{1}{2}p_2x_2^{AG} = w_2^{AG}$ and hence, from (8), $V^{AB} \geq V^{AG}$. Hence A will also be willing to match with Bs: $G \in \Omega^A \Rightarrow B \in \Omega^A$. A symmetric argument demonstrates that $G \in \Omega^B \Rightarrow A \in \Omega^B$.

(v) As and Bs must match ($\Omega^k \neq \emptyset$ for $k = A, B$)

This result follows because tasks A and B are gross complements in production. Hence, As or Bs who work in isolation do not produce positive output and earn no income.\footnote{Gross complements, $\sigma \leq 1$, is a (strong) sufficient condition for this result but is not necessary. If $\sigma > 1$, then positive production becomes possible when a specialist works alone. Nevertheless, it can be shown that, with $\sigma > 1$, As and Bs still prefer to match in equilibrium so long as $c > (1/2)^{\frac{1}{\sigma-1}}$, i.e., matching occurs as long as coordination costs are not too severe ($c$ is not too small) or the elasticity of substitution between tasks is not too great ($\sigma$ is not too large). The paper focuses on the case of $\sigma \leq 1$ to enhance tractability, brevity and intuition.}

With these five properties, the only remaining, possible equilibrium matching policies are \{$\Omega^A, \Omega^B, \Omega^G$\} = \{\{B\}, \{A\}, \{\emptyset\}\} or \{$\Omega^A, \Omega^B, \Omega^G$\} = \{\{B, G\}, \{A, G\}, \{A, B\}\}. ■
Proof of Lemma (Balanced Specialists)

Proof. (I) First consider the case where \( L^A > 0 \) and \( L^B > 0 \).

1. In equilibrium \( V^A = V^B \). Let \( \Omega^A, \Omega^B, \Omega^G \) = \( \{B, G\}, \{A, G\}, \{A, B\} \). Equating \( V^A = V^B \) using (11) implies
   \[ 0 = \Pr(A) - \Pr(B) \left[ w^A_{2B} + \frac{1}{2r} \Pr(G) (w^A_{2B} - w^A_{2G}) \right] \]
   Hence \( \Pr(A) = \Pr(B) \) in equilibrium. If, alternatively, \( \Omega^A, \Omega^B, \Omega^G \) = \( \{B\}, \{A\}, \emptyset \), it follows directly from \( V^A = V^B \) using (11) that \( \Pr(A) = \Pr(B) \).

2. Next we show that \( \Pr(A) = \Pr(B) \) implies \( L^A = L^B \). The probability of meeting a worker of type \( j \) is \( \Pr(k) = \frac{L^k}{\sum_{j} L^j \Pr(j)} \).

   (a) There are \( L^k \) people in the population of type \( k \). In steady state, they are born at rate \( rL^k \) and survive to their graduation with probability \( e^{-rs} \). The rate at which new graduates enter the matching pool is therefore \( rL^k e^{-rs} \).

   (b) There are \( L^k e^{-rs} - L^k_p \) type \( k \) workers currently matched in teams. Since workers die at rate \( r \), the rate of reentry into the matching pool is \( rL^k e^{-rs} - L^k_p \).

   (c) Type \( k \) workers in the matching pool die at rate \( rL^k_p \).

   (d) Type \( k \) workers in the matching pool match other unpaired workers at rate \( \lambda L^k_p \sum_{j \in \Omega} \Pr(j) \).

   Summing up these routes in and out of the matching pool, we have
   \[ \dot{L}^k_p = 2rL^k e^{-rs} - 2rL^k_p - \lambda L^k_p \sum_{j \in \Omega} \Pr(j) \]  \( (20) \)

   In steady-state, \( \dot{L}^k_p = 0 \), which implies that
   \[ L^k_p = \left[ 1 + \frac{\lambda}{2r} \sum_{j \in \Omega} \Pr(j) \right]^{-1} e^{-rs}L^k. \]

   The ratio of probabilities for an A and B meeting is therefore
   \[ \frac{\Pr(A)}{\Pr(B)} = \frac{1 + \frac{\lambda}{2r} \sum_{i \in \Omega} \Pr(i) L^A}{1 + \frac{\lambda}{2r} \sum_{i \in \Omega} \Pr(i) L^B} \]  \( (21) \)

   It then follows directly, given the allowable matching rules defined by Lemma 1, that \( \Pr(A) = \Pr(B) \) implies \( L^A = L^B \).

(II) Second, consider the case where \( L^A > 0 \) and \( L^B = 0 \).

We rule this case out by contradiction. Since As earn zero if they work alone, As must match in equilibrium. Hence an equilibrium with \( L^A > 0 \) and \( L^B = 0 \) would require \( L^G > 0 \) with As and Gs matching. In equilibrium, common \( V \) then implies from (11) that
\[ rV = \frac{\frac{\lambda}{2r} \Pr(G) \frac{1}{2} p^G_{2x} w^G_{2G} \Pr(G)}{1 + \frac{\lambda}{2r} \Pr(G)} \]  \( (22) \)

Now consider a player who deviates to type B. This player could choose to match only
with Gs and earn the same $V$.\(^{53}\) Hence, when meeting an A, the B deviator would have no worse outside option than $V$. Hence, if B chose to match with an A, $w_{AB}^B \geq \frac{1}{2} p_2 x_{AB}^2$.

Hence if the B deviator chose to match with As or Gs then

$$rV_B \geq \frac{\frac{\lambda}{2\gamma} \Pr(A) \frac{1}{2} p_2 x_{AB}^2 + \frac{\lambda}{2\gamma} \Pr(G) \frac{1}{2} p_2 x_{AG}^2}{1 + \frac{\lambda}{2\gamma} \Pr(A) + \frac{\lambda}{2\gamma} \Pr(G)} > rV$$

where the strict inequality follows because $x_{AB}^2 > x_{AG}^2$. Therefore, by contradiction, there is no equilibrium with $L_A > 0$, $L_B = 0$. By a symmetric argument there is no equilibrium where $L_A = 0$, $L_B > 0$.

Hence in equilibrium the model must feature $L_A = L_B$.

**Proof of Proposition (Knowledge Traps)**

**Proof.** Consider the "generalist", "specialist", and "mixed" cases in turn.

(I) The "generalist" case, where \( \{L_A, L_B, L_G\} = \{0, 0, L^s\} \).

In this case,

$$rV = w_G^2$$

where $w_G^2 = p_2 x_G^2$.

Now consider whether an (infinitesimal) individual would deviate to a specialist type, say type A. The type A worker earns $w_A^2 = 0$ when working alone. Hence from (11) $rV_A = \left[\frac{\lambda}{2\gamma} \Pr(A) \right] w_{AG}^2$, where $w_{AG}^2 = \frac{1}{2} p_2 x_{AG}^2 - \frac{1}{2} r(V - V^A)$ from the Nash Bargaining Solution. Solving these to eliminate $w_{AG}^2$ gives

$$rV_A = \frac{\frac{\lambda}{2\gamma}}{2 + \frac{\lambda}{2\gamma}} (p_2 x_{AG}^2 - w_G^2)$$

Workers won’t deviate if $rV \geq rV_A$, or (after some algebra)

$$x_{AG}^2 \leq 2 x_G^2 \left(1 + \frac{2r}{\lambda}\right)$$

If this condition holds, the "generalist" case is an equilibrium. With full employment, $\lambda \to \infty$, the "generalist" case is an equilibrium iff $x_{AG}^2 \leq 2 x_G^2$.

(II) The "specialist" case, where \( \{L_A, L_B, L_G\} = \{\frac{1}{4} L^s, \frac{1}{4} L^s, 0\} \).

In this case,

$$rV = \frac{\frac{\lambda}{2\gamma}}{2 + \frac{\lambda}{2\gamma}} w_{AB}^2$$

where $w_{AB}^2 = \frac{1}{2} p_2 x_{AB}^2$.

\(^{53}\) If a player deviates to type B and chooses $\Omega^B = \{G\}$, then $rV_B = \frac{\lambda}{2\gamma} \Pr(G) w_{BG}^G$. Nash Bargaining implies $w_{BG}^G = \frac{1}{2} p_2 x_{BG}^2 - \frac{1}{2} r(V - V^B)$. With $V$ given in (22), and noting $x_{BG}^2 = x_{AG}^2$, it then follows that $rV_B - rV = 0$. In this setting, deviating to be a player of type B and using the same matching policy as the existing As provides the same income as the existing players receive.
The "specialist" case is an equilibrium if $rV \geq rV^G$. If you deviate to be a generalist and don’t match with specialists, then $rV^G = w^G = p_2x_2^G$. If you do match with specialists, then $rV^G = (w^G + \frac{\lambda}{2}w^G_A)/ (1 + \frac{\lambda}{2r})$, where $w^G_A = \frac{1}{2}p_2x_2^AG - \frac{1}{2}r(V-V^G)$ from the Nash Bargaining Solution.

Assuming Gs match with As and Bs the condition that $rV \geq rV^G$ is therefore (after some algebra)

$$x_2^{AB} \geq \left( \frac{2 + \frac{\lambda}{r}}{1 + \frac{\lambda}{r}} \right) \left( \frac{4r}{\lambda} x_2^G + x_2^{AG} \right)$$

Assuming alternatively that Gs do not match, the condition that $rV \geq rV^G$ is

$$x_2^{AB} \geq \left( 1 + \frac{4r}{\lambda} \right) 2x_2^G$$

So the condition for the specialist case to be an equilibrium is

$$x_2^{AB} \geq 2x_2^G \max \left[ 1 + \frac{4r}{\lambda}, \left( \frac{2 + \frac{\lambda}{2r}}{1 + \frac{\lambda}{2r}} \right) \left( \frac{2r}{\lambda} + \frac{x_2^{AG}}{2x_2^G} \right) \right]$$

As $\lambda \rightarrow \infty$, the specialist case is an equilibrium if $x_2^{AB} \geq \max [2x_2^G, x_2^{AG}]$. Noting that $x_2^{AB} > x_2^{AG}$, the binding condition can therefore only be $x_2^{AB} \geq 2x_2^G$ with full employment.

(III) The "mixed" case, where $\{L^A, L^B, L^G\} = \{L', L', L^s - 2L'\}$. There are two sub-cases: (i) Gs do not match with As and Bs and (ii) Gs do match with As and Bs (see Lemma 1).

(i) If Gs do not match, then the equivalence of $rV$ across worker types in equilibrium requires, using (11), that

$$\frac{\lambda}{2r} P w^{AB} = w^G$$

where $P = \Pr(A) = \Pr(B)$, $w^G = p_2x_2^G$, and with the Nash Bargaining Solution $w^{AB} = \frac{1}{2}p_2x_2^{AB}$.

(ii) If Gs do match, then the equivalence of $rV$ across worker types in equilibrium requires that

$$\frac{\lambda}{2r} \left[ Pw^{AB} + (1 - 2P)w^{AG} \right] = \frac{w^G + \frac{\lambda}{2r}2Pw^{GA}}{1 + \frac{\lambda}{2r}2P}$$

where $w^{AB}$ and $w^G$ are as in (i) and, with the Nash Bargaining Solution, $w^{AG} = \frac{1}{2}p_2x_2^{AG}$.

Deviating to another worker type has no effect on payoffs, since players are infinitesimal. These cases thus exist as equilibria if (a) a player would not change her matching policy and (b) there exists a $P \in [0, 1/2]$ that satisfies equality of income between specialists and generalists.

Comparing a Gs payoff when he doesn’t match with the payoff when he does (the RHS of equations (23) and (24)), it is clear that $x_2^{AG} \geq 2x_2^G$ is necessary for G to match in equilibrium, and $x_2^{AG} \leq 2x_2^G$ is necessary for G not to match in equilibrium. Rearranging
(23), we can define an equilibrium value \( P^* \) as
\[
P^* = \frac{2r}{\lambda(x_{AB}^2 - 1)}
\]
where \( P \in [0, 1/2] \) is necessary for an equilibrium to exist. Thus the "mixed" case where Gs do not match is an equilibrium iff \( x_{2G}^A \leq 2x_{2G}^G \) (Gs do not want to match), \( x_{2B}^A \geq 2x_{2B}^G \) \( (P^* \geq 0) \), and \( \lambda \geq 4r \left[ \frac{1}{2} x_{2B}^A / x_{2G}^G - 1 \right]^{-1} \) \( (P^* \leq 1/2) \).

As \( \lambda \to \infty \) (full employment), \( P^* \to 0 \), so that this "mixed" equilibrium converges towards the "generalist" equilibrium.

If G does match in equilibrium, then rearranging (24) produces a quadratic in \( P \), with either 0, 1, or 2 roots such that \( P_2 \in [0, 1] = 2 \). With some algebra, we can define an equilibrium value \( \hat{P} \) as
\[
\hat{P} = -\frac{2r}{4 \left( x_{2G}^A - 4x_{2G}^G \right) + 1} \pm \sqrt{\left( \frac{2r}{4} \right)^2 - \left( x_{2G}^A - 4x_{2G}^G + 1 \right)^2 + 8r \left( x_{2B}^A - x_{2G}^A \right) (2r + 1 - x_{2G}^A)}
\]

The "mixed" case where Gs do match with As and Bs is an equilibrium iff \( x_{2G}^A \geq 2x_{2G}^G \) (Gs match with As and Bs) and \( \hat{P} \in [0, 1/2] \). It can be shown that as many as 2 such equilibria are possible for some parameter values.

As \( \lambda \to \infty \) (full employment), it follows directly from (25) that \( \hat{P} \to 0 \), so that any such "mixed" equilibrium also converges towards the "generalist" equilibrium.

**Proof of Corollary (Gains from Specialization)**

**Proof.** Output per specialist is \( \frac{1}{2} p_2 x_{2B}^A = p_2 mc 2^{-1+\sigma} z h \) and output per generalist is \( p_2 x_{2G}^G = p_2 2^{-1+\sigma} z h \), so that the ratio of these outputs is \( \frac{1}{2} p_2 x_{2B}^A / (p_2 x_{2G}^G) = mc \). Hence the first part. For the second part, recall that the condition for the generalist equilibrium to be stable is \( x_{2G}^A \leq 2x_{2G}^G \) with full employment. Using the production function (10), this condition is equivalently written in terms of underlying parameters as \( mc \leq \left( \frac{2}{\frac{1}{2} + \frac{1}{\sigma}} \right)^{1-\sigma} \).

Recalling that tasks A and B are gross complements in production \( (\sigma \leq 1) \), it follows that
\[
\lim_{m \to \infty} \left( \frac{2}{\frac{1}{2} + \frac{1}{\sigma}} \right)^{1-\sigma} = \infty.
\]

Hence the maximum possible \( mc \) for which generalists exist in a stable equilibrium is unbounded from above.

**Proof of Lemma (Log-Linear Wages)**

**Proof.** Given that individuals have the same choice set at birth and maximize income, they must be indifferent across career choices so that \( W^k = W \) for all worker types. With full employment, this income arbitrage means from (5) that
\[
\int_0^\infty w_1^n e^{-rt} dt = \int_s^\infty w_2^n e^{-rt} dt
\]
where $w_1^u = rV^u$ is the wage paid in the unskilled sector and $w_2^s = rV$ is the wage paid in the skilled sector. Integrating (26) gives $w_2^s = w_1^u e^{rs}$. 

Proof of Lemma (Mincer Accounting as Lower Bound)

Proof. In the model, $H_{AB}/H_G = Y_{AB}/Y_G$. Skilled workers are $mc > 1$ times more skilled in the AB case than the G case. From (17) and (16), we write

$$H_{AB} = \frac{L_1^{AB}}{L_1^G} \left( 1 + e^{rs} \frac{L_1^{AB}}{L_1^G} \right) \frac{e^{rs}}{1 + e^{rs} \frac{L_1^{AB}}{L_1^G}}$$

Recalling that $H_{Mincer}^u = L_1^u + e^{rs} L_2^u$, we can manipulate (27) to write the ratio of the true human capital ratio to the Mincerian calculation, $R_H = \frac{H_{AB}}{H_{Mincer}^u}$, as

$$R_H = \left( \frac{1 + e^{rs} \frac{L_1^{AB}}{L_1^G}}{1 + e^{rs} \frac{L_1^{AB}}{L_1^G}} \right) \frac{e^{rs}}{1 + e^{rs} \frac{L_1^{AB}}{L_1^G}}$$

Consider the case where $\varepsilon \in [1, \infty]$. From (19), $L_1^{AB}/L_1^G \geq L_2^G/L_1^G$, with strict inequality if $\varepsilon > 1$. Given the observed labor allocations, it follows that $\lim_{\varepsilon \to 1} R_H = \infty$ and that $R_H$ declines in $\varepsilon$. Further $\lim_{\varepsilon \to \infty} R_H = 1$. Hence, $R_H \geq 1$ given $\varepsilon > 1$.

Consider the case where $\varepsilon \in [0, 1]$. From (19), $L_1^{AB}/L_1^G \leq L_2^G/L_1^G$, with strict inequality if $\varepsilon < 1$. Given the observed labor allocations, it follows that $\lim_{\varepsilon \to 1} R_H = \infty$ and that $R_H$ increases in $\varepsilon$. Further $\lim_{\varepsilon \to 0} R_H > 1$. Hence, $R_H > 1$ given $\varepsilon \leq 1$.

In sum, over $\varepsilon \in [0, \infty]$ it follows that $R_H \geq 1$. Moreover, for a given labor allocation $L_1^G/L_1^{AB} \neq 1$, $\lim_{\varepsilon \to 1} R_H = \infty$. 

Proof of Corollary (Immigrant Workers)

Proof. The low-skilled immigrant earns a higher real wage by moving to the rich country because, from (17)

$$\frac{w_1^{AB}/p^{AB}}{w_1^G/p^G} = \frac{y_{AB}}{y_G} > 1$$

Hence an unskilled worker who migrates from a poor to a rich country will earn a higher real wage.

Now consider skilled immigrants.

Note first that the skilled generalist who migrates will never team with a specialist in the rich country. Rather, he would always prefer to work alone, since he must give up too much of the joint product to convince a specialist to partner with him. In particular, he would earn $p_2^{AB} x_2^G$ alone, while in a team (with full employment) he would earn $p_2^{AB} (x_2^{AG} - \frac{1}{2} x_2^{AB})$, and there are no parameter values where $x_2^G < x_2^{AG} - \frac{1}{2} x_2^{AB}$. To see this, write this condition
as $1 < x_2^{AG}/x_2^G - \frac{1}{2} x_2^{AB}/x_2^G$. Note that $\frac{1}{2} x_2^{AB}/x_2^G = mc$ and that $x_2^{AG}/x_2^G$ can be no greater than $mc + c$.\textsuperscript{54} Hence the condition is equivalently $1 < c$, which contradicts the assumption of the model that there are coordination costs in production, $c < 1$.

Next, note that working alone as a generalist in the rich country is never preferred to staying in the poor country. In either country, the generalist produces $x_2^G$ units of output per unit of time. Given that this good is relatively expensive in the poor country (i.e. recall that $p_2^G/p_1^G = mc(p_2^{AB}/p_1^{AB})$), the real income is higher working as the generalist in the poor country.

Lastly, note that the generalist may still prefer to migrate and work in the unskilled sector. This occurs when the real wage gain across countries for unskilled work $\frac{w_2^{AB}/p_2^{AB}}{w_2^{AB}/p_1^{AB}}$ (see above) is larger than the real wage gain locally for skilled work, $e^r$, which is more likely the greater the income differences between the countries; for example, the greater the gains from specialization, $mc$.

In sum, skilled generalists may or may not be better off migrating to rich countries, but if they do they will work in the unskilled sector. ■

\textbf{Proof of Corollary (Brain Drain)}

\textbf{Proof.} The specialist who moves to the poor country will earn a wage $w_2^G = p_2^G(x_2^{AG} - x_2^G)$. Since the poor country is in a generalist equilibrium, we must have $x_2^{AG} \leq 2x_2^G$ which implies that $w_2^G \leq p_2^G x_2^G = w_2^G$. Hence, the skilled worker who moves from the rich to the poor country will earn a wage no greater than the skilled worker wage in the poor country. Now note that skilled workers receive a higher real wage in the rich country than the poor country because, from (14) and (17),

$$\frac{w_2^{AB}/p_2^{AB}}{w_2^G/p_2^G} = \frac{y_2^{AB}}{y_2^G} > 1$$

Hence, specialists in the rich country will prefer to stay. ■

\textbf{Proof of Corollary (Desirable Cheap Specialists)}

\textbf{Proof.} Think of the firm as a specialist in the rich country. He earns $w_2^{AB} = \frac{1}{2} p_2^{AB} x_2^{AB}$. If he can alternatively form a cross-border team by locating an (off-equilibrium) specialist in the poor country, then he can earn at least $w_2 = p_2^{AB} x_2^{AB} - p_2^{AB} x_2^G$, where he need provide the specialist in the poor country no more than $x_2^G$, the going rate for generalists in that country. Hence, hiring a specialist in the poor country makes sense iff $x_2^{AB} - x_2^G \geq \frac{1}{2} x_2^{AB}$ or $x_2^{AB} \geq 2x_2^G$, which is just the condition for specialists to exist in the first place in the rich country. ■

\textbf{Data and Analysis for Figure 4}

\textsuperscript{54}This follows because $x_2^{AG}/x_2^G$ is increasing in $\sigma$, attaining a maximum $x_2^{AG}/x_2^G = mc + c$ as $\sigma \to \infty$. 41
Data on wages and occupations is taken from the 1% microsample of the 2000 United States census, which is available publicly through www.ipums.org.\textsuperscript{55} There are 2.8 million individuals in this sample, including 320 thousand individuals who immigrated to the United States.

The wage-schooling relationships in Figure 4a are the predicted values from the following regression

\[
\ln w_i = \alpha + \beta MALE + \text{Age}_{fe} + \text{English}_{fe} + \text{Group}_{fe} + \text{Education}_{fe} + \text{Group}_{fe} \times \text{Education}_{fe} + \varepsilon_i
\]

where \( w_i \) is the annual wage, \( MALE \) is a dummy equal to 1 for men and 0 for women, \( \text{Age}_{fe} \) are fixed effects for each individual age in years, \( \text{English}_{fe} \) are fixed effects for how well the individual speaks English (the IPUMS "speakeng" variable which has 6 categories), \( \text{Education}_{fe} \) are fixed effects for highest educational attainment (the IPUMS "educ99" variable, which has 17 categories) and \( \text{Group}_{fe} \) are fixed effects for three different groups: (1) US born, (2) immigrants who arrive by age 17, (3) immigrants who arrive age 30 or later. Figure 4a plots predicted values from this regression, plotting the log wage against educational attainment for each of the three groups. For comparison purposes, the predicted values focus on males between the ages of 30 and 40 who speak English at least well.

To construct Figure 4b, the modal educational attainment is first determined for each of the 511 occupational classes in the data (using the IPUMS variable "occ"). Occupations are then grouped according to modal educational attainment. For example, lawyers are grouped with doctors as typically having professional degrees, and taxi drivers are grouped with security guards as typically having high school degrees. For each of the three groups defined for the \( \text{Group}_{fe} \) above, Figure 4b shows the propensity of individuals with professional or doctoral degrees to work in occupations with the given modal educational attainment.

References


**Table 1: Access to Advanced Knowledge: An Example**

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Undergraduate, Masters, Ph.D.</th>
<th>Undergraduate, Masters, Ph.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faculty</td>
<td>20 (14 professors, 6 lecturers)</td>
<td>160 (93 professors, 62 lecturers, 5 technical instructors)</td>
</tr>
<tr>
<td>Subfields</td>
<td>1 (M.Sc. in Energy Engineering)</td>
<td>7 “Areas” and 17 different course groups</td>
</tr>
<tr>
<td>Courses</td>
<td>51 (Complete list)</td>
<td>175(b) (Examples)</td>
</tr>
</tbody>
</table>

**Undergraduate Courses (34 courses)**
- Thermodynamics I, II, III; Fluid Mechanics I, II
- Mechanics of Materials I, II; Manufacturing Process I, II
- Mechanical Engineering Laboratory I, II, III
- Mechanical Engineering Design I, II, III
- Hydraulic Machines I, II, Thermal Power Engineering I, II
- Dynamics of Mechanical Systems I, II
- Mechanics of Machines, Machine Elements
- Exposing Information Technology, Computer Applications
- Heat Transfer, Heat and Mass Transfer
- Engineering Economics, Engineering Management, Industrial Management
- Refrigeration and Air Conditioning, A/C Systems
- Gas Dynamics

**M.Sc. in Energy Engineering (17 courses)**
- Energy Science, Numerical Techniques & Computations
- Energy Economics & Management,
- Instrumentation & Experimental Techniques
- Hydro Power Plants, Advanced I.C. Engines
- Steam Power Plants, Gas Turbine Power Plants
- Solar and Wind Power Plants, Nuclear Power Plants(b)
- Rocket and Aircraft Propulsion(b), Novel Power Systems
- Energy Systems Control, Advanced Heat Transfer
- Storage and Transportation of Energy
- Computational Fluid Dynamics, Combustion Engineering

Notes: (a) Count of MIT mechanical engineering faculty does not include 56 research scientists and post-docs; (b) MIT provides 97 further courses in a separate department, “Aeronautics and Astronautics”, whereas Khartoum provides one course on that topic, Rocket and Aircraft Propulsion, listed within mechanical engineering. Similarly, MIT provides 44 further courses in a separate department, “Nuclear Science and Engineering”, whereas Khartoum provides one relevant course, Nuclear Power Plants, listed within mechanical engineering. Sources: MIT Bulletin (2007-2008) and www.uofk.edu (accessed, 9/2011).
Figure 4a: Do Skilled Immigrants Experience Wage Penalties?  
The Wage-Schooling Relationship

Figure 4b: Do Skilled Immigrants use their Education?  
Occupations of Workers with Professional or Doctoral Degrees