Strategic Investment
and Industry Risk Dynamics

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Abstract

This paper characterizes how firms’ strategic interaction in product markets affects the industry dynamics of investment and expected returns. Under imperfect product market competition, the investment strategy of each firm depends on the intra-industry standard deviation in firms’ market to book ratios or *intra-industry value spread*. The insight by asset pricers that a firm’s exposure to systematic risk is significantly related to its own investment is incomplete in industries with high value spread, in which a firm’s exposure to systematic risk is also explained by the investments of others. In the model and the data, firms’ betas and excess returns correlate more positively in industries with low value spread, low dispersion in operating mark-ups, and low concentration.

Keywords: expected returns, investment, imperfect competition, strategic interaction, value spread, industry concentration.

JEL codes: L11, L22, G11, G12, G31.

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Introduction

In imperfectly competitive industries, the ability of firms to affect market prices induces them to invest strategically. The value of each firm depends not only on its own assets in place and investment opportunities, but also on the ability of its competitors to expand capacity and affect market prices. As a result, under imperfect competition, the insight by asset pricers that a firm’s exposure to systematic risk or *beta* is significantly affected by its own investment decisions is incomplete. When firms invest strategically, a firm’s exposure to systematic risk may depend significantly on the investments of other firms in the same industry.

The production based asset pricing literature focuses on the impact of corporate investment on expected returns in perfectly competitive or perfectly monopolistic industries.¹ We explore the intermediate case of imperfectly competitive industries, in which firms’ strategic interaction affects the dynamics of investment and risk. The study of firms’ intra-industry interactions is relevant in the light of the existing empirical evidence, which suggests that commonly studied asset pricing regularities are predominantly intra-industry.² Our model rationalizes existing findings on the cross section of returns, and provides additional testable predictions which we find support for in our empirical section.

We motivate our study with several research questions. How does a firm’s relative position in its product market influence its investment decisions, and the conditional dynamics of its expected returns? In which types of industries are the stylized predictions of investment based asset pricers for monopolies or perfectly competitive industries still appropriate? How does strategic interaction affect the intra-industry correlation of firms’ investments and their exposure to systematic risk? And lastly, how do specific industry characteristics such as demand elasticity, demand growth or demand volatility affect the industry dynamics of investment and risk?

The core prediction of our model is that the dynamics of firms’ investments and expected returns depend critically on the intra-industry standard deviation in market to book ratios, or *intra-industry value spread*. Under imperfect competition, a firm’s market to book ratio reflects its comparative advantage to increase its market share. As a result, in industries with low value spread, firms have more similar investment strategies, they invest at more similar points in time, and their betas and excess returns correlate more positively. The model also

predicts that firms’ betas and excess returns may correlate more positively in industries with low standard deviation in mark-ups, and low concentration.$^3$

We obtain these predictions in partial equilibrium, real options model of duopoly in which heterogeneous firms compete in capacity, with costly production and irreversible investment. We solve for the investment strategies of firms that differ in either their production technologies, and have a single growth option to increase capacity. This represents a significant departure from earlier dynamic models of imperfect competition, which focus on identical firms and hence are silent about the intra-industry cross section of investments and risk.$^4$ Given that the setting is fairly complex, we spend substantial effort in deriving firms’ investment strategies in equilibrium. We derive testable implications on the impact of firms’ strategic interaction on expected returns by examining their betas.

We group our contributions into three different sets of results. Our first set of results relates to the predictions of the model on the dynamics of investment under imperfect competition. In neoclassical models, the investment of each firm solely depends on its own marginal product of capital or $q$.$^5$ In contrast, we find that under imperfect competition the investment strategy of each firm depends on the marginal product of capital or $q$ of all firms in the same industry. In our model, a firm’s $q$ reflects its comparative advantage to increase its market share relative to other firms in the industry. As a result, firms’ strategic behavior is such that the investment strategy of each firm depends on the intra-industry standard deviation in $q$.

The corresponding testable implication is that firms’ investment strategies depend on the intra-industry value spread. In industries with low value spread, firms are closer competitors with similar market to book ratios, and hence the cost of preempting each other by investing aggressively is too high. Firms’ investment strategies are more similar: firms increase their capacity by similar amounts, and their investments cluster. Conversely, in industries with high value spread, firms are more distant competitors, and firms with higher market to book invest earlier and more than their competitors.

Our second set of results relates to how firms’ strategic interaction affects the dynamics of expected returns. Consistent with the predictions for monopolies discussed in Carlson et al (2004), we find that under imperfect competition firms undergo a period of high expected returns before their own investment, and a period of low expected returns upon investment.

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$^3$We elaborate on the link between industry dynamics and these static measures of competition below.


$^5$See, for instance, Hayashi (1982).
Yet we add that, in industries with high value spread, firms also undergo a period of low expected returns before their competitors invest. The insight that a firm’s exposure to systematic risk is significantly related to its own investment is incomplete in industries with high value spread, and also in industries with low demand growth, low demand volatility, and high demand elasticity.

The model provides several empirical predictions on the relation between firms’ expected returns and market to book ratios. In the model, a firm’s market to book ratio reflects its ability to increase its market share in subsequent periods. As a result, the market to book sorts used in the empirical asset pricing literature are effectively aggregating firms according to their relative position in the industry.\textsuperscript{6} Similarly, we find that the dynamics of the expected returns of each firm are affected by the intra-industry value spread. This suggests why Cohen et al (2003) find that the value spread of US firms is predominantly intra-industry.

The core testable asset pricing implication is that firms’ betas and excess returns correlate more positively in industries with low value spread. Firms’ strategic interaction affects the intra-industry correlation of their expected returns, even when all firms in the industry are subject to no idiosyncratic shocks, and there is a single source of systematic risk. In industries with low value spread, firms have similar investment dynamics, and their expected returns correlate positively over time. Conversely, in industries with high value spread, there are leaders which invest earlier and more than other firms, and the betas of leaders and laggards correlate negatively over time.

The model also shows that those industries with low value spread usually have lower standard deviation in mark-ups, and lower concentration as measured by the Herfindahl-Hirshman Index (HHI). This allows us to formulate testable predictions on how the spread in mark-ups and the concentration of an industry affect the dynamics of expected returns. In particular, our testable implications on the HHI rationalize the recent evidence in Hoberg and Phillips (2010). Consistent with our model, Hoberg and Phillips (2010) find that in less concentrated industries have more predictable average industry returns, in which periods of high market to book ratios, high returns, high betas and high investment, are followed by periods of lower market to book ratios, lower investment, lower returns and lower betas.

A related implication is yet that commonly used measures of competition such as the spread in mark-ups or the HHI may prove insufficient to study industry dynamics, precisely

\textsuperscript{6}See Fama and French (1992) on market to book sorts.
because they are static. In our model, industries with low HHI may have low intra-industry value spread, but this need not apply to all cases; for instance, a deconcentrating industry may have a high value spread, and a concentrating industry may have a lower value spread. As a result, the key sorting variable to test for the impact of firms’ strategic interaction on industry dynamics is the intra-industry value spread. The intra-industry value spread is forward looking, and captures current and expected future differences in mark-ups and market shares.

The third main contribution of our paper relies on its empirical evidence. We first document that firms’ investments are significantly related to the intra-industry value spread, both at the firm and industry level. To test our predictions on industry dynamics, we construct a measure of comovement which captures the average pairwise correlation in firms’ investments, betas and excess returns by industry. Consistent with the underlying assumption that corporate investment affects firms’ expected returns, we report a significant relation between the intra-industry comovement in investment and the intra-industry comovement in betas or excess returns. More importantly, we report that betas and excess returns correlate more positively in industries with lower value spread, lower standard deviation in mark-ups, and lower concentration.

Finally, while we focus on a duopoly to derive all the testable implications of our paper, we provide an extension with three firms to show that our testable implications remain as the number of firms increases, and to explore how an increase in the number of firms affects firms’ values and betas in our model. As in Grenadier (2002) and Aguerrevere (2009), a higher number of firms erodes the values and betas of all firms in the industry. Yet in contrast with their studies which focus on identical firms, we find that a higher number of firms erodes more severely the betas of those firms with lower \( q \). Also, the stylized prediction in Grenadier (2002) that a higher number of firms induces firms to accelerate investment need not apply to all industries. In industries with high value spread, a higher number of firms may induce firms with lower \( q \) to delay their investment.

The paper is organized in four sections. Section 1 describes the basic model and its core predictions on investment and risk dynamics. Section 2 provides the testable implications of the basic model and related extensions. Section 3 reports the supporting empirical evidence. Section 4 concludes.

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7 See discussion in Section 2.
8 Our methodology follows Khanna and Thomas (2009). See Section 3.
Related literature

The model relates closely to Carlson et al (2004), who study the dynamics of firms’ exposure to systematic risk in a cross section of monopolistic firms. We build on their framework to study how firms’ strategic interaction under imperfect competition affects investment and expected returns. The model is related to Fundenberg and Tirole (1985), Grenadier (1996), Weeds (2002), and Mason and Weeds (2010), whose studies focus on duopolies in which firms only decide when to invest. Our solution approach with sorting conditions relates to Maskin and Tirole (1988), and is consistent with the observation in Back and Paulsen (2009) that dynamic investment models of oligopoly should account for firms’ incentives to preempt each other.

The model also relates to Carlson et al (2012) and Bena and Garlappi (2012), whose models of investment timing show that the beta of the leader is dampened by the expected reduction in profits once the follower invests. We show that in industries with high value spread the beta of any operating firm is dampened by the investments of its competitor. The theme of the paper is close to Grenadier (2002) and Aguerrevere (2009). While they study how a finite number of firms in symmetric oligopolies affects investment and betas, our focus is on the strategic interaction of heterogeneous firms.


1 Basic model

We begin by studying the simplest type of industry which conveys the core predictions of our paper. We elaborate on the corresponding empirical implications in Section 2.

1.1 Main assumptions

We build on the framework by Carlson et al (2004), and use their notation where possible. We consider an industry with two firms \( j = L, M \). Each firm has assets in place, and a single
growth option to increase their capacity. Each firm is all equity financed and run by a manager who is the single shareholder.

Firms compete in capacity and produce an homogeneous good which they sell in the market at a price $p_t$. Firms operate at full capacity at any point in time. The demand function requires that the product market price $p_t$ equals

$$p_t = X_t Y_t^{-\frac{1}{\varepsilon}} \quad (1)$$

where $\varepsilon > 1$ is the elasticity of demand and $X_t$ is a systematic multiplicative shock, and the industry output $Y_t$ is the sum of the production at time $t$. The demand shock $X_t$ follows a geometric Brownian motion with drift $\mu_x$ and volatility $\sigma_x$ such that

$$dX_t = \mu_x X_t dt + \sigma_x X_t dz_t \quad (2)$$

where $z_t$ is a standard Wiener process, and $X_0$ is strictly positive.

Managers maximize shareholder value by choosing when and how much to invest. Managers decide when to invest by determining the critical value $x_j$ for the stochastic demand shock $X_t$ at which the firm exercises its growth option. They decide how much to invest by determining the optimal scale of production $j$ upon investment. The resulting investment strategy is given by $\Gamma_j \equiv \{x_j; \Lambda_j\}$.

We assume that both firms have the same initial installed capacity before investment $K$. Firms increase their installed capacity from $K$ to $\Lambda_j K$ upon investment. We further set the output of each firm $j$ to be equal to its installed capacity, such that the total production $Y_t$ in (1) is the sum of the installed capacity of both firms at any point in time.

The decision to invest is irreversible, and entails benefits and costs. Upon investment, firms benefit from a lower instantaneous marginal cost of production. We assume that firms have the same instantaneous marginal cost of production $\bar{c}$ before investment, and have lower

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9Firms under imperfect competition do not operate in the range where $\varepsilon < 1$.
10We further assume that $X_0$ is sufficiently low such that the growth options of all firms in the industry are strictly positive at time $t = 0$. Hence $X_0 < x_L^*$ in the basic model; we define $x_L^*$ in Section 1.3.
11We relax this assumption in Section 2.
12Hence $Y_t = 2K$ if both firms have not invested, $Y_t = (1 + \Lambda_j) K$ if one of the two firms has invested, and $Y_t = (\Lambda_L + \Lambda_F) K$ if both firms have invested.
13The irreversibility of investment implies a commitment by firms not to adjust their capacity upon a reduction in market prices.
and different marginal costs of production \( c_j < c \) upon investment. We denote firm \( L \) as the firm with the lowest instantaneous marginal costs after investment, such that \( c_L < c_M \).

Upon investment, firms incur a fixed cost \( fK \). We do not consider variable costs of investment for the sake of tractability; the qualitative predictions of the model remain unchanged if firms are subject to linear cost of investment. We also assume that the costs of production are linear in \( X_t \); this allows us to compare more easily firms’ capacity choices in our model to those predicted by static games of strategic interaction.

Given all our assumptions, the instantaneous profits of firm \( j \) before its own investment \( \pi^-_{jt} \) are given by

\[
\pi^-_{jt} = (p^-_t - \pi X_t) K
\]

where the superscript \( - \) denotes the cashflows before investment. The instantaneous profits of firm \( j \) after its own investment \( \pi^+_{jt} \) are equal to

\[
\pi^+_{jt} = (p^+_t - c_j X_t) \Lambda_j K
\]

where the superscript \( + \) denotes the cashflows after investment.

### 1.2 Valuation

The value of any firm \( j \) at time \( t \) \( V_{jt} \) equals the expected present value of its risky profits. Using a similar argument in Carlson et al (2004), we assume that demand shocks are perfectly hedgeable, and determine the value of the firm using a replicating portfolio with weights on a risk free and a risky asset.

We let \( B_t \) denote the price of a riskless bond with dynamics \( dB_t = r B_t dt \), and we let \( S_t \) be a risky asset with dynamics \( dS_t = \mu_s S_t dt + \sigma_s S_t dz_t \). The risky asset \( S_t \) has a drift \( \mu_s - \mu_x \equiv \delta > 0 \), and we assume that the returns on \( S_t \) are perfectly correlated with percentage changes in demand shocks such that \( \sigma_x \equiv \sigma_s \). We use the traded assets \( B_t \) and \( S_t \) to define a risk neutral measure, under which the demand shock \( X_t \) follows a geometric Brownian motion with drift \( r - \delta \) and volatility \( \sigma_x \).\[^{15}\]

\[^{14}\]We can extend the model to incorporate a linear cost of capital \( p_I (\Lambda - 1) K \) as in neoclassical investment models, with \( p_I > 0 \) as the purchase price of capital. For optimal investment timing, adding this cost operates as redefining fixed costs as \( fK \), with \( f \equiv f + p_I (\Lambda - 1) \). Optimal capacity decisions would depend on the alternative marginal cost \( \tilde{c}_j = c_j X_t + p_I \).

\[^{15}\]As in Carlson et al (2004), this assumption does not affect the qualitative implications of the model.

\[^{16}\]The dynamics of the demand shock under the risk neutral measure are \( dX_t = (r - \delta) X_t + \sigma_x X_t d\tilde{z}_t \), where \( \tilde{z}_t = z_t + \frac{\mu_x - r}{\sigma_x} t \).
In our model, all firms sell their products at the common market price $p_t$. At any point in time, the market price $p_t$ at which firm $j$ sells its production depends on the capacity decisions of all its competitors. Whenever a competitor of firm $j$ invests, the market price $p_t$ goes down, and the current and expected future profits of firm $j$ are also lower. We denote by $\Delta \pi_{jt}^-$ the expected change in instantaneous profits of firm $j$ due to investments by other firms before firm $j$ invests. We denote by $\Delta \pi_{jt}^+$ the expected change in instantaneous profits of firm $j$ due to investments by other firms after firm $j$ invests.

**Proposition 1** [Firm value under imperfect competition] The value of firm $j$ at time $t$ for any investment strategy $\Gamma_j = \{x_j; \Lambda_j\}$ is given by

$$V_{jt} = \begin{cases} \frac{\pi_j^+}{\delta} + \frac{\Delta \pi_{jt}^-}{\delta} + \left[ \frac{\pi_j^+}{\delta} + \frac{\Delta \pi_{jt}^+}{\delta} - \frac{\Delta \pi_{jt}^-}{\delta} \right]_{X_t = x_j} - fK \left( \frac{X_t}{x_j} \right)^v & \text{if } X_t \leq x_j \\ \frac{\pi_j^+}{\delta} + \frac{\Delta \pi_{jt}^+}{\delta} & \text{if } X_t > x_j \end{cases}$$

(5)

where $v > 1$ is defined in the Appendix.

**Proof.** See Appendix A. ■

Equation (5) shows that $V_{jt}$ contains three components. The first is the value of a growing perpetuity of cash flows generated by its assets in place. The second is the value of its investment opportunities or growth options. This provides the standard prediction in real options models that the value of the firm depends on its lifestage. By considering $V_{jt}$ under imperfect competition, we obtain a third component which reflects the impact of firms’ strategic interaction on their value. The investments of the competitor of firm $j$ affect $V_{jt}$ through the expected reductions in future profits denoted by $\Delta \pi_{jt}^-$ and $\Delta \pi_{jt}^+$ in (5). We characterize the signs and magnitudes of $\Delta \pi_{jt}^-$ and $\Delta \pi_{jt}^+$ conditional on the equilibrium outcome in the next subsection.

We illustrate numerically how firms’ strategic interaction affects their values in Figure 1. We simulate multiple paths of the Brownian demand shocks, we compute firms’ values using the definition in Proposition 1, and we report the average firm value at each instant $t$. In Figure 1, we consider the special case in which firms invest sequentially such that firm $L$ invests earlier than firm $M$ ($x_L < x_M$).\textsuperscript{17} We observe that the value of each firm goes above the value

\textsuperscript{17} A strategy in which $x_L < x_M$ need not be an equilibrium outcome; other equilibria may exist where $x_L = x_M$ or $x_L > x_M$. As we discuss in Section 1.3, there exists indeed an equilibrium outcome in which $x_L < x_M$, another in which $x_L = x_M$, and no equilibrium with $x_L > x_M$. Figure 1 actually uses the sequential equilibrium strategies $\Gamma_j^s$ described in Section 1.3.
of its assets in place when its own growth option is in the money, and yet it goes below the
value of its assets in place when its competitor is about to invest. This second effect is entirely
due to firms’ strategic interaction.

1.3 Equilibrium investment strategies

1.3.1 Equilibrium concept

The equilibrium concept is Bayes-Nash. The state of the industry is described by the history
of the stochastic demands shocks $X_t$. At any point in time, a history is the collection of
realizations of the stochastic process $X_s$, $s \leq t$, and the actions taken by all firms in the
industry. The investment strategy $\Gamma_j$ maps the set of histories of the industry into the set of
actions $\{x_j; A_j\}$ for firm $j$. Before investment, firm $j$ responds immediately to its competitor’s
investment decision. This yields Nash equilibria in state dependent strategies of the closed-loop
type.\footnote{A closed-loop equilibrium is a Nash equilibrium in state-dependent strategies. See Chapter 13 of Fundenberg
and Tirole (1991), Weeds (2002) and Back and Paulsen (2009) for related discussions on closed-loop strategies.}
Upon investment, firm $j$ cannot take any other action.

We follow Weeds (2002) and we assume that firms follow Markov strategies such that their
actions are functions of the current state $X_t$ only. As discussed in Weeds (2002), other non-
Markov strategies may also exist; however, if one firm follows a Markov strategy, the best
response of the other firm is also Markov. We consider the set of subgame perfect equilibria
in which each firm’s investment strategy, conditional on its competitor’s strategy, is value
maximizing. A set of strategies that satisfies this condition is Markov perfect. The initial
demand shock $X_0$ is sufficiently low to focus on equilibria in pure strategies.\footnote{When firms are identical, the equilibrium may involve mixed strategies, whose formulation is complicated
by the continuous time nature of the game, as noted by Fundenberg and Tirole (1985) and Weeds (2002). When
firms have different production technologies, however, Mason and Weeds (2010) shows that a sufficient condition
to avoid these concerns is to assume that $X_0$ is sufficiently low. $X_0$ is assumed strictly lower than the lowest
optimal investment threshold in the industry. Hence $X_0 < x^*_L$ for $N = 2$; we define $x^*_L$ in subsection 1.3.2.}
Subgame
perfection requires that each firm’s strategy maximizes its value conditional on its competitor’s
strategy.

1.3.2 Equilibrium outcome

We begin by stating the equilibrium outcome of the basic model; we elaborate on the corre-
spoding solution approach and interpretation below. In a nutshell, there exist two alternative
industry equilibria in pure strategies: a \textit{sequential equilibrium} and \textit{clustering equilibrium}. We denote by $\Gamma_j^s$ the investment strategy of any firm $j$ in the sequential equilibrium with leaders and followers. We denote by $\Gamma_j^c$ the investment strategy of any firm $j$ in the clustering equilibrium in which firms invest simultaneously.

The equilibrium outcome depends on the cross sectional differences in firms’ production technologies. In a duopoly, these cross sectional differences are summarized by the standard deviation of firms’ instantaneous marginal costs of production $\sigma_c \equiv \frac{|c_M - c_L|}{2}$. We elaborate on the intra-industry value spread as a general measure of intra-industry heterogeneity in Section 2.

\textbf{Proposition 2} [Investment under imperfect competition] The subgame perfect industry equilibrium for $N = 2$ with $c_L < c_M$ is such that

\begin{itemize}
  \item if $\sigma_c < \underline{\sigma}_c$, firms invest simultaneously, $x_j = x^c$, $\Lambda_M^c < \Lambda_L^c$, and $\Delta \pi_j^{c} = 0$;
  \item if $\sigma_c \geq \underline{\sigma}_c$, firms invest sequentially, $x_L^* < x_F^*$, $\Lambda_M^s < \Lambda_L^s$, and $\Delta \pi_j^{s} \leq 0$.
\end{itemize}

where the threshold $\underline{\sigma}_c$ is determined endogenously.\footnote{We solve for $\underline{\sigma}_c$ in subsection 1.3.3 below.}

\textbf{Proof.} See Appendix B.

Proposition 2 characterizes the two alternative subgame perfect equilibria which arise in our basic model. When $\sigma_c < \underline{\sigma}_c$, firms are close competitors with similar future costs of production; neither firm has incentives to lead, and their market shares are similar upon investment. Furthermore, since both firms invest simultaneously, we have that $\Delta \pi_j^{c} = 0$ and the expression for firms’ values in (5) resembles that of monopolistic firms in the real options literature. Conversely, when $\sigma_c \geq \underline{\sigma}_c$ and firms are more distant competitors, firm $L$ invests earlier than firm $F$, and $\Delta \pi_j^{s} \leq 0$ such that the dynamics of firms’ values are affected their strategic interaction.

\subsection*{1.3.3 Solution approach}

To solve for the equilibrium outcome, we use \textit{sorting conditions} and \textit{incentive compatibility constraints} (ICCs). The sorting conditions of the multiple action strategy $\{x_j; \Lambda_j\}$ indicate which firms in the industry have the comparative advantage to invest earlier and have a larger market share than their competitors. When firms differ in their future costs of production $c_j$,
we prove in Appendix B that for any strategy \( \Gamma \) more efficient firms find it less costly to invest earlier and more since

\[
\frac{\partial}{\partial c_j} \left[ \frac{\partial V_{M}}{\partial x_j} \right] > 0, \quad \frac{\partial}{\partial c_j} \left[ \frac{\partial V_{L}}{\partial x_j} \right] < 0 \tag{6}
\]

The sorting conditions in (6) have important implications for firms’ strategic behavior. First, (6) implies that there are no sequential equilibria in which less efficient firm \( F \) invest earlier than the more efficient firm \( L \). Since firm \( L \) has a comparative advantage to invest earlier and more, firm \( M \) does not become a leader in equilibrium even if it has incentives to preempt firm \( L \). Consequently, firm \( L \) is the only potential leader when firms invest sequentially. Second, the sorting conditions imply that firms invest simultaneously when the more efficient firm has incentives to do so. If firm \( L \) does not have an incentive to become a leader, neither does firm \( M \), whose ability to invest earlier and more is comparatively lower.

In equilibrium, we account for firms’ incentives to preempt each other using ICCs. Due to the differences in firms’ production technologies, the sorting conditions in (6) show that more efficient firms with lower marginal costs of production \( c_j \) find it less costly to invest earlier and more than their competitors. Yet less efficient firms may still want to invest as if they had lower future costs of production. In particular, firm \( M \) has incentives to invest earlier and more than firm \( L \) whenever its value as a leader is higher than its value as a follower.

To express this intuition more formally, we denote by \( V_{M}^{*} \) the value of firm \( M \) in a Stackelberg game in which firm \( L \) is the leader by assumption; and we denote by \( x_{L}^{*} \) the investment threshold of firm \( L \) in such Stackelberg game. We denote by \( \bar{V}_{M} \) the value of the less efficient firm \( M \) when it deviates and pursues the optimal investment strategy of firm \( L \) as a leader.\(^{21}\) We conclude that firm \( M \) has incentives to become a leader whenever

\[
\bar{V}_{M} \big|_{x_{i}=x_{L}^{*}} \geq V_{M}^{*} \big|_{x_{i}=x_{L}^{*}} \tag{7}
\]

The inequality in (7) provides an upper bound \( \overline{\sigma}_{c} \) such that firm \( M \) has no incentives to become a leader and preempt firm \( L \) if \( \sigma_{c} > \overline{\sigma}_{c} \). When \( \sigma_{c} > \overline{\sigma}_{c} \), the investment strategy of firm \( L \) is that of a standard Stackelberg game in which firm \( L \) always invests first. Conversely, when \( \sigma_{c} < \overline{\sigma}_{c} \), the inequality in (7) imposes a constraint to the maximization problem of firm \( L \), and the investment strategy of firm \( L \) is affected by the preemptive behaviour of firm \( M \).

Given (7), firm \( M \) is indifferent between following the strategy of firm \( L \) or following its own when its value is the same under the two alternative strategies. The corresponding

\(^{21}\)Due to the sorting conditions in Appendix B, the strategies to deviate in either timing or capacity only are dominated by the strategy to deviate in both timing and capacity.
complementary slackness condition is given by
\[ \lambda \left[ V^s_M - \bar{V}^s_M \right]_{x_t = x_L^*} = 0 \] (8)
where the multiplier \( \lambda \) in (8) relates to Posner (1975), and measures to what extent the contest for monopoly power between firms \( L \) and \( M \) hinders the value of firm \( L \).\(^{22}\)

Put together, the conditions in (6) and (7) imply that there are two different types of sequential equilibria. The exists one sequential equilibrium in which \( \sigma_c < \bar{\sigma}_c \) and \( \lambda > 0 \); and there exists an alternative sequential equilibrium in which \( \sigma_c > \bar{\sigma}_c \) and \( \lambda = 0 \). Since the focus of our paper is on the impact of strategic interaction on investment and expected returns, we assume for simplicity \( \sigma_c \) is sufficiently low such that (7) holds throughout the paper. The sequential equilibrium in which \( \lambda = 0 \) has qualitatively the same properties of that in which \( \lambda > 0 \).

As a remark, we note that the impact of the assumption in (7) on the equilibrium outcome is captured through the multiplier \( \lambda > 0 \). The threshold \( \bar{\sigma}_c \) implied by (7) and the multiplier \( \lambda \) in (8) are mechanically related, and the economic intuition behind both concepts is the same. \( \bar{\sigma}_c \) defines the threshold up to which firm \( M \) has incentives to preempt firm \( L \); conversely, \( \lambda \) captures how costly it is for firm \( L \) to deter the investments by firm \( M \).\(^{23}\)

We solve for the remaining threshold \( \bar{\sigma}_c \) in Proposition 2 by considering the incentives of firm \( L \) to become a leader. Firm \( L \) may become a market leader, enjoy early monopoly rents and yet pay the shadow cost of preemption \( \lambda > 0 \). Alternatively, firm \( L \) may allow the follower to invest simultaneously, attain lower duopoly rents from the start, and yet avoid any cost of preemption. Hence \( \sigma_c \) is the threshold at which firm \( L \) is indifferent between pursuing the strategies \( \Gamma^s_L \) and \( \Gamma^c_L \) at \( X_t = x^s_L \), such that
\[ V^s_L|_{X_t = x^s_L} = V^c_L|_{X_t = x^s_L} \] (9)
where (9) is evaluated at \( x^s_L \) since the firm \( L \) invests earlier in the sequential equilibrium (i.e. \( x^s_L < x^c \)).

\(^{22}\)Due to the sorting conditions of the game in (6), the ICC of firm \( M \) in (8) is binding as long as (7) holds. The ICC of firm \( L \) is not binding in equilibrium.

\(^{23}\)We provide the comparative statics of \( \lambda \) with respect to \( \sigma_c \) in Figure 1. We discuss the comparative statics of \( \lambda \) with respect to \( \varepsilon, \mu_x, \sigma_x \) and \( N \) in Section 2.
1.3.4 Discussion of the equilibrium outcome

We illustrate the main properties of the equilibrium outcome by means of numerical examples. Figure 2 shows the values of firms $L$ and $F$ in equilibrium as a function of the parameter $\sigma_c$. In the upper left hand side panel, firm $L$ is more valuable in the clustering equilibrium when $\sigma_c < \sigma_c^*$, and that it is more valuable in the sequential equilibrium otherwise. In the upper right hand side panel, firm $M$ is more valuable under simultaneous investment for any $\sigma_c$. Hence the clustering equilibrium obtains when $\sigma_c < \sigma_c^*$, i.e. when both firms strictly prefer to invest simultaneously.

The middle and bottom charts of Figure 2 illustrate how the model predicts endogenously that the more efficient firm $L$ becomes the market leader if $\sigma_c$ is sufficiently high. Firm $L$ invests significantly earlier under sequential investment ($\sigma_c > \sigma_c^*$) than under simultaneous investment ($\sigma_c < \sigma_c^*$). Similarly, firm $L$ invests more under sequential investment ($\sigma_c > \sigma_c^*$) than under simultaneous investment ($\sigma_c < \sigma_c^*$). Also, the shadow cost of preemption for firm $L$ or $\lambda$ is decreasing in $\sigma_c$ - i.e. it is less costly for firm $L$ to deter the investment of firm $M$ as $\sigma_c$ increases.

Table 1 complements Figure 2 as it provides a numerical example of the sequential and simultaneous investment strategies for the same $\sigma_c$. The purpose of Table 1 is two-fold. First, since we consider all possible investment strategies for the same $\sigma_c$, Table 1 compares the value of all firms under each strategy for a given $\sigma_c$ and predict the equilibrium outcome. In the example, the equilibrium outcome is such that all firms invest simultaneously since $\sigma_c$ is relatively low.

Second, Table 1 illustrates how firms’ optimal investment strategies affect their values for the same $\sigma_c$. When firms invest sequentially, firm $L$ behaves more aggressively, and invests earlier and more compared to Stackelberg games. The binding ICC in (8) makes firm $L$ invest more aggressively to deter firm $M$. Consistent with Figure 2, Table 1 also shows that firm $L$ invests earlier in the sequential equilibrium than in the clustering equilibrium, while firm $M$ does the reverse.

Lastly, Table 1 shows how the magnitude and sign of the expected reduction in profits $\Delta \pi_{jt}$ depend on firms’ investment strategies in equilibrium. When $\sigma_c < \sigma_c^*$, firms’ invest simultaneously, $\Delta \pi_{jt}^c = 0$, and the characterization of firms’ values is the same as that predicted

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24 In Stackelberg games, firm $L$ does not have to deter firm $M$ from investing earlier as in our model; firm $L$ invests earlier than firm $M$ by assumption.
for idle firms in standard real options models.\textsuperscript{25} Conversely, when $\sigma_c > \underline{\sigma_c}$, firms invest sequentially, and each firm expects a reduction in its profits when its competitor invests such that $\Delta \pi_{jt}^s \leq 0$. Table 1 compares such expected reductions in profits $\Delta \pi_{jt}$ at $X_t = X_0$.

### 1.4 Firms’ betas

We study the impact of imperfect competition on firms’ expected returns by analyzing their exposure to systematic risk or betas. The conditional CAPM holds since firms are subject to a single source of systematic risk. The source of systematic risk is given by the demand shock in (1). The riskless rate of return $r$ is exogenously specified, and the market price of risk is constant and exogenously given.

While several papers in the literature consider single factor models to explain the evidence on the cross section of returns\textsuperscript{26}, Fama and French (1992) and subsequent papers show that firms’ betas are a poor measure of firms’ exposure to systematic risk, and that asset pricing models with multiple risk factors may have higher explanatory power. We address this concern empirically in Section 3, by testing our asset pricing predictions on both firms’ betas and returns.

We denote the beta of firm $j$ at time $t$ by $\beta_{jt}$. To determine $\beta_{jt}$, we follow Carlson et al (2004) and infer expected returns from replicating portfolios composed of a risk free asset and a risky asset that exactly reproduce the dynamics of firm value. The proportion of the risky asset held in the replicating portfolio at any time $t$ yields $\beta_{jt}$.

**Proposition 3** [Firms’ betas under imperfect competition] For any strategy $\Gamma$, the beta of firm $j$ at time $t$ is given by

$$\beta_{jt} = 1 + I_t (v - 1) \left[ 1 - \frac{1}{\delta} \frac{\pi_{jt}}{K} \times \frac{K}{V_{jt}} \right]$$

where $I_t$ is an indicator function which is equal to 0 if all firms have invested at time $t$ and is equal to 1 otherwise.

**Proof.** See Appendix C.

The identity in (10) for firms’ betas under imperfect competition resembles that in Carlson et al (2004) for monopolistic firms. As in their paper, the exposure to systematic risk of any firm $j$ depends on the relative contribution of its own growth opportunities to total firm

\textsuperscript{25}See, for instance, Dixit and Pindyck (1994).

\textsuperscript{26}See, for instance, Berk et al (1999), Carlson et al (2004), Zhang (2005), and Aguerrevere (2009).
value. Under imperfect competition, however, we add that $\beta_{jt}$ also depends on the growth opportunities of other competing firms in the industry.

The terms $\pi_{jt}$ and $V_{jt}$ in (10) are affected by firms’ strategic interaction in product markets. Whenever one firm in the industry invests, total production increases, the market price $p_t$ goes down, and there is a discrete change in the beta of all firms in the industry. This explains why the indicator function $I_t$ in (10) equals zero only when all firms in the industry have invested.

1.5 Industry risk dynamics

The remaining prediction of the basic model relates the dynamics of investment in Proposition 2 to the definition of firms’ betas in Proposition 3.

**Proposition 4** [Intra-industry correlation of betas] Given $X_t < x_s^*$, the equilibrium dynamics of $\beta_{jt}$ depend on $\sigma_c$ such that:

- if $\sigma_c < \sigma_c^*$, firms’ betas correlate positively;
- if $\sigma_c > \sigma_c^*$, the betas of leaders and followers correlate negatively.

**Proof.** See Appendix D.

Consider first the dynamics of $\beta_{jt}$ when $\sigma_c < \sigma_c^*$. When firms are close competitors, they invest simultaneously, the value of each firm is larger than the value of its assets in place before investment, and equal to the value of its assets in place thereafter. This implies that $\beta_{jt}^c$ is higher than one before investment and equal to one thereafter. This is illustrated numerically in the example of Table 1.

More importantly, the dynamics of $\beta_{jt}^c$ are qualitatively the same as those that obtain in a real options model in which firms do not invest strategically, such as Carlson et al (2004). This holds because strategic interaction has no equilibrium effects when firms are close competitors (i.e. $\Delta \pi_{jt}^c \equiv 0$). $\beta_{jt}^c$ increases before the investment of firm $j$ and decreases thereafter, as if firm $j$ were an idle firm.

Consider now the dynamics of $\beta_{jt}$ when $\sigma_c > \sigma_c^*$. When firms are distant competitors, there are leaders and followers, and strategic interaction affects the conditional dynamics of betas since firms expect a reduction in their profits once their competitors invest (i.e. $\Delta \pi_{jt}^s \leq 0$). When one firm expects to increase its market share, the other expects a reduction in its own. Hence $\beta_{Lt}^s$ and $\beta_{Mt}^s$ correlate negatively over time.
We illustrate the equilibrium dynamics firms’ betas in Figure 3. We simulate multiple paths of the Brownian demand shocks, compute firms’ betas using the definition in (10), and report the average firm beta at each instant \( t \). We consider the limit case in which \( \sigma_c = \sigma_c \); the limit case in which \( \sigma_c = \sigma_c \) highlights that even for the same set of parameters, and when all firms in the industry are subject to the same systematic shock, the intra-industry correlation of betas depends on the dynamics of investment. The corresponding average intra-industry correlation in firms’ betas equals \( \approx 0.9 \) when firms invest simultaneously, and equals \( \approx -0.9 \) when firms invest sequentially.\(^{27}\)

2 Empirical Implications

We can reinterpret the results in Section 1 more generally in the context of neoclassical investment models. This gives us the core empirical implication that the intra-industry dynamics of investment and risk in imperfectly competitive industries are driven by the intra-industry value spread.

2.1 Investment and the intra-industry value spread

The benchmark neoclassical model by Hayashi (1982) predicts that the optimal investment of any firm depends on its own marginal product of capital or \( q = V_K \). When firms behave strategically, we add that firms with a higher marginal product of capital or \( q \) have the ability to invest earlier and more than their peers, and that the investment strategy of each firm also depends on the intra-industry standard deviation in \( q \).

These results hold not only in industries in which firms differ in their marginal costs of production after investment \( c_j \), but also in industries in which firms differ in their installed capacity before investment \( K_j \). Our choice of \( K_j \) and \( c_j \) as relevant sources of heterogeneity across firms is both relevant and analytically convenient. The intra-industry heterogeneity in \( K_j \) relates broadly to industries in which firms differ in their assets in place. The intra-industry heterogeneity in \( c_j \) relates broadly to industries in which firms differ in their growth opportunities.

To state our results, we denote the intra-industry standard deviation in \( q \) at \( X_t = X_0 \) by \( \sigma_{q,0} \). We evaluate \( q \) at \( X_t = X_0 \) below for the sake of simplicity; all we need is to compare

\(^{27}\)We compute the average intra-industry correlation in betas over the entire time span of Figure 3.
firms’ marginal products of capital before firms invest (i.e. \( X_t < x^s_L \)). We denote by \( \sigma_{q,0} \) the value of \( \sigma_{q,0} \) when firm \( L \) is indifferent between becoming a leader or investing simultaneously with firm \( F \) at (9).

**Proposition 5** [\( q \)-theory of imperfect competition] Under imperfect competition, firms’ investment strategies are such that:

- if \( \sigma_{q,0} < \sigma_{q,0} \), firms invest simultaneously such that \( x_j^c = x^c, \Lambda_M^c < \Lambda_L^c, \Delta \pi_{jt}^c = 0 \);
- if \( \sigma_{q,0} \geq \sigma_{q,0} \), firms invest sequentially such that \( x_L^s < x_F^s, \Lambda_M^s < \Lambda_L^s, \Delta \pi_{jt}^s \leq 0 \).

**Proof.** See Appendix E. ■

When firms differ in \( K_j \) or \( c_j \), the key finding behind Proposition 3 is that the sorting conditions of the game can be restated such that firms with higher \( q \) have the ability to invest earlier and more than their peers. To see this, consider first the special case of the basic model. When firms differ exclusively in their marginal costs of production after investment \( c_j \), the more efficient firm has the ability to invest earlier and more. The marginal product of capital of any firm \( j \) is strictly decreasing in \( c_j \). Consequently, for any strategy \( \Gamma \), firms with a higher \( q \) at \( X_t = X_0 \) have the ability to invest earlier and more. Table 1 illustrates that firm \( L \) has the highest \( q \) at \( X_t = X_0 \).

As we show in Appendix E, a similar intuition holds when firms differ exclusively in their installed capacity before investment such that firm type is given by \( K_j \). In this case, firms differ in the value of their assets in place and have the same growth option. The option to invest is relatively more valuable for the smaller firms, and hence smaller firms are willing to invest earlier and more. Given that the marginal product of capital \( q \) is strictly decreasing in \( K_j \), those firms with a lower \( K_j \) have a higher \( q \), and have the ability to invest earlier and more.

We thus redefine firm type in terms of firms’ \( q \) at \( X_t = X_0 \) and characterize the equilibrium outcome as a function of the intra-industry standard deviation \( \sigma_{q,0} \). When firms differ in either \( K_j \) or \( c_j \), we show formally in Appendix E that firms invest simultaneously when \( \sigma_{q,0} < \sigma_{q,0} \) and sequentially otherwise. We also show in Appendix E that the same qualitative results of Proposition 3 hold when firms differ in both in \( K_j \) and \( c_j \); also in this case, the sorting conditions of the game are such that firms with high \( q \) have the ability to invest earlier and more.
Since firms’ marginal $q$ is not observable, we restate Proposition 3 in terms of testable predictions by considering the identity between $q$ and the market value to book ratio $\frac{V}{K}$. For any strategy $\Gamma$ and $X_t < x_j$, it is straightforward to show that

$$q_{jt} = \frac{V_{jt}}{K_{jt}} - \frac{1}{\xi \delta} \left[ \frac{p \bar{p}}{Y_t^{-}} + \left( \frac{p_{t}^{+} + p_{t}^{-}}{Y_t^{+} - Y_t^{-}} \right) \left( \frac{X_t}{x_j} \right)^{v-1} \right]$$  \hspace{1cm} (11)

The marginal $q$ of firm $j$ in (11) consists of two terms. The first term equals the market to book ratio. The second term is consistent with Hayashi (1982) and captures the net present value of the marginal extraordinary income per unit of capital due to firms’ market power.\(^{28}\)

Given that the second term in (11) is common to all firms in the same industry, the observable measure of the cross sectional variation in $q$ within an industry is given by $\sigma_{V,K,t} \equiv \sigma_{q,t}$. We refer to $\sigma_{V,K,t}$ as the intra-industry value spread at time $t$.

**Corollary 1** [Investment equations] Under imperfect competition, firms’ investment strategies depend on their own market to book ratio, their current and future expected extraordinary profits, and the intra-industry value spread.

The intra-industry value spread captures to what extent firms’ investment prospects are affected by firms’ strategic interaction. A marginal increase in the capacity of firm $j$ does not only reduce the future market price for firm $j$, as seen in the second term of (11); it also affects the market share and market prices of all its competitors. As a result, Corollary 1 states that $\sigma_{V,K,t}$ is significant in explaining investment. We provide the corresponding empirical evidence in Section 3.

### 2.2 Firms’ betas and the intra-industry value spread

We can also reinterpret the predictions on firms’ betas in Section 1 to obtain the core testable implication of the model on industry risk dynamics and the intra-industry value spread.

**Corollary 2** [Industry beta dynamics and value spread] Firms’ betas correlate positively in industries with low value spread, and negatively in industries with high value spread.

In a related empirical study, Cohen et al (2003) show how the value spread of the entire cross section of US public firms depends on the standard deviation in expected returns, and

\(^{28}\)The identity in (11) indicates that $K_{jt}$ may vary by firm type since it applies all types of industries, including those in which firms differ in their installed capacity.
the standard deviation in firms’ profits. We derive a similar identity for the intra-industry standard deviation of the logarithm of firms’ book to market ratios $\sigma_{\ln \frac{K}{V},t}$. We show in Appendix G that our definition of the value spread $\sigma_{\ln \frac{K}{V},t}$ is positively and mechanically related to $\sigma_{\ln \frac{K}{V},t}$.

**Proposition 6 [Industry $\sigma_{\beta,t}$ and value spread] Under imperfect competition, $\sigma_{\ln \frac{K}{V},t}$ is mechanically related to $\sigma_{\beta,t}$ and $\sigma_{\frac{V}{K},t}$ such that

$$
\sigma_{\ln \frac{K}{V},t}^2 \approx \gamma_t \sigma_{\beta,t}^2 - \sigma_{\ln \frac{V}{K},t}^2 - 2 \rho_t
$$

where $\gamma_t$ is defined in Appendix F, and $\rho_t$ is the covariance between $\ln \frac{K}{V}$ and $\ln \frac{V}{K}$.

**Proof.** See Appendix F. ■

Proposition 6 indicates that the intra-industry value spread is mechanically related to the cross section of expected returns and the cross sectional differences in firms’ cashflow to assets ratios. The fundamental difference between (12) and the identity for the value spread in Cohen et al (2003) is that our measure is purely intra-industry, and it is based on the premise that firms’ betas are interrelated under imperfect competition.

Cohen et al (2003) show that the value spread of US public firms in multiple industries is mostly explained by the intra-industry cross sectional variation in returns, and the intra-industry cross sectional variation in firms’ earnings. This result is highly consistent with the predictions of our model. We argue that the intra-industry value spread determines the dynamics of firms’ investments and firms’ betas in equilibrium.

The findings in Cohen et al (2003) and the predictions of our model provide an empirical interpretation on the role of book to market sorts commonly used in the asset pricing literature.29 Given that the value spread is predominantly intra-industry, market to book ratios reflect the relative ability of one firm to increase market share relative to its competitors. Consequently, the book to market sorts typically used by asset pricers aggregate stocks into portfolios according to firms’ relative position in the industry.

A related and yet mechanical testable prediction is that industries with high value spread have higher intra-industry standard deviation in betas or returns. We find supporting empirical evidence on this in Section 3. Figure 3 shows that, even at the threshold $\sigma_c = \sigma_{c'}$, the cross sectional variance of firms’ betas is higher in industries with leaders and followers.

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29See Fama and French (1992) and related papers.
2.3 Standard deviation in mark-ups and the HHI

While the intra-industry value spread is the key variable driving the dynamics of investment and betas, the model also yields testable implications based on commonly used measures of product market competition. We denote by \( m_{jt} \equiv \frac{\pi_{jt}}{p_t} > 0 \) the mark-up in profits of firm \( j \) at time \( t \).

When firms only differ in \( c_j \), industries with leaders and followers are weakly more concentrated and have a weakly higher intra-industry standard deviation in mark-ups \( \sigma_{m,t} \) than industries in which invest simultaneously. This is illustrated in Table 1. By construction and when \( X_t < x^*_L \), \( \sigma_{m,t} \) and the HHI of both industries is the same. Similarly, by construction, when the leader in the more heterogeneous industry invests at \( X_t = x^*_L \), it holds that \( \sigma^a_{m,t} \) and HHI\(^a\) are higher than \( \sigma^c_{m,t} \) and HHI\(^c\) thereafter.

In contrast, when we allow firms to differ in \( K_j \), the implied positive correlation between \( \sigma_{\Pi,t} \), \( \sigma_{m,t} \) and HHI need not hold. A deconcentrating industry may have a high value spread, and a concentrating industry may have a lower value spread. For instance, when firms differ exclusively in \( K_j \), the smaller firm finds it more profitable to invest earlier and more, and catches up in market share with the larger firm upon investment.\(^{30}\) Hence if the amount invested by leading small firms is sufficiently large, the HHI of a deconcentrating industry with high value spread may be higher than the HHI of a deconcentrating industry with low value spread, before all firms invest.\(^{31}\)

The corresponding empirical implication is that standard measures of competition such as \( \sigma_{m,t} \) and the HHI may prove insufficient to capture the degree of competition in an industry, since they are static. Firms’ investment decisions depend not only on the current spread in mark-ups or market shares, but also on the expected future changes in mark-up and market shares. In contrast, the intra-industry value spread is an observable industry characteristic which captures the unobserved heterogeneity in firms’ production technologies over time.

We can only extrapolate the predictions on industry dynamics and the intra-industry value

\(^{30}\)See Appendix E.

\(^{31}\)More formally, assume two industries in which firms differ exclusively in \( K_j \). In one industry the value spread is high and firms follow \( \Gamma^a \); in the other the value spread is low and firms follow \( \Gamma^c \). As smaller firms catch up with larger firms, the HHI in all industries decreases. At \( X_t > x^*_F \), the HHI given \( \Gamma^c \) or HHI\(^a\) is strictly lower than the HHI given \( \Gamma^a \) or HHI\(^a\). Yet if HHI\(^a\) falls significantly when firm \( L \) invests (i.e. \( \Lambda^L_t \) is large), then HHI\(^a\) \(<\)HHI\(^c\) for \( x^*_L < X_t < x^*_F \).
spread to $\sigma_{m,t}$ and the HHI when all of these measures are positively correlated. For instance, static measures of competition such as $\sigma_{m,t}$ and the HHI may sort industries in the same way as the intra-industry value spread when there is persistence in firms’ relative position in the product market - i.e. leaders remain leaders, while followers remain followers over time. We explore the empirical relation between $\sigma_{V,t}$, $\sigma_{m,t}$ and the HHI in Section 3.

2.4 Predicting industry betas

Consistent with Carlson et al (2004), firms’ betas increase before firms exercise their own investment opportunity, and decrease upon investment. It need not follow, however, that the same dynamics apply to the average industry beta $\mu_{\beta,t}$. The stylized real options prediction that a firm’s beta increases before investment and decreases upon exercise only applies to $\mu_{\beta,t}$ in industries with low value spread.

Corollary 3 [Predictability in industry betas] In industries with low value spread, periods of high market to book ratios, high investment and high betas are followed by periods of lower market to book ratios, lower investment and lower betas.

In industries with low value spread, average industry betas are more predictable, as a period of high market to book ratios, high investment, and high betas is followed by a period of lower market to book ratios, lower investment and lower betas. This pattern does not hold in industries with high value spread, in which the dynamics of the average industry beta are not representative of the dynamics of the beta of each firm in the industry.

A related testable implication is that average industry expected returns should be more predictable in less concentrated industries, unless these industries are undergoing deep transitions from high to low competition or vice versa. This is consistent with the evidence in Hoberg and Phillips (2010), who find that in less concentrated industries periods of high market to book ratios, high returns, high betas and high investment, are followed by periods of lower market to book ratios, lower investment, lower returns and lower betas. They also find no predictable pattern in the average industry returns of industries with high HHI.

2.5 Inter-industry implications

In Section 1, we keep all exogenous parameters related to the industrial organization constant except for $\sigma_c$. As a result, our focus is on how the *intra-industry* cross sectional variation in firms' technologies affects the industry dynamics of investment and risk. In practice however, industries differ in multiple characteristics which also influence these dynamics. We hereby explore how *inter-industry* differences in the demand growth $\mu_x$, demand volatility $\sigma_x$, demand elasticity $\varepsilon$, and the number of firms $N$ affect our main empirical implications.

2.5.1 Product Market Demand

The dynamics of investment and expected returns in Section 1 depend critically on the threshold $\sigma_c$. Figure 4 shows that $\sigma_c$ is decreasing in $\varepsilon$, and increasing in demand growth $\mu_x$ and volatility $\sigma_x$. The rationale behind these results is consistent with previous studies. Ivaldi et al (2003) show that tacit coordination is more likely in growing industries, in which current profits are low relative to future profits. Boyer et al (2001) suggest that demand uncertainty induces coordination as it boosts the growth option values of all firms. Similarly, Ivaldi et al (2003) and Motta (2004) argue that coordination is more likely with low demand elasticity.

Figure 4 also provides comparative statics on the shadow cost of preemption $\lambda$. These comparative statics show to what extent the underlying determinants of demand affect the preemptive behavior of firm $M$. $\lambda$ is increasing in $\mu_x$ and $\sigma_x$; since $\mu_x$ and $\sigma_x$ boosts the growth option value of all firms, leaders find it more costly to deter the investments of their competitors in industries with high demand growth and high demand volatility. $\lambda$ is decreasing in $\varepsilon$; since an increase in $\varepsilon$ implies a reduction in the market power of firm $M$, leadership is relatively less costly for firm $L$ as $\varepsilon$ increases.

The corresponding empirical implication is that in industries with low demand elasticity $\varepsilon$, high demand growth $\mu_x$, and high demand volatility $\sigma_x$ firms have more similar investment patterns, firms' betas correlate more positively, and the average industry beta is more predictable. Furthermore, the insight that a firm's beta is mainly affected by its own investment decisions is incomplete in industries with high demand elasticity $\varepsilon$, low demand growth $\mu_x$, and low demand volatility $\sigma_x$.

The comparative statics of $\sigma_c$ with respect to $\varepsilon$, $\mu_x$ and $\sigma_x$ also have an important indirect empirical implication. Firms' investment strategies and betas correlate more positively in industries with low value spread *not only* when we compare industries with a different $\sigma_c$ (as
in Proposition 2), but also when we compare heterogeneous industries with differences in \( \varepsilon, \mu_x \) and \( \sigma_x \). The practical implication of the comparative statics in Figure 4 is yet that we need not control for differences across industries in \( \varepsilon, \mu_x \) and \( \sigma_x \) when assess empirically if the intra-industry correlation in firms’ betas is higher in industries with low value spread. This is relevant for the sake of empirical tests in Section 3.

To see this, note that the threshold \( \sigma_c \) is decreasing in \( \varepsilon \); industries are more likely to have leaders and followers as \( \varepsilon \) increases. By construction, and all else equal, the intra-industry value spread is higher under sequential investment. Hence our prediction of more comovement in betas in industries with low value spread holds for industries with different demand elasticities \( \varepsilon \). Likewise, the threshold \( \sigma_c \) is increasing in \( \mu_x \) and \( \sigma_x \); industries are less likely to have leaders and followers as \( \mu_x \) and \( \sigma_x \) increase. Hence our prediction of more comovement in betas in industries with low value spread also holds for industries with differences in \( \mu_x \) and \( \sigma_x \).

### 2.5.2 Industries with \( N > 2 \) firms

While we focus on a model with \( N = 2 \) firms, in practice industries need not be duopolies. By construction, a higher number of firms increases the set of potential industry equilibria. When \( N > 2 \), some firms may invest sequentially, while some others may find it optimal to cluster instead.

The approach to solve the game is the same as in Section 1, and relies on sorting conditions and ICCs. As in Cho and Sobel (1990), the sorting conditions are the same for any value of \( N \), and hence also in oligopolies firms with higher \( q \) have the ability to invest earlier and more. Furthermore, the sorting conditions facilitate the analysis when \( N > 2 \), insofar they constrain the set of possible equilibria to those in which firms with high \( q \) invest earlier or in tandem with firms with lower \( q \).

Given that the sorting conditions of the game are the same, the core testable prediction for duopolies that firms’ investment patterns are more similar in industries with low value spread also holds when \( N > 2 \). In oligopolies, firms decide when and how much to invest based on the difference between their market to book ratio and that of their competitors. Hence when firms have more similar market to book ratios, firms have more similar investment patterns, and the intra-industry comovement in firms’ betas is higher.

As we discuss in Appendix G, however, while the intra-industry value spread is still key
in determining the equilibrium outcome with \( N > 2 \), in oligopolies the *entire* distribution of firms’ market to book ratios is necessary to characterize the equilibrium. Intuitively, in an industry with \( N \) firms, the intra-industry value spread captures how distant each firm is from the average firm in the industry. In the model, however, each firm is effectively constrained by the behavior of their closest competitor. Consequently, if the distribution of firms’ market to book ratios is fairly skewed, the equilibrium outcome may combine early investments by leaders with late, clustered investments by laggards.

We illustrate these arguments by means of an example with three firms. We label firms by \( L \), \( M \) and \( F \), where firm \( F \) can be interpreted as a single firm or more broadly as a fringe of firms with very similar costs of production whose investments cluster in equilibrium. We assume that firms have the same installed capacity \( K \) before investment, and have uniformly distributed marginal costs of production after investment such that \( c_L < c_M < c_F \). We report all potential equilibria in pure strategies in Tables 2 and 3. Firms may invest sequentially or simultaneously as in the duopoly case (Table 2). Furthermore, two of the three firms may cluster, and the remaining firm may either lead or follow (Table 3).

The example in Tables 2 and 3 provides several important insights. Consistent with our findings as in Section 1, we observe that since the intra-industry value spread is relatively low, the three firms optimally invest simultaneously (Table 2). Also, since the model has a single stochastic shock, the instantaneous correlation between the betas of any pair of firms is either 1 or \(-1\). However, the average intra-industry correlation in firms’ betas over time is affected by \( N \).

To see this, we use the parametrization in the numerical example of Table 2, we compute firms’ betas over time using simulated Brownian paths, and we compute the average intra-industry correlation in firms’ betas in the fully sequential equilibrium with \( N = 3 \) and \( x_L < x_M < x_F \). We compare our results to those in the fully sequential equilibrium for \( N = 2 \) depicted in Figure 3. When \( N = 2 \), the average intra-industry correlation in firms’ betas equals \( \approx -0.9 \). When \( N = 3 \), the average intra-industry correlation in firms’ betas is \( \approx -0.32 \). The absolute average intra-industry correlation is lower when \( N = 3 \), since the betas of two non-investing firms are always positively correlated over time.\(^{33}\)

The numerical example also illustrates how a higher number of firms \( N \) affects firms’

\(^{33}\)When \( x_L < x_M < x_F \), and firm \( L \) invests, the betas of firms \( M \) and \( F \) correlate positively as they both expect a reduction in their profits. When firm \( M \) invests, the betas of \( M \) and \( F \) comove positively for the same reason. Lastly, when firm \( F \) invests, the betas of \( L \) and \( M \) comove positively as well.
investment strategies and betas. For this sake, we compare our numerical examples for \( N = 2 \) in Table 1 and \( N = 3 \) in Table 2. Consistent with Grenadier (2002), Aguerrevere (2009) and Bulan et al (2009) for symmetric oligopolies, Tables 1 and 2 show that a higher \( N \) erodes the values and betas of all firms in the industry. We add to the their findings that a higher \( N \) need not erode the values and betas of all firms evenly. In asymmetric oligopolies, an increase in \( N \) affects more severely the betas of those firms with lower marginal \( q \) in the industry. The beta of firm \( M \) in Panel (B) of Table 2 at \( X_t = X_0 \) is lower than the corresponding beta of firm \( M \) at \( X_t = X_0 \) in Panel (C) of Table 1.

We also find that an increase in \( N \) need not induce all firms to accelerate investment. In Grenadier (2002) and in the industries with low value spread in this paper, firms optimally invest earlier and less as \( N \) increases. However, in industries with high value spread, an increase in \( N \) may induce firms with high \( q \) to invest earlier and more to preserve their position as leaders, forcing firms with lower \( q \) to delay their investment. In Tables 1 and 2, and when firms invest sequentially (Panels B and A, respectively), firm \( L \) invests earlier when \( N = 3 \); conversely, firm \( M \) invests earlier when \( N = 2 \).

3 Empirical evidence

Sections 2 provides qualitative testable implications on how firms’ strategic interaction affect the intra-industry dynamics of investments and betas. A reasonable concern, however, is whether these effects are economically significant. Given the nature of the problem of study, it is reasonable to argue that strategic interaction plays a bigger role in determining the dynamics of investment and risk in some industries and not in others.

A natural experiment to tackle this concern would be to calibrate the model to match the investment and risk dynamics of different industries. Complicating the task of calibration, however, the parameters which characterize the organization of an industry in our model are empirically unobservable, or require at least a thorough empirical study to infer their magnitude. These parameters include firms’ marginal costs of production, firms’ costs of investment, and the underlying determinants of product market demand \( \varepsilon \), \( \mu_x \), and \( \sigma_x \).

We pursue an alternative approach and assess whether the main testable predictions of our model hold on average for the cross section of US industries. Our tests rely on similar datasets used in previous studies such as Hou and Robinson (2006) and Hoberg and Phillips (2010).

\[ \text{See Panel B in Table 1 and Panel A in Table 2.} \]
Our empirical tests provide supporting empirical evidence on the following predictions:

- Firms’ investment strategies are significantly related to the intra-industry value spread;
- Firms’ betas and returns correlate more positively in industries with low intra-industry value spread; and
- Firms’ betas and returns correlate more positively in industries with low intra-industry standard deviation in mark-ups, and low HHI.

3.1 Dataset and empirical approach

We define an industry by its four-digit SIC code. This is the finest available industry classification that is available in our merged CRSP-COMPUSTAT dataset. We prefer such measure as opposed to a broader industry definition since our testable predictions rely on the impact of firms’ strategic interaction in product markets.

We include all NYSE, AMEX, and NASDAQ-listed firms in the intersection of the CRSP monthly returns file and the COMPUSTAT annual file between January 1968 and December 2008. We use data at annual frequency to run the tests on investment equations. We use data at monthly frequency to run the asset pricing tests. We elaborate on the database construction in Appendix H. We report the summary statistics of the working sample in Table 4.

We denote the relevant variables in our tests as the equity beta \( \beta \); the excess return or stock return in excess of the risk free rate \( R \); the market to book asset ratio \( \frac{V}{K} \); the book leverage ratio \( \frac{B}{K} \); the market to book equity ratio \( \frac{V-B}{B} \); the cashflow to assets ratio \( \frac{E}{K} \); the investment rate \( \frac{I}{K} \); and the mark-up in profits \( m \). We follow Khanna and Thomas (2009) and construct a measure of comovement which captures the average pairwise correlation in firms’ investments, market to book equity ratios, market to book asset ratios, betas and excess returns by industry. We denote the intra-industry comovement of variable \( x \) in month-year \( t \) as \( \omega_{x,t} \).

We also consider the two static measures of competition discussed in Section 1. One is the intra-industry deviation in mark-ups or \( \sigma_{m,t} \), which we construct using the COMPUSTAT annual files. The other is the logarithm of the HHI index by four-digit SIC code reported by the US Census Bureau or \( \lnHHI \), which is limited to manufacturing industries only.\(^{35}\) In line

\(^{35}\)We use logs for scaling purposes only; results are qualitatively the same when we use the HHI.
with Ali et al (2009), we do not compute the HHI using CRSP-COMPUSTAT sales data since such index is not highly correlated with the US Census Bureau concentration index.\footnote{In untabulated tests, we also consider the proxy of the HHI recently proposed by Hoberg and Phillips (2010). While we obtain similar results, the industry definition of their proxy is noisy for the sake of our study, as it relies on 3-digit SIC codes.}

We apply the same empirical methodology to test all our implications on investment and risk. Given that in our model the underlying industry determinants of demand and the number of firms are constant, we run all tests using cross sectional regressions as in Fama and MacBeth (1973). To account for serial correlation, we consider Newey West standard errors.\footnote{We have also run all empirical tests in this section using OLS regressions with year dummies. Results are qualitatively similar to those reported in the paper.}

Finally, the model assumes that firms are unlevered, while most firms in our working sample are levered. We thus run our tests on (equity) betas and stock returns using two alternative definitions of the intra-industry value spread: one based on the asset value spread or $\sigma_{V^K,t}$, and another based on the equity value spread $\sigma_{V^B,K,t}$.

### 3.2 Investment, betas and returns

The core asset pricing prediction that the firm’s betas comove more positively in industries with low value spread relies on three important results in the model. The first is that firms’ investments relate significantly to the intra-industry value spread. We provide the corresponding empirical evidence in Table 5. We find that the intra-industry value spread is significant in explaining investment, both at the firm level (Panels B and C) and industry level (Panel E and F). We obtain similar results when using the intra-industry asset value spread (Panels B and E), and the intra-industry equity value spread (Panels C and F).

The second result is that firms’ investment decisions affect their exposure to systematic risk. We provide the supporting empirical evidence of this result in Panels A to F of Table 6, by showing that the intra-industry comovement in betas and excess returns are significantly related to the intra-industry comovement in investment. Similarly, the intra-industry comovement in betas and excess returns are significantly related to the intra-industry comovement in market to book ratios.

Finally, the predictions of our single factor model apply to both betas and excess returns. Like other papers, we acknowledge that our single factor model does not explain why there exist value and size premia in excess returns. However, both in the model and in the data, the
intra-industry comovement in betas is significantly related to the intra-industry comovement in excess returns. The average R-square in Panel G of Table 6 indicates that the intra-industry comovement in betas explains on average 37% of the intra-industry comovement in excess returns.\footnote{The corresponding adjusted R-square using an OLS regression with year dummies is 30\%.}

### 3.3 Industry risk dynamics and product markets

The model predicts a negative and significant correlation between the intra-industry comovement in betas and excess returns and the intra-industry value spread. Table 7 provides the corresponding empirical evidence. We find a negative and significant correlation between the intra-industry comovement in betas and the intra-industry value spread (Panels A and B). We also find a negative and significant relation between the intra-industry comovement in excess returns and the intra-industry value spread (Panels E and F).

The model further suggests that those industries with low value spread may also have low standard deviation in mark-ups, and low HHI; this holds when the intra-industry value spread is positively correlated with the intra-industry standard deviation in mark-ups, and with the HHI. In our dataset, we observe a significant and positive correlation between the intra-industry asset value spread, the equity value spread, the standard deviation in mark-ups, and the log of the HHI. The pairwise correlation between the asset (equity) value spread and the dispersion in mark-ups is 17.55% (resp. 19.73%). The pairwise correlation between the asset (equity) value spread and the logarithm of the HHI is 16.51% (resp. 16.46%).

The corresponding testable implication is that of a negative and significant correlation between these static measures of competition and the intra-industry comovement in betas or excess returns. As suggested by the model, we report in Table 7 a negative and significant relation between the comovement in betas, and the static measures of competition given by $\sigma_{m,t}$ and lnHHI (Panels C and D). We also find a negative and significant relation between the comovement in excess returns, and the static measures of competition given by $\sigma_{m,t}$ and lnHHI (Panels G and H). Using an alternative empirical approach, Hoberg and Phillips (2010) also show that returns comove more positively in industries with low HHI.
4 Conclusion

This paper provides a model of industry equilibrium to study how strategic interaction affects the intra-industry dynamics of corporate investment and expected returns. Under imperfect competition, the fundamental insight in the asset pricing literature that a firm’s exposure to systematic risk or beta is affected by its own investment decisions is incomplete. In industries with high value spread, low demand growth, low demand volatility, and high demand elasticity, a firm’s beta is sometimes better explained by the investment of its peers.

In imperfectly competitive industries, we predict that the investment strategy and exposure to systematic risk of each firm is affected by marginal product of capital of all its competitors; this suggests why the value spread in Cohen et al (2003) is predominantly intra-industry. We find theoretically and empirically that firms’ betas and excess returns correlate more positively in industries with low value spread. We also document and explain why firms’ betas and excess returns correlate more positively in industries with low HHI, and low intra-industry standard deviation in mark-ups.

To conclude, we highlight that the fundamental insight of our paper is that product markets have non trivial effects on firms’ investment decisions and their expected returns. In this context, dynamic models of strategic interaction typically studied in the industrial organization literature become a useful tool to explain empirical regularities in the cross section of returns. The model can be extended in many ways.
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Appendix

A Proposition 1

For any strategy $\Gamma_j = \{x_j; \Lambda_j\}$, we denote $\Lambda_{jt}^+ = \frac{\bar{\pi}^+}{\pi^+} + \frac{\Delta \pi^+}{\pi^+}$ the value of the assets in place of firm $j$ before investment, $\Lambda_{jt}^- = \frac{\bar{\pi}^-}{\pi^-} + \frac{\Delta \pi^-}{\pi^-}$ the value of the assets in place of firm $j$ after investment. At the investment threshold $X_t = x_j$, the firm can pay $fK$ to increase the value of its assets in place from $\Lambda_{jt}^-$ to $\Lambda_{jt}^+$. Given exercise at $X_t \geq x_j$, the value of the growth option to invest is calculated as a perpetual binary option with payoff $V_0^\Delta^+ - V_0^\Delta^- - fK$. We then observe\(^{39}\) that the expected value of the growth option to invest is given by $G_{jt} \equiv \left[ \Lambda_{jt}^- - \Lambda_{jt}^+ - fK \right] \left( \frac{X_t}{x_j} \right)^u$, where $\left( \frac{X_t}{x_j} \right)^u$ is the price of a contingent claim that pays 1 if the firm invests and 0 otherwise, and the parameter $u > 1$ is such that

$$v = \frac{1}{2} - \frac{\bar{\pi}^+}{\pi^+} + \left[ \frac{\bar{\pi}^-}{\pi^-} - \frac{1}{2} \right]^2 + \frac{2\Delta \pi}{\pi^+} \left( 1 - \frac{\bar{\pi}^-}{\pi^-} \right)^u$$

For any strategy $\Gamma_j = \{x_j; \Lambda_j\}$, we conclude that $V_{jt}$ equals $A_{jt}^- + G_{jt}$ if $X_t < x_j$, and $A_{jt}^+$ if $X_t \geq x_j$.

B Proposition 2

B.1. Sorting conditions

The strategy pursued by firms is a multiple-action pair such that $\Gamma_j = \{x_j; \Lambda_j\}$. The proof of the sorting conditions on $\Gamma_j$ follows Bustamante (2012) and consists of two steps. The first step is to show that if the value function $V_{jt}$ complies the conditions in Cho and Sobel (1990), then the sorting condition of the action pair $\Gamma_j$ corresponds to the sorting conditions of each action in isolation. The second step is to derive the sorting conditions of $x_j$ and $\Lambda_j$ separately.

We denote by $X_t \bar{Y}_j^{\frac{1}{2}}$ the expected price by firm $j$ at time $t$. In equilibrium, $X_t \bar{Y}_j^{\frac{1}{2}}$ is equal to the market price $p_t$ when $\Delta \pi_{jt} = 0$; we use a more general notation since the sorting conditions should hold for any strategy $\Gamma_j$. In the basic model, the value function $V_{jt}$ firm $j$ given the set of actions $\Gamma_j$ is such that

$$V_{jt} = \left( \bar{Y}_j^- \right)^{-\frac{1}{2}} (\bar{K} \frac{X_t}{x_j} - \tau \frac{X_t}{x_j} \Lambda_j K) + \left[ \left( \bar{Y}_j^- \right)^{-\frac{1}{2}} \frac{X_t}{x_j} \Lambda_j K - \left( \bar{Y}_j^- \right)^{-\frac{1}{2}} \frac{X_t}{x_j} \Lambda_j - (c_j \Lambda_j - \bar{\pi}) \frac{X_t}{x_j} \Lambda_j K - fK \right] \left( \frac{X_t}{x_j} \right)^u$$

We consider the case of $N = 2$ in which $K_j = K$ and $c_L < c_M$. In line with Cho and Sobel (1990), $V_{jt}$ is continuous in $\Gamma_j$ and for any type $j$. Furthermore, if $x_L < x_M$ and $\Lambda_L > \Lambda_M$, then it must be the case that $V_{MT} \leq \bar{V}_{MT}$ implies $V_{LT} > \bar{V}_{LT}$. This last condition ensures that if firm $M$ has incentives to deviate, firm $L$ will pay a cost to ensure incentive compatibility. Denote $d = \frac{X_t}{x_M} < 1$. We also denote by $\bar{Y}_{ji}$ the expected production of the industry when firm $j$ deviates and pretends to be firm $i$. Then the condition $V_{MT} \leq \bar{V}_{MT}$ implies $c_M \leq \Omega_e$ where $\Omega_e$ is given by

$$\Omega_e = (\Lambda_L - \Lambda_M d^{u-1})^{-1} \left[ \Lambda_L \left( \bar{Y}_{ML}^+ \right)^{-\frac{1}{2}} - d^{u-1} \Lambda_M \left( \bar{Y}_{ML}^- \right)^{-\frac{1}{2}} - (1 - d^{u-1}) \left( \bar{Y}_{ML}^- \right)^{-\frac{1}{2}} - \bar{\pi} + \frac{fK}{x_M} \right]$$

\(^{39}\)See, for example, Dixit and Pindyck (1994). The details of the derivation of the parameter $u > 1$ are provided in Chapter 5.
Similarly, the condition \( V_{Lt} > \overline{V}_{Lt} \) implies \( c_L \leq \Omega_c \). Therefore if \( c_L < c_M \) and \( c_M \leq \Omega_c \), it holds that \( c_L < \Omega_c \) for any parameter value.

Consider now the sorting condition for each action \( x_j \) and \( \Lambda_j \) separately. The sorting conditions reflect that, all else equal, more efficient firms find it less costly to invest earlier and more, namely

\[
\frac{\partial}{\partial x_j} \left[ \frac{\partial V_j}{\partial x_j} \right] = -\left( \frac{1-\nu}{\delta} \right) \Lambda_j K \left( \frac{x_j}{s_j} \right)^\nu > 0 \quad \text{and} \quad \frac{\partial}{\partial \Lambda_j} \left[ \frac{\partial V_j}{\partial \Lambda_j} \right] = -\frac{1}{\delta} K \left( \frac{x_j}{s_j} \right)^\nu < 0
\]

Put together, these inequalities ensure that the incentive compatibility constraint of firm \( M \) is binding and that there exists a sequential equilibrium when \( N = 2 \).

The sorting conditions described for \( N = 2 \) also apply for the more general case of \( N > 2 \). This is because the sufficiency conditions in Cho and Sobel (1990) apply for games with \( N \) types. All conditions above hold when \( c_L < \ldots < c_j < \ldots < c_N \).

### B.2. Sequential equilibrium

In the sequential equilibrium, the manager of firm \( j \) chooses the strategy \( \Gamma_j^* \) to maximize \( V_j^* \), where \( j = L, M \).

The functional form of \( V_j^* \) is that provided in Proposition 1, where the terms \( \Delta \pi_j^{\pi} \) can be explicitly defined given \( x_L^* < x_M^* \). Given \( x_L^* < x_M^* \), it holds that \( \Delta \pi_{L,t}^* = \Delta \pi_{M,t}^* = 0 \), and the expressions for \( \Delta \pi_{L,t}^* < 0 \) and \( \Delta \pi_{L,t}^{++} < 0 \) are such that

\[
\Delta \pi_{M,t}^* = \left[ (1 + \Lambda_L^*)^{-\frac{1}{2}} - 2^\frac{1}{2} \left( \frac{x_L^*}{x_T^*} \right)^\nu \right] \quad \text{if } X_t \leq x_L^*
\]

\[
\Delta \pi_{L,t}^{++} = \left[ (\Lambda_M^* + \Lambda_L^*)^{-\frac{1}{2}} - (1 + \Lambda_M^*)^{-\frac{1}{2}} \right] x_M^* \Lambda_L^* K^{1-\frac{1}{2}} \left( \frac{x_L^*}{s_M^*} \right)^\nu \quad \text{if } x_L^* < X_t \leq x_M^*
\]

For the sake of convenience, we also use the notation in Appendix A, and we denote \( A_{j,t}^{++} \) as assets in place before investment, \( A_{j,t}^* \) as assets in place after investment, and \( G_{j,t} \) as the value of the growth option to invest of firm \( j \) in the sequential equilibrium. Then \( V_j^* \) equals \( A_{j,t}^{++} + G_{j,t}^* \) if \( X_t < x_j^* \), and \( A_{j,t}^* \) if \( X_t \geq x_j^* \), where

\[
A_{M,t}^* = \frac{x_M^*}{x_T^*} + \frac{\Delta \pi_{M,t}^*}{2}, \quad A_{L,t}^* = \frac{x_L^*}{x_T^*}, \quad A_{M,t}^{++} = \frac{x_M^*}{x_T^*} + \frac{\Delta \pi_{M,t}^{++}}{2}
\]

Consider first the optimization problem of firm \( M \). To ensure that \( x_M^* \) is chosen optimally, the derivative of \( G_{M,t}^* \) with respect to \( x_j \) must be zero for all values of \( X_t \). To ensure that \( \Lambda_M^* \) is chosen optimally, the derivative of \( A_{M,t}^* \) with respect to \( \Lambda_M^* \) must be zero for all values of \( X_t \). The corresponding optimal investment strategy \( \Gamma_M^* \) is such that

\[
x_M^* = f \left[ \frac{\delta u}{1-u} \right] \left[ (\Lambda_M^* + \Lambda_L^*)^{-\frac{1}{2}} \Lambda_M^* - (1 + \Lambda_L^*)^{-\frac{1}{2}} K^{-\frac{1}{2}} \right] K^{-\frac{1}{2}} \left( 1 - \frac{1}{\delta} \frac{\Lambda_M^*}{\Lambda_M^* + \Lambda_L^*} \right)^{-1} \quad \text{and} \quad c_M = (\Lambda_M^* + \Lambda_L^*)^{-\frac{1}{2}} K^{-\frac{1}{2}} \left( 1 - \frac{1}{\delta} \frac{\Lambda_M^*}{\Lambda_M^* + \Lambda_L^*} \right)
\]

Consider now the optimization problem of firm \( L \). The optimality conditions of firm \( L \) are different from those of firm \( F \) since in the sequential equilibrium firm \( L \) is subject to the complementary slackness condition in (8). We solve for \( \Gamma_L^* \) using Kuhn-Tucker. The value function of the manager of firm \( L \) at \( X_t = x_L^* \) is such that

\[
\mathcal{L} = V_{Lt}^* - \lambda \left( V_M^* - \overline{V}_M^* \right)
\]

and the corresponding optimality conditions are given by
further than its own optimal threshold. As a result, the equilibrium threshold corresponds to the optimal threshold for the less efficient firm.

Proposition 1 is given by

\[ B.3. \text{ Clustering equilibrium} \]

in place before investment, at investment thresholds. A priori, this might lead to a range of potential equilibrium thresholds. Given the asymmetry in firms’ production technologies, each firm would attain its maximum value by clustering as in Fundenberg and Tirole (1985) and Weeds (2002). Second, the optimal scale upon investment \( \Lambda_L^c > 1 \) is such that

\[
(1 - \kappa^c) \left( 1 - \frac{1}{2} \frac{\Lambda_L^c}{\Lambda_M^c} \right) (1 + \Lambda_L^c)^{-\frac{1}{2}} K^{-\frac{1}{2}} + \kappa^c (1 - v) \left( \frac{\partial \Lambda_M^c}{\partial x_L^c} \right) \left[ (\Lambda_M^c + \Lambda_L^c)^{-\frac{1}{2}} - (1 + \Lambda_L^c)^{-\frac{1}{2}} \right] K^{-\frac{1}{2}} = \varphi_{CL}
\]

where \( \kappa^c = \left( \frac{x_L^c}{x_L} \right)^{v-1} \).

B.3. Clustering equilibrium

We denote the value of firm \( j \) in the clustering equilibrium as \( V_{jL}^c \). Given \( x_j^c = x^c \), we characterize \( V_{jL}^c \) as in Proposition 1 such that \( \Delta \pi_{jL}^c = \Delta \pi_{jL}^{c+} = 0 \). Also, using the notation in Appendix A, we denote \( A_{jL}^{c-} \) as assets in place before investment, \( A_{jL}^{c+} \) as assets in place after investment, and \( G_{jL}^c \) as the value of the growth option of firm \( j \) in the clustering equilibrium. Hence \( V_{jL}^c \) equals \( A_{jL}^{c-} + G_{jL}^c \) if \( X_L < x_j^c \), and \( A_{jL}^{c+} \) thereafter.

To obtain the optimal cluster equilibrium strategies of firms in the clustering equilibrium, the proof consists of two steps. First, we show that the Markov-perfect clustering equilibrium is such that both firms are better off by investing simultaneously. This is consistent with Fundenberg and Tirole (1985) and Weeds (2002). Second, we derive the optimal investment thresholds \( x^c \) and the corresponding assets in place after investment, and the increases in capacity

\[
\frac{\partial \pi_{jL}^c}{\partial x_L^c} \bigg|_{x_L^c} = 0; \quad \frac{\partial \pi_{jL}^c}{\partial \Lambda_L^c} \bigg|_{x_L^c} = 0 \quad \text{and} \quad \frac{\partial \pi_{jL}}{\partial x_L^c} \bigg|_{x_L^c} = 0
\]

where the Lagrange multiplier \( \lambda > 0 \) due to the sorting conditions of the game. The optimal threshold \( x_L^c \) is given by

\[
x_L^c = f \frac{\partial \pi_{jL}}{\partial x_L^c} \bigg|_{x_L^c = x_L^c} = \left[ (1 + \Lambda_L^c)^{-\frac{1}{2}} \Lambda_L^c - \frac{(2K)^{-\frac{1}{2}} - \varphi_{CL}}{1 - \frac{1}{2} \frac{\Lambda_L^c}{\Lambda_M^c}} \right]^{-1}
\]

where \( \varphi = \frac{c_L - \lambda c_M}{c_L (1 - \lambda)} < 1 \). The optimal scale upon investment \( \Lambda_L^c > 1 \) is such that

\[
(1 - \kappa^c) \left( 1 - \frac{1}{2} \frac{\Lambda_L^c}{\Lambda_M^c} \right) (1 + \Lambda_L^c)^{-\frac{1}{2}} K^{-\frac{1}{2}} + \kappa^c (1 - v) \left( \frac{\partial \Lambda_M^c}{\partial x_L^c} \right) \left[ (\Lambda_M^c + \Lambda_L^c)^{-\frac{1}{2}} - (1 + \Lambda_L^c)^{-\frac{1}{2}} \right] K^{-\frac{1}{2}} = \varphi_{CL}
\]

where \( \kappa^c = \left( \frac{x_L^c}{x_L} \right)^{v-1} \).

The optimal scale \( \Lambda_L^c > 1 \) that maximizes the assets in place \( A_{jL}^{c+} \) of each firm \( j \) equals

\[
c_J = (\Lambda_M^c + \Lambda_L^c)^{-\frac{1}{2}} K^{-\frac{1}{2}} \left( 1 - \frac{1}{2} \frac{\Lambda_L^c}{\Lambda_M^c + \Lambda_L^c} \right)
\]

34
C Proposition 3
The derivation of $\beta_{jt}$ follows that in Carlson et al (2004). Applying Ito’s lemma to $V$, we note that the exposure to systematic risk of the firm equals the proportion of the replicating portfolio invested in the risky asset, such that $\beta = \frac{x^s}{x^Q}$. The exact expression for $\beta_{jt}$ depends on the equilibrium outcome. If $\sigma_c < \sigma_{c, t}$, $\beta_{jt}$ equals $1 + (v - 1) \frac{1}{5} (p_t^c - \bar{c}X_t) \frac{K}{V_{Lt}} > 1$ if $X_t \leq x^e$ and 1 otherwise, where $p_t^c = (2K)^{-\frac{1}{2}}$. If $\sigma_c > \sigma_{c, t}$, $\beta_{jt}$ equals $1 + (v - 1) \frac{1}{5} (p_t^c - \bar{c}X_t) \frac{K}{V_{Mt}} > 1$ if $X_t \leq x^e$ and is equal to 1 otherwise, where $p_t = (K + \Lambda^t K)^{-\frac{1}{2}}$. If $\sigma_c > \sigma_{c, t}$, $\beta_{jt}$ equals $1 + (v - 1) \frac{1}{5} (p_t^c - \bar{c}X_t) \frac{K}{V_{Mt}} < 1$ if $X_t \leq x^e$ and is equal to 1 otherwise.

D Proposition 4
The sign of the covariance between the beta of firm $L$ and that of firm $M$ depends on $\sigma_c$. For any investment strategy $\Gamma$, the definition of firms’ betas in (10) implies that the covariance in firms’ betas depends on the covariance in firms’ cashflow to value ratios, and hence

$$\text{sign} \left[ \text{cov} (\beta_{Lt}, \beta_{Mt}) \right] = \text{sign} \left[ \text{cov} \left( \frac{V_{Lt} - \frac{\sigma_c}{\sigma_{c, t}}}{V_{Lt}^2}, \frac{V_{Mt} - \frac{\sigma_c}{\sigma_{c, t}}}{V_{Mt}^2} \right) \right]$$

When $\sigma_c < \sigma_{c, t}$, both firms expect an increase in value upon investment, and $\Delta \pi^c_{jt} = \Delta \pi^+_{jt} = 0$. This implies that, before investment, $V_{jt} - \frac{x^c}{6} = G_{jt}$, where $G_{jt}$ is the value of the growth option of firm $j$, and

$$\text{cov} (G_{Lt}, G_{Mt}) = \Psi^s_L \times \Psi^s_M \times \sigma^2_X X^Q > 0$$

where we define $\Psi_j > 0$ such that $G_{jt} \equiv \Psi_j X^Q_j$. Hence $\text{cov} (\beta_{Lt}, \beta_{Mt}) < 0$ if $X_t < x^e$.

Conversely, when $\sigma_c > \sigma_{c, t}$, each firm expects a reduction in its profits upon the investment of its competitor, where $\Delta \pi^-_{Lt} < 0$ and $\Delta \pi^+_{Mt} < 0$. Consider first the interval $x^e_L < X_t < x^e_M$. In this case, firm $L$ only expects a reduction in its profits, while firm $M$ only expects an increase in its profits upon investment. As a result, $V_{Lt} - \frac{x^c}{6} = \Delta \pi^-_{Lt} < 0$, while $V_{Mt} - \frac{x^c}{6} = G_{Mt}$. Put together, this implies that $\text{cov} (\beta_{Lt}, \beta_{Mt}) < 0$ if $x^e_L < X_t < x^e_M$ since

$$\text{cov} (\Delta \pi^+_{Lt}, G_{Mt}) = \Psi^s_L \times \Theta^s_M \times \sigma^2_X X^Q < 0$$

where $\Psi^s_L = \Delta \pi^+_{Lt} X^-_L X^Q < 0$. Similarly, since $\Delta \pi^-_{Mt} < 0$, the same argument applies to show that $\text{cov} (\beta_{Lt}, \beta_{Mt}) < 0$ when $X_t < x^e_L$.

E Proposition 5
We define a scalar $q_j$ such that $q_j$ is the marginal product of capital of firm $j$, evaluated at $X_t = X_0$ and some strategy $\Gamma = \{x; \Lambda\}$. For any strategy $\Gamma = \{x; \Lambda\}$, the marginal product of capital $q$ equals (11). The choice of the strategy $\Gamma$ to define $q_j$ is without loss of generality; we use the same $\Gamma$ for all firms and do not affect the sorting of $q_j$. Similarly, we use $X_0$ for the sake of simplicity; any $X_t \leq x^*_L$ is suitable to define $q_j$. In all cases, the sufficiency conditions described for $N = 2$ also apply for $N > 2$.
E.1. Sorting conditions

We derive the sorting conditions with respect to \( q_j \) for three difference cases: one in which firms differ only in \( c_j \), another in which firms only differ on \( K_j \), and then when firms differ both in \( K_j \) and \( c_j \).

**Firms differ in \( c_j \)** The sorting conditions with respect to \( c_j \) are provided in (6) and derived in Appendix B. Equation (11) implies that for any investment strategy \( V_K \) is a monotone, strictly decreasing function of \( c_j \); hence firms with higher marginal costs of production are willing to invest less. Put together, we get that firms with higher \( q_j \) have the ability to invest earlier and more, namely

\[
\frac{\partial}{\partial q_j} \left[ \frac{\partial V_{1j}}{\partial x_j} \right] = \frac{\partial}{\partial c_j} \left[ \frac{\partial V_{1j}}{\partial x_j} \right] \frac{\partial q_j}{\partial c_j} > 0, \quad \text{and} \quad \frac{\partial}{\partial q_j} \left[ \frac{\partial V_{1j}}{\partial x_j} \right] = \frac{\partial}{\partial c_j} \left[ \frac{\partial V_{1j}}{\partial x_j} \right] \frac{\partial c_j}{\partial q_j} < 0
\]

**Firms differ in \( K_j \)** When \( K_L < K_M \) and \( c_j = c \), the functional form of \( V_{jt} \) is the same as in Appendix B. We first prove that if \( x_L < x_M \) and \( \Lambda_L > \Lambda_M \), then \( V_{Mt} \leq \tilde{V}_{Mt} \) implies \( V_{Lt} > \tilde{V}_{Lt} \). The condition \( V_{Mt} \leq \tilde{V}_{Mt} \) implies \( c \leq \Omega_e (K_M) \), where in \( \Omega_e (K_M) \) the definition of \( \tilde{Y}_{M,L}^+ \) is such that when firm \( M \) deviates its production upon investment is \( \Lambda_L K_M \). The condition \( V_{Lt} > \tilde{V}_{Lt} \) implies \( c \leq \Omega_e (K_L) \), where in \( \Omega_e (K_L) \) the definition of \( \tilde{Y}_{L,M}^+ \) is such that when firm \( L \) deviates its production upon investment is \( \Lambda_M K_L \). Since the market demand in (1) is strictly decreasing in \( K_L \), it holds that if \( K_L < K_M \) then \( \Omega_e (K_M) < \Omega_e (K_L) \). Therefore if \( K_L < K_M \), \( c \leq \Omega_e (K_M) < \Omega_e (K_L) \), it holds that \( c < \Omega_e (K_L) \) for any parameter value.

The expressions for the marginal sorting conditions on \( x_j \) and \( K_j \) are not as simple as those with respect to \( c_j \) in Appendix B. As a result, we set additional assumptions to ensure that the corresponding sorting conditions always have the same sign. To ease on exposition, we first provide their expression and required sign; we then consider sufficient conditions under which the indicated sign holds for any investment strategy. The economic rationale behind the sorting conditions with respect to \( K_j \) relates to the study by Boyer et al (2001).

We denote the market share of firm \( j \) before investment by \( s_j^- \equiv \frac{K_j}{Y_j} \), and upon investment by \( s_j^+ \equiv \frac{\Lambda_j K_j}{Y_j} \).

All else equal, firms with more \( K_j \) wait longer to invest, namely

\[
\frac{\partial}{\partial K_j} \left[ \frac{\partial V_{1j}}{\partial x_j} \right] = (v - 1) \left[ \left( \tilde{Y}_j^- \right)^{-\frac{1}{2}} \left( 1 - \frac{1}{2} s_j^- \right) - \right] - \left( \tilde{Y}_j^+ \right)^{-\frac{1}{2}} \left( 1 - \frac{1}{2} s_j^+ \right) - c \Lambda_j \left( \frac{X_j}{X_j} \right)^{\frac{1}{2}} > 0
\]

This sorting condition shows that the net gain from investing in capital for firm \( j \) is decreasing in \( K_j \). Since the relative gain from investing is larger for smaller firms, smaller firms are willing to invest earlier. A sufficient yet not necessary assumption such that this sorting condition is always positive is that the net decrease in marginal costs upon investment \( \frac{\partial V_{1j}}{\partial x_j} \) is relatively large.

All else equal, firms with less \( K_j \) are willing to invest more, namely

\[
\frac{\partial}{\partial K_j} \left[ \frac{\partial V_{1j}}{\partial x_j} \right] = \left( \tilde{Y}_j^+ \right)^{-\frac{1}{2}} \left[ 1 - \frac{1}{2} s_j^+ - \frac{1}{2} \left( 1 - \frac{1}{2} s_j^+ \right) \left( 1 - s_j^+ \right) \right] - \Lambda_j \frac{X_j}{X_j} \left( \frac{X_j}{X_j} \right)^{\frac{1}{2}} < 0
\]

This sorting condition implies that the marginal product of capital of any firm is decreasing in \( K_j \). When there are decreasing returns to scale, smaller firms are willing to invest more than larger firms. A sufficient yet not necessary assumption such that this sorting condition holds is that the elasticity of demand \( \varepsilon > 1 \) is relatively low. The positive relation between returns to scale and demand elasticity is discussed in neoclassical investment papers such as Hayashi (1982).

Given the sorting conditions, \( V_K \) is a monotone strictly decreasing function in \( K_j \) such that \( V_{KK} < 0 \). Hence firms with higher \( q_j \) have the ability to invest earlier and more, namely
Firms differ in $K_j$ and $c_j$. Firms with lower installed capacity before investment $K_j$ and lower costs of production after investment $c_j$ have the ability to invest earlier and more. To see this, we first show that if $x_L < x_M$ and $A_L > A_M$, then $V_{MT} < \tilde{V}_{MT}$ implies $V_{LT} > \tilde{V}_{LT}$. The condition $V_{MT} \leq \tilde{V}_{MT}$ implies $c_M \leq \Omega_e(K_M)$. Moreover, the condition $V_{LT} > \tilde{V}_{LT}$ implies $c_L \leq \Omega_e(K_L)$. Since $D_K < 0$ and also $K_L < K_M$, we know that $\Omega_e(K_M) < \Omega_e(K_L)$. Therefore if $c_L < c_M$, $K_L < K_M$, $c_M \leq \Omega_e(K_M) < \Omega_e(K_L)$ it holds that $c_L < \Omega_e(K_L)$.

We use our previous results on the marginal sorting conditions on $x_j$ and $A_j$ to show that, when firms differ in $K_j$ and $c_j$, the corresponding marginal sorting conditions are given by

$$\frac{\partial}{\partial c_j} \left[ \frac{\partial V_{LT}}{\partial x_j} \right] + \frac{\partial}{\partial K_j} \left[ \frac{\partial V_{LT}}{\partial x_j} \right] > 0 \quad \text{and} \quad \frac{\partial}{\partial c_j} \left[ \frac{\partial V_{LT}}{\partial A_j} \right] + \frac{\partial}{\partial K_j} \left[ \frac{\partial V_{LT}}{\partial A_j} \right] < 0$$

The pair which determines firm type $\{K_j; c_j\}$ can restated in terms of firms' marginal product of capital $q \equiv V_K$ in (11). The sorting conditions with respect to $K_j$ ensure that $V_K$ is strictly decreasing in the capacity of firms such that $V_{KK} < 0$. The sorting conditions on $c_j$ also imply that $V_K$ is strictly decreasing in $c_j$: firms with higher marginal costs of production are willing to invest less. Consequently, firms with more $K_j$ and higher $c_j$ have a lower $q$ before investment, for any strategy $\Gamma$. Furthermore, using our previous results, we get that

$$\frac{\partial}{\partial q_j} \left[ \frac{\partial V_{LT}}{\partial x_j} \right] > 0, \quad \text{and} \quad \frac{\partial}{\partial q_j} \left[ \frac{\partial V_{LT}}{\partial A_j} \right] < 0$$

**E.2. Equilibrium outcome**

When firms differ in $c_j$ only as in Section 1, we derive firms' investment strategies in equilibrium as in Appendix B. We get the investment threshold $\sigma_e$ at which firm $L$ is indifferent between investing simultaneously or sequentially using (9). The threshold $\sigma_{q,0}$ is given by the function $\sigma_{q,0}$ when $\sigma_e$ equals $\bar{\sigma}_e$. Since $q_j$ is a monotone strictly decreasing function in $c_j$, we restate the equilibrium outcome derived in Appendix B in terms of $q_j$. We solve for the equilibrium strategies when firms differ in $K_j$ only in the same way, since $q_j$ is a monotone strictly decreasing function in $K_j$.

When firms differ in both $K_j$ and $c_j$, the sorting conditions show that firms with higher $q_j$ have the ability to invest earlier and more, and hence the qualitative predictions of the basic model also apply here. Denote by $\sigma_K$ the standard deviation in firms' installed capacities before investment. There exist multiple combinations of the pairs of $\sigma_e$ and $\sigma_K$ such that (9) is satisfied. Hence there exists a range of values of $\sigma_{q,0}$ such that (9) is satisfied. We obtain a clustering equilibrium when $\sigma_{q,0}$ is sufficiently low (i.e. $\sigma_e$ or $\sigma_K$ are sufficiently low), and obtain a sequential equilibrium if $\sigma_{q,0}$ is sufficiently high (i.e. $\sigma_e$ or $\sigma_K$ are very high). Yet with multiple sources of heterogeneity there is no unique correspondence between (9) and $\sigma_{q,0}$.

**F Proposition 6**

We apply the variance operator to $\beta_{jt}$ in (10) such that $\sigma_{\beta,0}^2 \equiv \delta^{-2}(v - 1)^2 \sigma_{y,j}^2$. We use the property that the first Taylor approximation of $\text{var} \left( g(y) \right)$ is such that $\text{var} \left( g(y) \right) \approx \left[ g'(E(y))^2 \right] \text{var} \left( y \right)$, where $\text{var} \left( y \right)$ is the cross sectional variance of $y$ in a given industry. Given $y = \frac{x}{t}$ and $f(y) = \ln(y)$, we apply this property such that $\sigma_{\ln \frac{x}{t},jt} \approx \mu_{\ln \frac{x}{t}}^2 \times \sigma_{\frac{x}{t},jt}^2$. The intra-industry variance at time $t$ of $\ln \frac{x}{t}$ is such that $\sigma_{\ln \frac{x}{t},jt}^2 \approx \mu_{\ln \frac{x}{t}}^2 \times \sigma_{\frac{x}{t},jt}^2$. 

37
We note that \( \sigma_{\ln y, \tau}^2 = \sigma_{\ln y, \tau}^2 + \sigma_{\ln y, \tau}^2 + 2\rho_{\tau} \), where \( \rho_{\tau} \) is the covariance between \( \ln \frac{y}{y^*} \) and \( \ln \frac{y}{y^*} \). Reordering terms, we get (12) where \( \gamma_{\tau} = \delta \mu^2 (v - 1)^2 \).

The property that \( \text{var} (g(y)) \approx \left[ g'(E(y)) \right]^2 \text{var} (y) \) follows the derivation of the delta method in Greene (2003) and relies on two equations. Equation 1 is the first order Taylor series expansion of \( g(y) \), around the mean of \( y \) or \( \mu_y \) such that \( g(y) = g(\mu_y) + g'(\mu_y)(y - \mu_y) + \Delta, \) where \( \Delta \approx 0 \) is the approximation error. Equation 2 is the property that \( E[g(y)] = g(\mu_y) \) when \( \Delta \approx 0 \). Using Equation 2, we approximate the variance of \( g(y) \) such that \( \text{var} [g(y)] \approx E \left[ (g(y) - g(\mu_y))^2 \right] \). We use Equation 1 and operate to get \( \text{var} (g(y)) \approx \left[ g'(E(y)) \right]^2 \text{var} (y) \).

G The case of \( N > 2 \) firms

The solution approach for \( N > 2 \) relies on sorting conditions and ICCs just at the case of \( N = 2 \). In oligopolies, the use of sorting conditions facilitates the analysis, insofar they constrain the set of possible equilibria to those in which firms with higher marginal \( q \) invest earlier or in tandem with other firms. Just as signalling games with multiple discrete types,\(^{40}\) each firm cares about its closest and strongest competitor. Consequently, if firm \( j \) invests earlier and more than its closest competitor firm \( i \) in equilibrium, the only binding ICC for firm \( j \) is that of firm \( i \).

To illustrate the main properties of the case of \( N > 2 \), we consider an the extension of the basic model with \( N = 3 \) firms. We assume that firms have the same installed capacity before investment, and they have heterogeneous marginal costs of production upon investment. The sorting conditions of the game are the same for any value of \( N \), and hence they are equal to those in Appendix B. We label firms by \( L, M \) and \( F \). We assume \( c_L < c_M < c_F, c_L = c_M - a, c_F = c_M + b, a > 0, \) and \( b > 0 \). Given this notation, \( c_M \) is the average marginal cost of production upon investment, and \( \sigma_c = \sqrt{\frac{a^2 + b^2}{3}} \) is the standard deviation in firms’ marginal costs of production.

The key departure of the case of \( N > 2 \) relative to the case of \( N = 2 \) is that firms’ strategies in equilibrium cannot be characterized uniquely in terms of \( \sigma_c \). In the example with \( N = 3 \), the parameters \( a \) and \( b \) are both necessary to determine the equilibrium outcome of the game. More intuitively, this implies that higher order moments of the distribution of \( c_i \) also matter—i.e. the skewness of firms’ marginal costs of production. An important exception is the case in which \( c_i \) is uniformly distributed such that \( a = b, \) and \( \sigma_c = \sqrt{\frac{a^2}{3}}. \) Just as in Proposition 2, however, even if \( a \) and \( b \) are different, firms’ investments are more clustered in equilibrium if \( a \) and \( b \) are relatively small (i.e. low \( \sigma_c \)). Table 2 illustrates an example with \( N = 3 \) in which \( \sigma_c \) is small and the equilibrium outcome is that all firms invest at the common investment threshold \( x^* \).

Put differently, the equilibrium outcome of the game with \( N = 3 \) depends on the ability of both firm \( L \) and \( M \) to invest earlier than their closest competitor. Even if firm \( L \) may find it more convenient to invest earlier than firm \( M \), the strategy of firm \( M \) also depends on the preemptive behavior of firm \( F \). For instance, if firms \( M \) and \( F \) are close competitors (i.e. \( b > 0 \) is relatively small), firm \( M \) may find it too costly to invest earlier than firm \( F \). This, in turn, may constrain the ability of firm \( L \) to invest earlier than firms \( M \) and \( F \) (i.e. if \( a > 0 \) is not sufficiently large).

Consider now the more general case in which firms differ in their production technologies before and after investment. Just as in Proposition 3, the more general testable implication of the model for the case of \( N > 2 \) firms is that firms’ investment dynamics are more similar in industries with low value spread. We can extend the insight in Proposition 3 to the case of \( N > 2 \) firms since the sorting conditions in Appendix D apply for any number of firms \( N \).

\(^{40}\) See, for instance, Fundenberg and Tirole (1991), Chapter 7.
H  Database construction

The working sample is drawn from a merged CRSP-COMPUSTAT database from 1968 to 2008. We estimate the beta of each firm as the sum of the coefficients of monthly returns on lagged, lead and contemporary market returns of the stock return of each firm in the sample. We compute betas at a monthly frequency, using five-year rolling windows containing the previous 60 observations. We compute stock returns in excess of the risk free rate reported in CRSP.

We estimate betas as the sum of the coefficients of monthly returns on lagged, lead and contemporary market returns of the stock return of each firm in the sample. We compute betas at a monthly frequency. We follow Fama and French (1992) and match each firm’s CRSP stock return and betas from July of year $t$ until June of year $t + 1$ to the corresponding accounting information in COMPUSTAT for the fiscal year ending in year $t + 1$. With the exception of lnHHI, we construct the remaining explanatory variables using COMPUSTAT tapes. lnHHI is the logarithm of the HHI for manufacturing industries reported by the US Census Bureau; since the HHI is reported every five years, we repeat the HHI of year $t$ over the next four years for every industry.

The market value of equity is the product of item PRCC_F times CSHO. The market value of assets $V$ is the market value of equity plus total liabilities. The total liabilities $B$ are computed as AT minus CEQ minus TXDB. Operating cashflows $\pi$ are the sum of SALE minus COGS minus XSGA. Investment $I = (\Lambda - 1)K$ is CAPX. We consider $K$ to be total assets AT, with the exception of $I_K$ where $K$ is set as lagged PPENT. The operating mark-up $m$ is the ratio of $\pi$ over SALE. All COMPUSTAT variables are winsorized at 1%.

We construct the intra-industry comovement in variable $x$ at time $t$ or $\omega_{x,t}$ as in Khanna and Thomas (2009). For variables $x = \{ \beta; R; \frac{V}{K}; \frac{V-B}{K-B}; \frac{I}{K} \}$, and for each month, we consider the average of the correlation coefficients $C_{ij}$ between the variable $x$ of each unrepeated pair of firms $i$ and $j$ within the same industry, such that

$$C_{ij} = \frac{Cov(i,j)}{\sqrt{Var(i) \times Var(j)}}$$

where $Cov(i,j)$ is the covariance between the variable $x$ of firms $i$ and $j$ during the window between month $t$ and month $t - 60$, $Var(i)$ is the variance of firm $i$’s variable $x$ in such window, and $Var(j)$ is the variance of firm $j$’s monthly variable $x$. To compute the comovement in the ratios $\frac{V}{K}$ and $\frac{V-B}{K-B}$, we compute the market value of equity at a monthly frequency, using the time series of PRCC and CSHO reported in CRSP.

I  Parameter choice in numerical examples

The parameters in Tables 1-3 are $\tau = 6.5\%$, $\delta = 2.5\%$, $\sigma = 25\%$, $\varepsilon = 2.35$, $f = 1$, $X_0 = 0.01$, $c_L = 0.1$, $c_M = 0.11175$, $c = 0.1105$, and $K = 1$. The technology of firm $M$ is such that $c_F = 0.1235$. The parameters in Figures 1 – 4 are the same with exception of $c_M = 0.1077$. The numerical example in Figure 1 uses the equilibrium investment strategies $\Gamma_j$. Figures 1 and 3 represent firms’ expected values and betas by reporting the average of firms’ values and betas out of 500 simulations of the Brownian demand shocks.
Table 1: Investment strategies when $N = 2$

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Stackelberg Strategies</th>
<th>Sequential Strategies</th>
<th>Clustering Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
</tr>
<tr>
<td>$x_j$</td>
<td>0.032 0.125</td>
<td>0.019 0.655</td>
<td>0.090 0.090</td>
</tr>
<tr>
<td>$\Lambda_j$</td>
<td>98.88 50.88</td>
<td>162.07 25.68</td>
<td>64.42 60.96</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.000 0.000</td>
<td>0.939 0.000</td>
<td>0.000 0.000</td>
</tr>
</tbody>
</table>

Valuation at $X_0$

<table>
<thead>
<tr>
<th></th>
<th>Firm Value</th>
<th>$\Delta \pi_{j,0}$</th>
<th>$q_{j,0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Firm Value$</td>
<td>0.585 0.202</td>
<td>-0.405 -0.169</td>
<td>0.543 0.160</td>
</tr>
<tr>
<td>$\Delta \pi_{j,0}$</td>
<td>-0.405 -0.169</td>
<td>-0.119 -0.205</td>
<td>0.199 0.011</td>
</tr>
<tr>
<td>$q_{j,0}$</td>
<td>0.543 0.160</td>
<td>0.000 0.000</td>
<td>0.371 0.333</td>
</tr>
</tbody>
</table>

Firms’ Betas

<table>
<thead>
<tr>
<th></th>
<th>At $X_0$</th>
<th>At $x^*_L$</th>
<th>At $x^c$</th>
<th>At $x^*_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$At X_0$</td>
<td>1.176 0.923</td>
<td>1.021 0.012</td>
<td>1.129 1.112</td>
<td></td>
</tr>
<tr>
<td>$At x^*_L$</td>
<td>0.828 1.291</td>
<td>0.943 1.313</td>
<td>1.157 1.139</td>
<td></td>
</tr>
<tr>
<td>$At x^c$</td>
<td>0.706 1.296</td>
<td>0.879 1.311</td>
<td>1.000 1.000</td>
<td></td>
</tr>
<tr>
<td>$At x^*_M$</td>
<td>1.000 1.000</td>
<td>1.000 1.000</td>
<td>1.000 1.000</td>
<td></td>
</tr>
</tbody>
</table>

Indicators at $x^*_M$

<table>
<thead>
<tr>
<th></th>
<th>$HHI$</th>
<th>$\sigma_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$HHI$</td>
<td>0.551 0.551</td>
<td>0.764 0.764</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.104 0.104</td>
<td>0.115 0.115</td>
</tr>
</tbody>
</table>

This table illustrates the potential industry equilibria when $N = 2$ and firms differ in their future production technologies. The investment strategy $\{x_j; \Lambda_j\}$ consists of the demand threshold at which firm $j$ invests $x_j$, and the scale of the firm $\Lambda_j > 1$ upon investment. The superscript $s$ corresponds to sequential strategies; the superscript $c$ corresponds to clustering strategies. $\lambda$ is the multiplier of the ICC of firm $L$. $\Delta \pi_{j,0}$ is the expected reduction in future profits of firm $j$ before other firms invest, evaluated at $X_t = X_0$. $q_{j,0}$ is the marginal product of capital of firm $j$ at $X_t = X_0$. $HHI$ is the Herfindahl-Hirshman Index. $\sigma_m$ is the standard deviation in the operating mark-ups. The outcome is the clustering equilibrium in Panel (C).
Table 2: Sequential and clustering strategies when \( N = 3 \)

<table>
<thead>
<tr>
<th>Strategies and Valuation</th>
<th>Sequential Strategies (A)</th>
<th>Clustering Strategies (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_j )</td>
<td>0.014 0.093 0.801</td>
<td>0.212 0.212 0.212</td>
</tr>
<tr>
<td>( \Lambda_j )</td>
<td>113.30 46.81 21.05</td>
<td>56.21 51.01 45.81</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.959 0.920 0.000</td>
<td>0.000 0.000 0.000</td>
</tr>
<tr>
<td>Firm Value at ( X_0 )</td>
<td>0.237 0.069 0.033</td>
<td>0.257 0.233 0.211</td>
</tr>
</tbody>
</table>

Firms’ Betas

At \( X_0 \) 1.035 0.360 -0.603 1.063 1.038 1.009
At \( x^*_L \) 0.835 1.058 -2.900 1.083 1.051 1.013
At \( x^*_M \) 0.927 0.916 1.309 1.000 1.000 1.000

Table 3: Other investment strategies when \( N = 3 \)

<table>
<thead>
<tr>
<th>Strategies and Valuation</th>
<th>Mixed Cases (A)</th>
<th>Mixed Cases (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_j )</td>
<td>0.023 1.113 1.113</td>
<td>0.092 0.092 0.447</td>
</tr>
<tr>
<td>( \Lambda_j )</td>
<td>159.45 19.19 12.74</td>
<td>120.41 34.97 22.79</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.918 0.000 0.000</td>
<td>0.000 0.702 0.000</td>
</tr>
<tr>
<td>Firm Value at ( X_0 )</td>
<td>0.314 0.054 0.049</td>
<td>0.338 0.216 0.126</td>
</tr>
</tbody>
</table>

Firms’ Betas

At \( X_0 \) 1.107 0.134 0.021 1.122 1.007 0.809
At \( x^*_L \) 0.942 1.013 1.004 0.974 0.997 1.060
At \( x^*_M \) 1.000 1.000 1.000

These tables illustrate the potential equilibrium outcomes with \( N = 3 \) when firms have the same parameters as those in Table 1. The investment strategy \( \{x_j; \Lambda_j\} \) consists of the demand threshold at which firm \( j \) invests \( x_j \), and the scale of firm \( j \) \( \Lambda_j > 1 \) after investment. The superscript \( s \) corresponds to sequential investment; the superscript \( c \) corresponds to clustered investment. \( \Lambda_j \) is the multiplier of the binding ICC on firm \( j \). \( \Delta \pi_{j,0} \) is the expected reduction in profits of firm \( j \) before other firms invest, evaluated at \( X_t = X_0 \). The outcome is the clustering equilibrium in Panel (B) of Table 2.
Table 4: Working sample statistics

<table>
<thead>
<tr>
<th>Firm level</th>
<th>Industry level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
</tr>
<tr>
<td>$\frac{I}{K}$</td>
<td>0.360</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.102</td>
</tr>
<tr>
<td>$R$</td>
<td>0.082</td>
</tr>
<tr>
<td>$\frac{V}{K}$</td>
<td>1.477</td>
</tr>
<tr>
<td>$\frac{V-B}{K-B}$</td>
<td>2.085</td>
</tr>
<tr>
<td>$\frac{B}{K}$</td>
<td>0.526</td>
</tr>
<tr>
<td>$\frac{V}{K}$</td>
<td>0.082</td>
</tr>
<tr>
<td>$m$</td>
<td>0.144</td>
</tr>
<tr>
<td>$\sigma_{\frac{I}{K}}$</td>
<td>0.274</td>
</tr>
<tr>
<td>$\sigma_\beta$</td>
<td>0.635</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.374</td>
</tr>
<tr>
<td>$\sigma_{\frac{V}{K}}$</td>
<td>0.530</td>
</tr>
<tr>
<td>$\sigma_{\frac{V-B}{K-B}}$</td>
<td>1.088</td>
</tr>
<tr>
<td>$\sigma_{\frac{B}{K}}$</td>
<td>0.178</td>
</tr>
<tr>
<td>$\sigma_{\frac{V}{K}}$</td>
<td>0.111</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.058</td>
</tr>
<tr>
<td>lnHHI</td>
<td>5.645</td>
</tr>
<tr>
<td>$\omega_{\frac{I}{K}}$</td>
<td>0.031</td>
</tr>
<tr>
<td>$\omega_\beta$</td>
<td>0.026</td>
</tr>
<tr>
<td>$\omega_R$</td>
<td>0.016</td>
</tr>
<tr>
<td>$\omega_{\frac{V}{K}}$</td>
<td>0.107</td>
</tr>
<tr>
<td>$\omega_{\frac{V-B}{K-B}}$</td>
<td>0.178</td>
</tr>
</tbody>
</table>

This table reports the summary statistics of our CRSP-COMPUSTAT working sample of US public firms from 1968 to 2008. $\frac{I}{K}$ is the investment rate; $\beta$ is the equity beta; $R$ is the excess stock return, which is annualized in this table since all statistics are reported in annual terms; $\frac{V}{K}$ is the market to book asset ratio; $\frac{V-B}{K-B}$ is the market to book equity ratio; $\frac{B}{K}$ is the book leverage ratio; $\frac{V}{K}$ is operating cashflows to assets; $m$ is the operating mark-up on profits; $\sigma_x$ denotes the intra-industry standard deviation in variable $x$; lnHHI is the logarithm of the US Census HHI; and $\omega_x$ denotes the intra-industry comovement in variable $x$. The details on database construction are provided in Appendix H.
Table 5: Investment and the intra-industry value spread

<table>
<thead>
<tr>
<th>Firm level Investment</th>
<th>Industry level Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>$\frac{V}{K}$</td>
<td>0.131***</td>
</tr>
<tr>
<td>(0.010)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\frac{\bar{x}}{K}$</td>
<td>-0.084*</td>
</tr>
<tr>
<td>(0.046)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>$\sigma_{\frac{V}{K}}$</td>
<td>0.090***</td>
</tr>
<tr>
<td>(0.012)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\sigma_{\frac{V-B}{K-B}}$</td>
<td>0.039***</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>N</td>
<td>107,749</td>
</tr>
<tr>
<td>Avg. $R^2$</td>
<td>0.053</td>
</tr>
</tbody>
</table>

This table reports the Fama and MacBeth (1973) regressions on the investment to capital ratios $\frac{I}{K}$ at the firm and industry level. The data used is in annual frequency. $\frac{V}{K}$ is the market to book asset ratio; $\frac{V-B}{K-B}$ is the market to book equity ratio; $\frac{\bar{x}}{K}$ is operating profits to assets; and $\sigma_x$ denotes the intra-industry standard deviation in variable $x$. Newey-West corrected standard errors are reported in parentheses. ***$p < 0.01$, **$p < 0.05$ and *$p < 0.1$. 

43
Table 6: Betas, excess returns and investment dynamics

<table>
<thead>
<tr>
<th>Comovement in Betas $\omega_{\beta}$</th>
<th>Comovement in Excess Returns $\omega_{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>$\omega_{\frac{I}{K}}$</td>
<td>0.055***</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\omega_{\frac{V}{K}}$</td>
<td>0.028***</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\omega_{\frac{V-B}{K-H}}$</td>
<td>0.020***</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\omega_{\beta}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>Avg. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>173,434</td>
<td>0.034</td>
</tr>
<tr>
<td>173,939</td>
<td>0.062</td>
</tr>
<tr>
<td>163,932</td>
<td>0.034</td>
</tr>
<tr>
<td>173,434</td>
<td>0.056</td>
</tr>
<tr>
<td>173,939</td>
<td>0.062</td>
</tr>
<tr>
<td>163,932</td>
<td>0.062</td>
</tr>
<tr>
<td>174,002</td>
<td>0.366</td>
</tr>
</tbody>
</table>

This table reports the Fama and MacBeth (1973) regressions on comovement in betas and excess returns. The data used is in monthly frequency. $\omega_x$ denotes the intra-industry comovement in variable $x$; $\beta$ is the equity beta; $R$ is the excess stock return; $\frac{I}{K}$ is the investment rate; $\frac{V-B}{K-H}$ is the market to book equity ratio; and $\frac{V}{K}$ is the market to book asset ratio. Newey-West corrected standard errors are reported in parentheses. $***p < 0.01$, $**p < 0.05$ and $*p < 0.1$. 
Table 7: Industry dynamics, the intra-industry value spread, and competition

<table>
<thead>
<tr>
<th>Comovement in Betas $\omega_\beta$</th>
<th>Comovement in Excess Returns $\omega_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>$\sigma_\frac{V}{K}$</td>
<td>-0.0024***</td>
</tr>
<tr>
<td>(0.0008)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\sigma_\frac{V-B}{K-B}$</td>
<td>-0.0016***</td>
</tr>
<tr>
<td>(0.0004)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>-0.0316***</td>
</tr>
<tr>
<td>(0.0018)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>lnHHI</td>
<td>-0.0004***</td>
</tr>
<tr>
<td>(0.0001)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>N</td>
<td>147,243</td>
</tr>
<tr>
<td>Avg. $R^2$</td>
<td>0.023</td>
</tr>
</tbody>
</table>

This table reports the Fama and MacBeth (1973) regressions on comovement measures as a function of the intra-industry value spread and static measures of competition. The data used is in monthly frequency. $\omega_x$ denotes the intra-industry comovement in variable $x$; $\beta$ is the equity beta; $R$ is the excess stock return; $\frac{V}{K}$ is the market to book asset ratio; $\frac{V-B}{K-B}$ is the market to book equity ratio; $m$ is the mark-up on operating profits; and lnHHI is the logarithm of the US Census HHI. Newey-West corrected standard errors are reported in parentheses. ***$p < 0.01$, **$p < 0.05$ and *$p < 0.1$. 

45
Figure 1. This figure illustrates how firms’ strategic interaction affect their values for the special case in which firm $L$ invests earlier than firm $M$ such that $x_L < x_M$. The total value of any firm consists of its assets in place, its growth options, and the expected reduction in future profits due to investments by its competitors.
Figure 2. This figure illustrates how the intra-industry standard deviation in costs of production $\sigma_c$ affects firms’ investment strategies in equilibrium. The red color relates to the sequential strategies $\Gamma_j^s$; the blue color relates to the clustering strategies $\Gamma_j^c$. The black dotted line reflects firms’ investment strategies in equilibrium. $V_j$ is the value of firm $j$; $x_j$ is the demand threshold at which firms invest; $\lambda$ is the multiplier of the ICC of firm $j$; $s_L$ is the market share of firm $L$ when all firms have invested. $\sigma_c$ is expressed in %. 
Figure 3. This figure shows the dynamics of the beta of firm $j$ at time $t$ or $\beta_{jt}$ in the basic model when $\sigma_c = \sigma_c$. $\beta^s_j$ is the beta of firm $j$ when firms invest sequentially in equilibrium; $\beta^c_j$ is the beta of firm $j$ when firms investments cluster in equilibrium.
Figure 4. This figure illustrates how the underlying determinants of market demand affect the threshold \( \sigma_x \) and the shadow cost of preemption \( \lambda \). The threshold \( \sigma_x \) is the minimum value of the intra-industry standard deviation in firms’ costs of production \( \sigma_c \) at which firm \( L \) prefers to cluster. \( \lambda \) is the Lagrange multiplier on the incentive compatibility constraint of firm \( M \) under sequential investment.