Abstract

I study the asset pricing implications of heterogeneity in the financial intermediary sector. Due to the variation in funding sources, financial constraints, and regulatory requirements, different intermediaries such as bank holding companies and security broker-dealers, exhibit starkly different behaviors during economic downturns. Motivated by the empirical evidence on balance sheet adjustments within the intermediary sector during the Great Recession, I propose a dynamic general equilibrium model with heterogeneous intermediaries and financial frictions. My model generates opposite cyclical dynamics of leverage for the two intermediary sectors, reconciling empirical evidence that has previously seemed contradictory through the lens of representative intermediary asset pricing models. I use my model to quantify the impact of observed financial flows between intermediaries on risk premia and volatility. Finally, I examine empirical implications of the model for time-series predictability and the cross-section of asset returns. An empirical measure of heterogeneity in the financial sector forecasts excess returns and is priced in the cross-section of assets.

KEYWORDS: Heterogeneous Intermediaries, Intermediary Asset Pricing, Leverage Cyclicality.

JEL CLASSIFICATION: G12, G11, G21.
1 Introduction

Intermediary asset pricing theories consider financial intermediaries to be the marginal investors in most asset markets. The literature typically features heterogeneous agents with a levered representative intermediary and unlevered households, and introduces different frictions to the ability of the levered agents to raise funds. In the presence of financial frictions, the intermediary sector’s wealth share plays a key role in determining equilibrium asset prices. Representative intermediaries, however, can only be modeled as either buyers or sellers of assets during a crisis, but not both. In this paper, I propose a general equilibrium model with heterogeneous intermediaries that face financial constraints. The model generates both procyclical and countercyclical intermediary leverage, reconciling heretofore contradictory empirical evidence. I study the implications of this heterogeneity for asset prices and return predictability.

Variation in funding sources and financial constraints lead different intermediaries to exhibit starkly different behaviors during times of aggregate financial sector distress. Depending on the type of intermediaries and, thus, the financial frictions considered (i.e. debt or equity constraints), models with representative intermediaries predict opposite cyclical dynamics for intermediary leverage: countercyclical in models with equity constraints (He and Krishnamurthy, 2012, 2013 and Brunnermeier and Sannikov, 2014) or procyclical if intermediary faces a debt constraint (Brunnermeier and Pedersen, 2009 and Adrian and Shin, 2014).

Empirical evidence from financial flows during the Great Recession, discussed in Section 2, are at odds with a representative intermediary framework. Over the period from the first quarter of 2008 to the fourth quarter of 2009, which includes the most dramatic episode of the subprime crisis in the fall of 2008, (i) broker-dealers reduced leverage by approximately 47% while holding companies increased leverage by about 72%, and (ii) broker-dealers substantially reduced asset holdings by about $2 trillion while commercial banks increased total asset by approximately $1.1 trillion. In addition, recent empirical evaluations of the representative intermediary-based models in Adrian, Etula, and Muir (2014a) (henceforth, AEM) and He, Kelly, and Manela (2017) (henceforth, HKM) find opposite signs for the estimated price of intermediary leverage shocks in the cross-section of asset returns (positive and negative, respectively). AEM’s positive price of risk suggests that intermediary leverage is procyclical, while HKM’s finding implies countercyclical leverage. AEM and HKM, however, study different financial intermediaries: security broker-dealers, and bank holding companies, respectively.

In this paper, I present a general equilibrium model with heterogeneous intermediaries and financial constraints that is consistent with the evidence discussed above. The economy is populated by three types of agents who differ in their risk aversion: two financial intermediaries (labeled $A$ and $B$) and a household sector ($C$ agents) in order of increasing risk aversion.

$^1$Agents could differ in productivity, patience, risk aversion, or optimism. More productive/optimistic (or less patient/risk averse) agents face financial frictions: in Brunnermeier and Pedersen (2009) and Adrian and Boyarchenko (2015) intermediaries face borrowing constraints, while in He and Krishnamurthy (2012, 2013) and Brunnermeier and Sannikov (2014) the financial sector faces limitations in raising equity financing.
The model has two main ingredients. First, I assume agents differ in their attitudes toward risk: In equilibrium, less risk averse intermediaries hold levered positions in the risky asset financed by borrowing from the more risk averse agents. Second, levered investors face financial frictions in the form of occasionally binding state-dependent margin constraints. In a frictionless setting, funds are liquid and it is not important who owns them, hence, the economy can be studied with a single representative agent. With financial constraints, however, the distribution of wealth among agents matters.

A key assumption of the model is that margin constraints are tighter for the least risk averse $A$ agents. I think of $A$ and $B$ intermediaries as broker-dealers and bank holding companies, respectively. Hedge funds and broker-dealers primarily depend on collateralized repo financing, while the commercial banking sector has access to more stable funding sources, such as insured deposits and discount window lending from a central bank.

Margin constraints cap the maximum funding that intermediaries can obtain, which in turn impacts their asset demand. This friction plays an important role in delivering intermediary leverage dynamics consistent with observed patterns. My model features occasionally binding, time-varying margin constraints: The level of margin required in the model is state-dependent; and (inversely) linked to endogenously determined return volatility that resembles an approximate Value-at-Risk (VaR) rule. Such an approach is consistent with empirical evidence that haircuts tend to rise in crises, as documented in Gorton and Metrick (2012).

I show that model’s equilibrium dynamics can be described by two endogenous state variables: (i) total wealth share of the financial sector (i.e. $A$ and $B$ agents) in the economy, and (ii) the least risk averse agent $A$’s wealth share as a fraction of the total financial sector. The former is the key state variable in many recent representative intermediary-based models such as He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2010), and Drechsler, Savov, and Schnabl (2018). The unique feature of the model is the second state variable. It emphasizes that the wealth distribution within the financial sector is key for determining asset prices, risk premia, and other equilibrium objects such as volatility, optimal leverage, and real interest rates. Models with representative intermediaries are silent about the wealth distribution in the financial sector and its implications for asset prices and the real economy.

My model with heterogeneous intermediaries reconciles the conflicting evidence in AEM and HKM. When the most risk tolerant $A$ types face binding margin constraints, they are forced to reduce leverage by selling assets. To clear the risky asset market, the more risk averse agents ($B$ and $C$ types) take on a larger portion of the asset than they would in the absence of constraints.

---

2In the third quarter of 2017, approximately 35% of total financial assets and 47% of total liabilities for security broker-dealers was due to lending and borrowing in the repo market, respectively.

3Commercial banks, who have to maintain a costly equity cushion but also rely on deposit insurance, are perhaps better described by models with equity constraints. Broker-dealers and hedge funds, on the other hand, who rely less on deposit insurance and equity capital, may be closer to the intermediaries described by models with debt constraints.

4In the baseline model, I assume only $A$ types face binding margin constraints and $B$ agents’ constraint do not bind in equilibrium. In the online appendix, I solve the model under a more general setting where any investor in the risky asset faces potentially binding margin constraints.
In order to entice them to buy more, the risk premium must increase. Consistent with empirical
evidence, since endogenous margin constraints are more likely to bind in states where intermediary
wealth shares are low, the model can generate opposite cyclical dynamics for optimal leverage of A
(procyclical) and B (countercyclical) intermediaries. As mentioned above, one can think of levered
A and B investors as different types of intermediaries: broker-dealers (BDs) and bank holding
companies (BHCs), respectively, as BDs are more likely to face binding borrowing constraints in
bad times than BHCs.

When margin constraints are slack, the risk tolerant A and B investors have levered balance
sheets by borrowing from more risk averse households. Leverage increases intermediaries’ exposure
to aggregate shocks: Positive shocks result in their wealth share to increase. In bad times, the
levered A and B types’ wealth falls faster than that of households, and hence their share of total
wealth, the first state variable in the model, declines. Moreover, since more risk tolerant intermedi-
aries (i.e. A agents) have higher leverage, their margin constraint is more likely to bind in bad times.
As such, their wealth share in the financial sector, the second state variable of my model, declines
as well. The key takeaway is that both state variables in the model exhibit procyclical dynamics:
In high marginal utility states, the wealth share of the financial sector and A types’ net worth
share in the intermediary sector are both low. These opposite leverage dynamics fit intermediary
leverage patterns observed in the data.

I then examine two quantitative implications of my model: I first show that approximately 20%
of the variation in risk premia can be attributed to the wealth distribution among intermediaries,
which is a measure of heterogeneity in the financial sector. Thus, failing to account for heterogeneity
among intermediaries can lead to missing a substantial portion of the variation in risk premia. I
also use my model to quantify the impact of observed financial flows between intermediaries during
the 2008 crisis on risk premia and volatility. Specifically, a dealer deleveraging episode comparable
in magnitude to the one observed during the recent financial crisis leads to an approximately 55%
increase in the risk premia and a 5% increase in endogenous risk.

Next, I show that the distribution of wealth in the intermediary sector matters for determining
risk premia. I define an empirical proxy for measuring the heterogeneity in the intermediary sector:
The ratio of book equity of security broker-dealers to sum of book equities of broker-dealers and
holding companies from the Financial Accounts of the United States (Flow of Funds). The wealth
distribution within the financial sector predicts future excess returns and is also priced in the cross-
section. It negatively forecasts future excess returns on market, Fama-French size/book-to-market
and momentum portfolios, as well as non-equity portfolios of sovereign bonds and options. This
is consistent with my model’s implications. My model’s second state variable, which precisely
captures the wealth distribution among intermediaries, exhibits procyclical dynamics: Times of
low dealer wealth share in the intermediary sectors coincide with high marginal utility states where
prices are low and future expected returns are high.

Shocks to wealth distribution among intermediaries are priced in the cross-section of asset
returns: the heterogeneous intermediary factor (HIFac) alone exhibits strong explanatory power
for 55 equity and bonds portfolios with cross-sectional $R^2$ of 62% and a positive estimated price of risk as implied by the model. HIFac retains its explanatory power even in the presence of representative intermediary-based factors in AEM and HKM, as well as market risk premium. The cross-sectional $R^2$ increases to 73% when the heterogeneous intermediary factor is combined with AEM’s leverage factor.

I provide further evidence that heterogeneity in the financial sector is an important risk factor. Stock portfolios sorted on their exposure to shocks to dealers’ wealth share in the financial sector exhibit monotonically increasing excess returns: the highest-beta quintile has approximately 5% higher annualized excess return relative to the lowest-beta portfolio. Two-way-sorted portfolios with my heterogeneous intermediary factor and either AEM or HKM representative intermediary factors provide additional support for asset pricing importance of heterogeneity in the financial sector. Even within portfolios sorted based on AEM or HKM factor betas, I observe a monotonic progression in returns from low- to high-beta portfolios. Existing representative intermediary asset pricing models are unable to capture these results.

Finally, I construct a mimicking portfolio for the heterogeneous intermediary factor from my model. Mimicking portfolios for representative intermediary factors in AEM and HKM are unable to fully span the heterogeneous intermediary factor-mimicking portfolio (FMP): I find large and highly significant alphas when I regress my model’s FMP on AEM’s and HKM’s FMPs both individually and in bivariate regressions. This corroborates my earlier results: the heterogeneity in the financial sector is an important source of risk and has pricing information beyond representative intermediary asset pricing factors.

### 1.1 Related Literature

My paper extends macroeconomic models with a financial sector (He and Krishnamurthy, 2012, 2013, Brunnermeier and Sannikov, 2014, and Gertler and Kiyotaki, 2010, for example) to a framework with heterogeneous intermediaries. This literature builds on financial accelerator models of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) which emphasize the importance of financial frictions and leverage for persistence and amplification of aggregate shocks. The literature traditionally modeled intermediaries as one representative sector. Such an approach does not allow for the heterogeneity in financial intermediaries, documented in the data, to play a role in equilibrium.

A few recent papers focus on the importance of a heterogeneous financial sector. Coimbra and Rey (2017) develop a model with intermediaries heterogeneous in their Value-at-Risk constraints and limited liability resulting in risk-shifting. In their model, monetary easing can lead to an increase or a decrease in financial stability depending on the level of interest rates. Gertler, Kiyotaki, and Prestipino (2016) extend Gertler and Kiyotaki (2010)’s framework by incorporating a

---

5 See Brunnermeier, Eisenbach, and Sannikov (2012) for a survey of macro-based models with financial frictions.

6 A few other recent models with explicit roles for financial intermediaries include Danielsson, Shin, and Zigrand (2012), Adrian and Shin (2014), Adrian and Boyarchenko (2015), Moreira and Savov (2017), and Drechsler et al. (2018).
“wholesale” (shadow) banking sector alongside retail banks. They allow the possibility of runs to provide insights into how the growth of wholesale banks led to the near collapse of the financial system during the recent crisis. These papers do not study the implications of heterogeneous intermediaries for risk premia and return predictability. I contribute to this literature by showing that heterogeneity in the financial sector has strong predictive power for excess return of many asset classes and it is also priced in the cross-section of equity and bond returns.

The paper closest to mine is Ma (2017). In independent, contemporaneous work, he shows that a log-linearized SDF estimated from a model with heterogeneous intermediaries exhibits better explanatory power for pricing the cross-section of asset returns than ones from representative intermediary models empirically evaluated in AEM and HKM. Similar to my paper, Ma (2017) explores the cross-sectional implications of heterogeneous intermediaries by semiparametrically estimating a pricing kernel from a model with heterogeneous intermediaries. I study a linear SDF implied by my model and nonetheless, can explain a substantial variation in the cross-section of equity and bond excess returns. I further explore forecasting power of intermediary heterogeneity for returns and show the wealth share of broker-dealers in the financial sectors, an empirical proxy for one of the two endogenous state variables in my model, negatively forecasts future excess returns on the market, size/book-to-market, momentum, sovereign bonds, and options portfolios. Unlike Ma (2017), I explicitly model financial frictions faced by different intermediaries. My model can therefore quantify the impact of observed financial flows between intermediaries during the 2008 crisis on risk premia and volatility. I find that a dealer deleveraging episode comparable in magnitude to the one observed during the Great Recession, leads to a 55% increase in the risk premia and a 5% rise in endogenous risk. I can also attribute 20% of the variation in risk premia to a measure of heterogeneity among financial intermediaries.

This paper also contributes to the recent empirical intermediary asset pricing literature. Two recent papers, Adrian et al. (2014a) and He et al. (2017), evaluate the explanatory power of models where representative intermediaries face, respectively, debt and equity constraints for cross-sectional variations in expected returns. They find opposite signs for the estimated price of risk (and thus conflicting cyclical dynamics) for intermediary leverage. I reconcile these seemingly contradictory evidence in a unifying general equilibrium framework where the financial sector is modeled as two heterogeneous sectors facing different margin constraints. I demonstrate that this heterogeneity, which was not considered in these studies, plays a key role in return predictability and has significant explanatory power for the cross-section of expected returns. The predictive power remains even in the presence of representative intermediary asset pricing factors presented in AEM and HKM.

Finally, this paper relates to the large literature on asset pricing implications of investor heterogeneity and portfolio constraints. Dumas (1989), Wang (1996), Chan and Kogan (2002), Bhamra and Uppal (2009), Longstaff and Wang (2012), Gârleanu and Panageas (2015), and Santos and Veronesi (2016) study equilibrium in frictionless economies with two heterogeneous agents with

---

7More recently, Lewis, Longstaff, and Petrasek (2017) find strong empirical support for intermediary asset pricing theories by relating the observed violations of the law of one price to intermediary capital, funding cost, balance sheet capacity, and liquidity effects.

The rest of the paper is organized as follows. Section 2 presents motivating evidence of the heterogeneity in the financial intermediary sector. Section 3 provides the theoretical framework for the general equilibrium heterogeneous-intermediary model. Sections 4 and 5 present model solution and parameter values used for calibrating the model. Section 6 provides model results. In Section 7, I study the empirical implications of the model for time-series return predictability and the cross-section of asset returns. Section 8 concludes.

2 Motivating Evidence

Before presenting the theoretical framework, in this section, I provide motivating evidence on heterogeneity of the intermediary sector. Empirical evidence from asset reallocations within the financial sector recorded during the Great Recession are at odds with models featuring a representative financial intermediary.\(^8\)

Intermediaries exhibit heterogeneous behavior in the cyclical properties of their leverage. Figure 1 presents time-series of leverage for different financial intermediaries: security brokers and dealers (BDs), and bank holding companies of New York Fed’s primary dealers (BHCs), intermediaries recently studied in AEM, and HKM, respectively. Broker-dealer’s (book) leverage is calculated from balance sheet data in Table L.130 of the Financial Accounts of the United States (Flow of Funds) from the Federal Reserve and is defined as the ratio total financial assets to total equity (total financial assets minus total liabilities). BHC leverage is defined as the ratio of total market assets (book debt plus market equity) to total market equity constructed for publicly-traded holding companies of the New York Fed’s primary dealer counterparties using data from CRSP/Compustat and Datastream. Over the period from the first quarter of 2008 to the fourth quarter of 2009, which includes the Lehman bankruptcy in the fall of 2008, broker-dealers reduced leverage by approximately 47% (from 35 to 19) while holding companies increased leverage by approximately 72% (from 22 to 38) during the same period.\(^9\) We observe opposite cyclical leverage patterns for

---

\(^8\)Evidence of intermediary heterogeneity during the crisis has also been recently documented in the literature. He, Khang, and Krishnamurthy (2010) and Begenau, Bigio, and Majerovitz (2017) document flows of financial assets within the intermediary sector during the Great Recession and show that broker-dealers and hedge funds reduced leverage by selling securitized assets to commercial banks who have access to stable deposits. Moreover, Ang, Gorovyy, and van Inwegen (2011) document that hedge funds decreased leverage prior to the onset of the financial crisis while the leverage of banks and the financial sector continued to increase. They also show hedge fund leverage is very negatively correlated to leverage of banks and the financial sector.

\(^9\)According to He et al. (2010), book leverage of commercial banks rose from 10 to between 20 and 32 over the period from 2007Q4 to 2009Q1.
different financial intermediaries: BD leverage is procyclical, while BHC leverage is countercyclical. Over the sample period from 1970Q1 to 2017Q4, shocks to broker-dealer leverage exhibit a positive correlation of 0.12 (t-stat of 1.82) to innovations in the real GDP, while BHC leverage shocks have a negative correlation of −0.19 (t-stat of −2.62). Correlations with GDP innovations become stronger post 2000 with coefficients of 0.37 (t-stat of 3.27) and −0.39 (t-stat of −3.56) for broker-dealer and holding company leverage, respectively.\footnote{As discussed in HKM, observed opposite cyclical properties for leverage of different intermediaries above are unlikely to be entirely attributed to the differences between book- and market-based values for calculating BD and BHC leverage, respectively. To see this, I calculate holding company book leverage by simply replacing market equity with book equity in the calculation above. I find that book and market BHC leverage are in fact strongly positively correlated over the sample period (1970Q1 to 2017Q4) with correlation coefficient of 0.64 (t-stat of 11.64). Also, both market and book leverage for BHCs are strongly negatively correlated with book leverage of broker-dealers over the sample period with correlation coefficients of −0.50 (t-stat of −7.89) and −0.24 (t-stat of −3.47), respectively.}

In Figure 2, I plot real financial assets for security broker-dealers and U.S. chartered depository institutions (commercial banks) from Tables L.130 and L.111 of the Flow of Funds, respectively. Over the period from the first quarter of 2008 to the fourth quarter of 2009, who mainly depend on collateralized repo financing, substantially reduced asset holdings by approximately $2 trillion (from $5.7 to $3.7 trillion), while commercial banks, who have access to more stable deposit financing, increased total asset by about $1.1 trillion (from $13.1 to $14.2 trillion).\footnote{From the fourth quarter of 2007 to the first quarter of 2009, the Federal Reserve and the GSEs increased holdings of securitized assets by approximately $350 billion. See He et al. (2010) for more details.}

Models with representative intermediaries are unable to capture this heterogeneity within the financial sector and study its implications for asset prices and the real economy. In the next section, I present a general equilibrium model with heterogeneous intermediaries and financial frictions that is consistent with opposite cyclical dynamics of leverage within the financial sector. My model implies that a dealer deleveraging episode comparable to one observed during the recent financial crisis, leads to an approximately 55% increase in the risk premia and a 5% increase in endogenous volatility. I then study model’s asset pricing implications for time-series predictability and the cross-section of expected returns.

## 3 Model

In this section I present a general equilibrium model featuring heterogeneous intermediaries and financial constraints. My model reconciles seemingly contradictory results for the sign of price of intermediary leverage shocks from recent empirical evaluations of representative intermediary-based models. The model nests key forces behind Brunnermeier and Pedersen (2009) and He and Krishnamurthy (2012) with two main ingredients: (i) agents differ in their attitudes toward risk, and (ii) different intermediaries face state-dependent leverage constraints with varying degree of tightness, while equity issuance is ruled out by assumption.

I consider an endowment economy in continuous time populated by a continuum of agents whose total mass is one.\footnote{In the Internet Appendix, I extend the model to an \( AK \) production economy which allows for capital accumu-}
ensure stationarity, I assume each agent faces an exogenous constant mortality rate \( \kappa > 0 \). New agents are born at the same rate \( \kappa \) per unit of time with a fraction \( \bar{u} \) as type \( A \), a fraction \( \bar{v} \) as type \( B \), and a fraction \( 1 - \bar{u} - \bar{v} \) as type \( C \). So, the total population is kept constant (normalized to one). In aggregate, the newborns inherit the wealth of their deceased parents on a per capita basis. Garleanu and Panageas (2015) show that under these conditions, agents’ effective time preference is increased by \( \kappa \).\(^{13} \)

3.1 Endowment and Agents

The aggregate endowment \( D_t \) evolves according to

\[
\frac{dD_t}{D_t} = \mu_D \, dt + \sigma_D \, dZ_t,
\]

where \( \mu_D \) and \( \sigma_D \) are constant parameters and \( Z_t \) is a standard Brownian motion defined on a fixed probability space \( (\Omega, \mathcal{F}, P) \) and a filtration \( \{\mathcal{F}_t, t \geq 0\} \) of sub-\( \sigma \)-algebras of \( \mathcal{F} \) satisfying the usual conditions, as defined by Protter (2004).\(^{14} \) The shock \( dZ_t \) is the only source of uncertainty in the model representing a permanent shock to the aggregate dividend. I assume that the growth rate of the endowment is positive, \( \mu_D - \sigma_D^2/2 > 0 \). Without loss of generality I set \( D_0 = 1 \). Similar to He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), I assume agents are unable to hedge the aggregate risk.\(^{15} \)

To separate the effects of elasticity of intertemporal substitution (EIS) and risk aversion, I assume all agents have stochastic differential utility as in Duffie and Epstein (1992), the continuous-time analog of recursive preferences of Epstein and Zin (1989). In particular, an agent of type \( i \) has the lifetime utility \( U_{i,t} \) at time \( t \) given by

\[
U_{i,t} = E_t \left[ \int_t^\infty f_i(C_{i,s}, U_{i,s}) \, ds \right],
\]

where

\[
f_i(C_{i,t}, U_{i,t}) = \left( \frac{1 - \gamma_i}{1 - 1/\psi_i} \right) U_{i,t} \left[ \left( \frac{C_{i,t}}{[(1 - \gamma_i)U_{i,t}]^{1/(1-\gamma_i)}} \right)^{1-1/\psi_i} - (\rho + \kappa) \right].
\]

Function \( f_i \) aggregates over current consumption \( C_{i,t} \) and future utility \( U_{i,t} \). Parameters \( \gamma_i \) and \( \psi_i \) denote agent \( i \)'s coefficient of relative risk aversion and EIS, respectively. These preferences reduce to standard power utility when \( \psi_i = 1/\gamma_i \). All agents are assumed to have a common subjective discount factor \( \rho \) increased by \( \kappa \) as mentioned above.

Agents are heterogeneous in their attitudes toward risk \( \gamma_i \). \( A \) agents are the most risk tolerant.
and $C$ agents are the most risk averse. $B$ agents are more risk tolerant than $C$ types: $\gamma_A < \gamma_B < \gamma_C$. I think of $A$ agents representing shadow banks (broker-dealers, hedge funds, etc.) and $B$ agents as traditional banks, and $C$ agents representing the household sector. In equilibrium $A$ and $B$ will have levered balance sheets by borrowing from $C$ agents. The financial sector ($A$ and $B$ agents) face time-varying margin constraints, which I discuss later in detail.

### 3.2 Financial Markets and Budget Constraints

All agents can trade a risky asset in fixed supply (normalized to one) and an instantaneous (from $t$ to $t + dt$) risk-free bond in zero net supply which pays the endogenously-determined interest rate $r_t$. The risky asset is a claim on the aggregate endowment $\{D_t\}$, so, the total return on the risky claim is

$$dR_t = \frac{dP_t + D_t dt}{P_t} \equiv \mu_t dt + \sigma_t dZ_t,$$

where $P_t$ is the price of the risky claim, $\mu_t$ is its expected return, and $\sigma_t$ is its volatility, all determined in equilibrium. I use the consumption good as the numeraire. I also denote the dividend-price ratio of the risky asset by $F_t = D_t/P_t$.

Let $W_{i,t}$ denote agent $i$’s wealth and assume $W_{i,0} > 0$ for $i \in \{A, B, C\}$. Let $w_{i,s}$ be the share of agent $i$’s wealth invested in the endowment claim. Then agent $i$’s financial wealth evolves according to the following standard dynamic budget constraint

$$\frac{dW_{i,t}}{W_{i,t}} = (r_t + w_{i,s,t}(\mu_t - r_t) - c_{i,t}) dt + w_{i,s,t}\sigma_t dZ_t,$$

where $c_t \equiv C_t/W_t$ is agent $i$’s consumption-wealth ratio. The agent earns the risk-free rate, earns the risk premium on the risky asset, and pays for consumption. The intermediary leverage is defined as the ratio of asset over equity. Thus, when portfolios weights $w_s^A$ or $w_s^B$ exceeds one, the intermediaries operate with leverage by raising debt from households $C$.

### 3.3 Financial Constraints

Financial intermediaries (agent types $A$ and $B$) are assumed to face a occasionally binding state-dependent margin constraint: At each moment in time, borrowers are restricted on how much leverage they can use on their balance sheets. In other words, lenders impose margin requirements to protect themselves against losses caused by adverse price movements. Margins are set to shield lenders against adverse price movements and are widely used in the financial sector to fund levered balance sheets. They have also been previously studied in the asset pricing literature (see

---

Footnotes:

16 In the paper, I use terms shadow banks and broker-dealers interchangeably.

17 Throughout the paper, I use terms net worth and equity interchangeably.

18 In this paper I abstract from the question of why the intermediaries face dynamic margin constraint and do not model the contracting problem among agents. Adrian and Shin (2014) provide a microfoundation for the Value-at-Risk constraint using a moral hazard problem in a static partial equilibrium setting. Nuño and Thomas (2017) extend their model to a dynamic general equilibrium framework where presence of risk-shifting gives rise to endogenous leverage constraints.

The tightness of the margin constraint can be determined by the regulators (e.g. Federal Reserve Regulation T) or by security broker-dealers (e.g. overcollateralization of repos by a hedge fund’s prime brokerage). I also rule out equity financing by the financial sector.\textsuperscript{19}

At any time $t$, I assume margin constraints restricts agent $i$’s portfolio weight $w^i_s$ to be below a certain state-dependent threshold $\bar{\theta}_t$

$$w^i_s \leq \bar{\theta}_t,$$

where, $\bar{\theta}_t$ determines the form of margin constraints, which is linked to endogenously-determined equilibrium objects (e.g. volatility of risk asset returns). Since equilibrium objects also depend on the state of the economy, margin requirements are state-dependent as well.

In particular, I assume margin requirements depend on the volatility of the risky asset return $\sigma_t$, and have the following functional form

$$\bar{\theta}_t = \bar{m} \left( \frac{1}{\bar{m} \alpha \sigma_t} \right) ^\nu,$$

where $\nu$, $\alpha$, and $\bar{m}$ are parameters that determine the type and tightness of the constraint, respectively. When $\nu = 0$, agents face a constant margin requirement: $\bar{\theta} = \bar{m}$. When $\nu = 1$, equation (7) resembles a Value-at-Risk rule.\textsuperscript{20} In the latter case, the level of margin constraint is endogenous since it is inversely linked to the return volatility, which is an equilibrium object.

### 3.4 Agents’ Optimization Problems

Since agents are identical within each type and have homothetic preferences, I consider the problem faced by a representative agent $i$ for $i \in \{A, B, C\}$. Each agent solves a standard Merton (1973) dynamic portfolio choice problem subject to margin constraints: agent $i$ starts with initial wealth $W_{i,0} > 0$, decides how much to consume as a fraction of her wealth, $c_{i,t}$, and what fraction of her net worth to invest in risky asset, $w^i_{s,t}$, in order to maximize her value function in (2), subject to the dynamic budget constraint (5) and endogenous margin constraints (6). So, agent $i$’s problem is

$$V_{i,t} = \max_{(c_{i,t}, w^i_{s,t})} U_{i,t}$$

s.t.: dynamic budget constraint (5) and margin constraint (6)

and a solvency constraint $W_{i,t} \geq 0$.

\textsuperscript{19}The model can be easily generalized by enabling intermediaries to issue outside equity while facing a “skin in the game” constraint, that is, they must retain a fraction of their equity to discourage them from diverting funds to a private account. See Brunnermeier and Sannikov (2014, 2016) and Di Tella (2017) for some recent examples.

\textsuperscript{20}Value-at-Risk constraints aim at limiting downside risk and maintaining an equity cushion large enough so that the default probability is kept below some benchmark level. They are common for banks and other leveraged financial institutions and are embedded in Basel II and Basel III regulatory frameworks. See Danielsson et al. (2012) and Adrian and Shin (2014) for recent examples.
3.5 Equilibrium

The definition of the competitive equilibrium is standard and is given below.

**Definition 1.** A competitive equilibrium is the set of aggregate stochastic processes adapted to the filtration generated by $Z_t$: the price of claim on the aggregate endowment $P_t$, and the risk-free interest rate $r$; and a set of stochastic processes for each agent $i$: net worth $W_i$, consumption $C_i$, and stock holdings $w_{s,t}^i$; such that:

i. Given the aggregate stochastic processes $(P_t, r_t)$, choices $(C_{i,t}, w_{s,t}^i)$ solve agent $i$’s optimization problem in (8).

ii. Markets clear

\[
C_{A,t} + C_{B,t} + C_{C,t} = D_t \quad \text{(goods market)}
\]
\[
w_{s,t}^A W_{A,t} + w_{s,t}^B W_{B,t} + w_{s,t}^C W_{C,t} = P_t \quad \text{(stock market)}
\]

The bond market clears by Walras’ law. Note that bond market clearing implies that the aggregate wealth in the economy is equal to the value of the endowment claim, i.e.

\[W_{A,t} + W_{B,t} + W_{C,t} = P_t.\]

4 Model Solution

In order to solve the model, I need to determine how prices, portfolio choices, and consumption processes for all agents depend on the historical paths of the aggregate shock $Z_t$. The equilibrium can be characterized in a recursive formulation where all equilibrium objects are functions of two endogenous state variables, defined below. The computation of equilibrium requires solving the Hamilton-Jacobi-Bellman (HJB) partial differential equations of $A, B,$ and $C$ agents simultaneously. Unfortunately, the system of nonlinear PDEs does not admit a closed-form solution and I have to rely on numerical techniques. In this section, I first define my model’s two endogenous state variables and derive their dynamics. I then characterize agents’ value functions and provide some intuition for their optimal portfolio and consumption policy functions. I define a recursive Markov equilibrium and finally briefly discuss the numerical algorithm used to solve the PDEs.

4.1 Endogenous State Variables

Because Epstein-Zin preferences are homothetic, the optimal control variables for an agent are all linear in her wealth. The linear property allows me to simplify the endogenous state space, from an infinite-dimensional into a two-dimensional space. More precisely, I only need to keep track of the share of aggregate wealth that belongs to types $A$ and $B$ (the financial sector), as well as, the
wealth share of $A$ agents in the financial sector. I can derive equilibrium conditions as functions of the following endogenous state variables:

$$x_t = \frac{W_{A,t} + W_{B,t}}{P_t}, \quad y_t = \frac{W_{A,t}}{W_{A,t} + W_{B,t}}.$$  \hfill (11)

Since the risk-free asset is in zero net supply, the aggregate wealth in the economy is equal to the risky asset price $P_t$. The state variable $x$ is the share of aggregate wealth that belongs to the financial sector (i.e. $A$ and $B$ agents), and $y$ is the type $A$ intermediaries’ wealth share as a fraction of the total financial sector.\footnote{Note that the definitions in equation (11) ensure that the domain of both state variables is $[0, 1]$. Moreover, the wealth shares of agents $A$ and $B$ as a fraction of aggregate wealth are equal to $xy$ and $x(1-y)$, respectively.}

The state variable $x$ (total wealth share of the financial sector in the economy), is the key state variable in recent intermediary asset pricing models with a representative financial sector (see He and Krishnamurthy, 2013, Brunnermeier and Sannikov, 2014, and Gertler and Kiyotaki, 2010, for example) If only intermediaries can invest in the risky asset, state variable $x$ represents the equity capital ratio of the financial sector.\footnote{In this case, because riskless bonds are in zero net supply and the risky asset is assumed to be in unit supply, total assets of the intermediary sector is equal to the risky asset price $P$.} HKM show shocks to capital ratio of intermediaries price the cross-section of expected return with a positive price of risk: intermediary’s marginal value of wealth rises when capital ratio $x$ falls.

State variable $y$, on the other hand, captures the wealth distribution within the intermediary sector. It represents heterogeneity among intermediaries in the sense that it would not be present in models with a representative financial sector. Distribution of wealth among different intermediaries clearly plays no role in the models with a representative financial sector. In contrast, in Section 7, I show that the distribution of wealth between broker-dealers and bank holding companies (proxies for $A$ and $B$ agents, respectively) can negatively forecasts future returns for many asset classes. I also demonstrate that shocks to $y$ are a priced risk factor in the cross-section of equity and bond returns with a positive estimated price of risk.

I restrict my attention to a Markov equilibrium (defined below) in the state space $(x, y) \in [0, 1] \times [0, 1]$, where all processes are functions of $(x_t, y_t)$ only. Proposition 1, characterizes the dynamics of the two endogenous state variables $(x, y)$.

**Proposition 1.** The laws of motion for endogenous state variables $x$ and $y$ are given by

$$dx_t = \kappa (\bar{x} - x_t) dt + x_t (\mu_{x,t} dt + \sigma_{x,t} dZ_t), \quad dy_t = \kappa (\bar{y} - y_t) dt + y_t (1 - y_t) (\mu_{y,t} dt + \sigma_{y,t} dZ_t)$$  \hfill (12)

where $\bar{x} = \bar{u} + \bar{v}$ and $\bar{y} = \bar{u} / (\bar{u} + \bar{v})$.

i. The drifts of $x$ and $y$ are given by

$$\mu_x = [yw_s^A + (1-y)w_s^B - 1] (\mu - r - \sigma^2) - yc_A - (1-y)c_B + F$$ \hfill (13)

$$\mu_y = (w_s^A - w_s^B) (\mu - r) - c_A + c_B - [yw_s^A + (1-y)w_s^B] (w_s^A - w_s^B) \sigma^2$$ \hfill (14)

ii. The diffusions of $x$ and $y$ are given by
\[ \sigma_x = \left[ y w_s^A + (1 - y) w_s^B - 1 \right] \sigma \] (15)

\[ \sigma_y = (w_s^A - w_s^B) \sigma \] (16)

**Proof.** See Appendix A.

Given the dividend-price ratio \( F \), the return process for the endowment claim in equation (4) can be rewritten as

\[ dR = \frac{d(D/F)}{D/F} + F dt = \mu dt + \sigma dZ, \]

where time subscripts are dropped for notational simplification.

Using Ito’s lemma, the expected return and volatility of the risky asset will be

\[ \mu = \mu_D + F - \frac{F_x}{F} [\kappa(\bar{x} - x) + x(\mu_x + \sigma_D \sigma_x)] - \frac{F_y}{F} [\kappa(\bar{y} - y) + y(1 - y)(\mu_y + \sigma_D \sigma_y)] \]

\[ + \left[ \left( \frac{F_x}{F} \right)^2 - \frac{1}{2} \frac{F_{xx}}{F} \right] x^2 \sigma_x^2 + \left[ \left( \frac{F_y}{F} \right)^2 - \frac{1}{2} \frac{F_{yy}}{F} \right] y^2 (1 - y)^2 \sigma_y^2 \]

\[ + \left[ 2 \left( \frac{F_x}{F} \right) \left( \frac{F_y}{F} \right) - \frac{F_{xy}}{F} \right] xy(1 - y) \sigma_x \sigma_y \] (17)

\[ \sigma = \sigma_D - \frac{F_x}{F} x \sigma_x - \frac{F_y}{F} y (1 - y) \sigma_y \] (18)

Note that from (18), a part of the risk from holding the risky asset is fundamental, \( \sigma_D dZ_t \), and a part is endogenous, \( \left( -\frac{F_x}{F} x \sigma_x - \frac{F_y}{F} y (1 - y) \sigma_y \right) dZ_t \). Equation (18) also implies that the volatility of returns \( \sigma \) exceeds the fundamental volatility \( \sigma_D \) when price-dividend ratio, \( 1/F \), and the state variables \( x \) and \( y \) are procyclical, i.e. \( F_x > 0, F_y > 0, \sigma_x > 0, \) and \( \sigma_y > 0 \), which is the case in equilibrium.

The following proposition provides the boundary conditions that the state variable diffusions satisfy.

**Proposition 2.** The diffusion for state variables \((x_t, y_t)\) satisfy the following boundary conditions:

\[ \lim_{x \to 0} x \sigma_{x,t} = \lim_{x \to 1} x \sigma_{x,t} = 0, \quad \forall y \in [0, 1] \]

\[ \lim_{y \to 0} y(1 - y) \sigma_{y,t} = \lim_{y \to 1} y(1 - y) \sigma_{y,t} = 0, \quad \forall x \in [0, 1] \]

**Proof.** See Appendix A.

These boundary conditions will be used later to solve agents’ HJB equations discussed below.

### 4.2 Hamilton-Jacobi-Bellman Equations

The recursive formulation of agent \( i \)'s optimization problem is given by the following HJB equation

\[ 0 = \max_{c_i, w_i} f_i(c_i W_i, V_i(W_i, x, y)) dt + E_t [dV_i(W_i, x, y)], \] (19)
where \( V_i \) is agent \( i \)'s value function. With homothetic preferences, the value functions have the power form. The following proposition characterizes agents’ value functions.

**Proposition 3.** The value function of agent \( i \in \{A, B, C\} \) has the form

\[
V_i(W, x, y) = \frac{W^{1-\gamma_i}}{1-\gamma_i} J_i(x, y)^{1-\gamma_i},
\]

where \( J_i \) is agent \( i \)'s consumption-wealth ratio, \( c_i = J_i \).

Furthermore, \( J_i \) solves the following second-order partial differential equation (PDE)

\[
\rho + \kappa = \frac{1}{\psi_i} J_i + \left( 1 - \frac{1}{\psi_i} \right) \left[ r + w_i^*(\mu - r) - \frac{\gamma_i}{2} (w_i^*)^2 \sigma^2 \right] - \frac{1}{\psi_i} \left[ \frac{J_{ix}}{J_i} [\kappa(\bar{x} - x) + x \mu_x] + \frac{J_{iy}}{J_i} [\kappa(\bar{y} - y) + y(1 - y) \mu_y] \right] + \left( 1 - \gamma_i \right) \left( \frac{J_{ix}}{J_i} x \sigma_x + \frac{J_{iy}}{J_i} y(1 - y) \sigma_y \right) w_i^* \sigma
\]

\[
+ \frac{1}{2} \frac{J_{ixy}}{J_i} x y(1 - y) \sigma_x \sigma_y + \frac{J_{ixx}}{J_i} x^2 \sigma_x^2 + \frac{J_{iyy}}{J_i} y^2(1 - y)^2 \sigma_y^2.
\]

**Proof.** See Appendix A. \(\square\)

Functions \( J_i \) capture agent \( i \)'s investment opportunity set. In particular, note that from (20) if \( \frac{1 - \gamma_i}{1 - \psi_i} > 0 \) (which holds in my calibration), marginal utility of wealth is increasing in \( J_i \).

The first-order conditions of agent’s recursive problem gives the optimal consumption and portfolio choice

\[
c_i = \frac{C_i}{W_i} = J_i
\]

\[
w_{i,^*} = \frac{\mu - r}{\gamma_i \sigma^2} + \frac{1}{\gamma_i} \left( \frac{1 - \gamma_i}{1 - \psi_i} \right) \left( \frac{J_{ix}}{J_i} x \sigma_x \sigma + \frac{J_{iy}}{J_i} y(1 - y) \sigma_y \sigma \right)
\]

The optimal unconstrained portfolio \( w_{i,^*} \) is the standard ICAPM result of Merton (1973): the first term in (23) is the myopic demand of a one-period mean-variance investor and the second term is the hedging demand capturing the variations in the agent’s investment opportunity set. The optimal consumption-wealth ratio in (22) comes from the standard envelope condition.

So, from the optimal portfolio in the absence of constraints in (23) and the margin constraint in equation (6), the optimal portfolio is

\[
w_{i,t} = \min \left( \tilde{\theta}_t, w_{i,t}^* \right),
\]

where the leverage upper bound \( \tilde{\theta}_t \) is defined in (7).
4.3 Recursive Markov Equilibrium

I derive a Markov equilibrium in state variables $x_t$ and $y_t$. That is, I look for an equilibrium where all equilibrium objects (prices, consumption, and portfolio choices) can be written as functions of these two state variables. Next I define the Markov equilibrium in state space $(x, y)$.

**Definition 2.** A Markov equilibrium in state variables $(x_t, y_t)$ is the set of functions: marginal value of wealth $J_i(x, y)$, dividend-price ratio $F(x, y)$, real interest rate $r(x, y)$ and policy functions $c_i(x, y), w_i(x, y)$ for $i \in \{A, B, C\}$, and laws of motion for endogenous state variables $\mu_x(x, y), \mu_y(x, y)$ and $\sigma_x(x, y), \sigma_y(x, y)$ such that

i. marginal value of wealth $J_i$ solves agent $i$’s HJB equation, and $c_i$ and $w_i$ are corresponding policy functions, taking $F, r$ and laws of motion for $x$ and $y$ as given.

ii. Markets for consumption good and risky asset clear

\[
xy c_A + x(1 - y)c_B + (1 - x)c_C = F \quad \text{(goods market)}
\]

\[
xy w_s^A + x(1 - y)w_s^B + (1 - x)w_s^C = 1 \quad \text{(stock market)}
\]

iii. The laws of motion for $x$ and $y$ satisfy (13)-(16).

4.4 Numerical Solution

The computation of equilibrium requires solving the HJB equations of the three types of agents simultaneously. Functions $J_A(x, y), J_B(x, y)$, and $J_C(x, y)$ can be found by solving a system of second-order partial differential equations (PDEs) in $(x, y)$. To do so, all equilibrium objects (e.g. $F, \sigma, \mu, \sigma_x, \mu_x, \sigma_y, \mu_y$, etc.) need to be expressed in terms of functions $J_i$ and their derivative. Unfortunately, the system of nonlinear differential equations does not admit a closed-form solution and I have to rely on numerical techniques. This is particularly challenging in the presence of model’s two endogenous state variables. I use projection methods, specifically orthogonal collocation using Chebyshev polynomials (Judd, 1992, 1998), to solve for equilibrium. Unlike a log-linearized representation around the steady state, this method provides a global solution and a full characterization of the whole dynamical system. In Appendix B, I explain the numerical procedure in detail.

5 Calibration and Parameters Choices

Table 1 lists the parameter choices used in calibrating the model. I calibrate parameters at a quarterly frequency. I choose the drift and diffusion of the aggregate endowment to match the mean and volatility of aggregate U.S. consumption data. To create demand for risk sharing among agents, I set the risk aversions of $A$, $B$, and $C$ agents to 2.5, 5.5, and 15, respectively. I chose a common value of EIS for all agent types and set $\psi_i = 1.5$.$^{23}$ The values of $\gamma_i$ and $\psi_i$ imply agent $i$’s preference for the early resolution of uncertainty and have been extensively used in the asset

$^{23}$Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) both use EIS value of 1.5 in their calibrations.
pricing literature to address a number of asset pricing puzzles. A value of EIS greater than one implies a decline in asset prices when the effective risk aversion in the economy increases. I set mortality rate $\kappa$ to 0.0038 which implies that agents on average live for 65 years which is consistent with the calibration in Gârleanu and Panageas (2015). The subjective discount rate $\rho$ is set to 0.0025 to achieve reasonable values for the real interest rates of between 2–3% annually.

The parameter $\bar{m}$ in equation (7) determines the constant margin requirements (when $\nu = 0$), and I set $\bar{m} = 4$. When $\nu = 1$, equation (7) resembles a Value-at-Risk constraint. In this case, parameter $\alpha$ determines the tightness of the constraint. I set $\alpha = 20$ (at quarterly frequency) which is approximately equal to the one-month Value-at-Risk at the 99% level.

6 Model Results

In this section, I first present additional properties of the equilibrium with margin constraints and compare them with the unconstrained economy with the same fundamentals and degree of heterogeneity among agents. The economy with margin constraints simultaneously exhibits higher risk premium and lower risk-free rate and volatility, compared to the frictionless benchmark. Although some of these effects have been previously documented in the literature, my analysis extends these results to an economy with three agents (households and two heterogeneous intermediaries) and recursive preferences. The equilibrium with three heterogeneous agents and two endogenous state variables is considerably more challenging to solve numerically.

In Section 6.2, I show, consistent with empirical evidence, the model can generate different cyclical dynamics for different intermediaries (i.e. $A$ and $B$ agent). Implications of the heterogeneity in the financial sector (as captured by the wealth distribution among intermediaries) for variation is risk premia beyond are discussed in Section 6.3. Section 6.4 studies the impact of a dealer deleveraging episode comparable to the one observed during the recent financial crisis, for risk premia and volatility. Finally, in Section 6.5, I show how the model reconciles seemingly contradictory evidence for the sign of price of intermediary leverage shocks in AEM and HKM.

6.1 Constrained vs. Unconstrained Economy

Complete Market Benchmark

As a benchmark, I first consider an economy with no margin constraints, that is $\tilde{\theta}(x_t, y_t) \to \infty$. In the absence of constraints, investors face complete markets and their Euler equations hold with

---

24See, for example, Bansal and Yaron (2004), Hansen, Heaton, and Li (2008), and Bansal and Shaliastovich (2013) for resolution of equity premium, value premium, and uncovered interest rate parity puzzles, respectively.

25The one-month (21 trading days) VaR is equal to $\sqrt{21} \times \Phi^{-1}(0.99) \approx 10$, where $\Phi(\cdot)$ is the CDF of the standard normal distribution.

26For example, Rytchkov (2014) adds margin constraints to an endowment economy with two heterogeneous agents and CRRA preferences (similar to the models in Longstaff and Wang, 2012 and Bhamra and Uppal, 2014) to show binding constraints reduce return volatility and risk-free rate, but increase expected returns and Sharpe ratio. Phelan (2016) reaches similar results in an economy with two intermediate goods populated by risk-neutral household and banks facing leverage constraints.
equality in equilibrium. Without constraint, the economy is very similar to Gârleanu and Panageas (2015) but with three heterogeneous agents. It can be shown that without heterogeneity in risk tolerance, the interest rate, consumption-wealth ratio of each agent \(J_i\’s\), price-dividend ratio, and the price of risk are constant.\(^{27}\)

Figures 3 presents various equilibrium variables (price-dividend ratio \(1/F\), volatility of the risky asset return \(\sigma\), Sharpe ratio \((\mu-r)/\sigma\), and risk premium on the endowment claim \(\mu-r\) as a function of the state variable \(x\) while the state variable \(y\) is fixed at 0.56 (its stochastic steady state).\(^{28}\) In both figures solid blue lines correspond to the frictionless economy. Along the horizontal axis in each panel is the state variable \(x_t\) (the wealth share of types \(A\) and \(B\)), which ranges from 0 to 1.

The top right panel of Figure 3 shows the volatility of returns. Even though fundamental volatility is constant \((\sigma_D = 3.5\%\) in my calibration), return volatility is time varying and it exceeds fundamental volatility in a hump-shaped pattern. As mentioned above and shown in equation 18, a part of the risk from holding the risky asset is fundamental and a part is endogenous. From equation (11), wealth shares of agents \(A\), \(B\), and \(C\) in the aggregate economy are equal to \(xy\), \(x(1-y)\) and \(1-xy\), respectively. Thus, when \((x, y) = (1, 1)\), \((x, y) = (1, 0)\), or \((x = 0, \forall y)\), the economy is populated by one type of agent \((A, B, \text{and} \ C\), respectively) and the volatility of the endowment claim coincides with the fundamental volatility \(\sigma_D\). This can be validated from three-dimensional plots in Figure G.2 in Appendix F. The Sharpe ratio and expected excess return \(\mu - r\) both show countercyclical behavior as expected: higher risk premium and price of risk during distressed states when the intermediary sector is undercapitalized (low-\(x\) states) and/or broker-dealers deleverage (when \(y\) is low) are low.

The bottom right panel of Figure 3 shows that the risk premium largely tracks the Sharpe ratio. Note that this is the risk premium on a claim to the aggregate endowment, which has a relatively low volatility \((3.5\%\) in the baseline calibration). By comparison, equity volatility is around 16\%. Therefore, the equity premium implied by the model is about five to six times larger than that of the endowment claim, putting it in the range of standard estimates in the literature.

When the EIS exceeds one, the substitution effect dominates the income effect so that greater risk aversion reduces asset demand and valuations fall. In this case the rise in the risk premium exceeds the fall in the real rate. In contrast, if the EIS is less than one, greater risk aversion counter-intuitively causes the valuations of risky assets to increase.

Figure 4 shows optimal portfolio weights in the risky asset for all three agents. For the most risk tolerant type \(A\) investors it always exceeds one and for the most risk averse type \(C\) agents it is always less than one. \(B\’s\) optimal portfolio is greater than one for most of the state space. Thus, without constraints, financial intermediaries (type \(A\) and \(B\) agents) borrow from type \(C\) investors (households) to take a levered positions in the endowment claim. As the wealth share of

\[^{27}\]These are very general results. For more details, see Gârleanu and Panageas (2015), Rytchkov (2014), and Longstaff and Wang (2012), for examples.

\[^{28}\]Three-dimensional plots for equilibrium objects in the unconstrained and constrained equilibria are provided in Figures G.1–G.2 in Appendix F. The bivariate stationary distribution of state variables \((x, y)\) is presented in Figure 2 in the online Appendix.
the financial sector get bigger and A agents have more relative wealth in the financial sector, they borrow from traditional banks (B agents) as well.

Importantly, in the absence of constraints, the optimal leverage of A and B investors are both countercyclical: they are higher in bad states when investment opportunities are more attractive. This means when margin constraints are imposed to limit leverage, they are more likely to bind in high marginal utility states.

The relationship between portfolio weights and the wealth shares of the financial sector \((x)\) and wealth share of broker-dealers in the financial sector \((y)\) in Figure 4 is the result of market clearing for the risky asset in Equation (26). When \(x\) is close to zero or both \(x\) and \(y\) are close to one, a single type of agent (type C in the first case and type A in the second case) dominates the economy, which reduces the opportunity for risk sharing. In the absence of constraints, agents of the dominant type must hold all their wealth in the risky asset, whereas agents of the vanishing type can be satisfied with only a small amount of borrowing or lending. Thus, when \(x\) is near zero, households (C agents) set prices and intermediaries (A and B types) take high leverage.

**Equilibrium with Dynamic Margin Constraints**

To study the impact of financial constraints on the equilibrium, I solve the model with the same fundamentals and heterogeneity as before, but now I assume investors face margin requirements in the form given in equation (7). As noted earlier, margin constraints are occasionally binding and state-dependent. For simplicity, I solve the model under the assumption that margin constraints occasionally bind only for the most risk tolerant A types and more risk averse agents (i.e. types B and C) do not face binding leverage constraints.\(^{29}\)

Consistent with Kogan et al. (2007) and Rytchkov (2014), the economy with margin constraints simultaneously exhibits higher risk premium and Sharpe Ratio and lower risk-free rate and volatility, compared to the frictionless benchmark.\(^{30}\)

I focus on two cases: (i) a constant margin constraint with \(\bar{\theta}_t = \bar{m} (\nu = 0 \text{ in equation 7})\), and (ii) the case where type A agents face endogenous time-varying constraints \((\nu = 1)\) in the form \(\bar{\theta}_t = \frac{1}{\alpha \sigma_t}\), where parameter \(\alpha\) determines the tightness of the constraint. In the second case, margin requirements are determined by a Value-at-Risk-type rule and the level of margins is endogenous because it is (inversely) related to the return volatility, an equilibrium object.

Figures 3 and 4 present various objects for equilibria with constant (dash-dotted purple line) and state-dependent Value-at-Risk-type (dashed red line) margin constraints. There are few important observations from these figures. First, the impact of both types of constraints are qualitatively similar and the only difference is the magnitude. With the choice of parameters presented in Table 1, the time-varying margins are more restrictive and the effects are stronger with \(\nu = 1\) in

---

\(^{29}\)This assumption can be easily relaxed. The equilibrium will be qualitatively similar if we assume the least risk averse A intermediaries face tighter margin constraints than more risk averse B types. In the online appendix, I solve the model under the general case where any agent investing in the risky asset could face binding margin constraints.

\(^{30}\)Both papers study a two-agent economy with CRRA preferences, whereas my model has three heterogeneous agents with recursive preferences and two endogenous state variables which is considerably more challenging to solve.
The top left panel of Figure 3 shows the impact of margin constraints on the price-dividend ratio. In my calibration, margin constraints do not substantially decrease asset’s valuation ratio when they bind, reducing the price-dividend ration by less than 1% at steady state relative to the complete-market benchmark.

The top right panel of Figure 3 plots the return volatility $\sigma$. The impact of constraints on volatility is unambiguous: portfolio constraints reduce the volatility of the risky asset return relative to the unconstrained economy. Also the volatility decreases in states where the constraint actually binds, although the point at which the constraint starts to bind depends on the form and severity of the constraint. The intuition behind this effect is as follows. In equilibrium, less risk averse $A$ and $B$ investors (the financial sector) borrow from more risk averse households and operate with leverage. It is well established in the macro-finance literature that leverage makes returns more volatile than the fundamental volatility: levered balance sheets amplify an aggregate shock to dividends. Binding margin constraints reduce dynamic risk sharing and leverage in equilibrium, thereby reducing the return volatility. In my calibration, binding dynamic margin constraints results in the reduction of the return volatility by approximately 6% at model’s stochastic steady state.

The bottom panels of Figure 3 demonstrate that portfolio constraints increase the Sharpe ratio and risk premium of the endowment claim. This is again a general effect and does not depend on the form of the constraints. The intuition is straightforward: because the leverage of the risk tolerant agent ($A$ type) is bounded in the part of the state space where the margin constraints bind, to clear the market, the more risk averse investors ($B$ and $C$ types) are forced to take on a larger portion of the risky asset that they would without constraints. To induce them to buy more, the risk premium should increase. Following negative risky asset returns, margin constraints bind and broker-dealers ($A$ types) are forced to sell assets. As a result, to clear the market, the expected returns must increase enough to entice more risk averse agents to take on a larger supply of the risky asset than before the shock. Since banks ($B$ types) are not facing binding constraints, as discussed above, they increase leverage following a negative shock. Thus the model can qualitatively match the empirical evidence on opposite cyclical patterns of intermediary leverage in the financial sector documented in Figure 1. At model’s stochastic steady state, binding margin time-varying constraints causes the Sharpe ratio and risk premium to rise by approximately 39% and 36%, respectively.

Figure 4 also shows the effect of margin constraints on optimal portfolio weights. As explained above, because $A$’s leverage is countercyclical in the unconstrained economy, the margin constraints will bind in states where $x$ and $y$ are low. In the model with margin constraints, type $A$ agents operate with leverage in all states of the economy, however when margin constraints bind, leverage is restricted by $\bar{\theta}_t$. In other words, the presence of binding leverage constraints makes broker-dealers’ leverage countercyclical: they are forced to sell assets and delever in bad states of the economy where constraints bind. To clear the risky asset market, both $B$ and $C$ investors need to absorb

\footnote{See Kiyotaki and Moore (1997), Bernanke et al. (1999), and Brunnermeier and Sannikov (2014), for example.}
this additional supply and increase their portfolio weights as we see from dashed and dotted lines in the top right and bottom left panels of figure 4.

Finally, the middle right panel of Figure 4 also shows that margin constraints (regardless of the form) reduce the risk-free rate. This result is also intuitive. In the absence of constraints, A and B investors operate with leverage by borrowing from type Cs. The upper bound for leverage for type As reduces the demand for credit, thus lowering the risk-free rate. In my calibration, when margin constraint binds, the risk-free rate decreases by approximately 8% in the stochastic steady state \((x_{ss}, y_{ss}) = (0.36, 0.56)\).

Figure 5 illustrates the evolution of model’s two endogenous state variables \(x\) and \(y\) in the constrained and frictionless economies. The drift of \(x_t(y_t)\) is positive for low levels and becomes negative for high values of \(x_t(y_t)\). The points where the drift crosses zero is the stochastic steady state of the endogenous state variable, the point of attraction of the system in the absence of shocks. Importantly, from the right panels in Figure 5, notice that the diffusion terms \(\sigma_x\) and \(\sigma_y\) are always positive.\(^{32}\) This implies that following a negative aggregate shock, both state variables decline, i.e. \(x\) and \(y\) both exhibit procyclical dynamics in the model. In Section 7.1 below, I verify this also holds in the data. As shown in Proposition 2, at the boundary points of the state space \((x = 0, x = 1, y = 0, \text{ and } y = 1)\), the diffusion of state variables \(x\) and \(y\) are zero. This can be verified from the top- and bottom-right panels of Figure 5.

The left panels of Figure 5 also illustrate that portfolio constraints of both types reduce the volatility of the state variables. Because the impact of the constraints on the sensitivity of the price-dividend ratio to the state variables is very small (as shown in the top left panel of Figure 3), a decrease in \(\sigma_x\) and \(\sigma_y\) translates into a decrease in the return volatility \(\sigma\) as presented in the top-right panel of Figure 3 (this follows directly from equation (18)).

### 6.2 Cyclical Properties of Intermediary Leverage

**Countercyclical Holding Company and Financial Sector leverage, Procyclical leverage for Broker-Dealers**

In this section, I show that the model is able to generate leverage patterns for different intermediaries that are consistent with the empirical evidence presented earlier. As mentioned above and illustrated in Figure 4, in the absence of margin constraints, broker-dealers and bank holding companies both exhibit countercyclical leverage: their optimal leverage is higher in bad states. However, when broker-dealers face state-dependent margin constraints inversely dependent on return volatility, their leverage exhibit an (almost) opposite cyclical behavior. Since return volatility is hump-shaped (see the top right panel of Figure 3), margin constraints cause A type’s leverage to be U-shaped when constraints bind. When the constraints are sufficiently tight, shadow bank leverage is procyclical, consistent with the empirical evidence from broker-dealers leverage presented in Figure 1 (the solid blue line) and also documented in AEM.

\(^{32}\)From three-dimensional plots in the Appendix Figures G.1 and G.2, we can confirm that \(\sigma_x\) and \(\sigma_y\) both stay positive in the entire state space.
Leverage of the financial sector (types A and B in the model), $w_{FS}^s$, is defined as the share of sector’s wealth held in the risky assets:

$$w_{FS}^s = \frac{w_A^s W_A + w_B^s W_B}{W_A + W_B} = yw_A^s + (1 - y)w_B^s$$  \hspace{1cm} (27)

where state variable $y = W_A/(W_A + W_B)$ is the wealth share of A types in the financial sector as defined in equation (11). We see that the financial sector leverage is the weighted average of A and B types’ optimal leverage with a time-varying weight equal to the state variable $y \in [0, 1]$: the wealth share of broker-dealers (A types) in the financial sector.

The left panel of Figure 6 presents financial sector’s leverage in the unconstrained equilibrium and in the model with endogenous margin constraints. As discussed above, margin constraints reduce financial sector’s leverage when they bind: binding constraints reduce A type’s leverage causing the return volatility to decrease relative to the frictionless benchmark.

The right panel of Figure 6 presents intermediary leverage in the equilibrium with a Value-at-Risk-type state-dependent margin constraint. In the model with margin constraints, financial sector and bank holding companies exhibit countercyclical leverage, while broker-dealers could have procyclical leverage when constraints bind. This is again consistent with the evidence presented in Figure 1 (dashed red line for holding company leverage and solid blue line for broker-dealer leverage) and also recently documented in the empirical intermediary asset pricing literature.  

6.3 Heterogeneous versus Representative Intermediaries

Since the aggregate endowment in equation 1 is independent and identically distributed (iid) with constant volatility, variation in risk premia is only due to wealth distributions and intermediation frictions captured by state variables $x$ and $y$. State variable $x$, representing the wealth share of the total financial sector in the economy, is the main determinant of time-varying risk premia in representative intermediary models. In my model with a heterogeneous financial sector, however, the wealth distribution among intermediaries (captured by state variable $y$) also contributes to the variation in risk premia. In this section, I answer the following question: What fraction of the variation in risk premia can be attributed to the state variable $y$, a measure of heterogeneity in the financial sector?  

To answer this question and investigate the role of heterogeneity in the financial sector, I compare the main results of the three-sector model with the ones from an economy with identical fundamentals but two heterogeneous agents instead: a household sector (C types identical to the main model) and a representative intermediary sector (I), where I agents face endogenous margin constraints as in the original model (equation 7 with $\nu = 1$).

Most of the parameters in the two-agent representative intermediary model are identical to the ones in the main model listed in Table 1: household’s risk aversion $\gamma_C = 15$, EIS for household

\[33\] See AEM for security broker-dealer leverage and HKM for leverage of bank holding companies.

\[34\] State variable $y$ is a measure of heterogeneity among intermediaries in the sense that it does not exist in models with a representative financial sector.
and the representative intermediary $\psi_C = \psi_I = 1.2$, rate of time preference $\rho = .001$, growth rate and volatility of the aggregate endowment $\mu_D = .022, \sigma_D = .035$, and agents birth and death rates $\kappa = .0154$ (exactly as in the original three-sector model). I set risk aversion of the representative intermediary sector to $\gamma_I = 3.3$: the wealth-weighted risk aversion of the financial sector (A and B investors) in the main model (with heterogeneous intermediaries) evaluated at the stochastic steady state for the two state variables.\footnote{The wealth-weighted risk aversion of the intermediary sector in main three-agent model is $\gamma_I = \left( \frac{\gamma_A}{\gamma_A} + \frac{1 - y}{\gamma_B} \right)^{-1}$. Using values for $\gamma_A = 2.5$ and $\gamma_B = 5.5$ from Table 1, at the stochastic steady state $y_{ss} = 0.56$, we get $\gamma_{I,ss} = 3.3$.} Population share of the representative intermediary sector is set to $\bar{x} = .12$: sum of population shares of two intermediary sectors in the main model, $\bar{u}$ and $\bar{v}$ from Table 1. Note that with a representative intermediary, there is only one endogenous state variable $x$: the wealth share of the intermediary sector.\footnote{The wealth share of the financial sector is the key state variable in existing models with a representative intermediary sector (see He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Di Tella (2017), and Drechsler et al. (2018), for example.)}

I simulate representative- and heterogeneous-intermediary models for 3,000 quarters 20,000 times and examine the distribution of risk premium volatility. Figure 7 shows these distributions. As expected, the model with heterogeneous intermediaries exhibits more variation in risk premia than the one with a representative financial sector. Since the total intermediary sector in both model are (almost) identical, any excess variation in risk premia in the heterogeneous intermediary model has to be due to state variable $y$. In my calibration, approximately 20\% of the variation in risk premia can be attributed to heterogeneity in the financial sector (state variable $y$). Therefore, failing to account for heterogeneity among intermediaries can lead to missing a substantial portion of the variation in risk premia.

### 6.4 Implications of Financial Sector’s Balance Sheet Adjustments

As discussed in Section 2 and documented in Figures 1 and 2, during the height of the financial crisis, broker-dealers substantially delevered (reduced leverage by approximately 47\% relative the previous quarter), while during the same period, holding companies increased leverage by 72\%. In this section, I measure the impact of this balance sheet adjustments within intermediaries on risk premia and endogenous risk.

When the least risk averse $A$ types face binding margin constraints, they are forced to reduce leverage by selling assets. To clear the risky asset market, the more risk averse agents ($B$ and $C$ types) need to take on a larger portion of the asset than they would in the absence of constraints. In order to entice them to buy more, the risk premium must increase.

I perform the following exercise: I try to match the aforementioned increase and decrease in leverage of broker-dealers and holding companies, respectively, by tightening the margin constraint faced by $A$ types in baseline calibration in Table 1. This is consistent with empirical evidence that the contraction in repo market financing during the recent financial crisis hit broker-dealers (represented by A agents in the model) particularly hard and forced them to deleverage (see Gorton and Metrick, 2012 and He et al., 2010, for example). As noted earlier in Section 6.1, tighter marg
constraints leads to a decrease in total leverage and lower volatility. In order to achieve an increase in volatility consistent with the empirical evidence, I also make households relatively more risk aversion than intermediaries.

Figure 8 presents results of this exercise. Tighter margin constraints is implemented by an increase in the parameter $\alpha$. I also increases risk aversion of households relative to the intermediary sector (lower $\gamma_I/\gamma_C$) to obtain an increase in volatility. The top left panel shows that at model’s stochastic steady state, increasing parameter $\alpha$ by 50% (consistent with the rise in repo-haircuts index during the 2008 crisis from Gorton and Metrick (2012)’s Fig. 4) and reducing $\gamma_I/\gamma_C$ by 26% (from 0.61 to 0.45), results in approximately 48% decline in A types’ leverage. This deleveraging is very close to what broker-dealers experienced during the crisis.

To clear the risky asset market, the risk premium must increase enough to entice more risk averse agents to take on a larger supply of the asset than before the shock. More risk averse holding companies increase leverage as a result of dealers’ deleveraging (the top right and middle left panels): $B$ type leverage rises by 79% and households’ holding of the risky asset remain relatively unchanged (see the middle left panel of Figure 8). As the middle right and bottom left panels of Figure 8 show, this dealer deleveraging results in an approximately 55% increase in the risk premium and Sharpe ratio a 5% rise in endogenous volatility.

More importantly, asset reallocations between intermediaries do not impact the aggregate wealth share of the financial sector, and thus do no affect risk premia and volatility in a model featuring representative intermediaries. My model with a heterogeneous financial sector captures the implications of these balance sheet adjustments for equilibrium objects.

6.5 Reconciling Empirical Evidence in AEM and HKM

Price of leverage shocks is positive for BDs and negative for BHCs

In this section, I use the model to reconcile the potentially conflicting empirical evidence for the sign of price of intermediary leverage risk in the cross section of asset returns, recently documented in AEM and HKM. As mentioned earlier, AEM and HKM find opposite signs for the estimated price of intermediary leverage shocks in the cross section of asset returns and thus conflicting cyclical dynamics of the intermediary leverage. They, however, study different financial intermediaries: AEM use shocks to book leverage of broker-dealers to construct an intermediary stochastic

---

37He et al. (2017) present a simple, one-period model in their appendix which was originally suggested by Alexi Savov in a conference discussion of the paper. The model can reconcile the contradiction between HKM’s results and the ones documented in AEM. This static framework, however, is unable to capture implications of balance sheet adjustments within the financial sector for risk premium, the price of risk, and volatility discussed above.

38As mentioned above, the data for security brokers-dealers are from Table L.130 of the Financial Accounts of the United States (Flow of Funds). The underlying source for this data comes from FOCUS and FOGS quarterly reports filed with the SEC by these broker-dealers in isolation from other parts of their holding companies which are not publicly available. Data for publicly-traded holding companies of primary dealers are from CRSP/Compustat and Datastream. For a more detailed description of data sources, see Appendix C.
discount factor (SDF) and show that it prices equity and bond portfolios with a *positive* price of risk implying *procyclical* intermediary leverage. HKM, on the other hand, find that shocks to leverage of bank holding companies for the New York Fed’s primary dealers price the cross-section of returns for many asset classes with a *negative* price of risk. In contrast to AEM, HKM’s negative price of risk suggests that intermediary leverage is *countercyclical*.

I show that these seemingly contradictory results can be reconciled in a model where the intermediary sector is modeled as two heterogeneous sectors facing different financial constraints. Moreover, in Section 7, I will demonstrate that this heterogeneity plays an important role for time-series predictability and also has significant explanatory power for the cross-section of expected returns.

The stochastic discount factor of unconstrained $B$ agents is their marginal value of wealth:

$$V_{B,W} = W_B - \gamma B J_B^{1-\gamma B},$$

computed from equation (20), where $V_{B,W}$ is the partial derivative of $B$’s value function with respect to his wealth. Therefore from Ito’s lemma, the expected excess return on the risky asset satisfies

$$\mathbb{E}_t[dR_t] - r_t dt = (\mu_t - r_t) dt = -\mathbb{E}_t \left[ (\gamma B \frac{dW_B}{W_B} + \frac{1 - \gamma B}{1 - \psi B} \frac{dJ_B}{J_B}) \frac{dP_t}{P_t} \right]$$  \hspace{1cm} (28)

According to equation (28), bank holding companies’ equity return, $dW_B/W_B$, directly enters into the pricing kernel. Since, type $B$’s wealth $W_B$ is the product of the aggregate wealth $W_t$ and bank’s wealth share $v_t = x_t(1-y_t)$ (i.e. $W_{B,t} = u_t W_t$), shocks to wealth share of banks should price the cross section of returns with a *positive* price of risk. This is essentially the asset pricing result presented in He et al. (2017) that assets’ exposure to the equity capital ratio shocks of BHCs (of New York Fed’s primary dealer counterparties) explains the cross-sectional differences in expected returns for many asset classes.

Moreover, since broker-dealers’ leverage shocks are positively correlated to the innovations in BHC’s wealth share (both exhibit procyclical behavior) they should also price the cross-section of returns with a *positive* price of risk. This is the main result in Adrian et al. (2014a): shocks to broker-dealers’ leverage posses a strong ability to price the cross-section of equity and bond portfolios with a positive price of risk.$^{39}$

### 7 Empirical Implications of the Model

In this section, I study empirical implications of the model and show that the heterogeneity in the intermediary sector matters for time-series return predictability and is also priced in the cross-section of expected returns. I focus on asset pricing implications of the model with time-varying margin constraints ($\nu = 1$ in equation 7) and show that an empirical measure of the state variable $y$, which measures the wealth distribution within the financial sector thus capturing this

$^{39}$It is important to note that the intermediary leverage is endogenous, and the fact that shocks to leverage are priced does not necessarily mean that intermediaries are the marginal investors. It could well mean that leverage is proxy for aggregate risk aversion. See Santos and Veronesi (2016) and Haddad and Muir (2018) for more details.
heterogeneity, has strong predictive power for returns on various asset classes. Moreover, a linear SDF from my heterogeneous intermediary model can price the cross-section of equity and bond portfolios. For a detailed description of the data, see Appendix C.

7.1 Measuring Heterogeneity in the Intermediary Sector

The price of risk in my model is time-varying and depends on wealth share of the financial sector, state variable $x$, as well as, broker-dealers’ wealth share in the intermediary sector, state variable $y$ in the model. Since financial sector’s wealth share, $x$, is the key state variable in many existing models with a representative intermediary sector, in this section, I emphasize the importance of wealth distribution within the intermediary sector, captured by state variable $y$, for forecasting future returns.\(^{40}\)

Since risk premium is decreasing in $y$ (see Figure G.2 in the appendix), an asset that pays well when $y$ is low is less risky. Thus, my model predicts that a higher wealth share for BDs in the financial sector forecasts higher prices and thus, negatively predicts future returns.

In the data, I compute wealth share of the financial sector as the ratio of their market equity of to the total market value of firms in the CRSP universe:

\[ x_{\text{data}}^t = \frac{\text{Market capitalization of the financial sector}_t}{\text{Total market capitalization of the CRSP universe}_t}. \] \(^{(29)}\)

I use monthly equity data from CRSP to compute $x^{\text{data}}$. The financial sector is identified as firms in the CRSP universe for whom the first two digits of the header standard industry classification (SIC) code equals 60 through 67.\(^{41}\)

Since I don’t have access to market data for broker-dealers, model’s second state variable, $y$ (wealth share of broker-dealers in the financial sector), is computed as the ratio of BD’s book equity to the sum of BHC and BD book equities from the Follow of Funds Tables L.130 and L.131:

\[ y_{\text{data}}^t = \frac{\text{Book equity of BDs}_t}{\text{Book equity of BHCs}_t + \text{Book equity of BDs}_t}, \] \(^{(30)}\)

where equity is computed by subtracting total liabilities (excluding miscellaneous liabilities) from total financial assets.\(^{42}\)

\(^{40}\)Adrian, Moench, and Shin (2014b) study return predictability in representative intermediary models and show book leverage of broker-dealers negatively forecasts future equity and bond returns. He et al. (2017) also run time-series predictive regressions and show the squared reciprocal of capital ratio for bank holding companies of NY Fed’s primary dealers positively predicts future returns for many asset classes.

\(^{41}\)This definition of the financial sector has been commonly used in the literature. See Acharya, Pedersen, Philippon, and Richardson (2017) and Giglio, Kelly, and Pruitt (2016), for example.

\(^{42}\)In the Internet Appendix, using 49 industry definitions from Ken French’s website, I identify publicly-traded broker-dealers as all US firms in the CRSP universe with standard industry classification (SIC) codes 6211 (Security brokers, dealers & flotation companies) or 6221 (commodity contracts brokers & dealers). I then equivalently define state variable $y$ using market data as $y_{\text{data,mkt}}^t = \frac{\text{Market cap of dealers}_t}{\text{Market cap of the financial sector}_t}$. During the sample period (1971Q1-2010Q4), time series of $y_{\text{data,mkt}}^t$ is highly positively correlated with $y_{\text{data}}^t$ computed from book values (in equation 30) with correlation coefficient of 0.55 ($t$-stat of 8.88). Prior to 2010, book and market series are even more strongly positively correlated (correlation coefficient of 0.68 with $t$-stat of 11.74). Post 2010, however, the two series become
Table 2 reports the mean, standard deviation, and autocorrelation of the state variables in the data, both in levels and innovations. The factors are autocorrelated in levels but not in changes. Figure 9 shows the time-series of \( x_{\text{data}} \) and \( y_{\text{data}} \) using the CRSP and Flow of Funds data confirming the dramatic growth of the sector from 1980 to the onset of the recent financial crisis. Consistent with the model, \( x_{\text{data}} \) and \( y_{\text{data}} \) are both procyclical: innovations in \( x_{\text{data}} \) and \( y_{\text{data}} \) are both positively correlated with the innovations in the real GDP with correlation coefficients of 0.24 (t-stat of 2.05) and 0.21 (t-stat of 2.99), respectively. It is worth pointing out that the decline in \( y_{\text{data}} \) post 2009 is because two of the largest broker-dealers (Goldman Sachs and Morgan Stanley) became bank holding companies in 2009Q1. Two other broker dealers were also acquired by bank holding companies: J.P. Morgan purchased Bear Sterns and Merrill Lynch became part of Bank of America.

7.2 Intermediary Heterogeneity and Time-Series Predictability

The risk premium in my model is time-varying due to its association with wealth share of the financial sector, state variable \( x \), as well as, broker-dealers’ wealth share in the intermediary sector, state variable \( y \) in the model. As a result, expected returns are time-varying in the model and are predictable using lagged state variables as predictors. Since financial sector’s wealth share, \( x \), is the key state variable in existing models with a representative intermediary sector, in this section, I emphasize the importance of wealth distribution within the intermediary sector, captured by state variable \( y \), for forecasting future returns.\(^{43}\)

Since state variables \( y \) is procyclical (see Figure G.3), following a negative shock when risk premium is high, \( y \) declines. Therefore an asset that pays well when \( y \) is low is less risky. As such, my model predicts that a higher wealth share for BDs in the financial sector forecasts higher prices (lower returns) thus, negatively predicting future returns. To test this hypothesis, I regress one-year-ahead holding period excess return (from quarter \( t+1 \) to \( t+4 \)) for asset \( i \) on the lagged wealth share of BDs in the financial sector, \( y_{\text{data}} \) defined in equation (30):

\[
R_{t+1→t+4}^{e,i} = \gamma_0^i + \gamma_y^i y_t + \epsilon_{t+1→t+4}^{i} \tag{31}
\]

where \( R_{t+1→t+4}^{e,i} \) is the average excess return on asset \( i \).

The model predicts a negative and significant coefficients \( \gamma_y^i \), because states where wealth share of broker-dealers in the financial sector is low are associated with bad times when asset prices are low and expected future returns are high. As test assets, I use value- and equally-weighted CRSP portfolios, mean excess return of 25 size/book-to-market and 10 momentum portfolios from Ken French’s data library, as well as, an equally-weighted portfolio of assets within each non-equity negatively correlated with coefficient of \(-0.5 \) (t-stat of \(-2.92 \). See Figure 4 in the Internet Appendix for time series of empirical proxies for state variable \( y \) using book and market information.

\(^{43}\)Adrian et al. (2014b) study return predictability in representative intermediary models and show book leverage of broker-dealers negatively forecasts future equity and bond returns. He et al. (2017) also run time-series predictive regressions and show the squared reciprocal of capital ratio for bank holding companies of NY Fed’s primary dealers positively predicts future returns for many asset classes.
class studied in HKM available form Asaf Manela’s website.\textsuperscript{44} The sample is quarterly starts in 1970Q1 and ends in 2017Q4 for equity portfolios and in 2012Q4 for non-equity assets (limited by data availability).

Table 3 provides results of the predictive regression in (31) for different test assets mentioned above. Consistent with model predictions, it reports negative and significant $\hat{\gamma}_y$ for six out of ten portfolios: value- and equally-weighted CRSP, size/book-to-market, momentum, sovereign bonds, and options portfolios. For all but one asset class (FX), $\hat{\gamma}_y$ is negative, as expected: shocks to wealth share of the financial sector negatively forecast future returns.

The predictor variable $y^{data}$ is highly persistent with AR(1) coefficient around 0.96 in quarterly data. I verify (in unreported regressions) that the absolute value of the regression coefficient $\hat{\gamma}_y$ and the $R^2$ both rise with the forecast horizon (see Cochrane (2005)’s Chapter 20 for more details). In Appendix D, I provide robustness checks for the time-series predictability regressions above. In particular, I rerun the forecasting regression in Table 3 by adding AEM’s broker-dealer leverage ratio, HKM’s intermediary capital ratio, and $cay$ variable from Lettau and Ludvigson (2001) as additional predictors. Adding these additional predictors, however, does not change the sign and significance of the coefficient $\gamma_y$.

7.3 Intermediary Heterogeneity and the Cross-Section of Asset Returns

As mentioned above, the model with heterogeneous financial intermediaries can reconcile the seemingly contradictory evidence for the sign of the estimated price of risk for intermediary leverage shocks documented in AEM and HKM. In this section, I explore the implications of a heterogeneous financial sector for the cross-section of returns.

As shown in Figure 3 (and also in the bottom right panel of Figure G.2 in Appendix F), risk premium on the endowment claim is decreasing in both state variables $x$ and $y$.\textsuperscript{45} This suggests assets that pay poorly when: (i) the financial sector is less capitalized (i.e. when $x$ is low), and/or (ii) wealth share of broker-dealers in the financial sector is small (i.e. when $y$ is low), are riskier and should command higher expected returns. I emphasize that point (ii) can only be made in a model with heterogeneous financial intermediaries.

7.3.1 Cross-sectional Asset Pricing Tests

Similar to He et al. (2017), I construct the growth rate to dealers’ wealth share in the financial sector, denoted $y_t^\Delta$, as follows. I estimate a shock to dealer wealth share in levels, $q_t$, as an AR(1) innovation in the regression: $y_t = \phi_0 + \phi y_{t-1} + \eta_t$.\textsuperscript{46} I then convert these innovations to a growth

\textsuperscript{44}Non-equity assets in HKM are mostly from previous studies. See Appendix C and He et al. (2017) for more details on test assets.

\textsuperscript{45}This is true even in the absence of margin constraints as shown in the bottom right panel of Figure G.1.

\textsuperscript{46}As noted above, $y$ is highly persistent with $\hat{\phi} \approx 0.96$ in quarterly data.
rate by dividing them by the lagged wealth share:

\[ \text{HIFac} = \frac{y_t^\Delta}{y_{t-1}} \]  

(32)

I call this wealth share growth rate the heterogeneous-intermediary factor (HIFac) and use it to perform cross-sectional asset pricing tests. For each asset \( i \), I first estimate betas from time-series regressions of portfolio excess returns on the risk factors:

\[ R_{i,t}^e = a_i + \beta_{i,f}' f_t + \vartheta_{i,t}, \quad i = 1, \ldots, N, \]  

(33)

where \( f \) represents the \( K \times 1 \) vector of risk factors. I consider four cases: (1) \( f_t = \text{HIFac}_t \), (2) \( f_t = [\text{HIFac}_t \quad \text{MktRF}_t]' \), (3) \( f_t = [\text{HIFac}_t \quad \text{AEM}_t]' \), and (4) \( f_t = [\text{HIFac}_t \quad \text{HKM}_t \quad \text{MktRF}_t]' \), where AEM is the broker-dealer leverage factor from Adrian et al. (2014a), HKM is the intermediary capital risk factor from He et al. (2017), and MktRF represents the market risk premium. For comparison, I also report the pricing performance of AEM and HKM factors.

Next, in order to estimate factor risk prices, \( \lambda_f \), I run a cross-sectional regression of average excess returns on the estimated risk exposures \( \hat{\beta}_{i,f} \):

\[ \mathbb{E} \left[ R_{i,t}^e \right] = \alpha_i + \hat{\beta}_{i,f}' \lambda_f + \zeta_i, \quad i = 1, \ldots, N, \]  

(34)

As mentioned above, the model predicts a positive and significant sign for the estimated price of risk \( \lambda_{\text{HIFac}} \).

I test the ability of the heterogeneous intermediary factor in pricing the cross-section of 55 equity and bond portfolios: The test assets are 25 size and book-to-market and 10 momentum portfolios from Ken French’s website, 10 maturity-sorted US government bond portfolios from CRSP’s Fama Bond dataset with maturities up to five years in six month intervals, and 10 US corporate bond portfolios sorted on yield spreads from Nozawa (2017) obtained from Asaf Manela’s website. I choose equity and bond portfolios as test assets due to the availability of longer time-series than others such as options and CDS.

Table 4 presents the main asset pricing results. Below estimated risk prices I report Shanken (1992) \( t \)-statistics that corrects for estimation error in betas and cross-correlations. I also report Fama and MacBeth (1973) \( t \)-statistics by running period-by-period cross-sectional regressions and computing standard errors of the time-series average of \( \lambda_s \). I report cross-sectional \( R^2 \) and the mean absolute pricing error (MAPE) as measures of model fit.\(^{47}\) I also report a \( \chi^2(N - K) \) statistic that tests if the pricing errors are jointly zero.

Column 1 of Table 4 reports the results of heterogeneous intermediary factor as a single pricing factor. The estimated price of risk is positive, which means assets that pay well in states with a low broker-dealer wealth share in the financial sector (i.e. assets with low betas on \( y_t \)) are valuable.

\(^{47}\)MAPE is calculated as \( \frac{1}{N} \sum |\zeta| \) where \( N \) is the number of test assets. For example, \( N = 10 \) for momentum portfolios and \( N = 55 \) for all assets.
hedges and have lower expected returns in equilibrium. This risk price estimate confirms the procyclicality of broker-dealer wealth share $y_t$ documented in Figure 9. The adjusted $R^2$ is 61% while the total MAPE is only 1.86%. The single-factor model can explains 62% of the variation in average returns in these cross-sections, with an average absolute pricing error around 1.8% per annum.

Figure 10 visually shows the HIFac’s pricing performance: The top panel plots the annualized realized against the predicted excess returns for the 55 equity and bond portfolios when HIFac is the only pricing factor (Column (1) in Table 4). Most of the portfolios line up closely to the 45-degree line. The bottom panel is similar to the top panel when HIFac and AEM are used as pricing factors, corresponding to Column (4) in Table 4. The model slightly outperforms the one in panel (a) as shown in above.

For robustness and comparison with recent empirical work, in Columns 2–6, I add additional pricing factors. In Column 2, I include market risk premium, MktRF, as an additional factor. However, the price of risk for MktRF is not statistically significant and in terms of almost all test statistics, the two-factor model is nearly identical to the single-factor model in Column 1. The market adds essentially no explanatory power to the intermediary heterogeneity factor.

In Columns 3 and 5, for reference, I present performances of the pricing factors in AEM and HKM, respectively. AEM use a leverage factor defined as the seasonally adjusted growth rate in broker–dealer book leverage level from Flow of Funds. A shown in Column 3, for the test assets mentioned above, HIFac outperforms AEM with 38% lower MAPE (1.83% versus AEM’s 2.96%) and 56% higher cross-sectional $R^2$ (61% compared to 39% in AEM).48

In HKM, the pricing factors are the market risk premium (MktRF) and shocks to intermediary capital ratio defined as the ratio of total market equity to total market assets (book debt plus market equity) for New York Fed’s primary dealer holding companies. A shown in Column 5, my model with a single pricing factor performs almost as well as HKM’s two factor model with nearly identical MAPE (1.83% vs. 1.89% for HKM) and cross-sectional $R^2$ (61% vs. HKM’s 63%).

In Column 4, I add leverage factor from AEM to evaluate a model with two pricing factors: HIFac and AEM. Addition of AEM’s leverage factor does not make price of HIFac risk insignificant or change its sign. This even raises the cross-sectional $R^2$ to 72%. Finally, in Column 6, I add two pricing factors from HKM: MktRF and shocks to intermediary capital ratio. Again, $\lambda_{\text{HIFac}}$ remains positive and significant. Note that since HIFac is positively correlated with both HKM and AEM factors, it is not surprising that $\lambda_{\text{HIFac}}$ has weaker statistical significance in the presence of these additional factors.49

---

48The pricing performance of AEM reported in Table III and shown in Figure 1 of their paper, is substantially better than the ones reported in Column 3 of Table 4 (their reported MAPE is only 1.31% and $R^2 = 0.77$). This difference can stem from two possible sources: (i) In 2015, the Federal Reserve substantially revised and updated Flow of Funds historical data for security broker-dealers, changing the way assets and liabilities were counted (specifically they changed their handling of using gross vs net repo). For more detail, see Z.1 Technical Q&As. (ii) The test assets used in this paper are different from AEM’s. I have the same 35 equity portfolios (25 size/book-to-market and 10 momentum portfolios) but use both Treasury and corporate bonds from Nozawa (2017), while AEM only use 6 Treasury bonds sorted by maturity from CRSP.

49HIFac has positive correlation of 13% and 9% with AEM’s leverage and HKM’s capital risk factors, respectively.
In summary, the results in Table 4 demonstrate that heterogeneity in the financial sectors has explanatory power for the cross-section of expected returns even in the presence of representative intermediary asset pricing factors presented in AEM and HKM.

### 7.3.2 Sorted Portfolios on Exposures to Heterogeneous Intermediary Factor

The positive price of risk associated with shocks to wealth share of dealers in the financial sector means assets that pay more in states with a low dealer wealth share (i.e. assets with low betas on $y_t$ shocks) are viewed as hedges thus have lower expected returns in equilibrium.

#### One-Way Sorted CRSP Portfolios

To empirically verify the positive price of risk for innovations in the wealth share of dealers in the financial sector, I sort stocks based on their exposures to these shocks and form portfolios by quintiles on a 10-year trailing window. I consider all common stocks (share codes 10 and 11) in the CRSP universe from Amex, NASDAQ, and NYSE (exchange codes 1, 2, and 3). For every stock $i$ at quarter $t$, I regress its quarterly excess return on constant and innovations in the heterogeneous intermediary factor (HIFac), defined in equation (32):

$$ R_{i,t}^e = \alpha_i + \beta_{i,HIFac} \text{HIFac}_t + \xi_{i,t} \quad (35) $$

The coefficient $\beta_{i,HIFac}$ measures the exposure of firm $i$’s stock to the factor’s innovations. I then sort stocks into quintiles every quarter according to their $\beta_{i,HIFac}$.

The average returns of the beta-sorted portfolios are reported in Table 5, along with return volatilities, average book-to-market ratio, average market cap, and alphas from CAPM and Fama-French three-factor model. Consistent with model’s implications, when sorted on $\beta_{HIFac}$, average risk premia are increasing from the portfolio of low-beta stocks to the high-beta quintile. Excess returns are monotonically increasing from quintile one to five and the top portfolio earns an approximately 5% premium over the lowest quintile.

#### Two-Way Sorted CRSP Portfolios

In this section I verify the results above are robust to double-sorting with asset pricing factors from recent models with representative intermediaries. In this exercise, I independently double-sort CRSP stocks into three-by-three portfolios on their exposures to the heterogeneous intermediary factor (HIFac) and either AEM or HKM representative intermediary asset pricing factors. Table 6 reports returns for double-sorted portfolios on exposures to HIFac and AEM and HKM betas. The return spread on HIFac-beta-sorted portfolios is 4.34% and 3.14% per year among stocks with low exposures to the AEM leverage and HKM capital factors, respectively.

This exercise demonstrates that the heterogeneity in the financial sector is an important risk factor and has pricing information above and beyond representative intermediary asset pricing.
factors in AEM and HKM: even within portfolios sorted based on AEM or HKM factor betas, I see a monotonic progression in returns from low- to high-HIFac beta portfolios.

### 7.3.3 The Heterogeneous Intermediary Factor-Mimicking Portfolio

As emphasized above, the main argument of the paper is that the heterogeneity in the intermediary sector has important implication for asset prices. To conduct additional robustness tests, in this section, I project the heterogeneous intermediary factor (HIFac) onto the space of traded returns to form a factor-mimicking portfolio that mimics the HIFac. To further verify that this heterogeneity an important source of risk, I evaluate the heterogeneous intermediary factor-mimicking portfolio (HIMP) relative to the mimicking portfolios for representative intermediary factors in AEM and HKM. I show that the mimicking portfolios for these representative intermediary factors cannot fully span the HIMP and there is more to be captured by the heterogeneity within the financial sector.

This approach also allows me to run tests using higher frequency data and longer time series. Moreover, since the mimicking portfolio is a traded excess return, I can evaluate the model by testing alphas in the time-series regression without the need to estimate the cross-section risk prices.

### Construction of HIMP

To construct mimicking portfolio of the heterogeneous intermediary factor (HIFac), I follow AEM and project this factor, onto the space of excess returns by running the following regression:

$$
\text{HIFac}_t = a_{\text{HI}} + b'_{\text{HI}}[\text{BL, BM, BH, SL, SM, SH, Mom, Bond}]_t + \varrho_t,
$$

(36)

where HIFac is the heterogeneous intermediary factor defined in equation (32), and BL, BM, BH, SL, SM, SH are, respectively, the excess returns of the six Fama-French portfolios on size (Small and Big) and book-to-market (Low, Medium, and High), and Mom is the momentum factor, obtained from Ken French’s data library. Bond is the first principal component (PC) of excess returns on six Treasury bond portfolios sorted by maturity from CRSP. The heterogeneous intermediary mimicking portfolio (HIMP) is then given by

$$
\text{HIMP}_t = \tilde{b}'_{\text{HI}}[\text{BL, BM, BH, SL, SM, SH, Mom, Bond}]_t,
$$

(37)

where \( \tilde{b}_{\text{HI}} = \frac{b'_{\text{HI}}}{\sum b'_{\text{HI}}} = [-0.34, 0.20, -1.04, -0.09, 0.41, 1.64, 1.04, -0.83] \) positively loading on the momentum factor.

---

50 See the Internet Appendix for tests at monthly frequencies with factor-mimicking portfolios.
HIMP vs. Mimicking Portfolios for AEM and HKM Factors

To further verify that my heterogeneous intermediary factor captures sources of risk beyond the factors from representative intermediary asset pricing models, in this section I evaluate the performance of HIMP with mimicking portfolios for AEM and HKM factors. I similarly construct mimicking portfolios for AEM’s broker-dealer leverage and HKM’s holding company capital factors using quarterly data for the two factors from Tyler Muir’s and Asaf Manela’s websites, respectively. The mimicking portfolio for the heterogeneous intermediary factor has Sharpe ratio of 0.45 over the sample period (1970Q1 to 2017Q3), much higher than Sharpe ratios for AEM and HKM factor-mimicking portfolios (0.21 and 0.27, respectively).

To evaluate the importance of heterogeneity in the financial sector above and beyond representative intermediary factors, I regress HIFac on mimicking portfolios for AEM and HKM factors in the following regression:

\[ HIMP_t = \alpha_{MP} + \beta_{FMP} FMP_t + \epsilon_t, \]  

(38)

where FMP is either the mimicking portfolio for broker-dealer leverage factor from AEM (AEM\_MP), or the mimicking portfolio for capital factor for primary dealers’ holding companies from HKM (HKM\_MP), or both AEM\_MP and HKM\_MP. Notice the mimicking portfolios are traded excess returns, thus I can evaluate the model by testing alphas in the time-series regression without the need to estimate the cross-section risk prices. If HIMP is fully “explained” by AEM\_MP, HKM\_MP, or both, I expect to see small and insignificant \( \alpha_{MP} \) in the regression above. I find the opposite to be true, however.

Table 7 presents the results. In Columns 1 and 2 I run univariate regression where the dependent variables are AEM\_MP and HKM\_MP, respectively. In both cases the intercept, \( \alpha_{MP} \) is statistically significant at 1% level and the \( R^2 \) of the regressions are relatively low at 0.14 and 0.32, respectively. In Column 3, I added value-weighted return from CRSP (MktRF) to HKM\_MP as independent variables which leads to very similar results to Column 3. In Column 4, I add both AEM and HKM factor-mimicking portfolios as right-hand-side variables in equation (38). We observe a large and significant \( \alpha_{MP} \) and relatively small \( R^2 \). Adding MktRF in Column 5 to the regression in Column 4, further strengthen the results.

Alternatively, I build factor-mimicking portfolios by projecting them instead onto the Fama-French three factors, the momentum factor, and the first PC of bond portfolios, and repeat the regressions in Table 7. I arrive at very similar results: time-series alphas are large and significantly different from zero with low \( R^2 \) in all regressions. See Table G.4 in Appendix D.

This exercise confirms my earlier results: the heterogeneity in the financial sector is an important risk factor and has pricing information above and beyond representative intermediary asset pricing factors in AEM and HKM.

\(^{51}\)The loadings for AEM and HKM factor-mimicking portfolios are \( \tilde{b}_{AEM} = [-0.98, 0.50, -0.03, -0.26, 0.96, 0.05, 0.16, 0.59] \) and \( \tilde{b}_{HKM} = [0.30, 0.03, 0.58, -0.06, -0.16, 0.25, 0.09, -0.03] \).
7.4 Vector Autoregression Method

As mentioned earlier, the entire variation in risk premia in my model is due to intermediation frictions captured in state variables $x$ and $y$. In the absence of these frictions, with an iid endowment process, the risk premia will be constant. In order to more directly examine model’s prediction in the data, in section, I impose a structural vector autoregression (SVAR), while respecting the relationship between the aggregate shock and endogenous state variables dynamics in the model. This allows me to extract the aggregate shock while maintaining the nonlinear interactions of shocks and state variable captured in $\sigma_{x,t}$ and $\sigma_{y,t}$ (See equations 15 and 16). I then use the identified structural shock to study return predictability.

Consider the following structural vector autoregression model of order one with three variables:

$$AY_{t+1} = F + GY_t + e_{t+1}, \quad t = 1, \ldots, T,$$

(39)

where $Y_t$ is a $3 \times 1$ vector of observed endogenous variables; $F$ is a $3 \times 1$ vector of constants; $G$ is a $3 \times 3$ matrix of coefficients; and $e_{t+1}$ are heteroskedastic unobservable shocks assumed to follow a multivariate normal distribution with $e_t \sim MN(0, \Sigma_t = H_tH_t')$ with matrix $H_t$ defined below.\footnote{For simplicity, I assume elements of vector $F$ and matrices $A$ and $G$ are constants to be estimated. One can assume a more flexible model with time-varying intercept vector and coefficient matrices. For an example of such a structural VAR with time-varying coefficients, see Primiceri (2005).}

I set $Y_t = (\log \Delta C_t, \log \Delta x_t, \log \Delta y_t)'$, where $\log \Delta C_t$ is log aggregate consumption growth, and $\log \Delta x_t$ and $\log \Delta y_t$ are changes in log of the two endogenous state variables measured in the data according to equations (29) and (30). Since my general equilibrium model presented in Section 3 has a single shock to the aggregate endowment (i.e. $dZ_t$ represented by the first element of the vector $e_{t+1}$), only the first column of $H_t$ would have non-zero elements. From equation (1), the aggregate dividend shock has volatility $\sigma_D$. Also from state variables dynamics in Proposition 1, shocks to $dx$ and $dy$ are related to the fundamental $dZ$ shock through state-dependent volatilities $\sigma_{x,t} = \sigma_x(x_t, y_t)$ and $\sigma_{y,t} = \sigma_y(x_t, y_t)$. In addition, I assume state variables $x$ and $y$ are measured with iid errors independent of other shocks. Thus, matrix $H_t$ has the following form:

$$H_t = \begin{bmatrix} \sigma_D & 0 & 0 \\ \sigma_{x,t} & \eta_x & 0 \\ (1-y_t)\sigma_{y,t} & 0 & \eta_y \end{bmatrix},$$

(40)

where $\sigma_x$ and $\sigma_y$ are nonlinear functions of state variables $x$ and $y$ (defined in equations (15) and (16) resulting in matrix $H$ to be time-varying), and $\eta_x$ and $\eta_y$ denote the standard deviations of iid measurement errors for $x$ and $y$, respectively. I approximate state variable diffusions with the following second order functions

$$\sigma_x(x_t, y_t) = \frac{x_t(1-x_t)}{ay_t + b}, \quad \text{and} \quad \sigma_y(x_t, y_t) = \frac{y_t(1-y_t)}{dx_t + e},$$

(41)

and estimate coefficients $a, b, d, \text{ and } e$. Consistent with Figure G.3, the above functional forms...
ensure that $\sigma_x(\sigma_y) = 0$ when $x(y)$ is at the boundaries of the state space (i.e. 0 and 1). I augment the VAR with a number of identifying restrictions for matrix $A$: I normalize diagonal elements to one and also assume $A_{12} = A_{13} = 0$ so that log consumption growth is not affected by levels of state variables $x$ and $y$ as it is the case in the model.

Pre-multiplying equation (39) by $A^{-1}$, we get the following VAR that can be estimated:

$$Y_{t+1} = c + BY_t + u_{t+1}, \quad t = 1, \ldots, T,$$

(42)

where $c = A^{-1}F$, $B = A^{-1}G$, and $u_t \sim \mathcal{MN}(0, \Omega_t)$ where $A\Omega_tA' = H_tH_t' = \Sigma_t$. I estimate the VAR with maximum likelihood and extract residuals for the endogenous variables. Maximum likelihood parameter estimates for the VAR are provided in Table G.5 in Appendix E.

Variance decomposition verifies that measurement errors in $x$ and $y$ (i.e. $\eta_x$ and $\eta_y$) are negligible relative the aggregate shock: Almost the entire forecast error variance (> 99.6%) is explained by the aggregate shock over forecasting horizon of 12 quarters. This is robust to a reordering of the VAR so that the change in log aggregate consumption appears last.

Figure 11 presents the impulse response due to a one standard deviation shock to the aggregate consumption. Consistent with model’s prediction and empirical proxies in Figure 9, both state variables respond positively to an aggregate shock and are hence, procyclical.

Table 8 repeats the predictive regressions in equation (31) with cumulative extracted shocks to $y$ (i.e. cumulative $\sigma_{y,t}e_{t}^1$ denoted $y^{VAR}$, where $e_{t}^1$ is the aggregate shock extracted from the VAR) as the predictor. Similar to results in Tables 3, we observe negative and significant coefficient for $y^{VAR}$ for most of the test assets. As expected, cumulative shocks to $x$ show negative and significant predictive power as well.53

The results in this section directly test model’s mechanism and further verify the predictive power of wealth share of broker-dealer in the financial sector for future returns. In a model with representative intermediaries, wealth distribution in the financial sector has no impact on future asset prices and the entire variation in risk premia is due to the aggregate wealth share of the intermediary sector. In my model with heterogeneous intermediaries, wealth distribution in the financial sector contributes to risk premia variation and time-series predictability.

8 Conclusion

This paper studies the asset pricing implications of heterogeneity among financial intermediaries. Evidence on asset reallocations within the intermediary sector during the Great Recession is at odds with existing models featuring representative intermediaries. In this paper, I propose a dynamic general equilibrium model with two heterogeneous intermediaries facing financial constraints. The model generates opposite cyclical dynamics for leverage of the two intermediary sectors, reconciling empirical evidence that has previously seemed contradictory through the lens of representative

53These predictability results are consistent with findings in Campbell and Cochrane (1999) and Møller and Rangvid (2015).
intermediary asset pricing models.

My model implies that a dealer deleveraging episode comparable to the one observed during the recent financial crisis, leads to an approximately 18% increase in the risk premia and a 6% increase in endogenous volatility. In contrast, since balance sheet adjustments among different intermediaries does not affect the wealth share of the aggregate financial sector, in models with representative intermediaries these asset reallocations have no impact on asset prices and the real economy.

I show that the wealth distribution among intermediaries accounts for a substantial portion of the variation in risk premia in the model. With an independent and identically distributed aggregate endowment, the variation in expected returns is entirely due to intermediation frictions captured by model’s two state variables. I show that the wealth distribution among intermediaries, model’s second state variable, accounts for approximately 20% incremental variation in risk premia over a representative intermediary model.

Finally, I examine the empirical implications of the model for time-series and cross-sections of returns. I provide strong evidence that wealth share of broker-dealers in the financial sector, an empirical proxy for model’s second state variable and a measure of heterogeneity among intermediaries, has strong predictive power for future returns of many assets and is also priced in the cross-section of equity and bond portfolios.
References


Adrian, Tobias, and Nina Boyarchenko, 2015, Intermediary leverage cycles and financial stability, Working Paper, Federal Reserve Bank of New York Staff Reports.


Adrian, Tobias, Emanuel Moench, and Hyun Song Shin, 2014b, Dynamic leverage asset pricing, Federal Reserve Bank of New York Staff Report No. 625.


Figure 1. Leverage of different financial intermediaries. This figure presents time-series of leverage for different financial intermediaries: security broker-dealers (BDs) and bank holding companies (BHCs). Leverage for broker-dealers (solid blue line) is defined as the ratio total financial assets to total equity (total financial assets minus total liabilities) from Table L.130 of the Financial Accounts of the United States (Flow of Funds). BHC leverage (dashed red line) is defined as the ratio of total market assets (book debt plus market equity) to total market equity constructed for publicly-traded holding companies of the NY Fed’s primary dealer counterparties using CRSP/Compustat and Datastream data, where market equity is outstanding shares times stock price and book debt is total assets minus common equity (AT − CEQ in Compustat). Data is quarterly from 1970Q1 to 2017Q4. The vertical shaded bars indicate NBER recessions.
Figure 2. Real financial asset of different financial intermediaries. This figure presents time-series of real financial assets for different financial intermediaries: security broker-dealers (BDs) and U.S. chartered depository institutions (DIs). BDs’ (solid blue line) and DIs’ (dashed red line) total financial assets are from Tables L.130 and L.111 of the Financial Accounts of the United States (Flow of Funds). Both series are normalized to $1 trillion in 2000Q4. Data is quarterly from 1970Q1 to 2017Q4. The vertical shaded bars indicate NBER recessions.
Figure 3. Risk premia, the price of risk, valuation, and volatility. This figure presents price-dividend ratio $1/F$, return volatility $\sigma$, Sharpe ratio and risk premium on the endowment claim in constrained and unconstrained equilibria as functions of state variable $x$ (wealth share of the financial sector i.e. type $A$ and $B$ agents) under the benchmark parameters in Table 1. Each quantity is plotted against state variable $x_t$ while the value of the second state variable $y_t$ (wealth share of type $A$ investors, i.e. broker-dealers, in the financial sector) is fixed at 0.56 (its value at the stochastic steady state). The solid blue line corresponds to the unconstrained economy, the dash-dotted purple line corresponds to the economy with a constant portfolio constraint ($\bar{\theta}_t = \bar{m}$), and the dashed red line corresponds to the economy with a Value-at-Risk (VaR)-type margin constraint ($\bar{\theta}_t = \frac{1}{\alpha\sigma_t}$). Three-dimensional plots are provided in Appendix F.
Figure 4. Optimal portfolios and the risk-free rate. This figure presents portfolio weights of each type of agent $w_s^A$, $w_s^B$, and $w_s^C$ as well as the real interest rate $r_t$ in constrained and unconstrained equilibria as functions of state variable $x$ (wealth share of the financial sector i.e. $A$ and $B$ agents) under the benchmark parameters in Table 1. Each quantity is plotted against state variable $x_t$ while the value of the second state variable $y_t$ (wealth share of type $A$ investors in the financial sector) is fixed at 0.56 (its value at the stochastic steady state). The solid blue line corresponds to the unconstrained economy, the dash-dotted purple line corresponds to the economy with a constant portfolio constraint ($\bar{\theta}_t = \bar{m}$), and the dashed red line corresponds to the the economy with a Value-at-Risk (VaR)-type margin constraint ($\bar{\theta}_t = \frac{1}{\alpha \sigma_t}$). Three-dimensional plots are provided in Appendix F.
Figure 5. Dynamics of the endogenous state variables. This figure presents dynamics of the state variables $x$ and $y$ (wealth share of the financial sector i.e. $A$ and $B$ agents, and wealth share of type $A$ investors in the financial sector, respectively) in constrained and unconstrained equilibria under the benchmark parameters in Table 1. Drift and volatility of state variable $x$ (i.e. $\mu_x, \sigma_x$) are plotted as functions of $x$ while the value of the state variable $y$ is fixed at 0.56. Drift and volatility of state variable $y$ (i.e. $\mu_y, \sigma_y$) are plotted as functions of $y$ while the value of the state variable $x$ is fixed at 0.25. The solid blue line corresponds to the unconstrained economy, the dash-dotted purple line corresponds to the economy with a constant portfolio constraint ($\bar{\theta}_t = \bar{m}$), and the dashed red line corresponds to the the economy with a Value-at-Risk (VaR)-type margin constraint ($\bar{\theta}_t = \frac{1}{\alpha \sigma_t}$). Three-dimensional plots are provided in Appendix F.
Leverage in the Constrained Equilibrium

Figure 6. Cyclical properties of intermediary leverage. This figure presents optimal intermediary leverage in the unconstrained and constrained equilibria under parameters listed in Table 1. The left panel plots leverage of the financial sector (equation 27) in the unconstrained equilibrium (dashed red line) and the model with time-varying margin constraints, $\bar{\theta}_t = \frac{1}{\sigma_t}$ (solid blue line). The right panel plots intermediary leverage in the main model with endogenous margin constraints. The solid blue line corresponds to leverage of the financial sector ($w^{FS}_t$), the dashed red line presents broker-dealers’ leverage ($w^A_t$), and the dash-dotted purple line corresponds to leverage of bank holding companies ($w^B_t$). Each quantity is plotted against state variable $x_t$ (wealth share of the financial sector i.e. $A$ and $B$ agents) while the value of the second state variable $y_t$ (wealth share of type $A$ investors in the financial sector) is held fixed at 0.56 (its stochastic steady state value).

Figure 7. Heterogeneous and Representative Intermediaries. This figure presents distribution of risk premia volatility in models with representative (red) and heterogeneous (blue) intermediaries. I simulate each model 20,000 times for 3,000 quarters.
Figure 8. Asset reallocation within the financial sector. This figure presents portfolio weights for dealers, holding companies, and households (A, B, and C types, respectively), as well as, the risk premium and Sharpe ratio of the risky claim on the aggregate endowment, and the volatility of the risky asset return in the baseline model (solid blue line) and a model with tighter margin constraints and less risk averse financial sector (dashed red line). The changes in tightness of the margin constraint (parameter $\alpha$) and relative risk aversion of the financial and household sectors ($\gamma_I/\gamma_C$) are such that leverage of A (B) types is reduced (increased) by approximately 47% (72%): changes documented during the Great Recession in Figure 1. Each quantity is plotted against state variable $x$ (wealth share of the financial sector i.e. A and B agents) while value of the state variable $y$ (wealth share of dealers i.e. type A investors in the financial sector) is fixed at 0.56, its value at the stochastic steady state. Parameters for the baseline model are presented in Table 1.
Figure 9. State variables $x$ and $y$ in the data. This figure presents the three-month moving average of monthly wealth share of the financial sector, $x^{data}$, and quarterly book equity share of the broker-dealers in the financial sector, $y^{data}$, defined in equations (29) and (30), respectively. Financial sector is identified as firms in the CRSP universe for whom the first two digits of the header SIC code (HSICCD in CRSP) equals 60–67. Book equity for BDs and BHCs are computed from the Flow of Funds Tables L.130 and L.131. Sample period is from 1970 to 2017. The vertical shaded bars indicate NBER recessions.
Figure 10. Realized versus predicted mean returns: intermediary heterogeneity factor. This figure presents the realized mean excess returns of 35 equity portfolios (25 size and book-to-market-sorted portfolios and 10 momentum-sorted portfolios) and 10 Treasury bond portfolios (sorted by maturity), and 10 US corporate bond portfolios (sorted by yield spread) against the mean excess returns predicted by the single heterogeneous intermediary risk factor when only the heterogeneous intermediary factor (HIFac) (panel a) and HiFac and AEM factors (panel b) are used as pricing factor, respectively. The sample is quarterly from 1970Q1 to 2017Q4. Returns are reported in percent per year (quarterly percentages multiplied by four).
Figure 11. Impulse response to an aggregate dividend shock. This figure presents the impulse response function to a one standard deviation shock to the aggregate consumption for the VAR in equation 39.
Table 1. Parameter values for the endowment economy model.
This table reports parameter values used in calibrating the model. The parameters are calibrated at a quarterly frequency.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preference Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion of type A</td>
<td>$\gamma_A$</td>
<td>2.5</td>
</tr>
<tr>
<td>Risk aversion of type B</td>
<td>$\gamma_B$</td>
<td>5.5</td>
</tr>
<tr>
<td>Risk aversion of type C</td>
<td>$\gamma_C$</td>
<td>15</td>
</tr>
<tr>
<td>EIS of type A</td>
<td>$\psi_A$</td>
<td>1.5</td>
</tr>
<tr>
<td>EIS of type B</td>
<td>$\psi_B$</td>
<td>1.5</td>
</tr>
<tr>
<td>EIS of type C</td>
<td>$\psi_C$</td>
<td>1.5</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>$\rho$</td>
<td>0.0025</td>
</tr>
<tr>
<td><strong>Endowment and Demography</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endowment growth rate</td>
<td>$\mu_D$</td>
<td>0.0055</td>
</tr>
<tr>
<td>Endowment volatility</td>
<td>$\sigma_D$</td>
<td>0.0175</td>
</tr>
<tr>
<td>Agents birth/death rate</td>
<td>$\kappa$</td>
<td>0.0038</td>
</tr>
<tr>
<td>Population share of type A</td>
<td>$\bar{u}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Population share of type B</td>
<td>$\bar{v}$</td>
<td>0.07</td>
</tr>
<tr>
<td><strong>Margin Constraint</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant leverage constraint</td>
<td>$\bar{m}$</td>
<td>4</td>
</tr>
<tr>
<td>Tightness of the dynamic margin constraint</td>
<td>$\alpha$</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2. State variables statistics.
This table reports statistics for empirical proxies for model’s two state variables in level and changes (Innov.). AC($j$) represent $j^{th}$ autocorrelation. Data is quarterly from 1970Q1 to 2017Q4.

<table>
<thead>
<tr>
<th></th>
<th>$x^{data}$</th>
<th></th>
<th>$y^{data}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Innov.</td>
<td>Level</td>
<td>Innov.</td>
</tr>
<tr>
<td>Mean</td>
<td>0.141</td>
<td>0.000</td>
<td>0.191</td>
<td>0.000</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.037</td>
<td>0.007</td>
<td>0.068</td>
<td>0.018</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.966</td>
<td>0.003</td>
<td>0.958</td>
<td>−0.185</td>
</tr>
<tr>
<td>AC(2)</td>
<td>0.933</td>
<td>0.046</td>
<td>0.930</td>
<td>0.087</td>
</tr>
<tr>
<td>AC(3)</td>
<td>0.899</td>
<td>0.105</td>
<td>0.896</td>
<td>−0.087</td>
</tr>
<tr>
<td>AC(4)</td>
<td>0.861</td>
<td>0.004</td>
<td>0.870</td>
<td>−0.075</td>
</tr>
<tr>
<td>AC(5)</td>
<td>0.825</td>
<td>−0.025</td>
<td>0.851</td>
<td>0.071</td>
</tr>
</tbody>
</table>
Table 3. Predictive regressions: $y_t^{data}$.

This table provides results for one-year ahead predictive regressions according to $R_{t+1→t+4} = \gamma_0 + \gamma_y y_t + \varepsilon_{t+1→t+4}$, using lagged equity share of broker-dealers in the financial sector, $y_t^{data}$ defined in (30), as the predictor. The dependent variables are excess holding period returns from quarter $t + 1$ to quarter $t + 4$ on the CRSP value-weighted (Mkt$^{vw}$) and equally-weighted (Mkt$^{ew}$) portfolios, mean excess return on 25 Fama-French size and book-to-market (FF25), 10 momentum (Mom) portfolios, 10 maturity-sorted US government and 10 US corporate bond portfolios sorted on yield spreads (US bonds), mean excess returns on six sovereign bonds (Sov. bonds), 54 portfolios of S&P 500 index options sorted on moneyness and maturity (Options), 20 CDS portfolios sorted by spreads (CDS), 23 commodity (Commod.) and 12 foreign exchange (FX) portfolios. Size/book-to-market and momentum portfolios and the risk-free rate data are from Ken French’s website. Data on sovereign bonds, options, CDS, commodities, and FX portfolios are from He et al. (2017). The sample quarterly from 1974Q1 to 2017Q3 for market, FF25 and momentum portfolios, and to 2012Q4 for HKM assets. Hodrick (1992) standard errors are reported in parentheses to adjust for the fact that overlapping quarterly observations are used to forecast annual returns.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Mkt$^{vw}$</th>
<th>Mkt$^{ew}$</th>
<th>FF25</th>
<th>Mom</th>
<th>US bonds</th>
<th>Sov. bonds</th>
<th>Options</th>
<th>CDS</th>
<th>Commod.</th>
<th>FX</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^{data}$</td>
<td>$-0.63^*$</td>
<td>$-1.16^{***}$</td>
<td>$-0.98^{***}$</td>
<td>$-0.77^{***}$</td>
<td>$-0.11$</td>
<td>$-1.09^{**}$</td>
<td>$-1.57^{**}$</td>
<td>$-0.04$</td>
<td>$-0.40$</td>
<td>$0.37$</td>
</tr>
<tr>
<td></td>
<td>$(0.33)$</td>
<td>$(0.38)$</td>
<td>$(0.28)$</td>
<td>$(0.33)$</td>
<td>$(0.14)$</td>
<td>$(0.46)$</td>
<td>$(0.61)$</td>
<td>$(0.13)$</td>
<td>$(0.45)$</td>
<td>$(0.31)$</td>
</tr>
<tr>
<td>Constant</td>
<td>$0.24^{***}$</td>
<td>$0.39^{***}$</td>
<td>$0.34^{***}$</td>
<td>$0.27^{***}$</td>
<td>$0.09^{**}$</td>
<td>$0.39^{***}$</td>
<td>$0.46^{***}$</td>
<td>$0.03$</td>
<td>$0.14$</td>
<td>$-0.08$</td>
</tr>
<tr>
<td></td>
<td>$(0.06)$</td>
<td>$(0.08)$</td>
<td>$(0.05)$</td>
<td>$(0.06)$</td>
<td>$(0.04)$</td>
<td>$(0.11)$</td>
<td>$(0.14)$</td>
<td>$(0.03)$</td>
<td>$(0.11)$</td>
<td>$(0.08)$</td>
</tr>
<tr>
<td>Observations</td>
<td>173</td>
<td>173</td>
<td>173</td>
<td>173</td>
<td>152</td>
<td>62</td>
<td>100</td>
<td>44</td>
<td>102</td>
<td>132</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.06</td>
<td>0.10</td>
<td>0.11</td>
<td>0.08</td>
<td>0.01</td>
<td>0.15</td>
<td>0.12</td>
<td>0.005</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.05</td>
<td>0.09</td>
<td>0.11</td>
<td>0.07</td>
<td>0.01</td>
<td>0.14</td>
<td>0.11</td>
<td>$-0.02$</td>
<td>0.003</td>
<td>0.05</td>
</tr>
</tbody>
</table>

*Note:* $^*p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
Table 4. Cross-sectional asset pricing tests.
This table presents pricing results for the 25 size and book-to-market, 10 momentum, 10 Treasury bond portfolios sorted by maturity from CRSP with maturities in six month intervals up to five years, and 10 US corporate bond portfolios sorted on yield spreads from Nozawa (2017). The table reports the prices of risk and test diagnostics, including mean absolute pricing errors (MAPEs), and adjusted $R^2$s, and a $\chi^2$ statistic and $p$-value that tests whether the pricing errors are jointly zero. Shanken (1992)-corrected and Fama and MacBeth (1973) $t$-statistics ($t$-Shanken and $t$-FM, respectively) are reported in parentheses. Heterogeneous intermediary factor (HIFac), $y_{\Delta}$, is defined as AR(1) innovations in the wealth share of dealers, scaled by their lagged wealth share according to equation (32). The AEM leverage factor (AEMLevFac) is defined as the seasonally-adjusted growth rate in broker-dealer leverage from Table L.130 of the Flow of Funds. HKM capital factor (HKMFac) is the shock to intermediary capital ratio in He et al. (2017), defined as the ratio of total market equity to total market assets (book debt plus market equity) for bank holding companies of New York Fed’s primary dealer counterparties. MktRF is the excess return on CRSP value-weighted portfolio from Ken French’s website. The sample is quarterly from 1970Q1 to 2017Q4. Returns and risk premia are reported in percentage per year (quarterly percentages multiplied by four).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIFac</td>
<td>38.27∗</td>
<td>57.35∗∗</td>
<td>34.87∗</td>
<td>52.47∗</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-Shanken</td>
<td>(1.83)</td>
<td>(2.11)</td>
<td>(1.78)</td>
<td>(1.74)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-FM</td>
<td>(2.24)</td>
<td>(3.11)</td>
<td>(2.02)</td>
<td>(2.82)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MktRF</td>
<td>3.86</td>
<td>6.93*</td>
<td>4.61</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-Shanken</td>
<td>(1.20)</td>
<td>(1.97)</td>
<td>(1.38)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-FM</td>
<td>(1.27)</td>
<td>(2.20)</td>
<td>(1.52)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AEMLevFac</td>
<td>32.50***</td>
<td>21.66**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-Shanken</td>
<td>(2.68)</td>
<td>(2.42)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-FM</td>
<td>(3.73)</td>
<td>(3.14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HKMFac</td>
<td>12.55**</td>
<td>11.80**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-Shanken</td>
<td>(2.57)</td>
<td>(2.12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-FM</td>
<td>(3.96)</td>
<td>(3.68)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>3.47***</td>
<td>4.14***</td>
<td>5.32**</td>
<td>3.01**</td>
<td>2.77*</td>
<td>3.99***</td>
</tr>
<tr>
<td>$t$-Shanken</td>
<td>(2.92)</td>
<td>(2.90)</td>
<td>(2.03)</td>
<td>(2.32)</td>
<td>(1.84)</td>
<td>(2.38)</td>
</tr>
<tr>
<td>$t$-FM</td>
<td>(3.62)</td>
<td>(4.32)</td>
<td>(2.68)</td>
<td>(3.11)</td>
<td>(2.90)</td>
<td>(4.19)</td>
</tr>
<tr>
<td>Observations</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.62</td>
<td>0.63</td>
<td>0.40</td>
<td>0.73</td>
<td>0.65</td>
<td>0.71</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.61</td>
<td>0.61</td>
<td>0.39</td>
<td>0.72</td>
<td>0.63</td>
<td>0.69</td>
</tr>
<tr>
<td>MAPE, %</td>
<td>1.83</td>
<td>1.84</td>
<td>2.96</td>
<td>1.58</td>
<td>1.89</td>
<td>1.67</td>
</tr>
<tr>
<td>$\chi^2(N - K)$</td>
<td>195.39</td>
<td>133.53</td>
<td>151.78</td>
<td>167.17</td>
<td>121.35</td>
<td>95.34</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01
Table 5. One-way sorted CRSP portfolios on exposures to the heterogeneous intermediary factor.

This table reports average excess returns, alphas, volatility, average book-to-market ratio, and average market capitalization for portfolios formed on their exposure to shocks to dealer wealth share in the financial sector. Shocks to dealer wealth share (HIFac) are defined as AR(1) innovations in the wealth share, scaled by the lagged wealth share as shown in equation (32). Data is quarterly from 1970Q1 to 2017Q3. Returns, volatilities, and alphas are annualized.

<table>
<thead>
<tr>
<th></th>
<th>L (1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>H (5)</th>
<th>HML (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Excess Return (%)</td>
<td>11.66</td>
<td>11.53</td>
<td>12.81</td>
<td>14.34</td>
<td>16.65</td>
<td>4.98</td>
</tr>
<tr>
<td>Volatility (%)</td>
<td>19.69</td>
<td>19.08</td>
<td>21.76</td>
<td>26.41</td>
<td>35.53</td>
<td>26.72</td>
</tr>
<tr>
<td>$\beta_{\text{HIFac}}$</td>
<td>-0.20</td>
<td>0.33</td>
<td>0.69</td>
<td>1.19</td>
<td>2.21</td>
<td>2.41</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>-0.89</td>
<td>1.50</td>
<td>2.77</td>
<td>4.07</td>
<td>5.89</td>
<td>10.67</td>
</tr>
<tr>
<td>$\alpha_{\text{CAPM}}$</td>
<td>4.77</td>
<td>4.33</td>
<td>4.56</td>
<td>4.75</td>
<td>4.44</td>
<td>-0.32</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>3.40</td>
<td>3.90</td>
<td>3.63</td>
<td>2.84</td>
<td>1.57</td>
<td>-0.10</td>
</tr>
<tr>
<td>$\alpha_{\text{FF3}}$</td>
<td>3.89</td>
<td>2.88</td>
<td>3.36</td>
<td>3.98</td>
<td>4.02</td>
<td>0.14</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>3.14</td>
<td>4.21</td>
<td>5.45</td>
<td>5.04</td>
<td>2.24</td>
<td>0.05</td>
</tr>
<tr>
<td>Average Market Cap ($\text{bn}$)</td>
<td>5.28</td>
<td>3.66</td>
<td>2.40</td>
<td>1.97</td>
<td>0.89</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6. Two-way sorted CRSP portfolios.

This table reports average excess returns for portfolios independently double-sorted on their exposure to shocks to dealer wealth share in the financial sector (HIFac) and beta to the AEM leverage factor (AEM LevFac), as well as, double-sorted portfolios on HIFac beta and HKM capital ratio factor (HKM CapFac) beta. Shocks to dealer wealth share (HIFac) are defined as AR(1) innovations in the wealth share, scaled by the lagged wealth share as shown in equation (32). AEM leverage and HKM capital factors are from Tyler Muir’s and Asaf Manela’s websites, respectively. Returns are annualized in percentage points. Data is quarterly from 1970Q1 to 2017Q3.

<table>
<thead>
<tr>
<th>AEM LevFac</th>
<th>HIFac</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3)–(1)</th>
<th>$t$-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>12.85</td>
<td>14.05</td>
<td>17.19</td>
<td>4.34</td>
<td>1.49</td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>11.47</td>
<td>12.75</td>
<td>14.63</td>
<td>3.16</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>11.42</td>
<td>12.19</td>
<td>14.67</td>
<td>3.24</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>(3)–(1)</td>
<td>-1.43</td>
<td>-1.86</td>
<td>-2.54</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-stat</td>
<td>-0.65</td>
<td>-0.85</td>
<td>-0.91</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HKM CapFac</th>
<th>HIFac</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3)–(1)</th>
<th>$t$-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>10.05</td>
<td>11.39</td>
<td>13.19</td>
<td>3.14</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>11.91</td>
<td>12.48</td>
<td>14.82</td>
<td>2.91</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>14.75</td>
<td>15.17</td>
<td>17.74</td>
<td>2.99</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>(3)–(1)</td>
<td>4.70</td>
<td>3.78</td>
<td>4.55</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-stat</td>
<td>1.48</td>
<td>1.30</td>
<td>1.48</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7. The heterogeneous intermediary mimicking portfolio (HIMP): Comparing models
This table presents time-series regression results of heterogeneous intermediary mimicking portfolio (HIMP) on mimicking portfolios for the representative intermediary factors in AEM and HKM according to: HIMP\(_t\) = \(\alpha_{MP} + \beta_{FMP}FMP_t + \epsilon_t\), where FMP is either the mimicking portfolio for broker-dealer leverage factor from AEM (AEM\(_{MP}\)), or the mimicking portfolio for capital factor for primary dealers’ holding companies from HKM (HKM\(_{MP}\)), or both AEM\(_{MP}\) and HKM\(_{MP}\). The factor-mimicking portfolios are constructed by projecting the heterogeneous intermediary, AEM’s leverage, and HKM’s capital factors unto the space of equity and bond returns according to equations (36) and (37). The sample is quarterly from 1970Q1 to 2017Q3. Standard errors are in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_{MP})</td>
<td>5.03***</td>
<td>4.03***</td>
<td>4.14***</td>
<td>3.74***</td>
<td>3.91***</td>
</tr>
<tr>
<td></td>
<td>(0.94)</td>
<td>(0.85)</td>
<td>(0.85)</td>
<td>(0.83)</td>
<td>(0.83)</td>
</tr>
<tr>
<td>AEM MP</td>
<td>0.72***</td>
<td>0.37***</td>
<td>0.49***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HKM MP</td>
<td>0.94***</td>
<td>0.68**</td>
<td>0.82***</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.27)</td>
<td>(0.10)</td>
<td>(0.30)</td>
<td></td>
</tr>
<tr>
<td>MktRF</td>
<td>0.27</td>
<td>0.65**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.27)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>191</td>
<td>191</td>
<td>191</td>
<td>191</td>
<td>191</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.14</td>
<td>0.32</td>
<td>0.33</td>
<td>0.35</td>
<td>0.37</td>
</tr>
<tr>
<td>Adjusted (R^2)</td>
<td>0.14</td>
<td>0.32</td>
<td>0.32</td>
<td>0.35</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01
Table 8. Predictive regressions: Shocks to \( y \) from VAR.

This table provides results for one-year ahead predictive regressions according to \( R_{t+1-t+4} = \gamma_0 + \gamma_y y_{VAR,t} + \varepsilon_{t+1-t+4} \), using lagged equity share of broker-dealers in the financial sector, \( y_{t-data} \) defined in (30), as the predictor. The dependent variables are excess holding period returns from quarter \( t+1 \) to quarter \( t+4 \) on the CRSP value-weighted (Mkt\(_{t+1}^{vw}\)) and equally-weighted (Mkt\(_{t+1}^{ew}\)) portfolios, mean excess return on 25 Fama-French size and book-to-market (FF25\(_{t+1}\)), 10 momentum (Mom\(_{t+1}\)) portfolios, 10 maturity-sorted US government and 10 US corporate bond portfolios sorted on yield spreads (US bonds\(_{t+1}\)), mean excess returns on six sovereign bonds (Sov. bonds\(_{t+1}\)), 54 portfolios of S&P 500 index options sorted on moneyness and maturity (Options\(_{t+1}\)), 20 CDS portfolios sorted by spreads (CDS\(_{t+1}\)), 23 commodity (Commod.\(_{t+1}\)) and 12 foreign exchange (FX\(_{t+1}\)) portfolios. Size/book-to-market and momentum portfolios and the risk-free rate data are from Ken French’s website. Data on sovereign bonds, options, CDS, commodities, and FX portfolios are from He et al. (2017). The sample quarterly from 1974Q1 to 2017Q3 for market, FF25 and momentum portfolios, and to 2012Q4 for HKM assets. Hodrick (1992) standard errors are reported in parentheses to adjust for the fact that overlapping quarterly observations are used to forecast annual returns.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>VW MKT (1)</th>
<th>FF25 (2)</th>
<th>Mom (3)</th>
<th>US bonds (4)</th>
<th>Sov. bonds (5)</th>
<th>Options (6)</th>
<th>CDS (7)</th>
<th>Commod. (8)</th>
<th>FX (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{t-VAR} )</td>
<td>-0.17**</td>
<td>-0.16*</td>
<td>-0.17**</td>
<td>-0.06**</td>
<td>-0.10</td>
<td>-0.31***</td>
<td>0.02</td>
<td>-0.01</td>
<td>0.10*</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.08)</td>
<td>(0.03)</td>
<td>(0.09)</td>
<td>(0.10)</td>
<td>(0.02)</td>
<td>(0.08)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.14***</td>
<td>0.16***</td>
<td>0.14***</td>
<td>0.08***</td>
<td>0.16***</td>
<td>0.18***</td>
<td>0.01**</td>
<td>0.05</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.05)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Observations</td>
<td>184</td>
<td>184</td>
<td>184</td>
<td>158</td>
<td>62</td>
<td>100</td>
<td>44</td>
<td>102</td>
<td>132</td>
</tr>
<tr>
<td>R(^2)</td>
<td>0.06</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.23</td>
<td>0.03</td>
<td>0.0002</td>
<td>0.09</td>
</tr>
<tr>
<td>Adjusted R(^2)</td>
<td>0.06</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
<td>0.03</td>
<td>0.22</td>
<td>0.004</td>
<td>-0.01</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note: \(*p<0.1; **p<0.05; ***p<0.01\)
Appendix

A Proof of Propositions

Proof of Proposition 1

Proof. State variable $x_t$ is the wealth share of financial sector: $x_t = \frac{W_A + W_B}{W}$ and state variable $y$ is wealth share of type $A$ agents in the financial sector: $y_t = \frac{W_A}{W_A + W_B}$, where $W$, $W_A$, and $W_B$ are the aggregate wealth, wealth of $A$, and $B$ agents, respectively.

Dynamics of $x_t$: From equation (5), $W_A$ has the following law of motion

$$dW_A = (r + w^A_s (\mu - r) - c_A) \ dt + w^A_s \sigma \ dZ, \tag{A.1}$$

and $W_B$ has the similar law of motion.

The law of motion for the numerator, $W_A + W_B$, will be

$$d(W_A + W_B) = \left[ r + (yw^A_s + (1 - y)w^B_s) (\mu - r) - (yc_A + (1 - y)c_B) \right] dt$$

$$+ (yw^A_s + (1 - y)w^B_s) \sigma dZ \tag{A.2}$$

Define wealth share of agents $A$ and $B$ as $u ≡ \frac{W_A}{W} = xy$ and $v ≡ \frac{W_B}{W} = x(1 - y)$, respectively. Since the aggregate wealth is $W = W_A + W_B + W_C$, the law of motion for the denominator is

$$dW = \left[ r + (xyw^A_s + x(1 - y)w^B_s + (1 - x)w^C_s) (\mu - r) - (xy c_A + x(1 - y) c_B + (1 - x) c_C) \right] dt$$

$$+ \left[ (yw^A_s + x(1 - y)w^B_s + (1 - x)w^C_s) \right] \sigma dZ \tag{A.3}$$

$$= [r + (\mu - r) - F] \ dt + \sigma \ dZ \tag{A.4}$$

From Ito’s lemma for ratio of two stochastic processes,$^{55}$

$$\frac{dx}{x} = \kappa(\bar{x} - x) \ dt + \left[ (yw^A_s + (1 - y)w^B_s - 1) (\mu - r - \sigma^2) - yc_A - (1 - y)c_B + F \right] \ dt$$

$$+ (yw^A_s + (1 - y)w^B_s - 1) \sigma \ dZ \tag{A.5}$$

$^{54}$Agent $C$’s wealth share will then be $1 - u - v$.

$^{55}$If two stochastic processes $X_t$ and $Y_t$ follow

$$\frac{dX_t}{X_t} = \mu_{X,t} \ dt + \sigma_{X,t} \ dZ_t \quad \text{and} \quad \frac{dY_t}{Y_t} = \mu_{Y,t} \ dt + \sigma_{Y,t} \ dZ_t,$$

then by Ito’s lemma the ratio of the two follows

$$\frac{d(X_t/Y_t)}{X_t/Y_t} = (\mu_{X,t} - \mu_{Y,t} + \sigma^2_{Y,t} - \sigma_{X,t} \sigma_{Y,t}) \ dt + (\sigma_{X,t} - \sigma_{Y,t}) \ dZ_t.$$

56
Thus from the dynamics of $x$ in equation (12) we have
\[
\begin{align*}
\mu_x &= (yw^A_s + (1 - y)w^B_s - 1) (\mu - r - \sigma^2) - yc_A - (1 - y)c_B + F \\
\sigma_x &= (yw^A_s + (1 - y)w^B_s - 1) \sigma
\end{align*}
\]

\textbf{Dynamics of $y$:} The numerator of $y$ is $W^A$, and its denominator is $(W^A + W^B)$ which its law of motion is calculated above. So from Ito’s lemma for a ratio, we get
\[
\frac{dy}{y} = \kappa(\bar{y} - y) dt + (1 - y) \left[ (w^A_s - w^B_s) (\mu - r) - c_A + c_B - (yw^A_s + (1 - y)w^B_s) (w^A_s - w^B_s) \sigma^2 \right] dt \\
+ (1 - y) (w^A_s - w^B_s) \sigma dZ_t
\]

Thus from the dynamics of $y$ in equation (12) we have
\[
\begin{align*}
\mu_y &= (w^A_s - w^B_s) (\mu - r) - c_A + c_B - (yw^A_s + (1 - y)w^B_s) (w^A_s - w^B_s) \sigma^2 \\
\sigma_y &= (w^A_s - w^B_s) \sigma
\end{align*}
\]

\textbf{Proof of Proposition 2}

\textit{Proof.} Since $\sigma_x$ and $\sigma_y$ are finite, we trivially get
\[
\lim_{x \to 0} x \sigma_x = 0, \forall y \quad \text{and} \quad \lim_{y \to 0} y(1 - y) \sigma_y = \lim_{y \to 1} y(1 - y) \sigma_y = 0, \forall x.
\]

We only need to show $\lim_{x \to 1} x \sigma_x = 0 \forall y$. The market clearing condition for the risky asset when $x \to 1$ becomes $yw^A + (1 - y)w^B = 1$.

So, from the expression for $\sigma_x$ in equation (15), we have:
\[
x \sigma_x = x [yw^A_s + (1 - y)w^B_s - 1] \sigma
\]
which goes to zero as $x \to 1$ for all $y$ from the stock market-clearing.

\textbf{Proof of Proposition 3}

\textit{Proof.} We can write agent $i$’s optimization problem in equation (8) as
\[
0 = \max_{c_i, w_i} \{ f_i(c_{i,t}, V_{i,t}) dt + \mathbb{E}_t [dV_{i,t}] \}
\]

Using Ito’s lemma we have
\[
\mathbb{E}_t [dV_{i}] = V_{i,t} \mathbb{E}_t [dW_{i}] + \frac{1}{2} V_{i,t} W_{i,t} \mathbb{E}_t [dW_{i}^2] + V_{i,t} J_{i,t} \mathbb{E}_t [dJ_{i}] + \frac{1}{2} V_{i,t} J_{i,t} J_{i,t} \mathbb{E}_t [dJ_{i}^2] + V_{i,t} W_{i,t} J_{i,t} \mathbb{E}_t [dW_{i} dJ_{i}]
\]

where $V_{i,t}$ and $V_{i,t} W_{i,t}$ are the first and second partial derivatives of $V_i$ with respect to $W_i$ (similarly for $V_{i,t} J_{i}, V_{i,t} J_{i} J_{i}$, and $V_{i,t} W_{i,t} J_{i}$.) Also posit the following Ito process for marginal value of wealth $J_i$:
\[
\frac{dJ_{i}}{J_{i}} = \mu_{J_{i,t}} dt + \sigma_{J_{i,t}} dZ_t,
\]

57
with adapted processes $\mu_{J_i,t} = \mu_{J_i}(x_t, y_t)$ and $\sigma_{J_i,t} = \sigma_{J_i}(x_t, y_t)$. I will drop $t$ subscripts for notational simplicity.

Using Ito’s lemma, we can find the drift and diffusions $\mu_{J_i}$ and $\sigma_{J_i}$

$$\mu_{J_i} = \frac{J_{i,x}}{J_i} \left[ \kappa(x) + \mu_x \right] + \frac{J_{i,y}}{J_i} \left[ \kappa(y) + y(1-y)\mu_y \right]$$

$$+ \frac{1}{2} \left( \frac{J_{i,xx}}{J_i} \right) \sigma_x^2 + \frac{J_{i,xy}}{J_i} \sigma_{x}\sigma_y + \frac{1}{2} \left( \frac{J_{i,yy}}{J_i} \right) y^2 (1-y)^2 \sigma_y^2$$

(A.1)

$$\sigma_{J_i} = \frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} y(1-y)\sigma_y$$

(A.2)

Plugging in the felicity function $f$ in (3) and the conjecture for value function $V_t$ in (20) into the HJB equation above, using the budget constraint in (5) and the law of motion for $J_i$ in (A.1) and (A.2), and dropping $W_t^{1-\gamma} J_i^{1-\psi_i}$ and $dt$ terms yields

$$0 = \max_{c_i, w_i} \frac{1}{1 - \psi_i} \left[ \frac{c_i}{J_i^{1/(1-\psi_i)}} ight]^{1/(1-\psi_i)} - (\rho + \kappa) \right] + \left[ r - c_i + \kappa + w_i^\mu (\mu - r) - \frac{\gamma_i}{2} (w_i^\sigma)^2 \right]$$

$$+ \left( \frac{1}{1 - \psi_i} \right) \left\{ \frac{J_{i,x}}{J_i} \left[ \kappa(x) + \mu_x \right] + \frac{J_{i,y}}{J_i} \left[ \kappa(y) + y(1-y)\mu_y \right] + (1-\gamma_i) \left( \frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} y(1-y)\sigma_y \right) w_i^\sigma \right\}$$

$$+ \frac{1}{2} \left( \frac{1}{1 - \psi_i} \right) \left[ \frac{\psi_i - \gamma_i}{1 - \psi_i} \right] \left( \frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} y(1-y)\sigma_y \right)^2 + \frac{J_{i,xx}}{J_i} \sigma_x^2 + 2 \frac{J_{i,xy}}{J_i} \sigma_x \sigma_y + \frac{J_{i,yy}}{J_i} y^2 (1-y)^2 \sigma_y^2$$

$$+ \lambda_i \left( \bar{\theta}_t - w_i^\gamma \right),$$

where $\lambda_i$ is proportional to the Lagrange multiplier on the time-varying margin constraint. The first-order condition for consumption-wealth ratio and portfolio share will lead to equations (22) and (24):

$$c_i = J_i$$

(A.3)

$$w_i^\gamma = \frac{\mu - r}{\gamma_i \sigma^2} + \left( \frac{1 - \gamma_i}{1 - \psi_i} \right) \left( \frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} y(1-y)\sigma_y \right) - \frac{1}{\gamma_i \sigma^2} \lambda_i$$

(A.4)

When the margin constraint for agent $i$ is slack, $\lambda_i = 0$ and we have

$$w_i^{\gamma,\text{const}} = \frac{\mu - r}{\gamma_i \sigma^2} + \left( \frac{1 - \gamma_i}{1 - \psi_i} \right) \left( \frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} y(1-y)\sigma_y \right)$$

When the margin constraint for agent $i$ is binding, $\lambda_i$ is strictly positive and $w_i^{\gamma,\text{const}} = \bar{\theta}_t$.

Plugging in the $w_i^{\gamma,\text{const}}$ into (A.4), we get the expression for the multiplier on the time-varying margin constraint:

$$\lambda_i = (\mu - r) + \left( \frac{1 - \gamma_i}{1 - \psi_i} \right) \left( \frac{J_{i,x}}{J_i} \sigma_x + \frac{J_{i,y}}{J_i} y(1-y)\sigma_y \right) \sigma - \gamma_i \sigma^2 \bar{\theta}_t$$

(A.5)

□

58
B Numerical Procedure

The computation of equilibrium is reduced to solving three second-order PDEs for functions $J_i$ for $i \in \{A, B, C\}$.\(^{56}\) I use Chebyshev orthogonal collocation method to solve the model.\(^{57}\) The HJB equation for agent $i$ can be written as the following functional equation:

$$\mathcal{H}_i(J_i) = 0.$$  \(^\text{(B.1)}\)

I express marginal value of wealth functions $J_A(x,y)$, $J_B(x,y)$ and $J_C(x,y)$ as bivariate Chebyshev polynomials of order $N$ (I use $N = 20$), that is, I approximate $J_i$ with tensor product of Chebyshev polynomials of order $N$:

$$\hat{J}_i(x,y) = \sum_{j=0}^{N} \sum_{k=0}^{N} a_{jk} \psi_j(\omega_j(x)) \psi_k(\omega_k(y)), \; i \in \{A, B, C\}.$$  \(^\text{(B.1)}\)

where $\psi_j$ is the Chebyshev polynomial of degree $j = 0, 1, \ldots, N$, called the basis function, and $\{a_{ij}\}_{i=1}^{N}$ and $\{b_{jk}\}_{j=1}^{N}$, are unknown coefficients, and $\omega_j$’s are the Chebyshev nodes (collocation points) defined below.

I then plug in $\hat{J}_i$ into the HJB equation for agent $i$ to form the residual equation:

$$\mathcal{R}(\cdot | a, b) = \mathcal{H}_i(\hat{J}_i),$$

and find the vector of coefficients $(a, b)$ that makes the residual equation as close to $0$ as possible given some objective function $\rho(\mathcal{R}(\cdot | a, b), 0)$:

$$(a, b) = \arg \min_{(a,b)} \rho(\mathcal{R}(\cdot | a, b), 0)$$

The most common objective function is a weighted residual given some weight functions $\phi_j : \Omega \to \mathbb{R}^m$:

$$\rho(\mathcal{R}(\cdot | a, b), 0) = \begin{cases} 
0 & \text{if } \int_{\Omega} \int_{\Omega} \phi_j(x)\phi_k(y)\mathcal{R}(\cdot | (a,b)) \, dx \, dy = 0, \text{ for } j, k = 1, \ldots, N \\
1 & \text{otherwise}
\end{cases}$$

In the pseudo-spectral (or collocation) method, the weight functions are chosen as: $\phi_j(x) = \delta(x-x_i)$ where $\delta$ is the dirac delta function and $x_i$’s are the collocation points. In the orthogonal collocation method, which I use to solve the model, the basis functions are a set of orthogonal Chebyshev polynomials and collocation points are given by the roots of the $N$\textsuperscript{th} polynomial.

Chebyshev polynomials of degree $n$ can be easily defined recursively:

$$\psi_0(\omega) = 1$$
$$\psi_1(\omega) = x$$
$$\psi_{n+1}(\omega) = 2\omega\psi_n(\omega) - \psi_{n-1}(\omega)$$  \(^\text{(B.2)}\)

\(^{56}\)Duffie and Lions (1992) show existence and uniqueness of infinite-horizon stochastic differential utility by partial differential equation techniques in a Markov diffusion setting.

\(^{57}\)For more details, see Judd (1992, 1998) and Computational Tools & Macroeconomic Applications, NBER Summer Institute 2011 Methods Lectures, Lawrence Christiano and Jesus Fernandez-Villaverde, Organizers.
As mentioned above, the collocation points are the \( N \) zeros of the Chebyshev polynomial of order \( N, (\psi_N(\omega_j) = 0) \), and are given by the following expression

\[
\omega_j = \cos \left( \frac{2j - 1}{2n} \pi \right), \ j = 1, \ldots, N.
\]

These roots are clustered quadratically towards \( \pm 1 \). Chebyshev polynomials are defined on \( \omega_i \in [-1, 1] \). Since the state variables \( x, y \in [0,1] \) in my model, I use the linear transformation \( x_j = (1 + \omega_j)/2 \).

I calculate the derivatives of these functions as well as the state variable dynamics, agents’ portfolio choice, risky asset return and volatility using the relevant equilibrium expressions. I then plug these quantities into the HJB equations (21) and project the resulting residuals onto the complete set of Chebyshev polynomials up to order \( N \). I use the built-in Matlab function \texttt{fsolve} to find the coefficients of \( J_i \) polynomials that make the projected residuals equal to zero. This results in a highly accurate solution for coefficients in the \( J_i \) functions with errors in the order of \( 10^{-20} \).

The numerical algorithm is summarized below.

1. From goods market-clearing conditions and differentiating it with respect to the state variable, we get expressions for dividend yield \( F \) and its derivatives with respect to \( x \) and \( y \).

\[
F = xyJ_A + x(1 - y)J_B + (1 - x)J_C,
\]

\[
F_x = xJ_A + (1 - y)J_B - JC + xyJ_{A,x} + x(1 - y)J_{B,x} + (1 - x)J_{C,x},
\]

\[
F_y = yJ_A - xJ_B + xyJ_{A,y} + x(1 - y)J_{B,y} + (1 - x)J_{C,y},
\]

\[
F_{xx} = 2yJ_{A,x} + 2(1 - y)J_{B,x} - 2J_{C,x} + xyJ_{A,xx} + x(1 - y)J_{B,xx} + (1 - x)J_{C,xx},
\]

\[
F_{yy} = 2xJ_{A,y} - 2xJ_{B,y} + xyJ_{A,yy} + (1 - y)J_{B,yy} + (1 - x)J_{C,yy},
\]

\[
F_{xy} = J_A - J_B + xJ_{A,x} - xJ_{B,x} + yJ_{A,y} + (1 - y)J_{B,y} - J_{C,y} + xyJ_{A,xy} + x(1 - y)J_{B,xy} + (1 - x)J_{C,xy},
\]

where \( J_{i,x} \) and \( J_{i,xx} \) are the first and second partial derivative of \( J_i \) with respect to \( x \), respectively, and similarly for \( J_{i,y}, J_{i,yy} \) and \( J_{i,xy} \).

2. Using market-clearing condition for the endowment claim, plugging in the expression for agent \( C \)’s optimal portfolio choice \( w_s^C \) from (23), and substituting for \( (\mu - r)/\sigma^2 \) from the expression for \( w_s^{A,*} \), we will get the first of the two equations that \( w_s^{A,*} \) and \( w_s^B \) have to satisfy:

\[
1 = xyw_s^{A,*} + x(1 - y)w_s^B + (1 - x)w_s^C
\]

\[
= xyw_s^{A,*} + x(1 - y)w_s^B
\]

\[
+ (1 - x) \frac{1}{\gamma_C} \left\{ \frac{\mu - r}{\sigma^2} + \frac{1}{1 - \psi_C} \left[ \frac{JC}{JC} x (yw_s^{A,*} + (1 - y)w_s^B - 1) + \frac{JC}{JC} y(1 - y) \left( w_s^{A,*} - w_s^B \right) \right] \right\}
\]

\[
= xw_s^{A,*} + yw_s^B + (1 - x) \frac{1}{\gamma_C} \left\{ \gamma_A w_s^{A,*} - \frac{1 - \gamma_A}{1 - \psi_A} \left[ \frac{J_{A,x}}{JC} x (yw_s^{A,*} + (1 - y)w_s^B - 1) \right. \right.
\]

\[
+ \left. \left. \frac{J_{A,y}}{JA} y(1 - y) \left( w_s^{A,*} - w_s^B \right) \right] \right\}
\]

\[
+ \left( \frac{1 - \gamma_C}{1 - \psi_C} \right) \left[ \frac{JC}{JC} x (yw_s^{A,*} + (1 - y)w_s^B - 1) + \frac{JC}{JC} y(1 - y) \left( w_s^{A,*} - w_s^B \right) \right] \right\}
\]

\[
\text{For a general state space } x \in [x_L, x_H], \text{ we use a linear transformation } x_j = x_L + 0.5(x_H - x_L)(1 + \omega_j).
\]
3. Since the return volatility can be written as

\[ \sigma = \frac{\sigma_D}{1 + \frac{F_x}{F} x (yw_s^A + (1 - y)w_s^B - 1) + \frac{F_y}{F} y (1 - y) (w_s^A - w_s^B)} \]

We can rewrite the systems of equation as

\[ a_{11} w_s^{A,*} + a_{12} w_s^B = b_1 \]
\[ a_{21} w_s^{A,*} + a_{22} w_s^B = b_2 \]

where

\[ a_{11} = xy + (1 - x) \frac{1}{\gamma_C} \left[ \frac{J_{B,x}}{J_B} x y + \frac{J_{A,y}}{J_A} y (1 - y) \right] + \left( \frac{1 - \gamma_B}{1 - \psi_B} \right) \left( \frac{J_{C,x}}{J_C} x y + \frac{J_{C,y}}{J_C} y (1 - y) \right) \]
\[ a_{12} = x (1 - y) + (1 - x) \frac{1}{\gamma_C} \left[ - \left( \frac{1 - \gamma_A}{1 - \psi_A} \right) \left( \frac{J_{A,x}}{J_A} x (1 - y) - \frac{J_{A,y}}{J_A} y (1 - y) \right) \right] + \left( \frac{1 - \gamma_B}{1 - \psi_B} \right) \left( \frac{J_{C,x}}{J_C} x (1 - y) - \frac{J_{C,y}}{J_C} y (1 - y) \right) \]
\[ a_{21} = \frac{1}{\gamma_B} \left[ \frac{\gamma_B}{\gamma_A} - \left( \frac{1 - \gamma_A}{1 - \psi_A} \right) \left( \frac{J_{A,x}}{J_A} x y + \frac{J_{A,y}}{J_A} y (1 - y) \right) + \left( \frac{1 - \gamma_B}{1 - \psi_B} \right) \left( \frac{J_{B,x}}{J_B} x y + \frac{J_{B,y}}{J_B} y (1 - y) \right) \right] \]
\[ a_{22} = -1 + \frac{1}{\gamma_B} \left[ - \left( \frac{1 - \gamma_A}{1 - \psi_A} \right) \left( \frac{J_{A,x}}{J_A} x (1 - y) + \frac{J_{A,y}}{J_A} y (1 - y) \right) \right] + \left( \frac{1 - \gamma_B}{1 - \psi_B} \right) \left( \frac{J_{B,x}}{J_B} x (1 - y) - \frac{J_{B,y}}{J_B} y (1 - y) \right) \]
\[ b_1 = 1 + (1 - x) \frac{1}{\gamma_C} \left[ - \left( \frac{1 - \gamma_A}{1 - \psi_A} \right) \frac{J_{A,x}}{J_A} x + \left( \frac{1 - \gamma_C}{1 - \psi_C} \right) \frac{J_{C,x}}{J_C} x \right] \]
\[ b_2 = \frac{1}{\gamma_B} \left[ - \left( \frac{1 - \gamma_A}{1 - \psi_A} \right) \frac{J_{A,x}}{J_A} x + \left( \frac{1 - \gamma_B}{1 - \psi_B} \right) \frac{J_{B,x}}{J_B} \right] \]

The system of equations above can be solved easily to get \( w_s^{A,*} \) and \( w_s^B \).
when the margin constrains for agent $A$ bind, from equation (7) with $\nu = 1$, we must have

$$w_{s, \text{const}}^A = \frac{1 - F_x x + \left( \frac{F_x}{F} x - \frac{F_y}{F} y \right) (1 - y) w_s^B}{\alpha \sigma_D - \left[ \frac{F_x}{F} x + \frac{F_y}{F} (1 - y) \right] y} \quad \text{(B.4)}$$

So, we have $w_s^A \leq w_{s, \text{const}}^A$. Then from (24) we can find $A$ and $B$’s portfolio weights in the risky asset

$$w_s^A = \min \left( w_s^{A,*}, w_{s, \text{const}}^A \right),$$

where $w_{s, \text{const}}^A$ is given in equation (B.4).

4. From stock market clearing, we can get $C$’s optimal portfolio weight

$$w_s^C = \frac{1 - xy w_s^A - x(1 - y) w_s^B}{1 - x}$$

5. Using the expression for the return volatility in equation (18) and plugging in expressions for $\sigma_x$ and $\sigma_y$ from equations (15) and (16), the expression for return volatility is

$$\sigma = \frac{\sigma_D}{1 + \frac{F_x}{F} x \left[ y w_s^A + (1 - y) w_s^B - 1 \right] + \frac{F_y}{F} y (1 - y) \left( w_s^A - w_s^B \right)}.$$

6. Using the expression for $\sigma$ above, state variable diffusions ($\sigma_x$ and $\sigma_y$) can be found from equations (15) and (16):

$$\sigma_x = \left[ y w_s^A + (1 - y) w_s^B - 1 \right] \sigma, \quad \text{and} \quad \sigma_y = \left( w_s^A - w_s^B \right) \sigma.$$

7. From the expression for $w_s^C, \sigma, \sigma_x$, and $\sigma_y$, the expected excess return (risk premium) on the risky asset is

$$\mu - r = \gamma_C w_s^C \sigma^2 - \left( \frac{1 - \gamma_C}{\psi_C} \right) \left( \frac{J_{C,x}}{J_C} x \sigma_x + \frac{J_{C,y}}{J_C} y (1 - y) \sigma_y \right) \sigma,$$

8. Using the optimal consumption-wealth ratios $c_i = J_i$, we can then compute drifts of the state variables $\mu_x$ and $\mu_y$ as

$$\mu_x = \left[ y w_s^A + (1 - y) w_s^B - 1 \right] \left( \mu - r - \sigma^2 \right) - y J_A - (1 - y) J_B + F,$$

$$\mu_y = \left( w_s^A - w_s^B \right) \left( \mu - r - J_A + J_B - \left[ y w_s^A + (1 - y) w_s^B \right] \left( w_s^A - w_s^B \right) \sigma^2 \right).$$

9. From equation (17) the expected return on the risky asset can be calculated

$$\mu = \mu_D + F - \frac{F_x}{F} \left[ \kappa (\bar{x} - x) + x (\mu_x + \sigma_D \sigma_x) \right] - \frac{F_y}{F} \left[ \kappa (\bar{y} - y) + y (1 - y) (\mu_y + \sigma_D \sigma_y) \right]$$

$$+ \left[ \left( \frac{F_x}{F} \right)^2 + \frac{1}{2} \frac{F_{xx}}{F} \right] x^2 \sigma_x^2 + \left[ \left( \frac{F_y}{F} \right)^2 + \frac{1}{2} \frac{F_{yy}}{F} \right] y^2 (1 - y)^2 \sigma_y^2 + \left[ 2 \left( \frac{F_x}{F} \right) - \frac{F_{xy}}{F} \right] x y (1 - y) \sigma_x \sigma_y.$$  

10. The real interest rate is

$$r = \mu - (\mu - r).$$
11. Plugging expressions above into agent $i$’s HJB equations in (21), we get the residual functions for agent $i$:

\[
0 = -\left(\rho + \kappa + \frac{1}{\psi_i} J_i + \left(1 - \frac{1}{\psi_i}\right) \left[r + u_i^\dagger (\mu - r) - \frac{\gamma_i}{2} \left(w_i^\dagger\right)^2 \sigma^2\right]\right) \\
- \frac{1}{\psi_i} \left[\frac{J_{i,x}^\dagger}{J_i} \left[\kappa(x - x) + x_\mu \right] + \frac{J_{i,y}^\dagger}{J_i} \kappa(y - y) + y(1 - y)\mu_y\right] + \left(1 - \gamma_i\right) \left(\frac{J_{i,x}^\dagger}{J_i} x \sigma_x + \frac{J_{i,y}^\dagger}{J_i} y (1 - y) \sigma_y \right) w_i^\dagger \sigma \\
- \frac{1}{2 \psi_i} \left[\left(\psi_i - \gamma_i\right) \left(\frac{J_{i,x}^\dagger}{J_i} x \sigma_x + \frac{J_{i,y}^\dagger}{J_i} y (1 - y) \mu_y\right)^2 + \frac{J_{i,xx}^\dagger}{J_i} x^2 \sigma_x^2 + 2 \frac{J_{i,xy}^\dagger}{J_i} xy (1 - y) \sigma_x \sigma_y + \frac{J_{i,yy}^\dagger}{J_i} y^2 (1 - y)^2 \sigma_y^2\right].
\]

C Data Sources

Broker-Dealer and Holding Company Data Balance sheet data for broker-dealers and bank holding companies are from Tables L.130 and L.131 of Financial Accounts of the United States (Flow of Funds) from Federal Reserves, respectively. As noted in the description of Table L.130,

Security brokers and dealers are firms that buy and sell securities for a fee, hold an inventory of securities for resale, or do both. The firms that make up this sector are those that submit information to the Securities and Exchange Commission on one of two reporting forms, either the Financial and Operational Combined Uniform Single Report of Brokers and Dealers (FOCUS) or the Report on Finances and Operations of Government Securities Brokers and Dealers (FOGS). The major assets of the sector are collateral repayable from funding corporations in connection with securities borrowing (included in miscellaneous assets), debt securities and equities held for redistribution, customers’ margin accounts, and security repurchase agreements (reverse repos). Sector operations are financed largely by net transactions with parent companies, customers’ cash accounts, loans for purchasing and carrying securities from depository institutions, and security repurchase agreements.

Also from Table L.131’s description for holding companies,

... the holding companies sector consists of all top-tiered bank holding companies, savings and loan holding companies, U.S. Intermediate Holding Companies (IHCs), and securities holding companies (collectively “holding companies”) that file the Federal Reserve’s Form FR Y-9LP, Parent Company Only Financial Statements for Large Holding Companies, FR Y-9SP, Parent Company Only Financial Statements for Small Holding Companies, or FR 2320, Quarterly Savings and Loan Holding Company Report. Holding companies required to file FR Y-9LP include those with total consolidated assets of $1 billion or more or meet other criteria, such as having a material amount of debt or equity securities outstanding that are registered with the Securities and Exchange Commission, being engaged in significant nonbanking activity, or conducting off-balance-sheet activities either directly or through a nonbank subsidiary. Those holding companies required to file FR Y-9SP have total consolidated assets less than $1 billion. Form FR 2320 must be filed by top-tier savings and loan holding companies exempt from initially filing the Y-9LP or Y-9SP, because even though they own a savings and loan institution, that is not their primary line of business. Mutual stock companies that file the FR 2320 are excluded because they do not hold any assets or liabilities at the holding company level. The major assets of holding companies, other
than small amounts of loans and securities, are net transactions with their subsidiaries; this includes equity investments in subsidiaries and associated banks and net balances due from subsidiaries and related depository institutions. The main source of funding for the sector is the issuance of corporate bonds and commercial paper.\footnote{The holding companies sector has a large increase in the level of assets and liabilities in the 2009:Q1 because a number of large financial institutions became bank holding companies. These companies (including Goldman Sachs, Morgan Stanley, American Express, CIT Group, GMAC, Discover Financial Services, and IB Finance) had not previously been included in the financial accounts.}

**Test Assets** Test assets for time-series and cross-sectional asset pricing tests are from two sources: (i) equity portfolios (25 portfolios formed on size and book-to-market and 10 momentum portfolios) are from Ken French’s Data Library, and (ii) non-equity assets are from HKM obtained from Asaf Manela’s website and include 10 maturity-sorted US government and 10 corporate bond portfolios sorted on yield spreads, 6 sovereign bond portfolios based on a two-way sort on a bond’s covariance with US equity market and bond’s S&P rating, 54 portfolios of S&P 500 index options sorted on moneyness and maturity split by contract type (27 calls and 27 puts), 20 CDS portfolios sorted by spreads using single-name 5-year contracts, 23 commodity portfolios with at least 25 years of return data, and 12 foreign exchange currency portfolios, six sorted on interest rate differentials and six sorted on momentum. Except for Treasury bond portfolios which are from CRSP, non-equity test assets in HKM are from previous studies.

**Intermediary Asset Pricing Factors** AEM and HKM factors are from Tyler Muir’s and Asaf Manela’s websites, respectively. AEM leverage factor is defined as the seasonally adjusted growth rate in broker-dealer book leverage from Table L.130 of the Flow of Funds, where leverage is defined as total financial assets divided by total financial assets minus total volatility. The intermediary capital ratio in HKM is the ratio of total market equity to total market assets (book debt plus market equity) of primary dealer holding companies of the New York Fed. Shocks to capital ratio (HKM capital factor) are defined as AR(1) innovations in the capital ratio, scaled by the lagged capital ratio. Data for publicly-traded holding companies of primary dealers are from CRSP/Compustat and Datastream. Primary dealers are large and sophisticated institutions and serve as trading counterparties of the NY Fed in its implementation of monetary policy. For the current and historical list of primary dealers see this link.

**D Robustness Checks for Empirical Results**

**D.1 Predictive Regressions**

**Include Broker-Dealer Leverage Ratio from AEM**

As a robustness check, I add broker-dealer leverage ratio from AEM as an additional forecasting variable and rerun the predictive regressions:

\[
R_{t+1-t+4} - r_f^t = \gamma_0 + \gamma_y y_t + \gamma_{Lev} BDL_{Lev} + \varepsilon_{t+1-t+4}
\]

Table G.1 presents the results. Note that for the most part, adding BD leverage ratio does not change the sign and significance of the coefficient $\gamma_y$. Since leverage of BDs are procyclical, we expect to see a negative sign on $\gamma_{Lev}$. However, the sign of $\gamma_{Lev}$ is positive for market and momentum portfolios and it is only statistically significant for sovereign bond portfolios.
Include Intermediary Capital Ratio from HKM

As a robustness check, I add capital ratio of primary dealer holding companies from HKM as an additional forecasting variable and rerun the predictive regressions:

\[ R_{t+1 \rightarrow t+4} - r_t^f = \gamma_0 + \gamma_y y_t + \gamma_{\text{CapRatio}} \text{CapRatio}_t + \epsilon_{t+1 \rightarrow t+4} \]

Table G.2 presents the results. Note that for the most part, adding primary dealer capital ratio does not change the sign of the coefficient \( \gamma_y \) but reduces its statistical significance. Since primary dealer capital ratio is procyclical, as expected, the sign on \( \gamma_{\text{CapRatio}} \) is negative. However, except US bonds, the coefficient is less statistically significant than \( \gamma_y \) for all assets.

Include Consumption-Wealth Ratio (\( cay \)) Variable from Lettau and Ludvigson (2001)

As a robustness check, I add fluctuations in the aggregate consumption-wealth ratio (\( cay \)) variable defined in Lettau and Ludvigson (2001) as an additional forecasting variable and rerun the predictive regressions:

\[ R_{t+1 \rightarrow t+4} - r_t^f = \gamma_0 + \gamma_y y_t + \gamma_{\text{cay}} cay_t + \epsilon_{t+1 \rightarrow t+4} \]

This variable has been argued to capture aggregate effective risk aversion of a representative agent. Table G.3 presents the results. Note that adding \( cay \) does not change the sign and significance of the coefficient \( \gamma_y \). As expected, \( \gamma_{cay} \) has a positive sign because variations in \( cay \) are countercyclical: \( cay \) tends to decline during expansions and rise just prior to the onset of a recession when expected future returns are high (see Lettau and Ludvigson, 2001, for more details).

D.2 Factor-Mimicking Portfolios (FMP)

Alternative Projections for FMPs

In this section, I repeat the exercise in Section 7.3.3 with an alternative set of returns. I construct a mimicking portfolio for the heterogeneous intermediary factor (HIFac) by projecting it onto the space of excess returns running the following regression:

\[ \text{HIFac}_t = a_{\text{HI}} + b_{\text{HI}}' [\text{MktRF}, \text{SMB}, \text{HML}, \text{Mom}, \text{Bond}]_t + \eta_t, \quad (D.1) \]

where \( \text{HIFac} \) is the heterogeneous intermediary factor defined in equation (32), MktRF, SMB, and HML are the Fama-French three factors, \( \text{Mom} \) is the momentum factor, and \( \text{Bond} \) is the first principal component (PC) of excess returns on six Treasury bond portfolios sorted by maturity from CRSP. The heterogeneous intermediary mimicking portfolio (HIMP) is then given by

\[ \text{HIMP}_t = \tilde{b}_{\text{II}}' [\text{MktRF}, \text{SMB}, \text{HML}, \text{Mom}, \text{Bond}]_t, \quad (D.2) \]

where \( \tilde{b}_{\text{II}} = \frac{b_{\text{HI}}}{\sum b_{\text{II}}} = [0.19, 0.51, 0.22, 0.29, -0.22] \) positively loading on the momentum factor.

I similarly construct mimicking portfolios for AEM’s broker-dealer leverage and HKM’s holding company capital factors using quarterly data for the two factors from Tyler Muir’s and Asaf Manela’s websites, respectively. The mimicking portfolio for the heterogeneous intermediary factor has Sharpe ratio of 0.43 over the sample period (1970Q1 to 2017Q3), much higher than Sharpe ratios for AEM and HKM factor-mimicking portfolios (0.24 and 0.28, respectively).

To evaluate the importance of heterogeneity in the financial sector above and beyond representative intermediary factors, I regress HIFac on mimicking portfolios for AEM and HKM factors in
the following regression:

\[ \text{HIMP}_t = \alpha_{\text{MP}} + \beta_{\text{FMP}}' \text{FMP}_t + \epsilon_t, \]  

(D.3)

where FMP is either the mimicking portfolio for broker-dealer leverage factor from AEM (AEM_M), or the mimicking portfolio for capital factor for primary dealers’ holding companies from HKM (HKM_M), or both AEM_M and HKM_M. If HIMP is fully “explained” by AEM_M, HKM_M, or both, I expect to see small and insignificant \(\alpha_{\text{MP}}\) in the regression above. I find the opposite to be hold in the data, however. Table G.4 presents the results. Time-series alphas are positive and significant at 1% level in all columns similar to the regressions in Table 7 from the main text.

E Parameter Estimates for the VAR

Table G.5 presents the maximum likelihood parameters estimates for the VAR in equation (39).

F Three-Dimensional Plots

Figure G.1 plots various objects the unconstrained equilibrium in the endowment model, where \(\hat{\theta}_t = \hat{m}\). All variables are functions of the two state variables in the model: \(x\) (wealth share of agents A and B, i.e. the financial sector) and \(y\) (wealth share of A agents in the financial sector). These are the same objects plotted in solid blue line in Figures 3 and 4 but in three dimensions.

Figure G.2 presents various variables in the equilibrium with time-varying margin constraint in the endowment model with \(\hat{\theta}_t = \frac{1}{\alpha \sigma_t}\) as functions of state variables \((x_t, y_t)\). These are the same equilibrium objects plotted in dashed red line in Figures 3 and 4 but in three dimensions.

Figure G.3 presents the diffusions of state variables \(x\) and \(y\) (\(\sigma_x\) and \(\sigma_y\) in equation 15 and 16) in the equilibrium with state-dependent margin constraint. From equation 15, \(\sigma_x\) always remains positive. Since, in my calibration, \(\sigma_y\) is also positive in the entire state space, both state variables are procyclical: following a negative \(dZ\) shock, both state variables go down.

G Appendix Figures and Tables
Figure G.1. Equilibrium in the unconstrained economy. This figure presents price-dividend ratio $1/F$, return volatility $\sigma$, Sharpe ratio and risk premium on the endowment claim, optimal portfolio weights of each type of agent ($w^A_s$, $w^B_s$, and $w^C_s$) as well as the real interest rate $r_t$ and the drift and diffusion of state variables $x$ and $y$ ($\mu_x$, $\sigma_x$, $\mu_y$, and $\sigma_y$, respectively) in the frictionless economy as functions of state variable $x$ (wealth share of the financial sector i.e. type $A$ and $B$ agents) and $y_t$ (wealth share of type $A$ agents in the financial sector) under the benchmark parameters in Table 1.
Figure G.2. Equilibrium in the economy with time-varying margin constraints. This figure presents price-dividend ratio $1/F$, return volatility $\sigma$, Sharpe ratio and risk premium on the endowment claim, optimal portfolio weights of each type of agent ($w^A$, $w^B$, and $w^C$) as well as the real interest rate $r$, and the drift and diffusion of state variables $x$ and $y$ ($\mu_x$, $\sigma_x$, $\mu_y$, and $\sigma_y$, respectively) in the economy with time-varying margin constraints as functions of state variable $x$ (wealth share of the financial sector i.e. type $A$ and $B$ agents) and $y$ (wealth share of type $A$ agents in the financial sector) under the benchmark parameters in Table 1.
Figure G.3. State Variable Diffusions. This figure presents the diffusions of state variables $x$ and $y$ ($\sigma_x$ and $\sigma_y$, respectively) in the economy with time-varying margin constraints as functions of state variable $x$ (wealth share of the financial sector i.e. type $A$ and $B$ agents) and $y_t$ (wealth share of type $A$ agents in the financial sector) under the benchmark parameters in Table 1.
Table G.1. Predictive regressions: $y_{\text{data}}$ and broker-dealer leverage ratio, BDLev.
This table provides results for one-year ahead predictive regressions according to $R_{t+1}^{\text{vw}} = \gamma_0 + \gamma_1 y_{t} + \gamma_{Lev} \cdot \text{BDLev}_{t+\varepsilon_{t+4}}$, using lagged equity share of broker-dealers in the financial sector, $y_{\text{data}}$ defined in (30) and lagged leverage ratio of security broker-dealers from AEM, as predictors. The dependent variables are excess holding period returns from quarter $t + 1$ to quarter $t + 4$ on the CRSP value-weighted (Mkt$^{\text{vw}}_{t+1}$) and equally-weighted (Mkt$^{\text{ew}}_{t+1}$) portfolios, mean excess return on 25 Fama-French size and book-to-market (FF25$_{t+1}$), 10 momentum (Mom$_{t+1}$) portfolios, 10 maturity-sorted US government and 10 US corporate bond portfolios sorted on yield spreads (US bonds$_{s_{t+1}}$), mean excess returns on six sovereign bonds (Sov. bonds$_{s_{t+1}}$), 54 portfolios of S&P 500 index options sorted on moneyness and maturity (Options$_{t+1}$), 20 CDS portfolios sorted by spreads (CDS$_{t+1}$), 23 commodity (Commod.$_{t+1}$), and 12 foreign exchange (FX$_{t+1}$) portfolios. Size/book-to-market and momentum portfolios and the risk-free rate data are from Ken French’s website. Data on sovereign bonds, options, CDS, commodities, and FX portfolios are from He et al. (2017). Broker-Dealer leverage is calculated using data from Table L.130 of Flow of Funds and is defined: Total Financial Assets/(Total Financial Assets – Total Liabilities). The sample is quarterly from 1974Q1 to 2017Q3 for market, FF25 and momentum portfolios, and to 2012Q4 for HKM assets. Regression coefficients on BDLev are multiplied by 100. Hodrick (1992) standard errors are reported in parentheses to adjust for the fact that overlapping quarterly observations are used to forecast annual returns.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Mkt$^{\text{vw}}$</th>
<th>Mkt$^{\text{ew}}$</th>
<th>FF25</th>
<th>Mom</th>
<th>US bonds</th>
<th>Sov. bonds</th>
<th>Options</th>
<th>CDS</th>
<th>Commod.</th>
<th>FX</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{\text{data}}$</td>
<td>$-0.76$</td>
<td>$-1.30^{***}$</td>
<td>$-0.89^{**}$</td>
<td>$-1.04^{**}$</td>
<td>$0.08$</td>
<td>$-0.86^{**}$</td>
<td>$-1.32^{*}$</td>
<td>$-0.0003$</td>
<td>$-0.06$</td>
<td>$0.48$</td>
</tr>
<tr>
<td></td>
<td>$(0.51)$</td>
<td>$(0.50)$</td>
<td>$(0.37)$</td>
<td>$(0.44)$</td>
<td>$(0.16)$</td>
<td>$(0.36)$</td>
<td>$(0.69)$</td>
<td>$(0.11)$</td>
<td>$(0.51)$</td>
<td>$(0.31)$</td>
</tr>
<tr>
<td>BDLev$_t$</td>
<td>$0.16$</td>
<td>$0.16$</td>
<td>$-0.10$</td>
<td>$0.31$</td>
<td>$-0.19^{**}$</td>
<td>$-1.02^{***}$</td>
<td>$-0.58$</td>
<td>$-0.06$</td>
<td>$-0.79$</td>
<td>$-0.12$</td>
</tr>
<tr>
<td></td>
<td>$(0.58)$</td>
<td>$(0.48)$</td>
<td>$(0.48)$</td>
<td>$(0.51)$</td>
<td>$(0.09)$</td>
<td>$(0.29)$</td>
<td>$(0.57)$</td>
<td>$(0.14)$</td>
<td>$(0.50)$</td>
<td>$(0.26)$</td>
</tr>
<tr>
<td>Constant</td>
<td>$0.24^{***}$</td>
<td>$0.38^{***}$</td>
<td>$0.35^{***}$</td>
<td>$0.27^{***}$</td>
<td>$0.09^{**}$</td>
<td>$0.59^{***}$</td>
<td>$0.54^{***}$</td>
<td>$0.04$</td>
<td>$0.24$</td>
<td>$-0.08$</td>
</tr>
<tr>
<td></td>
<td>$(0.06)$</td>
<td>$(0.08)$</td>
<td>$(0.06)$</td>
<td>$(0.06)$</td>
<td>$(0.04)$</td>
<td>$(0.11)$</td>
<td>$(0.16)$</td>
<td>$(0.04)$</td>
<td>$(0.16)$</td>
<td>$(0.08)$</td>
</tr>
</tbody>
</table>

| Observations        | 173           | 173           | 173           | 173          | 152         | 62          | 100       | 44          | 102          | 132       |
| R$^2$               | 0.06          | 0.10          | 0.11          | 0.08         | 0.04        | 0.28        | 0.15      | 0.01        | 0.08         | 0.06      |
| Adjusted R$^2$      | 0.05          | 0.09          | 0.10          | 0.07         | 0.02        | 0.26        | 0.13      | $-0.03$     | 0.07         | 0.04      |

*Note:* $^*_{p<0.1}$; $^{**}_{p<0.05}$; $^{***}_{p<0.01}$
Table G.2. Predictive regressions: $y^\text{data}$ and primary dealer capital ratio, $\text{CapRatio}_t$.
This table provides results for one-year ahead predictive regressions according to $R^e_{t+1-t+4} = \gamma_0 + \gamma y^\text{data}_t + \gamma \text{CapRatio}_t + \varepsilon_{t+4}$, using lagged equity share of broker-dealers in the financial sector, $y^\text{data}_t$ defined in (30) and lagged intermediary capital ratio for New York Fed’s primary dealer counterparties from HKM, as predictors. The dependent variables are excess holding period returns from quarter $t + 1$ to quarter $t + 4$ on the CRSP value-weighted ($\text{Mkt}^\text{vw}_{t+1}$) and equally-weighted ($\text{Mkt}^\text{ew}_{t+1}$) portfolios, mean excess return on 25 Fama-French size and book-to-market ($\text{FF25}_{t+1}$), 10 momentum ($\text{Mom}_{t+1}$) portfolios, 10 maturity-sorted US government and 10 US corporate bond portfolios sorted on yield spreads ($\text{US bonds}_{t+1}$), mean excess returns on six sovereign bonds ($\text{Sov. bonds}_{t+1}$), 54 portfolios of S&P 500 index options sorted on moneyness and maturity ($\text{Options}_{t+1}$), 20 CDS portfolios sorted by spreads ($\text{CDS}_{t+1}$), 23 commodity ($\text{Commod.}_{t+1}$), and 12 foreign exchange ($\text{FX}_{t+1}$) portfolios. Size/book-to-market and momentum portfolios and the risk-free rate data are from Ken French’s website. Data on sovereign bonds, options, CDS, commodities, and FX portfolios are from He et al. (2017). The capital ratio for New York Fed’s primary dealer holding companies are downloaded from Asaf Manela’s website. The sample is quarterly from 1974Q1 to 2017Q3 for market, FF25 and momentum portfolios, and to 2012Q4 for HKM assets. Hodrick (1992) standard errors are reported in parentheses to adjust for the fact that overlapping quarterly observations are used to forecast annual returns.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Mkt$^{\text{vw}}$</th>
<th>Mkt$^{\text{ew}}$</th>
<th>FF25</th>
<th>Mom</th>
<th>US bonds</th>
<th>Sov. bonds</th>
<th>Options</th>
<th>CDS</th>
<th>Commod.</th>
<th>FX</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^\text{data}_t$</td>
<td>-0.42</td>
<td>-1.04*</td>
<td>-0.86*</td>
<td>-0.46</td>
<td>0.07</td>
<td>-1.03</td>
<td>-1.47</td>
<td>-0.16</td>
<td>-0.21</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.57)</td>
<td>(0.47)</td>
<td>(0.45)</td>
<td>(0.15)</td>
<td>(0.77)</td>
<td>(0.93)</td>
<td>(0.22)</td>
<td>(0.61)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>CapRatio$_t$</td>
<td>-0.81</td>
<td>-0.47</td>
<td>-0.48</td>
<td>-1.20</td>
<td>-0.63**</td>
<td>-0.16</td>
<td>-0.23</td>
<td>0.35</td>
<td>-0.44</td>
<td>-0.28</td>
</tr>
<tr>
<td></td>
<td>(1.36)</td>
<td>(1.23)</td>
<td>(1.06)</td>
<td>(1.20)</td>
<td>(0.32)</td>
<td>(1.20)</td>
<td>(1.55)</td>
<td>(0.40)</td>
<td>(1.14)</td>
<td>(0.65)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.25***</td>
<td>0.39***</td>
<td>0.35***</td>
<td>0.29***</td>
<td>0.09***</td>
<td>0.38***</td>
<td>0.45***</td>
<td>0.04</td>
<td>0.13</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.056)</td>
<td>(0.06)</td>
<td>(0.04)</td>
<td>(0.12)</td>
<td>(0.16)</td>
<td>(0.03)</td>
<td>(0.11)</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

| Observations        | 172               | 172               | 172   | 172  | 152      | 62        | 100     | 44   | 102     | 132 |
| R$^2$               | 0.06              | 0.10              | 0.11  | 0.09  | 0.06     | 0.16      | 0.12    | 0.04  | 0.02    | 0.06 |
| Adjusted R$^2$      | 0.05              | 0.09              | 0.10  | 0.08  | 0.04     | 0.13      | 0.10    | -0.005| -0.003  | 0.04 |

Note: *p<0.1; **p<0.05; ***p<0.01
Table G.3. Predictive regressions: \( y_{data} \) and \( cay_t \).
This table provides results for one-year ahead predictive regressions according to \( R_{t+1}^{\text{CRSP}} = \gamma_0 + \gamma_{y} y_t + \gamma_{cay} cay_t + \varepsilon_{t+1} \), using lagged equity share of broker-dealers in the financial sector, \( y_{data} \) defined in (30) and lagged \( cay \) variable of Lettau and Ludvigson (2001), as predictors. The dependent variables are excess holding period returns from quarter \( t+1 \) to \( t+4 \) on the CRSP value-weighted (\( \text{Mkt}_{t+1}^{\text{vw}} \)) and equally-weighted (\( \text{Mkt}_{t+1}^{\text{ew}} \)) portfolios, mean excess return on 25 Fama-French size and book-to-market (\( \text{FF25}_{t+1} \)), 10 momentum (\( \text{Mom}_{t+1} \)) portfolios, 10 maturity-sorted US government and 10 US corporate bond portfolios sorted on yield spreads (\( \text{US bonds}_{t+1} \)), mean excess returns on six sovereign bonds (\( \text{Sov. bonds}_{t+1} \)), 54 portfolios of S&P 500 index options sorted on moneyness and maturity (\( \text{Options}_{t+1} \)), 20 CDS portfolios sorted by spreads (\( \text{CDS}_{t+1} \)), 23 commodity (\( \text{Commod.}_{t+1} \)), and 12 foreign exchange (\( \text{FX}_{t+1} \)) portfolios. Size/book-to-market and momentum portfolios and the risk-free rate data are from Ken French’s website. Data on sovereign bonds, options, CDS, commodities, and FX portfolios are from He et al. (2017). The \( cay \) data is downloaded from Martin Lettau’s website. The sample is quarterly from 1974Q1 to 2017Q3 for market, FF25 and momentum portfolios, and to 2012Q4 for HKM assets. Hodrick (1992) standard errors are reported in parentheses to adjust for the fact that overlapping quarterly observations are used to forecast annual returns.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Mkt(^{\text{vw}})</th>
<th>Mkt(^{\text{ew}})</th>
<th>FF25</th>
<th>Mom</th>
<th>US bonds</th>
<th>Sov. bonds</th>
<th>Options</th>
<th>CDS</th>
<th>Commod.</th>
<th>FX</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{data} )</td>
<td>-0.94***</td>
<td>-1.43***</td>
<td>-1.23***</td>
<td>-1.09***</td>
<td>-0.14</td>
<td>-0.90**</td>
<td>-1.06*</td>
<td>-0.02</td>
<td>-0.37</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.35)</td>
<td>(0.29)</td>
<td>(0.35)</td>
<td>(0.15)</td>
<td>(0.45)</td>
<td>(0.59)</td>
<td>(0.10)</td>
<td>(0.46)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>( cay_t )</td>
<td>3.21***</td>
<td>2.84*</td>
<td>2.63***</td>
<td>3.26***</td>
<td>0.49</td>
<td>1.66*</td>
<td>4.97***</td>
<td>0.81**</td>
<td>0.44</td>
<td>2.06*</td>
</tr>
<tr>
<td></td>
<td>(0.88)</td>
<td>(1.55)</td>
<td>(0.98)</td>
<td>(0.92)</td>
<td>(0.46)</td>
<td>(0.87)</td>
<td>(1.15)</td>
<td>(0.36)</td>
<td>(1.11)</td>
<td>(1.22)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.29***</td>
<td>0.43***</td>
<td>0.39***</td>
<td>0.33***</td>
<td>0.09***</td>
<td>0.33***</td>
<td>0.29*</td>
<td>0.03</td>
<td>0.13</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.05)</td>
<td>(0.07)</td>
<td>(0.04)</td>
<td>(0.11)</td>
<td>(0.13)</td>
<td>(0.02)</td>
<td>(0.11)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Observations</td>
<td>173</td>
<td>173</td>
<td>173</td>
<td>173</td>
<td>152</td>
<td>62</td>
<td>100</td>
<td>44</td>
<td>102</td>
<td>132</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.15</td>
<td>0.14</td>
<td>0.16</td>
<td>0.17</td>
<td>0.03</td>
<td>0.20</td>
<td>0.30</td>
<td>0.15</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.14</td>
<td>0.13</td>
<td>0.15</td>
<td>0.16</td>
<td>0.02</td>
<td>0.17</td>
<td>0.29</td>
<td>0.11</td>
<td>-0.01</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Note: *\( p < 0.1 \); **\( p < 0.05 \); ***\( p < 0.01 \)
Table G.4. The heterogeneous intermediary mimicking portfolio (HIMP): Comparing models with alternative projections

This table presents time-series regression results of heterogeneous intermediary mimicking portfolio (HIMP) on mimicking portfolios for the representative intermediary factors in AEM and HKM according to: \( \text{HIMP}_t = \alpha_{\text{MP}} + \beta'_{\text{FMP}} \text{FMP}_t + \epsilon_t \), where FMP is either the mimicking portfolio for broker-dealer leverage factor from AEM (AEM\_MP), or the mimicking portfolio for capital factor for primary dealers’ holding companies from HKM (HKM\_MP), or both AEM\_MP and HKM\_MP. The factor-mimicking portfolios are constructed by projecting the heterogeneous intermediary, AEM’s leverage, and HKM’s capital factors unto the space of equity and bond returns according to equations (36) and (37). The sample is quarterly from 1970Q1 to 2017Q3. Standard errors are in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{\text{MP}} )</td>
<td>1.21***</td>
<td>0.96***</td>
<td>0.95***</td>
<td>0.88***</td>
<td>0.92***</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.22)</td>
<td>(0.23)</td>
<td>(0.22)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>AEM MP</td>
<td>0.55***</td>
<td></td>
<td>0.23**</td>
<td>0.32***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td></td>
<td>(0.09)</td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>HKM MP</td>
<td>0.38***</td>
<td>0.39***</td>
<td>0.34***</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.11)</td>
<td>(0.04)</td>
<td>(0.14)</td>
<td></td>
</tr>
<tr>
<td>MktRF</td>
<td>-0.01</td>
<td></td>
<td></td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td></td>
<td></td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>191</td>
<td>191</td>
<td>191</td>
<td>191</td>
<td>191</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.16</td>
<td>0.35</td>
<td>0.35</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.15</td>
<td>0.35</td>
<td>0.35</td>
<td>0.37</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Note: *\( p<0.1; **p<0.05; ***p<0.01 \)
Table G.5. VAR parameter estimates.
This table presents Maximum likelihood estimates and t-statistics for the VAR in equation (39): $AY_{t+1} = F + GY_t + H_t e_{t+1}$, with restrictions $A_{11} = A_{22} = A_{33} = 1$ and $A_{12} = A_{13} = A_{23} = 0$. I assume the following quadratic functions for state variable diffusions: $\sigma_x = x(1 - x)/(ay + b)$ and $\sigma_y = y(1 - y)/(dx + e)$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>0.0053</td>
<td>0.02</td>
</tr>
<tr>
<td>$F_2$</td>
<td>0.0057</td>
<td>0.01</td>
</tr>
<tr>
<td>$F_3$</td>
<td>0.0053</td>
<td>0.77</td>
</tr>
<tr>
<td>$G_{11}$</td>
<td>0.3807</td>
<td>1.76</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>-0.0048</td>
<td>-0.15</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>0.0041</td>
<td>0.24</td>
</tr>
<tr>
<td>$G_{21}$</td>
<td>1.4549</td>
<td>0.83</td>
</tr>
<tr>
<td>$G_{22}$</td>
<td>-0.0281</td>
<td>0.35</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>0.0745</td>
<td>2.01</td>
</tr>
<tr>
<td>$G_{31}$</td>
<td>3.3587</td>
<td>2.38</td>
</tr>
<tr>
<td>$G_{32}$</td>
<td>0.0106</td>
<td>-0.27</td>
</tr>
<tr>
<td>$G_{33}$</td>
<td>-0.1628</td>
<td>-2.65</td>
</tr>
<tr>
<td>$A_{21}$</td>
<td>1.3206</td>
<td>0.69</td>
</tr>
<tr>
<td>$A_{31}$</td>
<td>3.1218</td>
<td>2.47</td>
</tr>
<tr>
<td>$A_{32}$</td>
<td>0.1459</td>
<td>1.05</td>
</tr>
<tr>
<td>$\eta_x$</td>
<td>0.0551</td>
<td>19.79</td>
</tr>
<tr>
<td>$\eta_y$</td>
<td>0.1086</td>
<td>18.89</td>
</tr>
<tr>
<td>$a$</td>
<td>7.2376</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>3.3396</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>-9.4334</td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>3.2996</td>
<td></td>
</tr>
</tbody>
</table>