Measuring Uncertainty

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Motivation

- How important is time-varying uncertainty for explaining macroeconomic fluctuations?
  - Theories (e.g., Bloom 2009): uncertainty typically defined as volatility of disturbance unforecastable to economic agents.
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- Proxies used instead: implied or realized stock market volatility; cross-sectional dispersion in profits, returns, productivity, subjective (survey-based) forecasts; newspaper keywords.

- Unfortunately, proxies may not be tightly linked to standard theoretical notions of uncertainty:
Stock market volatility may fluctuate for lots of reasons other than uncertainty, e.g., leverage, risk-aversion, sentiment.
Limitations of Common Uncertainty Proxies

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  - VIX has large component driven by factors associated with time-varying risk-aversion (Bekaert, Hoerova, Duca, 2012)
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- Subjective survey-based forecasts found to be biased and omit relevant forecasting information (So, 2012)

- Disagreement in survey point forecasts could be more reflective of differences in opinion than uncertainty (Diether et. al, 2002, Mankiw et. al., 2003) and do not in general equal average forecast uncertainty across analysts (Lahiri and Sheng, 2010).
Goal: Superior Econometric Estimates of Uncertainty

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- Goal: superior econometric estimates of uncertainty that are:

  1. As free as possible from structure of specific theoretical models
  2. Free from dependencies on any single (or small number) of observable economic indicators

Premise: key for decision making is not whether economic indicators have become more or less variable or disperse... But rather whether economy has become more or less predictable, that is more or less uncertain.
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- But rather whether economy has become more or less predictable, that is more or less uncertain.
Defining Uncertainty

- Define $h$-period ahead uncertainty in variable $y_{jt}$ for $j = 1, \ldots, N_y$:

$$U_{jt}^y(h) \equiv E \left[ (y_{j,t+h} - E[y_{j,t+h} | I_t])^2 | I_t \right]$$

$I_t$ is information set of economic agents.
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- Aggregate across $j$ to get a measure of macroeconomic uncertainty:

$$U_t^y(h) \equiv \text{plim}_{N \to \infty} \sum_{j=1}^{N_y} w_j U_{jt}^y(h) \equiv E_w[U_{jt}^y(h)].$$
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  One example of weights $w_j$: eigenvector corresponding to the largest eigenvalue of the $N_y \times N_y$ covariance matrix of $\mathcal{U}_{jt}(h)$. If this eigenvalue diverges as $N_y \to \infty$, macro uncertainty interpreted as *common latent factor* in individual measures of uncertainty.
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\[ U_{jt}(h) \equiv E \left[ (y_{j,t+h} - E[y_{j,t+h}|I_t])^2 | I_t \right] \]  

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- Emphasize two features of this definition:
  1. Proper measurement of uncertainty requires *removing forecastable component* \( E[y_{j,t+h}|I_t] \).

Either way, expect to find evidence of macroeconomic uncertainty in many indicators, across sectors, markets, regions.
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  - Proper measurement of uncertainty requires *removing forecastable component* \( E[y_{j,t+h}|I_t] \).
   - Failure to do so lead to estimates that erroneously characterize forecastable variations as “uncertain”.
   - Nearly all measures of stock market volatility or dispersion do not take this into account.
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  2. Macro uncertainty **not** equal to uncertainty in any single series.
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    - It is the *common* variation in uncertainty across many series.
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     - It is the common variation in uncertainty across many series
     - Eliminates dependence of uncertainty on any particular series.
     - Uncertainty theories of b. cycle require common countercyclical variation in uncertainty across large number of series.

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- Other theories: common countercyclical variation in volatilities of idiosyncratic firm-level shocks.
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Either way, expect to find evidence of macroeconomic uncertainty in many indicators, across sectors, markets, regions.
Econometric Objective

- Objective: obtain estimates of objects in previous 2 equations.

1. Estimate of $E[y_{jt} + h | I_t]$. Use large set of predictors $\{X_{it}\}_{i=1}^N$ and linear diffusion index forecasts appropriate for data-rich environments.

2. Require estimate of volatility of $h$-step ahead forecast error $\nu_{yt}(h) \equiv y_{jt} + h - E[y_{j,t+h} | I_t]$, for a large number of $y_{jt}$:
   - Parametric stochastic volatility model for $\{\nu_{yt}(1)\}$, then recursively compute the values of $E[\nu_{yt}(h) | I_t]$ implied by the SV model for $h > 1$. This delivers an estimate of $U_{yt}(h)$.

3. Estimate of $U_{yt}(h)$ from $U_{yt}(h)$. Base-case: common latent factor of individual uncertainties estimated using the method of principal components (PCA) for large datasets.
Econometric Objective

- **Objective**: obtain estimates of objects in previous 2 equations.
- **To make operational**, require 3 key ingredients:
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  3. **Estimate of** $\mathcal{U}_{jt}^{y}(h)$ from $\mathcal{U}_{jt}^{y}(h)$. Base-case: common latent factor of individual uncertainties estimated using the method of principal components (PCA) for large datasets.
Two Datasets

- **Macro dataset**: *Monthly* data on mostly macro series + financial market data
  - Monthly data on 132 mostly macro series + aggregate stock and bond market returns
  - Monthly data on 147 financial indicators

- **Firm-level dataset**: *Quarterly* data on 155 firm-level observations on profit growth normalized by sales.

- Measures **macro uncertainty** constructed from both datasets:
  - Common macro uncertainty
  - Common firm-level uncertainty
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1. Far fewer uncertainty episodes than suggested by popular proxies.

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  2. Our estimates *more persistent* than common proxies
  
  3. Although fewer, have *larger* effects on real activity.
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  - Although fewer, have larger effects on real activity.

- Conclude: lots of movements in uncertainty proxies not attributable to genuine uncertainty.
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    - We find just 3 such episodes of macro uncertainty and common-firm level uncertainty.
  - Our estimates *more persistent* than common proxies
  - Although fewer, have *larger* effects on real activity.

- Indeed, *most* movements in stock market volatility and XS dispersion are *not* associated with broad-based movement in economic uncertainty.
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3. Although fewer, have *larger* effects on real activity.

Common proxies are, at best, very *noisy* measures of uncertainty.
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- Positive shocks to macro uncertainty lead to sizable declines in real activity, but no “overshooting” effect.
Key Results


- Averaged across all uncertainty horizons, $h$, **2007-09 recession the most striking episode** heightened uncertainty since 1960, with 1981-82 a close second.

- Macro uncertainty much higher in **recessions** than non-recessions.

- Positive shocks to macro uncertainty lead to sizable declines in real activity, but no “overshooting” effect.

- When proxies are “purged” of noise, uncertainty has **larger, more protracted** effects on real activity.
Related Literature

- **Theory papers** on uncertainty: Bloom ’09; Bloom et. al., ’10; Gilchrist et. al., ’10; Arellano et al., ’11; Bachmann and Bayer ’11; Baker et. al., ’11; Basu and Bundick, ’11; Knotek and Khan, ’11; Villaverde et. al., ’11; Bachmann et. al., ’12; Bloom et. al., ’12; Leduc and Lui, ’12; Nakamura et. al., ’12; Orlik and Veldkamp; ’13.
Related Literature

- **Stock market volatility as a proxy for uncertainty**: Romer, ’90; Leahy and Whitehead, ’96; Hassler, ’01; Bloom et. al., ’07; Greasley and Madsen, ’06; Bloom, ’09; Basu and Bundick, ’12.

- **Cross-sectional dispersion** in firm profits, productivity as proxy: Bloom, ’09; Bloom, et. al., ’12; Christiano, Motto, and Rostagno, ’13.

- **Dispersion in subjective forecasts** as proxy: Bachman, Elstner, and Sims, ’12; Scotti, ’12.

- **Diffusion index forecasts** and factor models: Stock and Watson ’02a, ’02b; Ludvigson and Ng ’07, ’09.
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- For each $y_{jt}, j = 1, \ldots, N_y$, we specify:

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forecastable

$$\nu_{j,t+1}^y = \sigma_{j,t+1}^y \varepsilon_{j,t+1}^y$$

unforecastable

stochastic vol

$$\log[(\sigma_{j,t+1}^y)^2] = \alpha_j^y + \beta_j^y \log(\sigma_{jt}^y)^2 + \tau_j^y \eta_{j,t+1}$$

where $\varepsilon_{j,t+1}$ and $\eta_{j,t+1}$ are iid $N(0, 1)$ random variables.
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\[
\begin{align*}
    y_{j,t+1} &= E[y_{jt+1} | I_t] + \varphi_{j,t+1}^y \\
    \varphi_{j,t+1}^y &= \sigma_{j,t+1}^y \varepsilon_{j,t+1}^y \\
    \log[(\sigma_{j,t+1}^y)^2] &= \alpha_j^y + \beta_j^y \log(\sigma_{jt}^y)^2 + \tau_j^y \eta_{j,t+1},
\end{align*}
\]

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Estimation:

1. $\hat{E}[y_{jt+1} | I_t]$ using diffusion index forecasts.
2. $\log(\hat{\sigma}_{jt}^y)^2$ stochastic volatility estimates, using Kim, Shephard, and Chib (1998, RES) algorithm on $\hat{\varphi}_{j,t}$. 

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Diffusion Index Forecasts

- Crucial step in analysis: estimate of conditional expectation, from which construct forecast error.

Most models of uncertainty: agents know economic state. Econometricians have less information. Important: predictive model be as rich as possible, so measured forecast error is purged of predictive content. Omitted information not used to form forecasts will lead to spurious estimates of uncertainty and its dynamics. Address this issue by using diffusion index forecasts: a relatively small number of factors estimated from large (e.g., hundreds or even thousands) economic time-series. Omitted information problem remedied by including estimated factors, possibly non-linear functions of factors, as predictors.
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- Important: predictive model be as rich as possible, so measured forecast error is *purged of predictive content*.

- Omitted information not used to form forecasts will lead to spurious estimates of uncertainty and its dynamics.

- Address this issue by using *diffusion index forecasts*: a relatively small number of factors estimated from large (e.g., hundreds or even thousands) economic time-series.
Diffusion Index Forecasts

- Crucial step in analysis: estimate of conditional expectation, from which construct forecast error.

- Most models of uncertainty: agents know economic state. Econometricians have less information.

- Important: predictive model be as rich as possible, so measured forecast error is purged of predictive content.

- Omitted information not used to form forecasts will lead to spurious estimates of uncertainty and its dynamics.

- Address this issue by using diffusion index forecasts: a relatively small number of factors estimated from large (e.g., hundreds or even thousands) economic time-series.

- Omitted information problem remedied by including estimated factors, possibly non-linear functions of factors, as predictors.
**Diffusion Index Forecasts**

- **Large number** of predictors: $X_{it}, i = 1, \ldots, N_X$. Estimate approximate factor model:

  $$X_{it} = \Lambda_i^{F'} F_t + e_{it}^X.$$  

- Latent factors $F_t$ are $r_F \times 1$ with $r_F \ll N_X$. Given additional predictors $W_t$ of dimension $r_W$, forecast $y_{j,t+1}$ according to:

  $$y_{jt+1} = \phi_j^y(L)y_{jt} + \gamma_j^F(L)\hat{F}_t + \gamma_j^W(L)W_t + \nu_{jt+1}^y.$$  

- Then set:

  $$\hat{E}[y_{j,t+1}|I_t] = \hat{\phi}_j^y(L)y_{jt} + \hat{\gamma}_j^F(L)\hat{F}_t + \hat{\gamma}_j^W(L)W_t,$$
  $$\hat{\sigma}_{j,t+1}^y = y_{j,t+1} - \hat{E}[y_{j,t+1}|I_t].$$

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Stochastic Volatility Estimates

- From the model, $v_{j,t+1}^y = \sigma_{j,t+1}^y \epsilon_{j,t+1}^y$. Take logs:
  \[
  \log[(v_{j,t+1}^y)^2] = \log[(\sigma_{j,t+1}^y)^2] + \log[(\epsilon_{j,t+1}^y)^2]
  \]
  \[
  \log[(\sigma_{j,t+1}^y)^2] = \alpha_j^y + \beta_j^y \log[(\sigma_{jt}^y)^2] + (\tau_j^y) \eta_{j,t+1}.
  \]
- Has the state-space representation
  \[
  z_{jt} = x_{jt} + \epsilon_{jt} \quad \text{observation equation}
  \]
  \[
  x_{jt} = \alpha_j + \beta_j x_{jt-1} + \tau_j \eta_{jt} \quad \text{state equation}
  \]
- **Difficulty**: $\epsilon_{j,t} \equiv \log(\epsilon_{j,t}^y)^2 \sim \log \chi^2(1)$.
- **Solution**: Kim, Shephard, and Chib (1998, RES) MCMC mixture of normals approximation:
  \[
  p(\epsilon) = \sum_{k=1}^{K} \pi_k \phi(\epsilon; m_k, s_k^2).
  \]
Computing Individual Uncertainty \((h = 1)\)

- Using its definition,

\[
\mathcal{U}_{jt}^y(1) = E[(\sigma_{j,t+1}^y)^2 (\varepsilon_{j,t+1}^y)^2 | I_t] \\
= E[(\sigma_{j,t+1}^y)^2 | I_t] \\
= \exp \left\{ \alpha_j^y + \beta_j^y \log(\sigma_{jt}^y)^2 + \frac{1}{2} (\tau_j^y)^2 \right\}.
\]

- The last equality follows from the AR(1) law of motion for

\(\log(\sigma_{j,t+1}^y)^2\), and the normality of \(\eta_{j,t+1}\).

- Given estimates: \(\hat{\alpha}_j^y, \hat{\beta}_j^y, (\hat{\tau}_j^y)^2\), and \(\left\{\log(\sigma_{jt}^y)^2\right\}_{t=1}^T\), compute \(\hat{\mathcal{U}}_{jt}^y(1)\) using this expression.
Computing Individual Uncertainty ($h \geq 1$)

- Define $q = \max(lags_y, lags_F, lags_w, h)$
- Let $\mathcal{Z}_t \equiv (\hat{F}_t', W_t')'$ and define $\mathcal{F}_t \equiv (\mathcal{Z}_t, \ldots, \mathcal{Z}_{t-q+1})'$ and $\mathcal{Y}_{jt} \equiv (y_{jt}, \ldots, y_{j, t-q+1})'$:

\[
\begin{pmatrix}
\mathcal{F}_t \\
\mathcal{Y}_{jt}
\end{pmatrix} =
\begin{pmatrix}
\Phi^F \\
\Lambda^j \\
\Phi^\gamma_j
\end{pmatrix}
\begin{pmatrix}
\mathcal{F}_{t-1} \\
\mathcal{Y}_{j, t-1}
\end{pmatrix} +
\begin{pmatrix}
V_{jt}^F \\
V_{jt}^\gamma
\end{pmatrix}
\]

\[
\mathcal{Y}_{jt} = \Phi^\gamma_j \mathcal{Y}_{j, t-1} + V_{jt}^\gamma.
\]
Computing Individual Uncertainty \((h \geq 1)\)

- Define \(q = \max (\text{lags}_y, \text{lags}_F, \text{lags}_w, h)\)
- Let \(\mathcal{Z}_t \equiv (\hat{F}_t', W_t')'\) and define \(\mathcal{F}_t \equiv (\mathcal{Z}_t, \ldots, \mathcal{Z}_{t-q+1})'\) and \(\mathcal{Y}_{jt} \equiv (y_{jt}, \ldots, y_{j,t-q+1})'\):

\[
\begin{pmatrix}
\mathcal{F}_t \\
\mathcal{Y}_{jt}
\end{pmatrix} = \begin{pmatrix}
\Phi^F & 0 \\
\Lambda_j' & \Phi_j^Y
\end{pmatrix}
\begin{pmatrix}
\mathcal{F}_{t-1} \\
\mathcal{Y}_{j,t-1}
\end{pmatrix} + \begin{pmatrix}
V_t^F \\
V_j^Y
\end{pmatrix}.
\]

\(\mathcal{Y}_{jt} = \Phi_j^Y \mathcal{Y}_{j,t-1} + V_j^Y.\)

- **Uncertainty** \(U_{jt}^Y(h) \equiv E_t[(\mathcal{Y}_{j,t+h} - E_t[\mathcal{Y}_{j,t+h}])^2].\) The following recursion holds (with \(U_{jt}^Y(0) \equiv 0\)):

\[
U_{jt}^Y(h) = \Phi_j^Y U_{jt}^Y(h-1) \Phi_j^Y' + E_t[V_{j,t+h}^Y V_{j,t+h}^{Y'}],
\]

- Then \(h\)-period ahead **uncertainty in** \(y_{jt}\) is

\[
U_{jt}^Y(h) = 1_j' U_{jt}^Y(h) 1_j.
\]

1\(_j\) a selection vector picks out the element for uncertainty in \(y_{j,t}\).
Sources of Uncertainty

- Uncertainty is \textit{not} equal to stochastic volatility in residuals $\nu_{jt}^Y$ unless $h = 1$.

\[ U_{jt}^Y(h) = \Phi_j^Y U_{jt}^Y(h-1) \Phi_j^Y' + U_{jt}^Z(h-1) + E_t(V_{jt+h}^Y V_{jt+h}^Y') \]

\[ + 2\Phi_j^Y U_{jt}^{YZ}(h-1) \]

- Autoregressive
- Factor
- Stochastic volatility $Y$
- Covariance
Sources of Uncertainty

- Uncertainty is *not* equal to stochastic volatility in residuals $v_{jt}^y$ unless $h = 1$.

$$U_{jt}^Y(h) = \Phi_j^Y U_{jt}^Y(h-1) \Phi_j^Y' + U_{jt}^Z(h-1) + E_t(V_{jt+h}^Y V_{jt+h}^Y')$$

  - **Autoregressive** component when $h > 1$
  - **Factor**
  - **Stochastic volatility $Y$**
  - **Covariance** $2\Phi_j^Y U_{jt}^{YZ}(h-1)$
Sources of Uncertainty

- Uncertainty is *not* equal to stochastic volatility in residuals $\nu_{jt}^Y$ unless $h = 1$.

$$U_{jt}^Y(h) = \Phi_j^Y U_{jt}^Y(h-1) \Phi'_j + U_{jt}^Z(h-1) + E_t(V_{jt+h}^Y V_{jt+h}^Y')$$

- **Autoregressive** component when $h > 1$
- **Factor** component: stochastic volatility in $F_t$ and $W_t$ contribute to uncertainty when $h > 1$
Sources of Uncertainty

- Uncertainty is *not* equal to stochastic volatility in residuals $\nu_{jt}^y$ unless $h = 1$.

$$
\mathcal{U}_jt(h) = \Phi_j^Y \mathcal{U}_jt(h - 1) \Phi_j^Y' + \mathcal{U}_jt(h - 1) + E_t(V_{jt+h}^Y V_{jt+h}^Y') \\
+ 2\Phi_j^Y \mathcal{U}_jt^{YZ}(h - 1)
$$

- **Autoregressive** component when $h > 1$
- **Factor** component: *stochastic volatility in $F_t$ and $W_t$ contribute to uncertainty when $h > 1$
- **Covariance** component: $\text{cov}(y_{t+h} - y_{t+h}|t, F_{t+h} - F_{t+h}|t)$, non-zero when $h > 2$. 
Macro Uncertainty

- Wanted: estimate of common *macro uncertainty* \( \mathcal{U}_t^y(h) \), some weighted average of \( \mathcal{U}_{jt}^y(h) \).

\[
\log \mathcal{U}_{jt}^y(h) = c \mathcal{U}_j(h) + \Lambda \mathcal{U}' h F \mathcal{U}_t(h) + e \mathcal{U}_{jt}(h)
\]

Choose weights to minimize volatility of \( e \mathcal{U}_{jt}(h) \) (PCA).

Although \( \mathcal{U}_t^y(h) > 0 \), PC estimates do not constrain estimated \( F \mathcal{U}_t(h) > 0 \). Use log specification.

Base-case estimate of macro uncertainty \( \mathcal{U}_t^y(h) \) is \( \hat{\mathcal{U}}_t^y(h) = \exp(\hat{F} \mathcal{U}_t(h)) \).
Macro Uncertainty

- Wanted: estimate of common *macro uncertainty* $\mathcal{U}_t^y(h)$, some weighted average of $\mathcal{U}_{jt}^y(h)$.

- Could equally weight, but may not be efficient. If uncertainty has a factor structure

  \[
  \log \mathcal{U}_{jt}^y(h) = c_j^U(h) + \Lambda_{hj}^U F_t^U(h) + e_{jt}^U(h)
  \]

  choose weights min volatility of $e_{jt}^U(h)$ (PCA).

Although $\mathcal{U}_{jt}^y(h) > 0$, PC estimates do not constrain estimated $F_t^U(h) > 0$. Use log specification.

Base-case estimate of macro uncertainty $\mathcal{U}_t^y(h)$ is $\hat{\mathcal{U}}_t^y(h) = \exp(\hat{F}_t^U(h))$. 

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Macro Uncertainty

- Wanted: estimate of common macro uncertainty $U^y_t(h)$, some weighted average of $U^y_{jt}(h)$.

- Could equally weight, but may not be efficient. If uncertainty has a factor structure

$$\log U^y_{jt}(h) = c^U_j(h) + \Lambda_{hj}^U F^U_t(h) + e^U_{jt}(h)$$

choose weights min volatility of $e^U_{jt}(h)$ (PCA).

- Although $U^y_{jt}(h) > 0$, PC estimates do not constrain estimated $F^U_t(h) > 0$. Use log specification.
Macro Uncertainty

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- Although $U_{jt}^y(h) > 0$, PC estimates do not constrain estimated $F_t^U(h) > 0$. Use log specification.

- Base-case estimate of macro uncertainty $U_t^y(h)$ is

$$\hat{U}_t^y(h) = \exp \left( \hat{F}_t^U(h) \right).$$
Importance of Macro Uncertainty in Total Uncertainty

- Uncertainty a function of a constant (homoskedastic) component, an idiosyncratic time-varying component, and a common (macro) time-varying component.

- To assess importance of macro uncertainty in total uncertainty:

$$R^2_{jt}(h) = \frac{(\hat{\Lambda}_{hj})' \Delta \hat{F}_t^U (h) \Delta \hat{F}_t^{U'} (h) \hat{\Lambda}_{hj}}{(\Delta \log U_{jt}^y)' (\Delta \log U_{jt}^y)}$$

average over $t$ for each $j$. 

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Data

- \( X_t^m \): 132 monthly macroeconomic series, 1960:01-2011:12.

- Updated from Ludvigson and Ng (2009, RFS).
- Output and income; hours, employment; retail, manufacturing, trade sales; housing starts; consumption, orders, inventories; money and credit; bond and exchange rates; prices; stock return.

- \( X_t^f \): 147 monthly financial series, 1960:01-2011:12.
- Updated from Ludvigson and Ng (2007, JFE).

- Change in pre-tax profits, normalized by 2 period MA of sales, as in Bloom '09.
- Use year-over-year version, due to seasonality.
Data

\[ X_t^m: 132 \text{ monthly macroeconomic series, 1960:01-2011:12.} \]
\[ \text{a. Updated from Ludvigson and Ng (2009, RFS).} \]
Data

  a. Updated from Ludvigson and Ng (2009, RFS).
  b. Output and income; hours, employment; retail, manufacturing, trade sales; housing starts; consumption, orders, inventories; money and credit; bond and exchange rates; prices; stock return.

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  a. Change in pre-tax profits, normalized by 2 period MA of sales, as in Bloom '09.
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Data

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  a. Updated from Ludvigson and Ng (2007, JFE).
Data

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  a. Updated from Ludvigson and Ng (2007, JFE).

  a. Change in pre-tax profits, normalized by 2 period MA of sales, as in Bloom ’09.
Data

  - a. Updated from Ludvigson and Ng (2009, RFS).
  - b. Output and income; hours, employment; retail, manufacturing, trade sales; housing starts; consumption, orders, inventories; money and credit; bond and exchange rates; prices; stock return.

  - a. Updated from Ludvigson and Ng (2007, JFE).

  - a. Change in pre-tax profits, normalized by 2 period MA of sales, as in Bloom ’09.
  - b. Use year-over-year version, due to seasonality.
Implementation: Macro Dataset

For the forecasting equation:

\[ y_{jt+1} = \phi^y_j(L)y_{jt} + \gamma^F_j(L)\hat{F}_t + \gamma^W_j(L)W_t + \sigma^y_{j,t+1}. \]

(1) \( \hat{F}_t \): 12 factors estimated from \( X_t = [X^m_t : X^f_t] \).
Implementation: Macro Dataset

For the forecasting equation:

\[ y_{j,t+1} = \phi^y_j(L)y_{jt} + \gamma^F_j(L)\hat{F}_t + \gamma^W_j(L)W_t + \nu^y_{j,t+1}. \]

(1) \( \hat{F}_t \): 12 factors estimated from \( X_t = [X^m_t : X^f_t] \).

(2) \( Y_t \equiv (y_{1t}, \ldots, y_{Nt})' = X^m_t \). Uncertainty constructed for \( y_{jt} = X^m_{jt} \).
Implementation: Macro Dataset

For the forecasting equation:

\[ y_{j,t+1} = \phi_j^y(L)y_{jt} + \gamma_j^F(L)\hat{F}_t + \gamma_j^W(L)W_t + \nu_{j,t+1}^y. \]

(1) \( \hat{F}_t \): 12 factors estimated from \( X_t = [X^m_t : X^f_t] \).
(2) \( Y_t \equiv (y_{1t}, \ldots, y_{Nt})' = X^m_t \). Uncertainty constructed for \( y_{jt} = X^m_{jt} \).
(3) \( W_t = [\hat{F}^2_{1t} : \hat{G}_t] \).

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Implementation: Macro Dataset

For the forecasting equation:

\[ y_{j,t+1} = \phi_j^y(L)y_{jt} + \gamma_j^F(L)\hat{F}_t + \gamma_j^W(L)W_t + \nu_{j,t+1}^y. \]

(1) \( \hat{F}_t \): 12 factors estimated from \( X_t = [X_t^m : X_t^f] \).

(2) \( Y_t \equiv (y_{1t}, \ldots, y_{Nt})' = X_t^m \). Uncertainty constructed for \( y_{jt} = X_{jt}^m \).

(3) \( W_t = [\hat{F}_{1t}^2 : \hat{G}_t] \).

(4) \( \hat{G}_t \): 1st factor estimated from \( X_t^2 \).
Implementation: Macro Dataset

For the forecasting equation:

\[ y_{j,t+1} = \phi_j^y(L)y_{jt} + \gamma_j^F(L)\hat{F}_t + \gamma_j^W(L)W_t + \nu_{j,t+1}^y. \]

(1) \(\hat{F}_t\): 12 factors estimated from \(X_t = [X^m_t : X^f_t]\).

(2) \(Y_t \equiv (y_{1t}, \ldots, y_{Nt})' = X^m_t\). Uncertainty constructed for \(y_{jt} = X^m_{jt}\).

(3) \(W_t = [\hat{F}^2_{1t} : \hat{G}_t]\).

(4) \(\hat{G}_t\): 1st factor estimated from \(X^2_t\).

(5) First-stage hard threshold for \([\hat{F}_t : W_t]\): \(t > 2.575\) (Bai and Ng ’08).
Implementation: Macro Dataset

For the forecasting equation:

\[ y_{jt, t+1} = \phi_j^y(L)y_{jt} + \gamma_j^F(L)\hat{F}_t + \gamma_j^W(L)W_t + \sigma^v_{jt, t+1}. \]

(1) \( \hat{F}_t \): 12 factors estimated from \( X_t = [X_t^m : X_t^f] \).
(2) \( Y_t \equiv (y_{1t}, \ldots, y_{Nt})' = X_t^m \). Uncertainty constructed for \( y_{jt} = X_{jt}^m \).
(3) \( W_t = [\hat{F}_t^2 : \hat{G}_t] \).
(4) \( \hat{G}_t \): 1st factor estimated from \( X_t^2 \).
(5) First-stage hard threshold for \( [\hat{F}_t : W_t] \): \( t > 2.575 \) (Bai and Ng ’08).
(6) Form estimate \( \hat{U}_t^y(h) \) as common latent factor in \( \hat{U}_{jt}^y(h) \) for each 132 \( y_{jt} = X_{jt}^m \) macro series.
Implementation: Firm-Level Dataset

For the forecasting equation:

\[ y_{jt+1} = \phi_j^y (L) y_{jt} + \gamma_j^F (L) \hat{F}_t + \gamma_j^W (L) W_t + \nu_{j,t+1}^y. \]

1. \( Y_t \equiv (y_{1t}, \ldots, y_{Nt})' = X_t^p. \) Uncertainty constructed for \( y_{jt} = X_{jt}^p \) series
2. \( \hat{F}_t: \) 2 factors estimated from \( X_t = [X_t^p], 3 \) from \( X_t = [(X_t^2)^p]. \)
3. \( W_t \) include macro forecasting factors.
4. First-stage hard threshold for \( [\hat{F}_t : W_t]: t > 2.575 \) (Bai and Ng ’08).
5. Form estimate \( \hat{U}_t^y (h) \) as common latent factor in \( \hat{U}_{jt}^y (h) \) for each \( y_{jt} = X_{jt}^p \) firm-level profit series.
Implementation

- Strong evidence of factor structure in individual uncertainties.

\[ \hat{U}^y_t(h) = \exp(\hat{F}^U_t(h)) \] taken as first common factor:
  - Problem determining number of factors non-standard b/c individual uncertainty measures are estimated. Existing criteria do not take into account first-step estimation error and will likely overestimate number of factors.

- Estimates of common factors (\( \hat{F}_t \) and \( \hat{F}^U_t(h) \)) can be treated as known–no need to correct standard errors in 2nd stage (Bai and Ng ’06).

- Put both \( \hat{F}_t(h) \) and \( \hat{U}^y_{jt}(h) \) in units of standard deviation: square root.
Results
Macroeconomic Uncertainty Factor

Uncertainty increases with horizon, $h$ months. Uncertainty is strongly countercyclical.

3 periods when uncertainty factor $> 1.65$ std above mean:
- 1973-74;
- 1981-82;
- 2007-09

On average across all $h$, 2007-09 most striking episode of heightened uncertainty, with 1981-82 a close 2nd.

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Macroeconomic Uncertainty Factor

- Uncertainty increases with horizon, $h$ months

![Graph showing uncertainty factor over time with shaded periods of increased uncertainty and correlation coefficients for different horizons.]

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Macroeconomic Uncertainty Factor

- Uncertainty is strongly countercyclical

![Graph showing the correlation between uncertainty and the industrial production index (IP)]

- U(1) with h=1, corr with IP = -0.66
- U(3) with h=3, corr with IP = -0.61
- U(12) with h=12, corr with IP = -0.57

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Macroeconomic Uncertainty Factor

- 3 periods when uncertainty factor > 1.65 std above mean:
  - 1973-74
  - 1981-82
  - 2007-09

![Graph showing macroeconomic uncertainty factor with dates and correlations.](attachment:image)
Macroeconomic Uncertainty Factor

On average across all $h$, 2007-09 most striking episode of heightened uncertainty, with 1981-82 a close 2nd.
Deepest, most protracted recessions in post-war period are those associated with heightened uncertainty.

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Importance of Macro Uncertainty

$R^2$ from series by series regressions log difference in uncertainty on log change in common uncertainty factor and constant.

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Importance of Macro Uncertainty

- Importance of macro uncertainty grows with $h$.

\[ R^2 \text{ from series by series regressions log difference in uncertainty on log change in common uncertainty factor and constant.} \]
Importance of Macro Uncertainty

- Macro uncertainty explains quantitatively large fraction, especially in recessions.

$R^2$ from series by series regressions log difference in uncertainty on log change in common uncertainty factor and constant.
Role of Predictors in Uncertainty

Plots of $h = 1$ month-ahead uncertainty, with and without removing forecastable component.
Failure to remove $E[y_{jt+h}|I_t]$ lead to estimates that erroneously categorize forecastable variations as “uncertain.”

Plots of $h = 1$ month-ahead uncertainty, with and without removing forecastable component.
Volatility vs. Uncertainty S&P 500 Index

$h = 1$ month-ahead uncertainty in S&P index, with and without removing forecastable components.

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Volatility v.s. Uncertainty S&P 500 Index

“Unconditional mean” most akin to VXO SM volatility index.

$h = 1$ month-ahead uncertainty in S&P index, with and without removing forecastable components.
Volatility v.s. Uncertainty S&P 500 Index

- Many spikes in SM volatility are not movements in uncertainty.

$h = 1$ month-ahead uncertainty in S&P index, with and without removing forecastable components.
Uncertainty vs. Stock Market Volatility

Macro uncertainty factor and VXO index, standardized units. Red horizontal lines give 17 uncertainty dates of Bloom ’09.
Uncertainty vs. Stock Market Volatility

- VXO index much more volatile than $\hat{\mathcal{U}}_t^y(h)$

Macro uncertainty factor and VXO index, standardized units. Red horizontal lines give 17 uncertainty dates of Bloom ’09.
Uncertainty vs. Stock Market Volatility

- Many spikes in VXO not present in $\hat{U}_t^y(h)$.

Macro uncertainty factor and VXO index, standardized units. Red horizontal lines give 17 uncertainty dates of Bloom ’09.
Bivariate VAR(12): VXO Index vs. $\hat{U}_t^{y}(12)$

- Dynamic relation between VXO and uncertainty in bivariate VAR

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.
Bivariate VAR(12): VXO Index vs. $\hat{U}_t^{y}(12)$

- Shocks to uncertainty lead to increase in VXO

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.
Bivariate VAR(12): VXO Index vs. $\hat{U}_t^{y}(12)$

- But shocks to VXO lead to decline in uncertainty

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.

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Bivariate VAR(12): VXO Index vs. $\hat{U}_t^y(12)$

- Substantial *independent variation* in uncertainty and stock market volatility

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.
Bivariate VAR(12): VXO Index vs. $\hat{U}_t^{y}(12)$

- Shocks to uncertainty much more *persistent* than shocks to VXO

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.

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Bivariate VAR(12): VXO Index vs. $\hat{U}_t^{y}(12)$

- Half-life VXO to its own shock is 3 months; half-life of $\hat{U}_t^{y}(12)$ to its own shock is 54 months.

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.
Estimate VAR(12) from 1960:01-2011:12 with endogenous variables:

\[
\begin{bmatrix}
\log(\text{S&P 500 Index}) \\
\text{uncertainty factor} \\
\text{federal funds rate} \\
\log(\text{wages}) \\
\log(\text{CPI}) \\
\text{hours} \\
\log(\text{employment}) \\
\log(\text{industrial production})
\end{bmatrix}.
\]

(1) federal funds rate: effective federal funds rate (FRED: FEDFUNDS).
(2) wage: average hourly earning in manufacturing (FRED: CES30000000008).
(3) CPI: all urban consumers, all items (FRED: CPIAUCSL).
(4) hours: average weekly hours in manufacturing (FRED: AWHMAN).
(5) employment: all employees in manufacturing (FRED: MANEMP).
Eight-variable VAR(12)

Shocks to uncertainty sharply reduce real activity. Magnitude of responses larger using $U_y(t)$ than VXO. No volatility "over-shoot" using $U_y(t)$ like when using VXO.

Responses more protracted using $U_y(t)$ than when using VXO.

Responses of variable in column to shock named in row, 4 standard deviation shocks.

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Shocks to uncertainty sharply reduce real activity

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### Variance Decomposition: 8 Variable VARs

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Fraction 12-month ahead VAR forecast error variance of variable in row explained by shocks to variable in column.

*Jurado, Ludvigson, and Ng*  
*Measuring Uncertainty*  
*May 2013*
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- Uncertainty associated with *less than 4%* of variation in VXO.

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Fraction 12-month ahead VAR forecast error variance of variable in row explained by shocks to variable in column.
Variance Decomposition: 9 Variable VAR

- Uncertainty associated with *less than 4%* of variation in VXO.
- Suggests *most movements in VXO* not associated with movement in broad-based economic uncertainty.

<table>
<thead>
<tr>
<th></th>
<th>SPX</th>
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Comparison of $\hat{U}_t^y (12)$ with Measures of Dispersion

Stock return dispersion, like VXO, lots of spikes not present in macro uncertainty. Profit dispersion suggests low level of uncertainty 1980-82 when macro uncertainty high, increases at end 1982, when $\hat{U}_t^y$ was low.

GDP forecast dispersion as high in many periods as in 2007-09, contrast to macro uncertainty.

Each series standardized.

Jurado, Ludvigson, and Ng

Measuring Uncertainty

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Much independent variation in dispersion and uncertainty

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.
Shocks to macro uncertainty more persistent

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.
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Positive shock to $\hat{U}_t^y(12)$ leads to *decline* in dispersion

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VAR: XS Dispersion GDP Forecasts \& \hat{U}_t^y (12)

Orthogonalized shocks with VXO ordered first. Responses of first variable to shock in second named variable plotted.
Common Firm-Level Uncertainty


\[\text{Jurado, Ludvigson, and Ng} \]

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Common Firm-Level Uncertainty

Unlike common macro uncertainty, sharp rise in “tech” bust 2000-01.
Comparison to Cross-Section Dispersion

- Like VXO, many movements in *XS dispersion firm profit growth not* driven by broad uncertainty across firms.

Both series in standardized units.

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Comparison to Cross-Section Dispersion

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Importance of Common Firm-Level Uncertainty

- Importance of common firm-level uncertainty grows with $h$.

$R^2$ from series by series regressions log difference in uncertainty on log change in common uncertainty factor and constant.

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Importance of Common Firm-Level Uncertainty

- Common uncertainty explains quantitatively large fraction, especially in recessions.

$R^2$ from series by series regressions log difference in uncertainty on log change in common uncertainty factor and constant.
Conclusion

- This paper: introduces new econometric measures of uncertainty.
  - Goal: insure measures are free as possible from restrictions imposed by theoretical models, and from empirical dependencies on handful economic indicators.

1. Far fewer large uncertainty episodes, just 3 since 1960.
2. Larger associations with real economic activity.
3. More persistent, more prolonged effects.

Macro uncertainty associated with much larger fraction of variability in production and hours, than VXO, other proxies.

Uncertainty strongly countercyclical, most striking episode 2007-09.

Conclude macro uncertainty shocks made a quantitatively important contribution to the observed historical variation in real activity.
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- Our measures of macro uncertainty fluctuate quite distinctly from popular proxies for uncertainty.
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- **Caution**: we do not directly address the direction of causality
  - (Is uncertainty the cause or effect of recessions?)

- But we have found that the economy is *objectively less predictable* in recessions than it is in normal times.
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- **Caution**: we do not directly address the direction of causality
  - (Is uncertainty the cause or effect of recessions?)

- But we have found that the economy is *objectively less predictable* in recessions than it is in normal times.

- Any theory for which uncertainty is entirely the *effect of recessions* would need to be consistent with this basic fact.
APPENDIX
Define log of squared forecast error as $z_t$.

$$ z_t = x_t + v_t $$

$$ x_t = \alpha + \beta x_{t-1} + \tau \eta_t $$

where $x_t = \log(\sigma_t^2)$ and $v_t \equiv \log(\epsilon_t^2)$.

Let $\theta \equiv (\alpha, \beta, \tau^2)$.

Given conjugate priors

$$ (\alpha, \beta | \tau^2) \sim \mathcal{N} (a, \tau^2 A) $$

$$ \tau^2 \sim \mathcal{IG}(b/2, B/2) $$

Posterior

$$ p(\theta | x, z) \sim \mathcal{N\mathcal{IG}} (a_T, A_T, b_T, B_T) $$

where parameters $a_T$ etc follow a recursion.

Goal: Create Markov chain with ergodic dist equal to posterior of interest $p(x, \theta | z)$. Markov chain is sequence of draws $x(s)$ and $\theta(s)$. 

---

Kim Shephard, Chib ’98 FFBS Algorithm

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Kim Shephard, Chib ’98 FFBS Algorithm

An efficient algorithm for sampling \( x^{(s+1)} \) from \( p(x|\theta^{(s)}, z) \) when this distribution is conditionally normal.

1. Initialize \( \theta^{(s)}, x^{(s)} \).

2. Draw indicators \( \kappa^{(s)} \) from
   \[
   \Pr(\kappa_t = i|x_t^{(s)}, z_t) \propto \pi_i \mathcal{N}(z_t; x_t^{(s)} + m_i, \omega_i^2).
   \]

3. **Forward filtering:** run the Kalman filter for \( t = 1, \ldots, T \) to get the moments \( (\mu_t^{(s)}, \Sigma_t^{(s)}) \) corresponding to the filtering densities
   \[
   p(x_t|\theta^{(s)}, z^t) \sim \mathcal{N}(\mu_t^{(s)}, \sigma_t^{(s)}).
   \]

4. **Backward sampling:** draw \( x_T^{(s+1)} \) from \( p(x_T|\theta^{(s)}, z^T) \sim \mathcal{N}(\mu_T^{(s)}, \sigma_T^{(s)}) \). Then draw \( x_t^{(s+1)} \) for \( t = T - 1, \ldots, 1 \) backward through time using the conditional smoothing densities
   \[
   p(x_t|x_{t+1}^{(s)}, \theta^{(s)}, z^t) \sim \mathcal{N}(\tilde{\mu}_t^{(s)}, \tilde{\sigma}_t^{(s)}).
   \]
(5) Draw $x_0^{(s+1)} \sim N(\mu_1^{(s+1)}, \Sigma_1^{(s+1)})$ where

$$\mu_1 = \Sigma_1 \left[ \Sigma_0^{-1} \mu_0 + \beta / \tau^2 (x_1 - \alpha) \right]$$

$$\Sigma_1^{-1} = \Sigma_0^{-1} + \beta^2 / \tau^2.$$ 

(6) Draw $\theta^{(s+1)}$ from a Normal – Inverse Gamma distribution.

(7) Return to Step (1) and repeat $S$ times. We choose $S = 50,000$ with 20,000 burn-in.
Different Measures of Aggregate Uncertainty, $h = 1$

- One-step ahead uncertainty

\[ U_{jt} \equiv E_t[(y_{jt+1} - y_{jt+1|t})^2] \]

- From SV model

\[
(y_{jt+1} - y_{jt+1|t})^2 = \exp(x_{jt+1})\varepsilon_{t+1}^2 \\
x_{jt+1} = \alpha_j + \beta_j x_{jt} + \tau \eta_{jt+1}
\]

- With $\varepsilon_{jt+1}, \eta_{jt+1} \sim \mathcal{N}(0, 1)$

\[
E_t[(y_{jt+1} - y_{jt+1|t})^2] = E_t[\exp(x_{jt+1})] \\
= \exp(\alpha_j + \beta_j x_{jt} + \tau_j^2 / 2)
\]
Different Measures of Aggregate Uncertainty, $h = 1$

\[ U_{jt} = \exp(\alpha_j + \tau_j^2 / 2 + \beta_j x_{jt}) \]
\[ U_{jst}(\theta_{js}, x_{jst}) = \exp(\alpha_{js} + \tau_{js}^2 / 2 + \beta_{js} x_{jst}) \]
\[ U_{jt}(\bar{\theta}_j, \bar{x}_{jt}) = \exp(\bar{\alpha}_j + \bar{\tau}_j^2 / 2 + \bar{\beta}_j \bar{x}_{jt}) \]

- Version 1: (blue) base-case
  \[ \hat{U}_{jt} = U_{jt}(\bar{\theta}_j, \bar{x}_{jt}) \quad \hat{U}_t = PC(\triangle \log(\hat{U}_{jt})). \]

- Version 2: (fuschia)
  \[ \hat{U}_{jt} = \frac{1}{S} \sum_{s=1}^{S} U_{jst}(\theta_{js}, x_{jst}) \quad \hat{U}_t = PC(\triangle \log(\hat{U}_{jt})). \]

- Version 3: (black)
  \[ \hat{U}_{jt} = U_{jt}(\bar{\theta}_j, \bar{x}_{jt}) \quad \hat{U}_t = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \hat{U}_{jt}}. \]
Different Measures of Aggregate Uncertainty

- Version 4: (red)

\[
\hat{U}_{jt} = \mathcal{U}_{jt}(\bar{\theta}_j, \bar{x}_{jt}) \quad \hat{U}_t = \frac{1}{N} \sum_{j=1}^{N} \sqrt{\hat{U}_{jt}}.
\]

- Version 5: (green)

\[
\hat{U}_{jt} = \frac{1}{S} \sum_{s=1}^{S} \mathcal{U}_{jst}(\theta_{js}, x_{jst}) \quad \hat{U}_t = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \hat{U}_{jt}}.
\]

- Version 6: (cyan)

\[
\hat{U}_{jt} = \frac{1}{S} \sum_{s=1}^{S} \sqrt{\mathcal{U}_{jst}(\theta_{js}, x_{jst})} \quad \hat{U}_t = \frac{1}{N} \sum_{j=1}^{N} \hat{U}_{jt}.
\]
Different Measures of Aggregate Uncertainty

\[ \frac{1}{\sqrt{N}} \sum_{j} \sum_{s} \sqrt{U_{jst}(\theta_{js}, x_{js})} \]

\[ \frac{1}{\sqrt{N}} \sum_{j} \sqrt{U_{jt}(\bar{\theta}_{j}, \bar{x}_{jt})} \]

\[ \sqrt{\frac{1}{N} \sum_{j} U_{jt}(\bar{\theta}_{j}, \bar{x}_{jt})} \]

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\[ \sqrt{\frac{1}{S} \sum_{j} \frac{1}{N} \sum_{s} U_{jst}(\theta_{js}, x_{js})} \]

Alternative Measures of \( U_t(1) \) (standardized units)

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Timing Differences

- The model presented above is equivalent to the following model:

\[ y_{j,t+1} = E[y_{j,t+1}|I_t] + v_{j,t+1}^y \]

\[ v_{j,t+1}^y = \sqrt{U_{jt}^y} \xi_{j,t+1}^y \]

\[ \log(U_{jt}^y) = \tilde{\alpha}_j^y + \beta_j^y \log(U_{j,t-1}^y) + \tilde{\tau}_j^y \eta_{j,t+1}, \]

where \( \xi_{j,t+1}^y \) and \( \eta_{j,t} \) are uncorrelated \( N(0, 1) \) random variables.

- To map back to the model above, set:

\[ \xi_{j,t+1}^y \equiv \exp \left\{ \frac{\tau_{j,t+1}^y}{2} \left( \eta_{j,t+1} - \frac{\tau_{j,t+1}^y}{2} \right) \right\} \varepsilon_{j,t+1}^y \]

\[ \tilde{\alpha}_j^y \equiv \alpha_j^y + \frac{(\tau_{j}^y)^2}{2}(1 - \beta_j^y) \]

\[ \tilde{\tau}_j^y \equiv \tau_j^y \beta_j^y. \]
Uncertainty Factor Calibration

- $\tilde{F}_t^U(h)$: estimated factor before calibration.
Uncertainty Factor Calibration

- $\tilde{F}_t^U(h)$: estimated factor before calibration.
- $\hat{U}_t^y(h) \equiv \frac{1}{N} \sum_{j=1}^{N} \hat{U}_{jt}^y(h)$: average uncertainty.
Uncertainty Factor Calibration

- $\tilde{F}_t^U(h)$: estimated factor before calibration.
- $\hat{U}_t^y(h) \equiv \frac{1}{N} \sum_{j=1}^{N} \hat{U}_{jt}^y(h)$: average uncertainty.
- For each horizon $h$, choose scalars $a_h, b_h \in \mathbb{R}$ such that:

$$\hat{U}_t^y(h) \equiv \exp \left\{ \frac{1}{2} (a_h \tilde{F}_t^U(h) + b_h) \right\}$$

satisfies two conditions:

$$\frac{1}{T} \sum_{t=1}^{T} \hat{U}_t^y(h) = \frac{1}{T} \sum_{t=1}^{T} \hat{U}_t^y(h)$$

$$\frac{1}{T} \sum_{t=1}^{T} \hat{U}_t^y(h)^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{U}_t^y(h)^2$$

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VXO Index Construction

- Introduced in 1993 based on S&P 100 options prices; dates back to 1986:01.

Black and Scholes (1973, JPE) model:

\[ P_t = f(\sigma; S_t, r_t, K, T - t) \]

Given \( P_t \), invert this to compute (model-) implied volatility \( \sigma \).
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Stochastic Volatility of $F_t$
Role of Predictors in Estimating Common Uncertainty

- **h = 1**
  - Correlation: 0.88

- **h = 3**
  - Correlation: 0.90

- **h = 12**
  - Correlation: 0.91

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Responses to a VXO Shock in VAR(12) Models

![Graphs showing responses of production and employment to VXO shocks in VAR(12) models with h = 1, 3, and 12.](image)

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Variance Decomposition

Production

Employment

$h = 1$

$h = 3$

$h = 12$

VXO Index

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Bivariate VAR(1): Cross-Sectional Dispersion of TFP vs. $\hat{U}_t^{y}(12)$
Balanced and Unbalanced Panels

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Uncertainty Factor Based on Recursive Forecasts

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Eight-Variable VARs

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Vector Autoregression

Estimate VAR(12) from 1960:01-2011:12 with endogenous variables:

\[
\begin{bmatrix}
\text{log(S&P 500 Index)} \\
\text{VXO Index} \\
\text{uncertainty factor} \\
\text{federal funds rate} \\
\text{log(wages)} \\
\text{log(CPI)} \\
\text{hours} \\
\text{log(employment)} \\
\text{log(industrial production)}
\end{bmatrix}
\]