Informational Black Holes in Financial Markets

ULF AXELSON        IGOR MAKAROV*

February 22, 2019

ABSTRACT

We study how well primary financial markets allocate capital when information about investment opportunities is dispersed across market participants. Paradoxically, the fact that information is valuable for real investment decisions destroys the efficiency of the market. To add to the paradox, as the number of market participants with useful information increases, a growing share of them fall into an “informational black hole,” making markets even less efficient. Contrary to the predictions of standard theory, social surplus and the revenues of an entrepreneur seeking financing can be decreasing in the size of the market, and collusion among investors may enhance efficiency.

JEL Codes: D44, D82, G10, G20.

*London School of Economics. We thank Philip Bond, James Dow, Mehmet Ekmekci, Alexander Gorbenko, Andrey Malenko, Tom Noe, James Thompson, Vish Viswanathan, John Zhu, and seminar participants at Cass Business School, Cheung Kong GSB, Chicago Booth, Frankfurt School of Management, INSEAD, London School of Economics, Luxembourg School of Finance, Stockholm School of Economics, Toulouse School of Economics, UBC, University of Oxford, University of Piraeus, University of Reading, Warwick Business School, Yale School of Management, the CEPR Gerzensee 2015 corporate finance meetings, European Winter Finance Conference 2015, the 2015 NBER Asset Pricing summer meetings, the 2015 NBER Corporate Finance fall meetings, the 2015 NBER fall entrepreneurship meetings, the 2015 OxFit meetings, the 12th Finance Theory Group meeting, the Financial Intermediation Research Society Conference 2015 (Reykjavik), and the Western Finance Association 2015 Seattle meetings for very helpful comments.
The main role of primary financial markets is to channel resources to firms with worthwhile projects. This process requires information about demand, technological feasibility, management, and current industry and macroeconomic conditions, as well as views on how to interpret such information. Today, a large and growing number of professional investors such as business angels, venture capitalists, and private equity firms alongside traditional commercial banks compete to invest in entrepreneurs with good investment opportunities. Because no single investor typically possesses all relevant information, the efficiency of the capital allocation process depends on how well markets aggregate this information.

One might expect that larger markets with more experts lead to better investment decisions and a lower cost of capital for entrepreneurs seeking financing. There are two compelling reasons from economic theory to support these expectations. First, increased competition between investors should reduce their informational rents and drive down the cost of capital for entrepreneurs. Second, when the market as an aggregate possesses more information about the viability of a project, investment decisions should become more efficient—which should further drive down the cost of capital.

Yet, the fact that periods of high growth in the financial sector coincided with episodes of large misallocation of capital in the dot-com bubble and the financial crisis of 2007-2008 has led many observers to question whether increasing the size of financial markets is socially useful. In this paper, we develop a model of information aggregation and capital allocation in primary financial markets and identify a new general economic mechanism that leads to a trade-off between competition and informational efficiency. We show that larger and more competitive markets can lead to worse information aggregation, and therefore less efficient investment decisions and lower revenues to the entrepreneur. More generally, the identified trade-off has normative implications for how entrepreneurs should maximize revenues that drastically contrast with common wisdom. We show that collusion among investors and policies restricting competition among investors may enhance efficiency.

In our model, informed investors compete for the right to finance an entrepreneur and only few of them can take a stake in the firm in return for providing financing. The stakes can be in the form of debt, equity, convertible debt, or any of the other securities that are used in real life. We allow for a wide set of market interactions between firms and investors. Our results hold for any capital raising process that results in financing from the most optimistic investors, and that does not involve transfers from or to potential investors who end up with no stake in the firm. There are few primary financial markets that do not satisfy these assumptions.¹

The important departure from the existing literature, is that in our setting the information generated in a financing mechanism is useful for deciding whether to start the project or not. In our setting, if an investor with a sufficiently pessimistic signal wins the right to finance the project, he assumes that the project is negative NPV and not worth investing in. Relatively pessimistic investors therefore abstain from bidding.² As a result, all their information is pooled

¹In a companion paper (Axelson and Makarov (2018)), we show that our results are also robust to modeling competition as a sequential search market in which an entrepreneur visits investors in sequence.
²Investors are free to submit negative bids, but never do so in equilibrium.
together and lost—they fall into an “informational black hole”. This loss of information is costly, and leads to investment mistakes of two types—some projects that would have been worth pursuing had all market information been utilized do not get financed, while some that are not worth pursuing get financed.

The problem is exacerbated as the market grows larger, because of the winner’s curse. In a larger market, even an investor with somewhat favorable information will conclude that the project is not worth investing in if he wins, since winning implies that all other investors are more pessimistic. Hence, the informational black hole grows with the size of the market, and we show that for some reasonable distributional assumptions the social surplus as well as the expected revenues to the entrepreneur decrease with the size of the market.

Our findings might explain why we often see entrepreneurs engage in so-called “proprietary transactions,” where they negotiate a financing deal with a single venture capitalist rather than engaging in a more competitive search. Similarly, in acquisition procedures investment banks working on behalf of a selling firm often restrict the set of invited bidders, and there is no evidence that this practice reduces seller revenues (see Boone and Mulherin (2007)).

When firms cannot commit to restrict the number of bidders, we show that the equilibrium size of the financial sector may be inefficiently large. This happens because the marginal investor does not internalize the negative externality he imposes on allocational efficiency when he enters the market. We show that social welfare can decrease with a decrease in the cost of setting up an informed intermediary, and that policies aimed at restricting the size of the market can lead to Pareto improvements.

We further show that in our setting, efficiency can be improved by allowing a sufficiently large number of investors to receive a stake in the project if this is practically feasible. This is in contrast to the standard setting, where revenues are maximized by concentrating the allocation to the highest bidder. In a multi-unit auction where the number of units grows with the number of bidders, a loser’s curse balances out the winner’s curse (as shown in Pesendorfer and Swinkels (1997) for standard multi-unit auctions) which in our setting leads to higher participation and a recovery of information aggregation, and hence a higher surplus. This may be one rationale for crowd-funding, in which start-ups seek financing on a platform that looks very much like a multi-unit auction. The finding may also explain why IPO allocations are rationed to increase the number of winning participants.

A related solution is to allow syndicates or consortia consisting of multiple investors to submit joint “club bids” in the auction. Club bids and syndicates are common practice among both angel investors, venture capitalists, and private equity firms, and have been the subject of investigation by competition authorities for creating anti-competitive collusion. Indeed, in a standard auction setting, club bids reduce the expected revenues of the seller. In our setting, the opposite may hold—because club bids reduce the winner’s curse problem, it encourages participation, which increases the efficiency of the market.

We also show that the famous “linkage principle” of Milgrom and Weber (1982) may fail in

---

3To commit to restrict the number of bidders, a firm needs to commit not to consider unsolicited offers, because ex post it is always optimal to consider all offers.
our setting. The linkage principle holds that any value-relevant information that can be revealed before an auction should be revealed in order to lower the informational rent of bidders. For example, if an entrepreneur can postpone seeking financing until some public information about market conditions is revealed, he should do so. In our setting, to the contrary, it is often better to attempt financing of the project before some value-relevant information is revealed. The reason is that residual uncertainty creates an option value to the project which makes less optimistic bidders participate, which in turn increases the information aggregation properties of the market.

Our paper is related to several different strands of literature. A few papers in auction theory show that restricting the number of bidders can be optimal. Samuelson (1985) and Levin and Smith (1994) consider auctions with participation costs and show that it may be optimal to restrict entry to reduce the wasteful expenditures in equilibrium. In both papers, efficiency increases as the costs decrease. Furthermore, the optimal size of the market goes to infinity as costs go to zero. In contrast, we show optimal market size can be finite even with zero costs and that lowering costs can lead to a decrease in social surplus. Thus, both the economics mechanism and implications of Samuelson (1985) and Levin and Smith (1994) are very different from those in our paper. At a more general level, our paper is also related to the literature on the social value and optimal size of financial markets. Several papers have argued that gains associated with purely speculative trading or rent-seeking activities can attract too many entrants into financial markets (see, e.g., Murphy, Shleifer and Vishny (1991) and Bolton, Santos and Scheinkman (2016)). We provide an alternative mechanism in which each market participant possesses valuable information for guiding real production, but competition inhibits the effective use of information.

Our paper is also related to the literature on information aggregation in auctions. Wilson (1977), Milgrom (1979), and Milgrom (1981) show that in first-price and second-price auctions the price aggregates information only under special assumptions about the signal distribution. In contrast, Kremer (2002) and Han and Shum (2004) show that the price in ascending-price auctions always aggregates information. Our paper complements these results in two ways. First, we show that once information is valuable for production, the ascending-price auction no longer aggregates information as the market grows large, and observing bids does not improve information aggregation in first- and second-price auctions. Second, we show that not only do the auctions not aggregate information as the market grows large, but the informational content can decrease with market size.

For multi-unit auctions, Pesendorfer and Swinkels (1997) show that the price converges to the true value of the asset in uniform-price auctions if the number of units sold also grows sufficiently large. In the paper closest to ours, Atakan and Ekmekci (2014) show that information aggregation can fail in a large uniform-price auction if the buyer of each object can make a separate decision about how to use it. Like in our paper, in Atakan and Ekmekci (2014) information aggregation fails because bidders with different signals submit the same pooling bid. However, the economics mechanism behind pooling is different. In their setting, winning at
the pooling bid involves rationing and is more informative than winning at a higher bid. This pooling bid can be sustained because the bidder’s value function is non-monotonic in his signal: a bidder with signal zero has higher value than the bidder with signal small positive signal. Neither the assumption of non-monotonicity nor the assumption that multiple winners take different actions, which are essential for their results, are natural in the project financing setting we are focusing on. Also, in Atakan and Ekmekci (2014) statistics other than price, such as the amount of rationing and bid distributions, are informative. Hence, whether these statistics are observed after an auction can affect how much information is aggregated in equilibrium. This is not the case in our setting.

More generally, the link between the informativeness of financial markets (such as stock markets) and real decisions by firms or governments is studied in the relatively recent “feedback” literature (for a summary of this literature, see Bond, Edmans and Goldstein (2012)). The closest to our work in this literature are the papers by Bond and Eraslan (2010), Bond and Goldstein (2014) and Goldstein, Ozdenoren, Yuan (2011) who show that when an economic actor takes real decisions based on the information in asset prices, they affect the incentives to trade on this information in an endogenous way that may destroy the informational efficiency of the market. None of these papers analyze the effect of market size on efficiency, which is one of our main objectives. Furthermore, our paper shows that informational and allocational efficiency can fail even in the primary market for capital, where investors directly bear the consequences of their actions.

Finally, like us, Broecker (1990) studies a project financing setting. He considers a special case of our model when first-price auctions are used, signals are binary, and investors who provide financing do not have the option to cancel a project after an offer is accepted. Broecker (1990) does not study information aggregation and surplus specifically and does not consider the effect of reducing the number of bidders, releasing information, revealing bids, or allowing bidders to endogenously decide on the investment after the auction is over.

1 Basic Setup

In this section, we present a model of a penniless entrepreneur or small business seeking outside financing for a new project from a set of \( N \) risk-neutral potential investors. There is initial uncertainty about the type \( \theta \) of the project, which can be either \( G \) (Good) or \( B \) (Bad). This uncertainty translates into uncertainty about what the right investment policy is—whether the project should be pursued at all, at what scale investment should be made, whether the firm should wait for more information to arrive, et cetera. We denote the investment policy of the firm by an action \( a \), which we assume belongs to some compact subset \( A \) of a metric space.

An action \( a \) leads to expected cash flow \( x(a, \theta) \geq 0 \) for a project of type \( \theta \), but requires funding of \( I(a) \) to be implementable. The expected net present value of a project conditional on type \( \theta \) and action \( a \) is
\[
v(a, \theta) = x(a, \theta) - I(a).
\]
Hence, if the probability of a good type is $\pi$ at the time that action $a$ is taken, the expected net present value is

$$\pi v(a, G) + (1 - \pi) v(a, B).$$

The firm has the possibility of abandoning the project completely without any cost, so that $v(\text{abandon}, \theta) = 0$. We also assume that if not abandoned, a good project has positive net present value and a bad project has negative net present value:

**Assumption 1**

(i) $\max_{a \in A} v(a, G) > 0,$

(ii) There exists a maximal $\pi^* > 0$ such that for all $\pi < \pi^*$ and all $a \neq \text{abandon},$

$$\pi v(a, G) + (1 - \pi) v(a, B) < 0.$$  

These assumptions ensure that with sufficiently negative information ($\pi < \pi^*$) the project should be abandoned, while for sufficiently positive information ($\pi \geq \pi^*$) the project should be started.

A particularly simple example of the project that satisfies Assumption 1 is the case in which the project is either started and some fixed investment is made or it is abandoned. Let $v_G > 0 (v_B < 0)$ is the net present value of the project after the investment is made in case the project is Good (Bad). Then the expected net present value of the project if it is started is

$$\pi v_G + (1 - \pi) v_B,$$

so the maximal $\pi^*$ below which project should be abandon is

$$\pi^* = -\frac{v_B}{v_G - v_B}.$$  

In the above example, investments are binary and cannot be postponed or made in stages as new information arrives. However, as we show in Section 5.2 none of these properties are required for a project to satisfy Assumption 1.

1.1 Capital Markets

Each investor $i \in \{1, ..., N\}$ gets a noisy private signal $S_i \in [0, 1]$ about the project type. Signals are drawn independently from a distribution with cumulative distribution function $F_G(s)$ and density $f_G(s)$ if the type is good, and from a distribution with cdf $F_B(s)$ and density $f_B(s)$ if the type is bad. To simplify the exposition, we assume that both $f_G(s)$ and $f_B(s)$ are continuous, and that signals satisfy the strict monotone likelihood ratio property:

$p^4$In the working paper version we showed that our results hold if functions $f_G(s)$ and $f_B(s)$ are left-continuous and have right limits everywhere, and signals satisfy weak MLRP: $\forall s \geq s', f_G(s)/f_B(s) \geq f_G(s')/f_B(s')$.  

5
Assumption 2  Strict MLRP:

\[ \forall s > s', \quad \frac{f_G(s)}{f_B(s)} > \frac{f_G(s')}{f_B(s')} \]

Assumption 2 ensures that higher signals are better news than lower signals, and that as \( N \to \infty \),
an observer of all signals would learn the true type with probability one and make the right investment decision.\(^5\) Denote

\[ \lambda = \lim_{s \to 1} \frac{f_G(s)}{f_B(s)} \quad \text{(3)} \]

In what follows, we focus on the most interesting case of \( \lambda < \infty \), and assume that \( f_B(1) > 0 \).

After investors privately observe their signals, they choose whether to participate in the market or not. Participation is “virtually” free: Investors incur a private participation cost \( \varepsilon \), but we study the limit of equilibria as \( \varepsilon \) goes to zero. Investors who remain compete against each other in bargaining with the entrepreneur. The outcome is either no financing, or an agreement in which the chosen investor funds an action \( a \). In exchange, the entrepreneur issues a security \( w \) to the chosen investor backed by the cash flows of the firm. Both the action \( a \) and the security \( w \) can be made contingent on any information extracted in the bargaining process.

Competition between investors can take many forms, ranging from structured auctions to informal negotiations with offers and counter offers. Rather than specifying a particular format, we will focus on fundraising mechanisms with symmetric equilibria satisfying the following natural properties:

(i) The participating investor with the highest signal offers the most attractive financing terms
and is chosen by the entrepreneur.\(^6\)

(ii) Investors who are not chosen receive or make no payments.

This class of mechanisms includes a wide range of structured and unstructured environments such as auctions and bidding processes where investors join as they see fit. In the empirical literature, these weak conditions have been used by, for example, Haile and Tamer (2003) and Gorbenko and Malenko (2014) to make inference in settings that resemble ascending price auctions but may not follow the analytically tractable “button” specification used in Milgrom and Weber (1982).

By the revelation principle, any such equilibrium can be obtained as a symmetric equilibrium in the following setting. Let

\[ \{a(\cdot), I(\cdot), w(\cdot, a, \theta)\} : [0, 1]^N \to \mathcal{A} \times \mathbb{R} \times \mathbb{R}^2 \]

be a map from the space of investor signals to the investment policy of the firm, the required funding, and the expected value of security given the project type \( \theta \) and the investment policy

\(^5\)This follows since \( E(S_i|G) > E(S_i|B) \), and since the average of the signals converges to \( E(S_i|\theta) \) as \( N \to \infty \) by the law of large numbers.

\(^6\)For the probability zero event when the two most optimistic investors share the same signal \( s \), we assume that they jointly finance the firm.
Each investor, given his signal $S_i$, first decides whether to participate. Denote the region of signals where investors do not participate as $B \subseteq [0, 1]$.

For each $i$ define $U(s', s_i, \varepsilon)$ as

$$U(s', s_i, \varepsilon) \equiv \begin{cases} 
0, & \text{if } s' \in B \\
E_{S^{-i}} \left[ 1_{s' \geq \max_{j \neq i} S_j \notin B} (w(s', S^{-i}) - I(s', S^{-i})) | S_i = s_i \right] - \varepsilon, & \text{if } s' \notin B,
\end{cases}$$

(4)

where $S^{-i}$ is the set of all investor signals excluding investor $i$’s signal. The expectation in (4) is taken over all realizations of $S^{-i}$ and project types conditional on signal $S_i$ being $s_i$. $U(s', s_i, \varepsilon)$ is the expected utility of investor $i$ who receives signal $S_i = s_i$ and reports his signal as being $s'$. It is equal to zero if the investor chooses not to participate in the fundraising process. Otherwise, the investor incurs a cost $\varepsilon$ and is awarded the project if and only if his report is higher than other participating investors’ signals. The winning investor is required to implement action $a$, fund this action, and receives $w$ in exchange. We first define an equilibrium with nonzero participation costs:

**Definition 1** The contract set $\{a(\cdot), I(\cdot), w(\cdot, a, \theta)\}$ and the non-participation region $B$ is an equilibrium with participation costs $\varepsilon$ if

1. (Incentive compatibility) For any $s_i$, $s_i = \arg \max_{s'} U(s', s_i, \varepsilon)$

2. (Individual rationality) For any $s_i$, $U(s_i, s_i, \varepsilon) \geq 0$

3. (Measurability) For any $i = 1..N$, $\{a(\cdot), I(\cdot), w(\cdot)\}$ is constant for $S_i \in B$

4. (Feasibility) $w(\cdot, a, \theta) \leq x(a, \theta)$.

The incentive compatibility condition requires investors report their signal truthfully. Since any investor can choose not to participate in the fundraising mechanism and receive zero the incentive rationality condition ensures that the expected utility of any investor is nonnegative. Because signals of nonparticipating investors are not observed we require no equilibrium objects can depend on them. Finally, the feasibility condition requires a security $w$ to be fully backed the project’s cash flows. Next we define an equilibrium with zero participation costs. To deal with potentially multiple equilibria we require an equilibrium to be robust with respect to the arbitrary small participation costs:

**Definition 2** The contract set $\{a(\cdot), I(\cdot), w(\cdot, a, \theta)\}$ and the non-participation region $B$ is a robust equilibrium if it is an equilibrium with zero participation costs and there is a sequence of $\varepsilon_k > 0$ converging to 0 as $k \to \infty$, and a sequence of maps $\{a_k(\cdot), I_k(\cdot), w_k(\cdot)\}$ and the non-participation region $B_k$ such that

1. For each $k$, $\{a_k(\cdot), I_k(\cdot), w_k(\cdot)\}$ and $B_k$ is an equilibrium with participation costs $\varepsilon_k$
2. \{a_k(\cdot), I_k(\cdot), w_k(\cdot)\} and \textbf{B}_k converge in probability to \{a(\cdot), I(\cdot), w(\cdot)\} and \textbf{B} as \(k\) goes to infinity.\(^7\)

2. Analysis: Informational black holes

In this section we study how well fundraising markets incorporate information into investment decisions. We show that even the maximum amount of information that can be learned leads to significant investment inefficiencies relative to the first best. The loss of information occurs because less optimistic investors do not expect to break even with an offer that an entrepreneur accepts, and hence choose not to participate. Their information therefore cannot be recovered.

We first show that in any robust market equilibrium the participation decision is of a threshold type:

**Lemma 1** The participation decision is of a threshold type: There is a certain threshold \(\hat{s}\) such that investors participate if and only if their signals are above \(\hat{s}\).

**Proof.** See the Appendix.

Intuitively, Lemma 1 follows from the fact that the expected profit of an investor is increasing in his signal \(s_i\), which in turn is implied by MLRP. Since investors with signals below \(\hat{s}\) do not participate their signals cannot be used for investment decisions. The potential for information aggregation is higher the lower the participation threshold \(\hat{s}\) is, and we now derive a lower bound on this threshold.

Consider an arbitrary small participation cost. Note that any participating investor can recoup this cost only if he starts the project, since losing investors get no transfers. Moreover, because of the entrepreneur’s limited liability, a winning investor can at most capture the full net present value of the project. When the marginal participating investor wins he only learns that his signal is the highest among all investors. Hence, in any robust equilibrium the marginal participating investor must perceive the project as positive NPV conditional on this information. Therefore, by the Assumption 1 his updated probability that the project is Good must be at least \(\pi^*\). Define a threshold \(\overline{s}_N\) as the maximal threshold such that

\[
P(G | \max_i S_i = \overline{s}_N) \leq \pi^*.
\]  

If all investors’ signals are below \(\overline{s}_N\) then the project will not be started. At the same, by the MLRP whenever there is at least one signal above \(\overline{s}_N\) there is an action \(a\) such that the project becomes positive NPV if the action \(a\) is taken. Thus, we have the following result:

**Lemma 2** In any robust equilibrium, the participation threshold cannot be lower than \(\overline{s}_N\). The most efficient feasible investment policy is to abandon the project if and only if no investor signal is above \(\overline{s}_N\).

\(^7\)We assume that the action space \(\mathcal{A}\) is a metric space, and define the distance between two sets \(\textbf{B}_k\) and \(\textbf{B}\) as \(\mu(\textbf{B}_k \triangle \textbf{B})\), where \(\mu\) is a Lebesgue measure on \([0, 1]\).
Lemma 2 shows that in any robust equilibrium, all information relevant for the decision to start or abandon the project is contained in the highest signal among investors. Whenever the highest signal is below the threshold \( \bar{s}_N \) the project is abandoned. In general, signals below \( \bar{s}_N \) are pooled together and lost. We therefore call the the non-participation region \( B = [0, \bar{s}_N] \) the informational black hole.

The existence of the informational black hole leads to inefficient investment behavior relative to the situation where all signals are observed because a winner will assume that all investors who do not participate have “average” signals. When all signals are in the black hole and close to the threshold, the project will not be undertaken even though it can be positive NPV. In contrast, some signals in the black hole can be very pessimistic implying that the project negative NPV but the project can still be started if there is at least one signal outside the informational black hole. This loss of efficiency leads to a reduced surplus, and hence lower expected revenues to the entrepreneur relative to the first best.

The constrained efficient investment policy can be implemented by issuing straight equity securities in an ascending price auction, very much in line with the typical venture capital setting observed in practice. Suppose the entrepreneur seeks to raise a fixed amount of capital \( I^* \) by selling a fraction \( \alpha \) of firm equity to investors. We assume that \( I^* \) is high enough to implement any action: \( I^* \geq \max_a I(a) \). For a given fraction \( \alpha \), denote the implied “price” of the firm’s equity by \( p = I^*/\alpha \)—this is what venture capitalist refer to as the post-money valuation of the firm.

The auction proceeds as follows. The price starts at \( p = I^* \), a price at which an investor would own the whole firm, and is gradually increased. Participating investors can drop out of the auction at any time but cannot reenter after dropping out, and all investors can observe the number of remaining participants. The auction stops at the price \( p \) where only one investor remains, who is then rewarded a fraction \( \alpha = I^*/p \) of the shares in exchange for supplying the capital \( I^* \).

The entrepreneur and the winning investor then jointly decide on the action \( a \) taking into account any information learnt from the auction. If the capital \( I(a) \) required to implement the action is lower than the firm capital \( I^* \), the excess cash \( I^* - I(a) \) is distributed back to shareholders. We have

\[ \text{Lemma 3} \quad \text{There is a unique robust symmetric equilibrium in the ascending price equity auction. Each investor participates if and only if her signal is above } \bar{s}_N \text{ defined in Equation (5).} \]

\[ \text{Proof.} \quad \text{See the Appendix.} \]

Lemmas 2 and 3 show that the threshold \( \bar{s}_N \) is a sharp bound. We next investigate its behavior as the number of investors goes to infinity. Lemma 4 shows that the informational black hole increases with the number of investors. The reason is the winners curse: In a larger market, winning with the same signal is bad news because winning implies that all other investors are more pessimistic.
Lemma 4 If $\pi^*/(1 - \pi^*) < \lambda \pi_0/(1 - \pi_0)$ then the threshold $\bar{s}_N < 1$ and goes to one with $N$. Furthermore, there exist limits

$$\lim_{N \to \infty} \Pr(\max_i S_i \leq \bar{s}_N | B) = e^{-\tau},$$

(6)

$$\lim_{N \to \infty} \Pr(\max_i S_i \leq \bar{s}_N | G) = e^{-\lambda \tau},$$

(7)

where $\tau > 0$ is a unique solution to Equation (8):

$$e^{-(\lambda - 1)\tau} = \frac{1}{\lambda} \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}.$$  

(8)

If $\pi^*/(1 - \pi^*) \geq \lambda \pi_0/(1 - \pi_0)$ then for any $N$, $\bar{s}_N = 1$ so the project is always abandoned.

Proof. See the Appendix.

The condition $\pi^*/(1 - \pi^*) < \lambda \pi_0/(1 - \pi_0)$ is necessary for the project to ever be started. Therefore, going forward we assume that it is satisfied. If it does hold then even an investor with the most optimistic signal perceives the project as a negative NPV one. Equations (6) and (7) show that that as long as the likelihood ratio $\lambda$ at the top signals is finite even in the limiting case of infinitely many investors who in aggregate have complete information investment mistakes persist in equilibrium. Furthermore, in the next section we show that the investment mistakes can be increasing in the number of investors. As a result, large financing markets can lead to strictly worse social surplus and revenues for entrepreneurs.

3 Smaller versus larger markets

We first continue with the case of binary action space, in which the project’s value takes the simple form (1). Assume that a fundraising mechanism delivers the lowest possible threshold $\bar{s}_N$ defined by Equation (5). The project is perceived positive NPV, and therefore is started, if and only if there is at least one investor with signal above $\bar{s}_N$. Thus, social surplus with $N$ investors is

$$\pi_0 V_G \Pr(\max_i S_i > \bar{s}_N | B) + (1 - \pi_0) V_B \Pr(\max_i S_i > \bar{s}_N | B).$$

(9)

Recall that the threshold $\bar{s}_N$ is set so that the marginally accepted project is zero NPV. Hence, $\bar{s}_N$ delivers the maximal possible surplus among all thresholds, and so it satisfies the first-order condition:

$$\frac{\pi_0}{1 - \pi_0} \frac{F_G(s_N)^{N-1} f_G(s_N)}{F_B(s_N)^{N-1} f_B(s_N)} = \frac{v_B}{v_G}.$$  

(10)

Suppose that signals are binary. If the project is good, investors get only high signal, while if the project is bad, they can get either high and low signal. This binary signal structure can be represented by setting $f_B(s) = 1$ for all $s \in [0, 1]$, and setting $f_G(s) = 0$ for $s \in [0, 1 - 1/\lambda]$ and $f_G(s) = \lambda$ for $s \in (1 - 1/\lambda, 1]$. Suppose for simplicity that the prior belief $\pi_0$ is such that is the project is break even in the absence of any investors’ information, i.e., $\pi_0 v_G = (1 - \pi_0) v_B$. 

10
The participation threshold $\bar{s}_N$, which solves (10), then

$$\bar{s}_N = \frac{1 - 1/\lambda}{1 - \frac{\lambda}{N-1}}. \tag{11}$$

Plugging (11) into (9) we obtain an explicit expression for social surplus:

$$\pi_0 V_G \left(1 \left(1 - \frac{1}{\lambda}\right)^N \left(1 - \frac{\lambda}{N-1}\right)^{N-1}. \tag{12}\right.$$

Figure 1 plots surplus (12) as a function of the market size $N$. We can see that social surplus declines with the market size for all $N$—surplus is maximized with a single investor. Next we show that the above case is not an isolated example and provide necessary and sufficient conditions for the surplus to be increasing or decreasing with the number of investors.

The number of investors affects social surplus in two ways. First, the number of signals increases (direct effect). Second, the participation threshold also increases (indirect effect). To isolate the first effect we use our prior result that all information relevant for the decision to start or abandon the project is contained in the highest signal among investors. Recall that the first-order statistic with $N$ investors has the cumulative distribution function (cdf) $F(s)$, where $F(s)$ is the cdf of one signal. Suppose we have $N \times M$ independent identically distributed signals, each with the cdf $F(s)^{1/M}$. Then the maximum of these $N \times M$ signals has the same distribution as the maximum of the original $N$ signals. Therefore, the participation threshold with $N \times M$ investors receiving each a signal with the cdf $F_G(s)^{1/M} (F_B(s)^{1/M})$ if the project is Good(Bad) is the same as the original threshold $\bar{s}_N$.

Consider a case of $M$ being very large. The likelihood ratio of such an infinitesimal signal is

$$\lim_{M \to \infty} \frac{f_G(s)}{F_G(s)^{(1-1/M)}} \frac{F_B(s)^{(1-1/M)}}{f_B(s)} = \frac{f_G(s)F_B(s)}{f_B(s)F_G(s)}. \tag{13}$$

Suppose the number of investors increase from $N \times M$ to $N \times M + 1$. Adding an extra infinitesimal signal has then only the direct effect that investment will now be made when the order statistic of $N \times M$ investors is below $\bar{s}_N$ but the extra signal $s$ is above $\bar{s}_N$ (the indirect effect of increasing the threshold is zero from the envelope condition). Conditional on this event $A$, we have

$$\Pr(G|A) = \frac{\pi_0 f_G(s)F_B(s)F_G(\bar{s}_N)^N}{1 - \pi_0 f_B(s)F_G(s)F_B(\bar{s}_N)^N} = \frac{v_B f_G(s)F_B(s)F_B(\bar{s}_N)F_G(\bar{s}_N)}{v_G f_B(s)F_G(s)F_B(\bar{s}_N)F_G(\bar{s}_N)}. \tag{14}$$

From (14) we can see that if the ratio

$$\frac{f_G(s)F_B(s)}{f_B(s)F_G(s)} \tag{15}$$

is a decreasing(increasing) function of $s$ at $\bar{s}_N$ then conditional on the event $A$ the project is negative(positive) NPV, and hence, social surplus decreases(increases) with the number of
Proposition 1 If \( \frac{f_G(s)}{f_B(s)} \) is an increasing function on \([s_1, 1]\) then social surplus increases with the number of investors. If there exists \( N \) such that \( \frac{f_G(s)}{f_B(s)} \) is a decreasing function on \([s_N, 1]\) then maximal social surplus is achieved with no more than \( N \) investors.

Proof: See the Appendix.

In the example with binary signals, the ratio (15) is equal to \( \frac{s}{s - (1 - 1/\lambda)} \). The function \( s/(s - (1 - 1/\lambda)) \) decreases on the interval \((1 - 1/\lambda, 1]\). Therefore, social surplus decreases with the number of investors, which confirms our direct calculations. More generally, the ratio (15) is a decreasing function if the likelihood ratio \( f_G/f_B \) is sufficiently flat at the top range of signals.

We next consider entrepreneurial revenues as a function of market size. If the entrepreneur has the power to pick the number of investors, he will do so in order to maximize revenues rather than surplus. The private optimum may differ from the social optimum if the entrepreneur captures only part of the surplus. When surplus itself is fixed, in many mechanism, in particular in all auction formats, the fraction of surplus captured by the entrepreneur usually goes to one with \( N \). Hence, if surplus increases with \( N \), there is no conflict between the private and social optimum—the entrepreneur will prefer the maximal number of investors.

The non-trivial case is when surplus decreases with \( N \). Will the entrepreneur find it optimal to restrict the number of investors even though this may entail surrendering a higher fraction of the surplus to investors? Our answer is a qualified “Yes”. The next proposition gives a sufficient condition for when this is the case.

Proposition 2 Suppose that there exists an \( \varepsilon > 0 \) such that \( f_G(s)/f_B(s) = \lambda \) for \( s \in [1 - \varepsilon, 1] \), and fundraising is done using issuing straight equity securities in an ascending price auction. Then, there exists some \( N^* \) such that revenue is strictly decreasing in \( N \) for \( N > N^* \).

Proof: See the Appendix.

To understand this result, note that surplus decreases with \( N \) when the top of the signal distribution is relatively flat, so that investors who draw high signals are informationally close to each other. But when this is the case, investors also capture little informational rent even for moderate levels of \( N \). In other words, increasing \( N \) beyond a certain level has little effect on the split of revenues but a large negative effect on surplus.

The conditions in Proposition 2 are sufficient but not necessary for the entrepreneur to prefer a smaller market. We provide an example in the next section, which shows that the entrepreneur will prefer a smaller market whenever the likelihood ratio does not increase too steeply at the top of the signal distribution. Our results provide one explanation for why so many capital raising situations involve negotiations with a restricted set of investors rather than an auction open to everyone.

3.1 Can financial markets be too big?

In the previous section we established that small markets may be preferable both from the entrepreneur’s and from a social surplus perspective. In this section we show that the equilibrium
size of the market can be too large relative to both the social and the entrepreneurial optimum, and can be Pareto inferior relative to a market with one less investor.

If the entrepreneur can commit to seek financing from a restricted set of investors, the market can obviously never be larger than what is optimal for the entrepreneur. However, restricting the set of potential investors may be difficult in practice because it is ex post optimal for the entrepreneur to consider any offer he receives, even if the offer is unsolicited. In this section we therefore assume no commitment so that investors can enter any auction.

So far, we have assumed that investors observe signals for free to make our results on the failure of information aggregation in large markets as striking as possible. In order to have a non-trivial equilibrium market size, we now assume that investors face some costs of gathering information. Assume that each potential investors $i$ has a cost $c_i$ of gathering information about the project, and that $c_i$ is strictly increasing. We focus on the case where $f_G(s)/f_B(s)$ is a decreasing function at $s = 1$ so that social surplus (gross of investor costs) is maximized at a finite market size. The socially optimal market size net of costs is then even smaller.

**Proposition 3** Suppose that $f_G(s)/f_B(s)$ is a decreasing function at $s = 1$. Then, there is $c > 0$ such that if sufficiently many investors have costs of gathering information below $c$, the equilibrium size of the market is larger than the socially optimal size. Lowering information gathering costs proportionally for all investors can lead to a decrease in both net and gross of fees social surplus.

**Proof:** See the Appendix.

The proposition shows that there is no reason to believe that markets will become more efficient as information technology improves. This is in contrast to the predictions of Samuelson (1985) and Levin and Smith (1994) who study information costs in an otherwise standard auction theory setting. In both papers, the optimal size of the market goes to infinity as costs go to zero. Proposition 3 shows that there can be too much entry in equilibrium relative to the social optimum. The next example shows that both investors and the entrepreneur can be better off if entry is restricted.

Suppose that $f_B(s) \equiv 1$ and $f_G(s)$ is a truncation to the interval [0, 1] of a normal distribution with mean 1 and standard deviation 0.75. The likelihood ratio $f_G(s)/f_B(s)$ is strictly increasing over [0, 1], so MLRP holds strictly. Also, because the derivative of the likelihood ratio is zero at $s = 1$, the ratio $f_G(s)/f_B(s)$ is a decreasing function around $s = 1$. We assume that the net present value for a good project is 0.8, while a bad project has an NPV of minus one.

Panel A of Figure 2 shows social surplus gross of investor costs and the expected revenues to the entrepreneur as a function of the size of the market. Social surplus is maximized at a market size of one, while the entrepreneur’s revenues are maximized at a market size of three. The entrepreneur prefers a somewhat larger market size than what maximizes social surplus because increased competition between investors reduces their share of the surplus.

Panel B shows expected gross profits to investors from participating in the auction as a function of market size, as well as a particular specification for the cost $c_i$ of information gathering.
for each investor. In equilibrium, investors will enter as long as expected profits cover their cost, so that for the specific costs drawn in the figure the first five investors will enter in equilibrium with the fifth investor indifferent between entering and staying out. Hence, the equilibrium market size is larger than both the social optimum and the entrepreneur’s optimum.

Now suppose that every investor’s cost was just slightly larger. This would be the case if, for example, tax rates on venture capitalist profits are increased slightly. The equilibrium market size would drop to four, which would constitute a Pareto improvement. Participating investors would make higher profits because of both reduced competition and more efficient investment decisions. The entrepreneur’s revenues would increase because the increased surplus from more efficient investment outweighs the loss from reduced competition. Finally, the investor who drops out of the market is no worse off since he was just breaking even before.

Our result that small markets may be optimal for entrepreneurs provides a new explanation for the phenomenon of “proprietary transactions” in venture capital and private equity in which entrepreneurs appear to voluntarily restrict competition when seeking financing. The results we derive for the case of stochastic bidders (see Section 5.1) identifies a further value of small markets—as we show, uncertainty about the number of bidders often leads to less efficient outcomes than when the number of bidders is known, and a proprietary negotiation removes this uncertainty.

4 Strategies for reducing the winner’s curse

The source of inefficiency in our model is the effect the winner’s curse has on the participation of pessimistic investors, an effect that becomes stronger as the market grows larger. In this section we discuss a number of strategies that can help to alleviate the winner’s curse. First, we show that it may be beneficial to raise capital before important information is learnt in order to increase the option value embedded in the project. Second, we show that allowing a larger set of investors to co-finance the project helps reduce the winner’s curse. Third, in contrast to results for standard auctions, we show that allowing investors to collude ex ante via bidding clubs can also improve efficiency and revenues. Finally, we discuss how adding an appropriately designed derivative market where investors can bet on project failures might eliminate the informational black hole. All these “fixes” rely on alternative trading mechanisms that may not always be implementable in practice.

4.1 Choosing when to finance and the linkage principle

Suppose that there is some exogenous signal $X$ that helps predict the value of the project. For example, this could be a signal about demand conditions for the products the project is meant to create, or in general, any relevant information the entrepreneur might have that can be credibly communicated to the investors. The question then is whether it is better to raise funds before or after signal $X$ is released.

For standard auctions, where all investors always participate, the linkage principle of Milgrom
and Weber (1982) suggests that it is better to raise funds after all value-relevant information is realized in order to lower the informational asymmetry between investors. However, in our setting there is a countervailing effect. Any signal, which is revealed after the funds are raised but before investments are made adds an extra real option value component to the project. Proposition 4 shows that the extra value component prompts investors even with low signals to participate in the hope that the project turns out to be positive NPV. As a result, the participation threshold is always lower compared to that when \( X \) is released before the fundraising process. As a consequence, social surplus is higher if funds are raised before \( w \) \( X \) is released.

We assume that \( X \) satisfies the following properties:

**Assumption 3**

(i) Investors’ signals \( S_i \) are symmetric and conditionally independent on \( X \) and the project type \( \theta \).

(ii) The conditional density \( P(S_i = s|X = x, \theta) \equiv f_{\theta_{X}}(s|x) \) exists and for any \( x \) satisfies the strict MLRP:

\[
\forall s > s', \quad \frac{f_G(s|x)}{f_B(s|x)} > \frac{f_G(s'|x)}{f_B(s'|x)}.
\]

Assumption 3 guarantees that conditional on \( X \) the participation policy is still of threshold type. Any signal \( X \), which is independent from investors’ signals trivially satisfies Assumption 3. In the extended model where the total number of investors is unknown, which we consider in Section 5.1, \( X \) can be a signal about the number of investors.

**Proposition 4** Suppose there is a signal \( X \) that satisfies Assumption 3. Suppose \( X \) is revealed after the funds are raised but before investments are made. Then, the participation threshold is lower than the participation threshold if \( X \) is released before the fundraising process.

**Proof:** See the Appendix.

### 4.2 Dispersed ownership

In the previous sections we assumed that only one investor ends up with a stake in the project. In this section we allow for the possibility that \( K > 1 \) investors can co-finance the project. Allowing for more investors to receive an allocation weakens the winner’s curse and hence encourages more investors to submit non-zero bids, which has a positive effect on efficiency. Pesendorfer and Swinkels (1997) show that the \( K \)-unit auction has a unique symmetric monotone equilibrium in the standard setting and that the auction fully aggregates information as \( N \to \infty \) if and only if \( K \) satisfies the “double largeness” condition: \( K \to \infty \) and \( N - K \to \infty \).

While there are multiple equilibria in our setting, we show that the aggregation properties of \( K \)-unit auction mirror those of Pesendorfer and Swinkels (1997). In particular, inefficiencies persist as long as \( K \) is finite, even if the bids are made known after the auction and are incorporated in the investment decision. The case of finite \( K \) seems reasonable in most corporate
finance situations. If $K$ is allowed to grow proportionately with $N$, we show that inefficiencies disappear in the limit.

Specifically, we assume that the $K$ highest bidders who submit nonzero bids share the investment costs and the project’s payoff. Each winner pays the bid submitted by the $K + 1$ highest bidder. If there are less than $K$ investors who submit nonzero bids the project is canceled. Otherwise the $K$ highest bidders get the right to finance the project. In principle, winning investors may disagree about the decision to start the project. When $K$ grows with $N$ we show that for large $N$ all winning investors agree on the investment decision. When $K$ is finite we consider the optimistic scenario in which all winning investors share their information with each other and jointly decide whether to start the project.

**Proposition 5** In the $K$-unit auction, for any finite $K$, the limiting surplus is strictly lower than the first-best expected surplus. If $K/N$ goes to some constant larger than zero and smaller than one, then the expected surplus converges to the first-best expected surplus.

**Proof:** See the Appendix.

Our results in this section can be used to explain why firms explicitly ration the allocation of shares in initial public offerings so that a larger number of investors receive an allocation. It can also explain why entrepreneurs often allow a number of venture capitalists to co-invest, and the increasing popularity of crowd-funding platforms.

In related work, Atakan and Ekmekci (2014) study a large multi-unit auction in which each unit can be put to a different use, and show that the price does not fully aggregate information. Their equilibria are specific to the multi-unit setting and fail to exist in a single-unit setting or when $K$ is finite. Our results are the reverse—information is aggregated when double-largeness holds but not when $K$ is finite. The non-revealing equilibria in Atakan and Ekmekci (2014) require that winners of different units take different actions. In contrast, we assume that winners have to take a joint action (start the project or not), which is the appropriate assumption in a project financing context.

### 4.3 Syndicates and club bids

We now study a setting in which investors can form consortia and submit a joint bid. A full analysis of club bidding is challenging for several reasons. First, club formation is an endogenous process which may lead to clubs of different size, which would require analysis of auctions with asymmetric bidders. Second, there may be incentive problems within the club that prevent full sharing of information among club members.

Dealing with these issues is beyond the scope of our paper and we therefore consider a simplified setting where we assume clubs are of equal and exogenously given size, the number of investors is large, and that information is freely shared within the club. We assume that there are $N \times M$ investors in the market. We will contrast two market settings. In the first, there is no collusion among investors and everyone submits bids independently. In the second, investors
are randomly allocated to $N$ symmetric clubs each consisting of $M$ investors, whereupon each club submits a joint bid in the auction.

In the standard setting, where the asset for sale is already in place, surplus is always the same so collusion among investors tends to lower seller revenues (see e.g., Axelson (2008)). There are countervailing two forces favoring club bidding in our setting. First, club bidding reduces the effective number of bidders, which is beneficial when markets are inefficiently large, even if the club would submit a bid based on the signal of only one member. Second, signals become more informative whenever there is some information sharing within the club. When these effects outweigh the reduced competition, the entrepreneur gains. Proposition 6 shows that for large $N$, information sharing channel always dominates the competition effect.

**Proposition 6** For large enough $N$, entrepreneur’s revenue in any of the standard format auction among $N$ clubs consisting of $M$ investors is higher than social surplus with $N \times M$ individual investors.

**Proof:** See the Appendix.

Proposition 6 provides a benign rationale for the prevalent use of club bids in private equity and the use of syndicates in venture capital that has come under scrutiny by competition authorities. Our theory predicts that some arrangement for sharing/selling of signals via co-ownership should develop when feasible. In general, the extent of co-investment/syndication depends on the trade-off between increased informational efficiency and the extra costs of adding more investors to the capital structure. In VC markets, practitioners view the cost of adding members to a syndicate as quite large. The main costs come from the difficulty of coordinating the exercise of control rights, free-riding problems in the governance of the firm, and the cost to the lead VC of sharing some of the surplus with other investors. As a result, when syndicates do exist, they tend to be of very limited size relative to the set of potential investors. In comparison, bank syndicates in loan markets tend to be of larger size, possibly reflecting the more limited role banks play in the governance of the firm. And at the other extreme is crowdfunding and initial public offerings, where investors play a very passive role in the running of the firm and the number of investors getting a stake can become quite large.

5 **Extensions and robustness**

5.1 **Stochastic number of investors**

In this section, we extend our theory to the stochastic number of investors and show that for a wide class of distributions of the number of investors, informational black holes not only continue to exist but lead to even less efficient investment decisions than in the deterministic case (Proposition 7). We also show that under some conditions, informational black holes can disappear and full information can be achieved (Proposition 8). Overall, our results suggest that the existence of informational black holes is a robust phenomenon.

---

8See Bailey (2007) for further discussion.
Consider the following extension of our main case. Consider a sequence of markets indexed by $N = 1, 2, \ldots$, and assume that the number of investors in market $N$ is $N\nu$, where $\nu$ is a non-negative random variable with a cumulative distribution function $F$ over $[0, \infty)$. Investors know $N$ and $F$ but not the realization of $\nu$. If $\nu$ is one with probability one we are back to the deterministic case considered in main part of the paper.\footnote{We allow the number of investors $N\nu$ to be non-integer. Our results would not change if we round $N\nu$ to the nearest integer, but formulas become cumbersome.} We make the following assumptions about distribution $F$:

**Assumption 4** $F$ has a continuous density at zero.

**Assumption 5** $\nu$ is smaller in the likelihood ratio ordering than $\lambda\nu$.

Assumption 4 implies that the probability that the market is populated by any finite number of investors as $N$ goes to infinity goes to zero, which is necessary for markets to ever become fully efficient in the limit. Assumption 5 is not important for our results on when markets feature informational black holes and investment inefficiencies. The main role of this assumption is to ensure the uniqueness of the informational black hole equilibrium. Without it, there could potentially be multiple black hole equilibria. The assumption is satisfied for many distributions. Examples include the uniform and exponential distributions.

**Proposition 7** Suppose that Assumptions 4 and 5 hold. Suppose that either (i) $\pi_0 < \pi^*$, or (ii) there is an $\hat{\nu} > 0$ such that $F(\hat{\nu}) = 0$. Then for large enough $N$, in each market $N$, there exists a robust equilibrium. Any robust equilibrium has the same participation threshold $\bar{\pi}_N$. The threshold $\bar{\pi}_N$ goes to one with $N$, and is a unique solution to Equation (16):

$$
\frac{E e^{-\lambda\nu}}{E e^{-\tau\nu}} = \frac{1}{\lambda} \frac{\pi^*}{1 - \pi^*} \frac{1 - \pi_0}{\pi_0}.
$$

Furthermore, there exist limits

$$
\lim_{N \to \infty} \Pr(\text{Project is started | } B) = 1 - E e^{-\tau\nu} > 0,
$$

$$
\lim_{N \to \infty} \Pr(\text{Project is not started | } G) = E e^{-\lambda\nu} > 0.
$$

The limiting social surplus is strictly lower than in the deterministic case.

**Proof:** See the Appendix.

Proposition 7 shows that the existence of informational black holes is robust to having a stochastic number of investors. Investors with sufficiently negative information will not want to participate, and the informational black hole grows with the expected size of the market, even when there is a possibility that the actual number of investors is small. Investment efficiency is lower than in the deterministic case because the inference of a winning investor about project quality is confounded with inference about the size of the market.
The randomness in the number of investors makes the winner’s curse weaker, because winning with a low bid is a signal that there may be fewer potential investors in the market. As we show in Proposition 7, this effect is not strong enough to eliminate the informational black hole if the unconditional NPV of the project is negative, or if there is a lower bound on the potential number of investors which grows with $N$. The following Proposition shows conditions under which the markets can aggregate information:

**Proposition 8** Suppose that $\pi_0 > \pi^*$, that $F(\nu)$ has a strictly positive continuous density at zero, and that $f_B(s)$ and $f_G(s)$ are continuously differentiable. Then there exists a robust equilibrium that leads to full efficiency as $N \to \infty$ if bids are revealed ex post.

**Proof:** See the Appendix.

The economics behind Proposition 8 are as follows. When $F$ has a strictly positive continuous density at zero, it may be possible to sustain equilibria with a participation threshold that does not go to one. In such an equilibrium, when the marginal investor wins the auction, he concludes that he is in a market with very few potential investors independent of how large $N$ is. If the unconditional NPV of the project is positive, he can then break even. Because the participation threshold is bounded away from zero, the set of observed bids generates a lot of information and investment inefficiencies are eliminated as $N \to \infty$.

Propositions 7 and 8 are derived under the assumption that the number of potential investors $N\nu$ does not depend on the quality of the project. There may be settings where it is more natural to assume that the expected number of potential investors is larger when the project is good. For example, this would be the case if VCs do a quick check initially and only acquire a serious signal if the initial check is positive. Our results can be easily extended to such a case by assuming that $\nu$ is drawn from a distribution $F_G(\nu)$ when the project is good and from a distribution $F_B(\nu)$ when the project is bad. One can show that informational black hole equilibria are then even easier to sustain, because the winner’s curse gets stronger. Winning with a low bid signals that the market is small, but this is now negative information for the NPV of the project.

### 5.2 Private values, scalability of investment, and real options

One could imagine a more general model in which investments are scalable and can be made in stages, and along with a common value component there is a private value one. All the analysis in the paper is straightforward to modify to cover these cases and all our results are robust to these extensions. For example, suppose that the NPV of the project for investor $i$ is $v(a, \theta) + \alpha S_i$, where as before, $v(a, \theta)$ is the common value component, and $\alpha S_i$ is an extra private value component, which is perfectly correlated with investor’s signal $S_i$. Provided that there exists a maximal $\pi^* > 0$ such that for all $\pi < \pi^*$ and all $a \neq \text{abandon}$,

$$\pi v(a, G) + (1 - \pi) v(a, B) + \alpha < 0,$$


the informational black hole and investment inefficiencies will continue to exist. In particular, the participation threshold will still solve

$$P(G | \max_i S_i = \bar{\pi}_N) = \pi^*.$$  

Propositions 2 and 3, which show that small markets can be more efficient and can create higher revenue than large markets, go through when the private value component $\alpha$ is not too large—increasing the size of the market now has the benefit that there is more likely to be an investor with a high private value, which acts as a countervailing force to the investment inefficiencies in large market.\footnote{One can also show that all our results are robust to investors having a private value component which is independent of their common value component.} The case of private values highlight that our results depend on “nonvariation” that arises from the option to abandon the project, and not from nonvariation in the NPV of the underlying project.

Next, we show that the existence of a critical value $\pi^*$ is robust to investments being scalable and to an option to make investments in stages. Consider the following simple model of investments. Suppose that the production function is state-dependent and equal to

$$a_\theta \ln(1 + I) - I,$$

where $I$ is investment, and $a_G > 1 > a_B$. Taking the first-order conditions it is straightforward to see that the investment is nonzero only if

$$\pi a_G + (1 - \pi) a_B > 1.$$

Hence, the critical value $\pi^* = (1 - a_B)/(a_G - a_B)$.

Finally, consider a typical Dixit-Pindyck type model that satisfies Assumption 1. Suppose that right after the fundraising process, investors can observe news about the project type. The news process evolves according to

$$dX_t = \mu_\theta dt + \sigma dB_t,$$

where $B_t$ is a standard Brownian motion, $\mu_\theta = \mu_G$ if the project is good, and $\mu_\theta = \mu_B$ otherwise. At each time $t$, the entire history of news, $\{X_s\}_{0 \leq s \leq t}$, is observable. The parameters $\mu_G$, $\mu_B$ and $\sigma$ are common knowledge. Without loss of generality, $\mu_G - \mu_B \geq 0$. Define the signal-to-noise ratio $\varphi = (\mu_G - \mu_B)/\sigma$. When $\varphi = 0$, the news is completely uninformative. Larger values of $\varphi$ imply more informative news. In what follows, we assume $\varphi > 0$.

At each time $t$, the winner of fundraising process faces the following decision tree. He can either start the project, postpone it, incur the cost and observe the news, or completely abandon the project; in the latter case the game ends. Suppose as before that the NPV of of the project conditional on the state is $v_G > 0$ and $v_B < 0$. Suppose also that postponing the project incurs cost $c > 0$ per unit of time. We have the following standar result:
Lemma 5 There exist probabilities $\pi^* > 0$ and $\pi^{**}$. For $\pi < \pi^*$ the project is abandoned, for $\pi \geq \pi^{**}$ the project is started immediately, and for $\pi^* \leq \pi \leq \pi^{**}$ the project is postponed.

Proof. See the Appendix.

As a numerical example, assume the following parameter values: $v_G = 1$, $v_B = -1$, $(\mu_G - \mu_B) = 10\%$, $\sigma = 20\%$. If the decision to invest has to be taken immediately after the auction then the project is started if and only if the winner has the belief $\pi_0 \geq 1/2$. The table below shows values $\pi^*$ and $\pi^{**}$ for different values of cost $c$.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$\pi^*$</th>
<th>$\pi^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.14</td>
<td>0.86</td>
</tr>
<tr>
<td>0.05</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>0.1</td>
<td>0.35</td>
<td>0.65</td>
</tr>
<tr>
<td>0.2</td>
<td>0.42</td>
<td>0.58</td>
</tr>
</tbody>
</table>

5.3 Assets in place and entrepreneurial wealth

We have assumed that the entrepreneur has no wealth of his own to finance the project, and no other assets that can be pledged to investors in exchange for financing. The model easily extends to the case of an existing firm raising financing for a new project, where the firm could either use some of its cash to co-finance the project or issue securities that are backed not only by the cash flows of the new project but also by the existing assets of the firm.

First, imagine that the entrepreneur has some wealth $w$, and issues an equity stake backed by a fraction $1 - w$ of the cash-flows of the project, where the winner invests $1 - w$ and the entrepreneur invests $w$ to start the project if they find it optimal to do so. It is easy to see that this leads to the exact same equilibria as when there is no wealth, except that all prices and bids are scaled down by a factor $1 - w$. Hence, surplus is exactly the same independent of the wealth of the entrepreneur. The only change is that revenues of the entrepreneur go up with wealth, since the fraction of surplus captured by investors goes down by a factor $1 - w$. This effect reinforces our result in Proposition 2 that revenues can go down with the size of the market: as $w$ goes to one, revenues will behave in exactly the same way as surplus.

One can also show that the entrepreneur would never want to subsidize investors by giving up a larger share of the project than $1 - w$. Doing so would lower equilibrium black-out levels, but only because investors sometimes would find it optimal to pursue negative NPV projects, which would lead to a destruction of surplus.

Now suppose that the entrepreneur does not have liquid wealth, but has an existing firm with assets that can be pledged to back the security issue. For example, suppose the firm has assets in place with random but positive cash flows $Z$ uncorrelated with the project’s cash flows and that the firm issues new shares backed by both the assets in place and the new project. Suppose the firm runs a security auction in which investors bid the fraction of shares $\alpha$ they are willing to accept in exchange for the capital needed to finance the project. The most pessimistic investors would then submit a bid of $1/E(Z + 1)$; this is the fraction of shares needed to break even on an investment of 1 if the project is not pursued and the money raised is kept within the
firm. The equilibrium black-out level below which investors submit this bid would be exactly the same as in our original model, so surplus would also remain the same. Again, as in the case of wealth, the entrepreneur would capture a larger share of the surplus the larger the value of the existing assets are, but investment efficiency would not be improved.

6 Conclusion

Our paper studies how well primary financial markets allocate capital when information is dispersed among market participants, and how the efficiency of the market is affected by market size. We show that financing offers made by investors fail to convey their private information once information has real value for guiding investment decisions, and that the resulting investment inefficiencies can grow larger with the size of the market. Our analysis shows that several intuitive prescriptions from standard theory need to be reexamined when information has a real allocational role: a more competitive, larger financial market may reduce welfare and entrepreneurial revenues, early releases of information may be suboptimal, and collusion among investors may be beneficial for an entrepreneur seeking financing.
References


Figure 1. Market size and social surplus. Figure 1 plots social surplus as a function of number of investors in the setting with binary signals: $f_B(s) = 1$ for all $s \in [0, 1]$, $f_C(s) = 0$ for $s \in [0, 1/2]$ and $f_C(s) = 2$ for $s > 1/2$.

Figure 2. Equilibrium market size. Panel A of Figure 2 shows social surplus gross of investor costs and the expected revenues to the entrepreneur as a function of the size of the market. Panel B shows expected gross profits to investors from participating in the auction as a function of market size, as well as a particular specification for the cost $c_i$ of information gathering for each investor. The parameters are as follows: The project is good or bad with equal probabilities. A good project has net present value of 0.8 and a bad project has net present value of -1; $f_B(s) \equiv 1$; $f_C(s)$ is the normal distribution with mean 1 and standard deviation 0.75 truncated to the interval [0, 1].
Proof of Lemma 1: To prove the lemma it is enough to show that if an investor with signal \( \hat{s} \) participates in the fundraising mechanism then any investor with signal above \( \hat{s} \) will also choose to participate. Note that by the incentive compatibility the expected profit of the investor with signal \( \hat{s} \) must be nonnegative:

\[
\Pr(\hat{s} \geq \max_{j \neq i} S_j \notin B | S_i = \hat{s}) E \left[ w(S, a, \theta) - I(S) | \hat{s} \geq \max_{j \neq i} S_j \notin B, S_i = \hat{s} \right] \geq 0. \tag{A1}
\]

Because signals are conditionally independent, we can rewrite the expected profit as

\[
\sum_{\theta \in \{G,B\}} \Pr(\theta | S_i = \hat{s}) \Pr(\hat{s} \geq \max_{j \neq i} S_j \notin B | \theta) E \left[ w(\hat{s}, S^{-i}, a, \theta) - I(\hat{s}, S^{-i}) | \theta, \hat{s} \geq \max_{j \neq i} S_j \notin B \right]. \tag{A2}
\]

Note that

\[
E \left[ w(\hat{s}, S^{-i}, a, \theta) - I(\hat{s}, S^{-i}) | B, \hat{s} \geq \max_{j \neq i} S_j \notin B \right] \leq 0
\]

since the bad type project is negative NPV and the security payout cannot exceed the project’s cash flows. Hence, it must be that

\[
E \left[ w(\hat{s}, S^{-i}, a, \theta) - I(\hat{s}, S^{-i}) | G, \hat{s} \geq \max_{j \neq i} S_j \notin B \right] \geq 0.
\]

Consider now an investor with signal \( s > \hat{s} \). From strict MLRP, \( \Pr(\theta | S_i = s) > \Pr(\theta | S_i = \hat{s}) \). Therefore, if he submits a report \( \hat{s} \) his expected payoff is strictly larger than that of the investor with signal \( \hat{s} \). From the incentive compatibility condition it then follows that such an investor will participate in the fundraising mechanism. Q.E.D.

Proof of Lemma 3: We first prove that the expected profit of an investor is increasing in his signal. Suppose an investor with signal \( s \) uses a strategy \( b \) (in the first- and second price auctions, strategies are bids; in ascending-price auctions, strategies are drop-out prices contingent on the history observed in the auction). Define the information learnt from winning with strategy \( b \) by \( \Omega(b) \). In the first- and second price auction, \( \Omega(b) \) is the bids of other investors. In the ascending price auction, \( \Omega(b) \) is the drop-out levels of other investors. Let the investment decision upon winning be a function \( I(\Omega(b), s) \in \{0,1\} \), where \( I(\Omega(b), s) = 1 \) if the winner decides to invest in the project. Because signals are conditionally independent, the expected profits for a bidder with signal \( s \) is

\[
\sum_{X \in \{G,B\}} \Pr(X | s) \Pr(\text{win} | X, b) \left( E[V - I | X] \Pr(I(\Omega(b), s) = 1 | X) - E[\text{price} | X, b] \right), \tag{A3}
\]

where \( E[\text{price} | X, b] \) is the expected price paid by a winner using strategy \( b \) conditional on the true state of the project being \( X \).
Suppose the bidding strategy $b$ and the investment rule $I(\Omega(b), s)$ is a best reply for a bidder with signal $s$. Note that

$$E[V - I|B]\Pr(I(\Omega(b), s) = 1|B) - E[\text{Price}|B, b] \leq 0$$

since the project does not break even in the bad state and since prices are always weakly positive. If the strategy $b$ is a best reply, it must be that

$$E[V - I|G]\Pr(I(\Omega(b), s) = 1|G) - E[\text{Price}|G, b] \geq 0$$

since it is always possible to not participate in the auction. From strict MLRP, it then follows that an investor’s expected revenues are always weakly increasing in his signal $s$, since he can always replicate the best reply he would use if he had a lower signal and make at least weakly higher profits. Furthermore, if expected profits are strictly positive for some signal $s$, expected profits are strictly higher for higher signals.

Thus, the participation is of the threshold type. The existence of equilibrium now follows from Milgrom and Weber (1982). For any sufficiently small cost $\varepsilon$ there exists a unique threshold $\hat{s}_\varepsilon > \bar{s}_N$ defined as the highest signal such that

$$\Pi(\hat{s}_\varepsilon) \leq \varepsilon.$$

Because the probability of winning the auction is bounded from below, it is clear that $\hat{s}_\varepsilon \rightarrow \bar{s}_N$ as $\varepsilon \rightarrow 0$. Q.E.D.

**Proof of Lemma 4:** Recall that the threshold $\bar{s}_N$ is defined as the highest signal such that

$$P(G|\max_i S_i = \bar{s}_N) \leq \pi^*.$$  \hspace{0.5cm} (A4)

Equation (A4) can be written explicitly as

$$\left( \frac{F_G(\bar{s}_N)}{F_B(\bar{s}_N)} \right)^{N-1} \frac{f_G(\bar{s}_N)}{f_B(\bar{s}_N)} \leq \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}.$$  \hspace{0.5cm} (A5)

From MLRP the left-hand side of equation (A5) is increasing $\bar{s}_N$ and is equal to $\lambda$ at $\bar{s}_N = 1$. Therefore, if $\pi^*/(1 - \pi^*) \geq \lambda \pi_0/(1 - \pi_0)$ then for any $N$, $\bar{s}_N = 1$ so the project is always abandoned. Suppose, therefore that $\pi^*/(1 - \pi^*) > \lambda \pi_0/(1 - \pi_0)$.

For any fixed $\pi_0$, the left-hand side of equation (A5) goes to zero as $N$ goes to infinity. Therefore, $\lim_{N \rightarrow \infty} \bar{s}_N = 1$. By assumption the densities $f_G$ and $f_B$ are continuous functions. Therefore, equation (A5) has a unique solution for large enough $N$. Taking the logarithm of both parts of equation (A5) we have

$$\lim_{N \rightarrow \infty} (N - 1) \ln \left( \frac{F_G(\bar{s}_N)}{F_B(\bar{s}_N)} \right) = \ln \left( \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)} \right) - \ln \lambda.$$  \hspace{0.5cm} (A6)
Since
\[
\lim_{s \to 1} F_G(s) = 1 - f_G(1 - s), \\
\lim_{s \to 1} F_B(s) = 1 - f_B(1 - s), \\
\lim_{s \to 1} \frac{f_G(s)}{f_B(s)} = \lambda,
\]
there exist limits
\[
\lim_{N \to \infty} -(N - 1) \ln(F_B(\bar{s}_N)) = \tau, \\
\lim_{N \to \infty} -(N - 1) \ln(F_G(\bar{s}_N)) = \lambda \tau.
\]

From Equation (A6) then \(\tau\) must solve
\[
(\lambda - 1)\tau = \ln \lambda - \ln \left( \frac{1 - \pi_0}{\pi_0} \frac{\pi^*}{1 - \pi^*} \right).
\]

By Theorem 4.2.3 of Embrechts, Klüppelberg and Mikosch (2012)
\[
\lim_{N \to \infty} \Pr(\max_i S_i \leq \bar{s}_N | B) = e^{-\tau}, \\
\lim_{N \to \infty} \Pr(\max_i S_i \leq \bar{s}_N | G) = e^{-\lambda \tau},
\]
which completes the proof of the lemma. Q.E.D.

**Proof of Lemma 5:** Let \(\pi_t\) be the probability that the project is good given the history of news up to the moment \(t\). It is well-known (see e.g., Liptser and Shiryaev (1978)) that \(\pi_t\) evolves according to
\[
d\pi_t = \varphi \pi_t (1 - \pi_t) d\tilde{B}_t, \quad \pi_0 = \hat{\pi}
\]
where
\[
\tilde{B}_t = (\mu_0/\sigma) t - \varphi \int_0^t \pi_s ds + X_t
\]
is a standard Brownian motion under investors’ filtration. Since \(\pi_t\) is a Markov process we can write the value of the project (which now includes the real option value to wait) as \(V(\pi_t)\). The standard result then is that in the continuation region (see e.g., Dixit-Pindyck (1994)) the value function satisfies the Bellman equation:
\[
\frac{1}{2} \varphi^2 \pi^2 (1 - \pi)^2 V''(\pi) - c = 0.
\]

A general solution to the above equation is given by
\[
V(\pi) = C_1 + C_2 \pi - \frac{c}{\varphi^2} (1 - 2\pi) \ln \left( \frac{\pi}{1 - \pi} \right),
\]
(A8)
where constants $C_1$ and $C_2$ are determined from the boundary conditions:

$$V(\pi^*) = 0, \quad (A9)$$
$$V'(\pi^*) = 0, \quad (A10)$$
$$V(\pi^{**}) = \pi^{**} v_G + (1 - \pi^{**}) v_B, \quad (A11)$$
$$V'(\pi^{**}) = v_G - v_B. \quad (A12)$$

Note that if $c > 0$, $V(\pi)$ in Equation (A8) goes to $-\infty$ as $\pi \to 0$. Therefore, it must be that $\pi^* > 0$. Q.E.D.

**Proof of Proposition 1:** Social surplus is

$$U(\bar{\pi}_N, N) = \pi_0 V_G \Pr(Y_{1,N} > \bar{\pi}_N | G) + (1 - \pi_0) V_B \Pr(Y_{1,N} > \bar{\pi}_N | B)$$
$$= \pi_0 V_G (1 - F_G(\bar{\pi}_N)^N) + (1 - \pi_0) V_B (1 - F_B(\bar{\pi}_N)^N)$$
$$= \pi_0 V_G \left( (1 - F_G(\bar{\pi}_N)^N) - \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)} (1 - F_B(\bar{\pi}_N)^N) \right). \quad (A13)$$

Recall that $\bar{\pi}_N$ solves

$$\frac{F_G(\bar{\pi}_N)^N - 1}{F_B(\bar{\pi}_N)^N - 1} = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}. \quad (A14)$$

Note that one can extend $U(\bar{\pi}_N, N)$ to real $N$ using the above formulas. Equation (A14) implies that $\frac{\partial}{\partial \bar{\pi}_N} U(\bar{\pi}_N, N) = 0$. Therefore, we have

$$\frac{d}{dN} U(\bar{\pi}_N, N) = \frac{\partial}{\partial \bar{\pi}_N} U(\bar{\pi}_N, N) \frac{d\bar{\pi}_N}{dN} + \frac{\partial}{\partial N} U(\bar{\pi}_N, N) = \frac{\partial}{\partial N} U(\bar{\pi}_N, N). \quad (A15)$$

From equation (A13),

$$\frac{\partial}{\partial N} U(\bar{\pi}_N, N) = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)} \ln(F_B(\bar{\pi}_N)) F_B(\bar{\pi}_N)^N - \ln(F_G(\bar{\pi}_N)) F_G(\bar{\pi}_N)^N. \quad (A16)$$

From equation (A14), we can rewrite equation (A17) as

$$\frac{\partial}{\partial N} U(\bar{\pi}_N, N) = f_G(\bar{\pi}_N) F_G(\bar{\pi}_N)^{N-1} \left( \ln(F_B(\bar{\pi}_N)) \frac{F_B(\bar{\pi}_N)}{f_B(\bar{\pi}_N)} - \ln(F_G(\bar{\pi}_N)) \frac{F_G(\bar{\pi}_N)}{f_G(\bar{\pi}_N)} \right). \quad (A17)$$

Thus, the surplus $U(\bar{\pi}_N, N)$ increases(decreases) with $N$ whenever $\ln(F_G(s)) / \ln(F_B(s))$ is an increasing(decreasing) function at $\bar{\pi}_N$. It is straightforward to verify that $\ln(F_G(s)) / \ln(F_B(s))$ is an increasing(decreasing) function on $(x, 1]$ if $\frac{f_G(s)}{f_G(s)} / \frac{f_B(s)}{f_B(s)}$ is an increasing(decreasing) function.
Proof of Proposition 2: By Lemma 4 there exists some $N^*$ such that $\bar{s}_N \geq 1 - \varepsilon$ for all $N > N^*$. Over the interval $[1 - \varepsilon, 1]$, the function $\frac{f_G(s)}{F_G(s)} / \frac{f_B(s)}{F_B(s)}$ is strictly decreasing, so by Proposition 1, surplus is decreasing in $N$ for $N > N^*$. All investor with signals in this interval must make the same expected profits because their signals have the same informational content. Investors with signals in $[1 - \varepsilon, \bar{s}_N]$ do not participate and hence makes zero profits, which implies that all bidders make zero profits. Hence, the entrepreneur captures all the surplus, and since surplus is maximized with a restricted number of investors, and so is entrepreneurial revenue. Q.E.D.

Proof of Proposition 3: Suppose all costs are zero. Then by Proposition 1 there is an $N^*$ such that surplus decreases with $N > N^*$. Thus, the optimal size of the market cannot be larger than $N^*$. Because the MLRP holds strictly all investors earn strictly positive expected profit. Fix any $N > N^*$. Let $p_N$ be the expected profit of an individual investor in the market with $N$ investors. It is clear then that if $c < p_N$ than the size of the market will be larger than socially optimal size $N^*$.

To show that a proportional decrease in costs for all investors can lead to a decrease in social surplus consider the following situation. Fix $N > N^*$ and let $\Delta$ be the difference in social surplus in the market with $N$ and $N + 1$ investors. Since $N > N^*$ this difference is positive. Suppose that gathering costs are such that $c_N < p_{N+1}$ and $c_{N+1} > p_{N+1}$. In this case, the market size is $N$. Let $C = \sum_{i=1}^{N+1} c_i$. Suppose $\gamma > 0$ is small enough so that $\gamma C < \Delta$ and $\gamma C < C_{N+1}$, and that $c_{N+1}(1 - \gamma) < p_{N+1}$. Then a proportional decrease in all costs by a factor of $\gamma$ will lead to a new new market size of $N + 1$ and a overall reduction in both net and gross of fees social surplus. Q.E.D.

Proof of Proposition 4: Let $Y = \max_i S_i$. As before, if $X$ is released before the fundraising process than the participation threshold $\bar{s}_N(x)$ solves as

$$\frac{P(G|Y = \bar{s}_N(x), X = x)}{P(B|Y = \bar{s}_N(x), X = x)} = \frac{\pi^*}{1 - \pi^*}. \quad (A18)$$

If $X$ is released after the fundraising process than investor will bid a positive amount as long as there is a nonzero probability that that after $X$ is released the updated probability that the project is good is above $\pi^*$. Hence, the participation threshold $\bar{s}_N$ solves

$$\frac{P(G|Y = \bar{s}_N)}{P(B|Y = \bar{s}_N)} \leq \sup_x \frac{P(X = x|G, Y = \bar{s}_N)}{P(X = x|B, Y = \bar{s}_N)} = \frac{\pi^*}{1 - \pi^*}. \quad (A19)$$

Note that

$$\frac{P(G|Y = \bar{s}_N(x), X = x)}{P(B|Y = \bar{s}_N(x), X = x)} = \frac{P(G|Y = \bar{s}_N)}{P(B|Y = \bar{s}_N)} \frac{P(X = x|G, Y = \bar{s}_N)}{P(X = x|B, Y = \bar{s}_N)}. \quad 31$$
Therefore, we can write Equation (A19) as

\[
\mathbb{E} \sup_x \frac{P(G|Y = \bar{\sigma}_N, X = x)}{P(B|Y = \bar{\sigma}_N, X = x)} = \frac{\pi^*}{1 - \pi^*}.
\]  

(A20)

By Assumption 3 the likelihood ratio

\[
\frac{P(G|Y = \bar{\sigma}_N, X = x)}{P(B|Y = \bar{\sigma}_N, X = x)}
\]

is increasing in \(\bar{\sigma}_N\). Therefore, for any \(x, \bar{\sigma}_N \leq \bar{\sigma}_N(x)\). Q.E.D.

**Proof of Proposition 5:** We first prove that the expected surplus in the \(K\)-unit auction with finite \(K\) is strictly lower than \(\pi_0 v_G\), even if winning investors share their signals before the decision to invest is made. As before, one can show that participation decision is monotone in investor’s signal. Let us denote by \(s_{K,N}\) such a participation threshold. We also denote the order statistics of the \(N\) signals received by investors by \(Y_{1,N},...,Y_{N,N}\) so that \(Y_{1,N}\) represents the highest signal, \(Y_{2,N}\) represents the second-highest signal, et cetera.

Consider an arbitrary small participation cost. Note that any participating investor can recoup this cost only if there is a nonzero probability that the project is started when this investor is among \(K\) investors with the most optimistic signals. Note that the most negative(positive) signal realization of the first \(K-1\) investors is when each of them gets the marginal(top) signal. Hence, in any robust equilibrium it must be that \(s_{K,N}\) lies between the \(\underline{s}_{K,N}\) and \(\bar{s}_{K,N}\) defined as

\[
P(G|Y_{K} = ... = Y_{1} = \bar{s}_{K,N}) = \pi^*
\]  

(A21)

and

\[
P(G|Y_{K} = \underline{s}_{K,N}, Y_{K-1} = ... = Y_{1} = 1) = \pi^*.
\]  

(A22)

We can write Equations (A21) and (A22) explicitly as

\[
\frac{F_G(\bar{s}_{K,N})^{N-K} f_G(\bar{s}_{K,N})^K}{F_B(\bar{s}_{K,N})^{N-K} f_B(\bar{s}_{K,N})^K} = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}.
\]  

(A23)

and

\[
\frac{F_G(\underline{s}_{K,N})^{N-K} f_G(\underline{s}_{K,N})^K}{F_B(\underline{s}_{K,N})^{N-K} f_B(\underline{s}_{K,N})^K} = \frac{1}{\lambda^{K-1}} \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}.
\]  

(A24)

Following similar steps as in the proof of Lemma 4 one can show that there exist limits

\[
\lim_{N \to \infty} -(N-K) \ln(F_B(\bar{s}_{K,N})) = \lim_{N \to \infty} -(N-K) \ln(F_B(\underline{s}_{K,N})) = \tau,
\]

\[
\lim_{N \to \infty} -(N-K) \ln(F_G(\bar{s}_{K,N})) = \lim_{N \to \infty} -(N-K) \ln(F_G(\underline{s}_{K,N})) = \lambda \tau.
\]

where \(\tau\) solves

\[
(\lambda - 1)\tau = K \ln \lambda - \ln \left(\frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}\right).
\]  

(A25)
By Theorem 4.2.3 of Embrechts, Klüppelberg and Mikosch (2012)

$$\lim_{N \to \infty} \Pr(Y_K \leq s_{K,N}|B) = e^{-\tau \sum_{i=0}^{K-1} \frac{\tau^i}{i!}},$$

$$\lim_{N \to \infty} \Pr(Y_K \leq s_{K,N}|G) = e^{-\lambda \sum_{i=0}^{K-1} (\lambda \tau)^i}. $$

Next, we prove that if $K/N \to (1 - \alpha)$, $\alpha \in (0, 1)$ as $N \to \infty$ then the expected surplus converges to $\pi_0 v_G$. We assume that the decision to start the project lies with the $K^{th}$ highest bidder. Note that as $N \to \infty$ and $K/N \to 1 - \alpha$, $Y_{K,N}$ becomes an $\alpha^{th}$ sample quantile. It is well-known that

$$\sqrt{N}(Y_{K,N} - s_\alpha) \xrightarrow{d} N(0, \alpha(1 - \alpha)/f(s_\alpha)^2),$$

where $f(x)$ and $F(x)$ are pdf and cdf of observations and $F(s_\alpha) = \alpha$. Let $s_{\alpha,G}$ and $s_{\alpha,B}$ be such that $F_G(s_{\alpha,G}) = \alpha$ and $F_B(s_{\alpha,B}) = \alpha$. Because of the MLRP $s_{\alpha,B} < s_{\alpha,G}$.

Consider a threshold $s_{K,N}$ that solves

$$P(G|Y_K = s_{K,N}) = \pi^*. \tag{A26}$$

Note that any investor with signal above $s_{K,N}$ will find it profitable to participate in the fundraising process provided that the cost of participation is small enough. We can write Equation (A26) implicitly as

$$\frac{F_G^{N-K}(s_{K,N})(1 - F_G(s_{K,N}))^{K-1} f_G(s_{K,N})}{F_B^{N-K}(s_{K,N})(1 - F_B(s_{K,N}))^{K-1} f_B(s_{K,N})} = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}. \tag{A27}$$

If $K/N = 1 - \alpha$ then we can write equation (A27) as

$$\left( \frac{F_G(s_{K,N})^\alpha(1 - F_G(s_{K,N}))^{1-\alpha}}{F_B(s_{K,N})^\alpha(1 - F_B(s_{K,N}))^{1-\alpha}} \right)^N \frac{(1 - F_B(s_{K,N})) f_G(s_{K,N})}{(1 - F_G(s_{K,N})) f_B(s_{K,N})} = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}. $$

As $N$ goes to infinity $s_{K,N}$ converges to the value $s_\alpha$, which solves

$$F_G(s_\alpha)^\alpha(1 - F_G(s_\alpha))^{1-\alpha} = F_B(s_\alpha)^\alpha(1 - F_B(s_\alpha))^{1-\alpha}. \tag{A28}$$

Notice that $x^\alpha(1 - x)^{1-\alpha}$ is a single-peaked function that reaches its maximum at $x = \alpha$. Therefore, $s_{\alpha,B} < s_\alpha < s_{\alpha,G}$, and so as $N \to \infty$ the probability of undertaking the project goes to one if the project is good and goes to zero if the project is bad. Q.E.D.

**Proof of Proposition 6:** In general, auctions with multi-dimensional signals are notoriously difficult to analyze. What makes our model amenable for analysis is that one can reduce a multi-dimensional signal to a one-dimensional one. To see this, for each club $i = 1, ..., N$ partition the signal space $[0,1]^M$ into a collection of $(M - 1)$-dimensional hyperplanes such that on each
hyperplane, the likelihood ratio of $M$ club signals is constant:

$$L_i(s) = \{s_{i1}, ..., s_{iM} \in [0, 1]^M : \prod_{j=1}^M \frac{f_G(s_{ij})}{f_B(s_{ij})} = \left( \frac{f_G(s)}{f_B(s)} \right)^M, \ s \in [0, 1]. \ (A29)$$

Note that because signals are conditionally independent $L_i$ is a sufficient statistics to update the likelihood that the project is good given club $i$’s investor signals $S_{i1}, ..., S_{iM}$. Requiring then club bids to be measurable with respect to the partition induced by hyperplanes $L_i$ reduces a fundraising process with multi-dimensional signals to a fundraising process with a one-dimensional signal, to which all theory we have developed in this paper applies.

For each $s \in [0, 1]$ define

$$\hat{F}_G(s) = \Pr(\prod_{j=1}^M \frac{f_G(S_j)}{f_B(S_j)} \leq \left( \frac{f_G(s)}{f_B(s)} \right)^M \mid G),$$

$$\hat{F}_B(s) = \Pr(\prod_{j=1}^M \frac{f_G(S_j)}{f_B(S_j)} \leq \left( \frac{f_G(s)}{f_B(s)} \right)^M \mid B).$$

It is straightforward to verify that functions $\hat{F}_G$ and $\hat{F}_B$ satisfy

$$\lim_{s \to 1} \hat{F}_G(s) = 1 - cf_G(1)^M (1 - s)^M,$$

$$\lim_{s \to 1} \hat{F}_B(s) = 1 - cf_B(1)^M (1 - s)^M,$$

where $c$ is some constant. Let $\bar{s}_{M,N}$ be a participation threshold with $N$ clubs where each club consists of $M$ investors. Following similar steps as in the proof of Lemma 4 one can show that $\bar{s}_{M,N}$ satisfies

$$\hat{F}_G(\bar{s}_{M,N})^{N-1} f_G(\bar{s}_{M,N})^M \hat{F}_B(\bar{s}_{M,N})^N f_B(\bar{s}_{M,N})^M = (1 - \pi_0)^\pi^* \pi_0 (1 - \pi^*),$$

and that there exist limits

$$\lim_{N \to \infty} -(N - 1) \ln(\hat{F}_B(\bar{s}_{M,N})) = \tau,$$

$$\lim_{N \to \infty} -(N - 1) \ln(\hat{F}_G(\bar{s}_{M,N})) = \lambda^M \tau.$$

From Equation (A30) then $\tau$ must solve

$$(\lambda^M - 1)\tau = M \ln \lambda - \ln \left( \frac{1 - \pi_0}{\pi_0} \cdot \frac{\pi^*}{(1 - \pi^*)} \right). \ (A31)$$

Thus, the limiting participation threshold with clubs is the same as the participation threshold with individual investors where the informativeness of the top signal is $\lambda^M$. Lemma 4 imply
that the asymptotic social surplus when the number of investor is large depends only on the
informativeness of the top signal $\lambda$, and is equal to

$$
\pi_0 V_G \left( 1 - \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{1 - \pi^*} + (\lambda - 1) \left( \frac{1 - \pi_0}{\lambda} \frac{\pi^*}{\pi_0 (1 - \pi^*)} \right)^{\frac{\lambda}{1 - \pi}} \right). 
$$

(A32)

It can be directly verified that (A32) is increasing in $\lambda$. Since the top club signal is more
informative than a top signal of a single investor social surplus with club bids is higher than
that with individual investors for large $N$. Therefore, for large $N$, the entrepreneur will obtain
higher revenues with club bids in any competitive fundraising mechanism where her share of the
total surplus goes to one with the number of investors. Q.E.D.

Proof of Proposition 7: Recall that the threshold $s_N$ is defined as the highest signal such
that if an investor with signal $s_N$ wins the auction he concludes that the project is zero NPV.
When the number of potential investors is stochastic, $s_N$ solves

$$
P(G|Y_{1,N\nu} = s_N) = \pi^*,
$$

(A33)

where $Y_{1,N\nu}$ is the first-order statistic from a random sample of size $N\nu$. Equation (A33) can be
written explicitly as

$$
\frac{f_G(s_N) \int_0^\infty F_G(s_N)^{N\nu} dF(\nu)}{f_B(s_N) \int_0^\infty F_B(s_N)^{N\nu} dF(\nu)} = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{1 - \pi^*}.
$$

(A34)

Let $\phi(\tau)$ denote the Laplace transform of the random variable $\nu$

$$
\phi(\tau) = \int_0^\infty e^{-\tau \nu} dF(\nu).
$$

Let

$$
\tau_G,N(s_N) = -N \ln(F_G(s_N)),
$$

$$
\tau_B,N(s_N) = -N \ln(F_B(s_N)).
$$

Equation (A34) can be written as

$$
\frac{f_G(s_N) \phi(\tau_G,N(s_N))}{f_B(s_N) \phi(\tau_B,N(s_N))} = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{1 - \pi^*}.
$$

(A35)

Consider a sequence of $\tau_{B,N}(s_N)$ as $N$ goes to infinity. There can be two cases: either $\tau_{B,N}(s_N)$
stays bounded or it goes to infinity. We will show that the latter case cannot realize. Suppose
that $\tau_{B,N}(s_N)$ goes to infinity. Note that by the MLRP $F_B(s) \geq F_G(s)$ for any $s$. Therefore, if
$\tau_{B,N}(s_N)$ goes to infinity then $\tau_G,N(s_N)$ also goes to infinity.

Suppose first that $F(\bar{\nu}) > 0$ for any $\bar{\nu} > 0$ and that $\pi_0 < \pi^*$. Since $\nu$ has a continuous
density at zero
\[ F(t) \sim t^\rho L \left( \frac{1}{t} \right), \quad t \to 0, \]
where \( L \) is some positive function varying slowly at \( \infty \) (see e.g., Feller (1970), ch 8 for a definition), and \( \rho \geq 1 \). Therefore, for large \( N \), by theorems 2 and 3 in Feller (1970), ch 8.5, we have
\[ \frac{\phi(\tau_{G,N}(\bar{s}_N))}{\phi(\tau_{B,N}(\bar{s}_N))} \sim \left( \frac{\tau_{B,N}(\bar{s}_N)}{\tau_{G,N}(\bar{s}_N)} \right)^\rho = \left( \frac{\ln(F_B(\bar{s}_N))}{\ln(F_G(\bar{s}_N))} \right)^\rho. \]
The condition \( \pi_0 < \pi^* \) implies that
\[ \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)} > 1. \]
Hence, if \( \tau_{B,N}(\bar{s}_N) \to \infty \) with \( N \) the limit of \( \bar{s}_N \) must solve
\[ \lim_{N \to \infty} \frac{f_G(\bar{s}_N)}{f_B(\bar{s}_N)} \left( \frac{\ln(F_B(\bar{s}_N))}{\ln(F_G(\bar{s}_N))} \right)^\rho = \frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)}. \quad (A36) \]
The above equation, however, has no solution. To see this, note that for \( 0 \leq x \leq y \leq 1 \)
\[ \frac{\ln y}{\ln x} \leq \frac{1 - y}{1 - x}. \]
Therefore,
\[ \frac{f_G(\bar{s}_N)}{f_B(\bar{s}_N)} \left( \frac{\ln(F_B(\bar{s}_N))}{\ln(F_G(\bar{s}_N))} \right)^\rho \leq \frac{f_G(\bar{s}_N)}{f_B(\bar{s}_N)} \left( \frac{1 - F_B(\bar{s}_N)}{1 - F_G(\bar{s}_N)} \right)^\rho \leq 1, \]
where the last inequality follows from the MLRP.
Suppose now that there exists \( \tau > 0 \) such that \( F(\tau) = 0 \). Then as \( \tau_{B,N}(\bar{s}_N) \) and \( \tau_{G,N}(\bar{s}_N) \) go to infinity
\[ \frac{\phi(\tau_{G,N}(\bar{s}_N))}{\phi(\tau_{B,N}(\bar{s}_N))} \to 0. \]
Hence, equation (A35) cannot have a solution.
Thus, we arrive at a contradiction and it must be that \( \tau_{B,N}(\bar{s}_N) \) stays bounded as \( N \) goes to infinity. Hence, there is a subsequence \( N_k \) such that there is a finite limit
\[ \tau = \lim_{N_k \to \infty} \tau_{B,N_k}(\bar{s}_{N_k}). \]
Since \( \tau_{B,N}(\bar{s}_N) = -N \ln(F_B(\bar{s}_N)) \) has a finite limit, \( \bar{s}_{N_k} \) must go 1 with \( N_k \). Since
\[ F_G(s) \sim 1 - f_G(1)(1 - s), \quad s \to 1 \]
\[ F_B(s) \sim 1 - f_B(1)(1 - s), \quad s \to 1 \]
\[ f_G(1)/f_B(1) = \lambda, \]
there also exists a limit
\[ \lambda \tau = \lim_{N_k \to \infty} \tau_{G,N_k}(\bar{s}_{N_k}). \]
Thus, $\tau$ must solve equation (8).

Note that the rhs of equation (8) is a continuous function of $\tau$. Since $\phi(\tau) \to 1$ as $\tau$ goes to 0, the rhs of Equation (8) goes to $\lambda$ as $\tau$ goes to 0. Suppose again that $F(\nu) > 0$ for any $\nu > 0$ and that $\pi_0 < \pi^*$. As $s_N$ goes to 1 and $\tau$ goes to $\infty$ the rhs of equation (8)

$$\lim_{N_k \to \infty} \frac{f_G(s_{N_k}) \phi(\tau_B N(s_{N_k}))}{f_B(s_{N_k}) \phi(\tau_G N(s_{N_k}))} = \lim_{N_k \to \infty} \lambda \left( \frac{\ln(F_B(s_{N_k}))}{\ln(F_G(s_{N_k}))} \right)^\rho = \lambda^{1-\rho}.$$  

If there exists $\nu > 0$ such that $F(\nu) = 0$ then the rhs of equation (8) goes to zero with $N_k$. If

$$\frac{(1 - \pi_0)}{\pi_0} \frac{\pi^*}{(1 - \pi^*)} < \lambda,$$

then equation (8) has a solution.

By Assumption 5 the rhs of equation (8) is a strictly decreasing function of $\tau$. Therefore, Equation (8) has, in fact, a unique solution. As a result, $\tau_{B,N}(s_N)$ converges to $\tau$ with $N$. The last fact implies

$$\pi_N = 1 - \frac{\tau}{N} + o\left(\frac{1}{N}\right)$$  \hspace{1cm} (A37)

By Theorem 4.3.2 of Embrechts, Klüppelberg and Mikosch (2012)

$$\lim_{N \to \infty} \Pr(Y_{1,N\nu} \geq \pi_N | B) = 1 - E e^{-\tau\nu},$$

$$\lim_{N \to \infty} \Pr(Y_{1,N\nu} \leq \pi_N | G) = E e^{-\lambda\tau\nu},$$

which are the same as equations (6) and (7). Therefore, to prove (6) and (7) it remains to show that if an investor with signal above $s_N$ wins the auction he always starts the project even after observing bids of other investors. We showed Section 2 that this is always the case when the number of investors is known. When the number of investors is stochastic this may not necessarily be the case because by observing the number of investors above the black-out level the winner may conclude that there have been many potential investors, and so his signal is actually not so good.

Assumption 5, however, ensures that if an investor with signal above $s_N$ wins the auction he always starts the project even after observing bids of other investors. Intuitively, under Assumption 5, the likelihood of the project being good increases with the observed number of investors. Formally, suppose an investor with signal $\hat{s} > s_N$ wins the auction. Fix $k$ signal thresholds

$$s_{(1)} = \hat{s} > s_{(2)} > s_{(3)} > \cdots > s_{(k-1)} > s_{(k)} = s_N.$$

Define the variables

$$B_i = \sum_{j=1}^{N\nu} \mathbf{1}_{s_j > s_{(i)}}, \quad i = 1, \ldots, k,$$

which count the number of investors with signals above the thresholds $s_{(i)}$, $i = 1, \ldots, k$. Define
numbers $\tau_i, i = 1, \ldots, k$ as

$$s_{(k)} = 1 - \frac{\tau_k}{N}. \quad (A38)$$

By Theorem 4.3.1 of Embrechts, Klüppelberg and Mikosch (2012), for all integers $l_i \geq 0, i = 1, \ldots, k$,

$$\lim_{N \to \infty} \Pr(B_1 = l_1, B_2 = l_1 + l_2, \ldots, B_k = l_1 + \cdots + l_k | B) = E \left[ \frac{(\nu \tau_1)^{l_1} (\nu (\tau_2 - \tau_1))^{l_2} (\nu (\tau - \tau_{k-1}))^{l_k}}{l_1! l_2! l_k!} e^{-\nu \tau} \right],$$

and

$$\lim_{N \to \infty} \Pr(B_1 = l_1, B_2 = l_1 + l_2, \ldots, B_k = l_1 + \cdots + l_k | G) = E \left[ \frac{(\lambda \nu \tau_1)^{l_1} (\lambda \nu (\tau_2 - \tau_1))^{l_2} (\lambda \nu (\tau - \tau_{k-1}))^{l_k}}{l_1! l_2! l_k!} e^{-\lambda \nu \tau} \right].$$

Therefore,

$$\lim_{N \to \infty} \frac{\Pr(B_1 = l_1, B_2 = l_1 + l_2, \ldots, B_k = l_1 + \cdots + l_k | B)}{\Pr(B_1 = l_1, B_2 = l_1 + l_2, \ldots, B_k = l_1 + \cdots + l_k | G)} = \frac{E \left[ \frac{(\lambda \nu \tau_1)^{l_1} (\lambda \nu (\tau_2 - \tau_1))^{l_2} (\lambda \nu (\tau - \tau_{k-1}))^{l_k}}{l_1! l_2! l_k!} e^{-\lambda \nu \tau} \right]}{E \left[ \frac{(\nu \tau_1)^{l_1} (\nu (\tau_2 - \tau_1))^{l_2} (\nu (\tau - \tau_{k-1}))^{l_k}}{l_1! l_2! l_k!} e^{-\nu \tau} \right]} = \frac{\lambda^L \phi^{(L)}(\lambda \tau)}{\phi^{(L)}(\tau)}, \quad (A39)$$

where $L = l_1 + \cdots + l_k$, and $\phi^{(L)}$ is the $L$-order derivative of $\psi$. By theorem 2.1 of Jarrahiferiz, Borzadaran, and Roknabadi (2016), for all $L \geq 0$, $\phi^{(L)}(\lambda \tau)/\phi^{(L)}(\tau)$ is decreasing in $\tau$. Therefore,

$$\frac{\lambda^L \phi^{(L)}(\lambda \tau)}{\phi^{(L)}(\tau)} \geq \frac{\lambda^{L-1} \phi^{(L-1)}(\lambda \tau)}{\phi^{(L-1)}(\tau)} \geq \cdots \geq \frac{\phi(\lambda \tau)}{\phi(\tau)}.$$

Thus, we have shown that the likelihood of the project being good increases with the observed number of investors. Therefore, if an investor with signal above $s_N$ wins the auction he always starts the project.

The limiting social surplus is equal to

$$\pi_0 V_G \left( 1 - \left( \frac{1 - \pi_0}{\pi_0} \right) \frac{\pi^*}{1 - \pi^*} + (\lambda - 1)E e^{-\lambda \tau \nu} \right),$$

where we used equations (9) and (8). Therefore, to find $\nu$ that maximizes the limiting social surplus one needs to solve

$$\sup_{\nu} E e^{-\lambda \tau \nu} \quad (A39)$$

$$s.t. \quad E(e^{-\lambda \tau \nu} - \xi e^{-\tau \nu}) = 0. \quad (A40)$$

where

$$\xi = \frac{1}{\lambda} \left( \frac{1 - \pi_0}{\pi_0} \right) \frac{\pi^*}{1 - \pi^*}.$$
Let us introduce a new random variable \( v = e^{-\tau \nu} \). We can rewrite the above problem as

\[
\sup_{v \in [0,1]} Ev \\
\text{s.t. } Eg(v) = 0, 
\]

where

\[
g(x) = x(x^{\lambda-1} - \xi). 
\]

Note that \( g(x) \) is a convex function. Therefore, by the Jensen’s inequality

\[
0 = Eg(v) \geq g(Ev).
\]

Hence, the maximum possible value of \( Ev \) is \( (\xi)^{\frac{1}{\lambda-1}} \) and is achieved when \( v \) takes the value \( (\xi)^{\frac{1}{\lambda-1}} \) with probability 1. Thus, the limiting social surplus is maximized when there is no uncertainty about the number of investors. \textit{Q.E.D.}

**Proof of Proposition 8:** Consider the following equation:

\[
\frac{f_G(\bar{s}_N) \ln(F_B(\bar{s}_N))}{f_B(\bar{s}_N) \ln(F_G(\bar{s}_N))} = \frac{\pi^*}{1-\pi^*} \frac{1-\pi_0}{\pi_0}. 
\]

The condition \( \pi_0 > \pi^* \) implies that

\[
\frac{\pi^*}{1-\pi^*} \frac{1-\pi_0}{\pi_0} < 1. 
\]

By Assumption 2 in the paper,

\[
\lim_{s_N \to 0} \frac{f_G(\bar{s}_N) \ln(F_B(\bar{s}_N))}{f_B(\bar{s}_N) \ln(F_G(\bar{s}_N))} = \frac{f_G(0)}{f_B(0)} < \frac{\pi^*}{1-\pi^*} \frac{1-\pi_0}{\pi_0}. 
\]

We showed in the proof of Proposition 7 that

\[
\lim_{s_N \to 1} \frac{f_G(\bar{s}_N) \ln(F_B(\bar{s}_N))}{f_B(\bar{s}_N) \ln(F_G(\bar{s}_N))} = 1 > \frac{\pi^*}{1-\pi^*} \frac{1-\pi_0}{\pi_0}. 
\]

Since

\[
\frac{f_G(\bar{s}_N) \ln(F_B(\bar{s}_N))}{f_B(\bar{s}_N) \ln(F_G(\bar{s}_N))}
\]

is a continuous function of \( s_N \), Equation (A43) has a solution. Let \( s^* \in (0,1) \) be the largest solution to Equation (A43) such that

\[
\left( \frac{f_G(\bar{s}_N) \ln(F_B(\bar{s}_N))}{f_B(\bar{s}_N) \ln(F_G(\bar{s}_N))} \right)'_{\bar{s}_N=s^*} \neq 0.
\]
Fix a small neighborhood $B(s^*)$ of $s^*$. For any $\hat{s} \in B(s^*)$ define
\[
\tau_{G,N}(\hat{s}) = -N \ln(F_G(\hat{s})),
\tau_{B,N}(\hat{s}) = -N \ln(F_B(\hat{s})).
\]

We showed in the proof of Proposition 7 that
\[
\lim_{N \to \infty} \frac{\phi(\tau_{G,N}(\hat{s}))}{\phi(\tau_{B,N}(\hat{s}))} = \frac{\ln(F_B(\hat{s}))}{\ln(F_G(\hat{s}))},
\]
where we use the fact that if $F$ has a strictly positive density at zero then $\rho = 1$. By the implicit function theorem, for large enough $N$, there is a solution $\bar{s}_N$ to Equation (A33):
\[
P(G|Y_{1,N} = \bar{s}_N) = \pi^*.
\]
Furthermore,
\[
\lim_{N \to \infty} \bar{s}_N = s^*.
\]
Since $s^* < 1$ as $N$ goes to infinity the number of signals above $\bar{s}_N$ goes to $\infty$ with $N$. Since bids are strictly increasing observing them leads to perfect knowledge of the project type in the limit. Q.E.D.