Volatility, Correlation, and Spread ETFs as Factors

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Abstract
In a multi-factor world, true diversification benefits are not related to correlation. Portfolios can be re-weighted so that risk profiles mimic one another. Consequently, diversification depends only on the (idiosyncratic) volatility that remains unexplained by the factors after re-weighting. This evinces the fundamental importance of measuring the underlying factors and estimating factor sensitivities for every asset. Several methods for measuring factors have been investigated in previous literature, but an easy-to-implement general method is simply to specify a group of heterogeneous indexes or traded portfolios. Exchange Traded Funds (ETFs), particularly the new “Spread” ETFs, are ideally suited to such a purpose.

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I. Diversified Portfolios, Volatility and Correlation

There are two striking facts about portfolios of assets. First, even really well-diversified portfolios are quite volatile. The volatility of a large positively-weighted portfolio is often around half as large as the average volatility of its constituents.¹ Second, although well-diversified portfolios are highly correlated within the same asset class, they are much less correlated across asset classes; i.e., across bond vs. equities vs. commodities or across countries or across industry sectors.²

The first empirical fact is sufficient to suggest the existence of common underlying systematic influences, (or “risk drivers” or “factors”) that limit diversification within an asset class; otherwise diversified portfolios would have much smaller volatilities. The second fact intimates the presence of multiple systematic factors; otherwise diversified portfolios would be more correlated across asset classes, countries, and sectors.

Simple correlation is one-dimensional. Consequently, correlations are not likely to reveal a multi-dimensional structure in the underlying systematic factors. To illustrate this most easily, consider the simplest possible multi-dimensional world wherein all asset returns are driven by just two common factors that affect every asset linearly at time t according to the following return generating model:

\[ R_{i,t} = E_i + \beta_{i,1} f_{1,t} + \beta_{i,2} f_{2,t} + \varepsilon_{i,t} \]

where \( f_1 \) and \( f_2 \) denote the two common factors that influence the return \( R \) on asset \( i \) through its sensitivity coefficients, the \( \beta \)s. By assumption and without loss of generality, the factors have zero means, as does the idiosyncratic risk, \( \varepsilon \), while the expected return on asset \( i \) is \( E_i \). Note that everything is specific to asset \( i \) (and thus carries an \( i \) subscript), except the common factors.

¹ For example, during the decade from 2001 through 2010, the monthly total return on the S&P 500 had an annualized volatility (standard deviation) of 16.3%. Over the same period, the average volatility for the S&P’s constituents was 36.1%.
² From 2001 through 2010, the monthly total return correlation between the S&P 500 and Barclay’s Bond Aggregate Index was -0.0426. The return correlations between these two indexes and the Goldman Sachs Commodity index were 0.266 and 0.0113, respectively.
Also, in this most elementary of all multi-dimensional models, the asset’s expected return and its sensitivities (\(\beta\)’s) are assumed to be time invariant constants.

Given this simple world, consider now the correlations of well-diversified portfolios across asset classes. For example, suppose that two asset classes, A and B, have broad, widely-followed, well-diversified market indexes. Let’s suppose initially that the indexes are so well-diversified that both have negligible remaining idiosyncratic volatility; i.e.,

\[
R_{A,t} = E_A + \beta_{A,1} f_{1,t} + \beta_{A,2} f_{2,t}, \\
R_{B,t} = E_B + \beta_{B,1} f_{1,t} + \beta_{B,2} f_{2,t},
\]

The returns on both of these indexes are explained entirely by the same two underlying systematic factors. What about their correlation? It turns out that their correlation will be perfect if and only if for some constant of proportionality, \(k \neq 0\), both \(\beta_{A,1} = k \beta_{B,1}\) and \(\beta_{A,2} = k \beta_{B,2}\). For any other set of sensitivity coefficients, the correlation will be imperfect.\(^3\) Conceivably, the correlation can be quite low even though both index A and index B are driven by the same two common influences and are thus perfectly “integrated” in the sense used by Pukthuanthong and Roll (2009.)

To illustrate the range of possibilities, let’s allow two possibly different constants of proportionality, \(k_1\) and \(k_2\), that relate the sensitivities as follows: \(\beta_{A,1} = k_1 \beta_{B,1}\) and \(\beta_{A,2} = k_2 \beta_{B,2}\). For ease of illustration, assume that the factors have the same variance and that they are uncorrelated. In this situation, the surface shown in Figure 1 plots the correlation between A and B for different values of \(k_1\) and \(k_2\) ranging between -1 and +1. As the Figure shows, the correlation is +1 when the constant are positive and equal while the correlation is -1 when they are negative and equal. There is a discontinuity when the constants are zero. Notice too that the correlation is exactly zero for an entire set of non-zero constants of proportionality with opposite signs.

\(^3\)The formal proof is delivered by the Cauchy inequality; it generalizes to any number of factors (greater than one.) The correlation is +1 (-1) when \(k > (\leq) 0\).
Within an asset class, portfolios have similar sensitivities to the underlying factors, so correlations are relatively high. But across asset classes, this is evidently not the case. One might well imagine why it’s not by using the example of equities and bonds. Suppose factor 1 is related to shocks in real output and factor 2 is related to shocks in expected inflation. Then a positive shock in factor 1 would increase equity returns but not affect bonds all that much. Conversely, a reduction (a positive shock) in expected inflation would drive up nominal bond prices but have a more attenuated impact on equities. The result over many periods, when there are shocks in both real output and expected inflation, is a relatively low correlation between stocks and bonds. Of course, this is just an example for discussion and does is not meant to imply that equities and bonds really are so divergently sensitive to the true underlying factors. Indeed, there could be other systematic factors, such as investor confidence, that drive them in the same direction.

Another example comes from the rather puzzling low correlation often observed between equity indexes in two countries. Hong Kong and Saudi Arabia might both be driven similarly by global shocks to investor confidence but driven differentially by global energy shocks. Saudi stocks are driven upward by energy price increases but the opposite is true for Hong Kong, an energy importer. These two countries could be very well integrated in the sense that they both depend on the same global factors, yet their simple correlation could be small or even negative depending on the relative volatilities of investor confidence and energy shocks.

The remarks above have profound implications for portfolio management. Following Markowitz, diversification has traditionally been thought to be the most effective when assets or portfolios are not very correlated. But this intuition is misleading! Low correlation between bundles of assets fails to properly measure the potential benefits of diversification.

To see why, consider two diversified portfolio/indexes, perhaps in different asset classes, whose returns are driven by the same underlying systematic factors but with diverse sensitivities. Assume their simple correlation is relatively low, for the reasons previously mentioned. Diversification into the two indexes might seem powerful because many allocations between
them (such as 50-50) appear to substantially reduce volatility. But this overstates the true benefit because the respective index compositions are held constant.

Imagine the possibility of structuring a different investment portfolio from the individual assets in the first class (A) that matches extremely well the factor sensitivities of the original index in the second class (B). This is straightforward when there is a large enough menu of available derivatives or when short positions are feasible and inexpensive. The resulting returns would then conform to the following generating models:

\[
R_{A,t} = E_A + \beta_{B,1} f_{1,t} + \beta_{B,2} f_{2,t} + \varepsilon_{A,t}
\]

\[
R_{B,t} = E_B + \beta_{B,1} f_{1,t} + \beta_{B,2} f_{2,t} + \varepsilon_{B,t}
\]

Notice that the sensitivity coefficients (\(\beta\)'s) from the restructured portfolio of A assets now match the original sensitivity coefficients of index B. To allow for generality, there is still some remaining idiosyncratic risk, as represented by the \(\varepsilon\)'s.

What, then, is the actual diversification benefit available from combining A and B? We can gain some insight about this question by finding the minimum variance portfolio from combining index B with the \(\beta\) re-structured portfolio composed of assets in class A. It is straightforward to show\(^4\) that this portfolio has a weighting \(w\) in index B (and \(1-w\) in the re-structured portfolio A) equal to

\[
w = \frac{\text{Var}(\varepsilon_{A,t})}{\text{Var}(\varepsilon_{A,t}) + \text{Var}(\varepsilon_{B,t})}.
\]

In words, if the re-structured portfolio from the class A assets has no idiosyncratic component, diversifying into B brings no benefit in terms of risk reduction alone; \(w\) is zero. This is true even when the correlation is weak between the original indexes of classes A and B. Any benefit from combining B with A would have to be in terms of enhanced return, not reduced risk.

The logic behind this result is that a mimicking portfolio for B can be engineered from assets in A to have the same \(\beta\)'s as B. This engineered A-asset-only portfolio was available within A all along. It could have been combined with the original A index. At an extreme, if the engineered

\(^4\) Assuming, as usual, that the idiosyncratic terms are uncorrelated with the factors and with each other.
portfolio has no idiosyncratic volatility, it strictly dominates index B provided that B does have at least a modicum of idiosyncratic volatility. Consequently, B provides no genuine risk-reducing benefit over what could have been obtained with the A assets by themselves.

If the engineered A-asset-only portfolio retains some idiosyncratic risk, there is still some diversification benefit. But that benefit has nothing to do with the correlation between the original indexes A and B. Also, if $E_B$ happens to be large compared to $E_A$, there would be a benefit from combining B with A, but not because of risk reduction.

II. Diversification Benefits: a Better Measure

If the $\beta_B$-structured B-mimicking portfolio composed of A assets has an r-square on the underlying factors close to 1.0, there will be negligible diversification benefits from combining B and A. (The same would be true going the other direction; i.e., restructuring B to match the factor sensitivities of the A index.)

The initial impression of strong diversification benefits, suggested by the simple correlation between the initial indexes from classes A and B, does not account for the entire range of possibilities. Thus, the simple correlation between portfolios of assets is a bit misleading. There is nothing wrong with the Markowitz efficient set math, but it doesn’t go far enough when dealing with portfolios as opposed to individual assets. There is virtually no benefit from diversification when factor r-squares are close to 1.0 (in either A or B) even when correlation appears to be weak between their indexes. The r-squares of the indexes on the portfolios, not their correlation, is a better (inverse) measure of potential diversification benefits; high r-square, low benefits

Of course, a full-blown mean/variance analysis of individual assets in A and B would yield a correct measure, but this is rarely considered for good reasons. It is impossible to estimate the covariance matrix for a large number of individual assets except with a very long time series sample, and then non-stationarity becomes a serious problem.
III. The diversification benefit of adding individual assets to diversified portfolios.

In the well-known Treynor/Black [1973] analysis, the impact of adding an individual asset to an existing portfolio depends on their relative expected returns and their correlation (individual asset with portfolio).\(^5\)

But suppose the individual asset is i has the same form of return generating equation, (where, for illustration, we again assume that there are only two factors):

\[
R_{i,t} = E_i + \beta_{i,1} f_{1,t} + \beta_{i,2} f_{2,t} + \varepsilon_{i,t},
\]

and suppose a (perfect) i-mimicking portfolio (P) with exactly matching sensitivities can be engineered from among the portfolio of assets already held; i.e.,

\[
R_{P,t} = E_P + \beta_{i,1} f_{1,t} + \beta_{i,2} f_{2,t}.
\]

There can be no genuine reduction in diversified risk from adding asset i, regardless of its correlation with the original portfolio. (Adding i to the portfolio would be beneficial only if \(E_i > E_P\).)

IV. But what are the Factors?

Given the importance of assessing the true diversification benefit when combining portfolios or when adding a single asset to an existing portfolio, it is clearly essential to develop estimates of the \(\beta\)'s in every situation. But what exactly are the factors, the underlying risk drivers? They cannot be the infrequently-published official numbers about macro-economic variables because market prices move around much too rapidly. Instead, they must be high-frequency changes in market perceptions of pervasive macro-economic conditions. Perceptions could include (a) rational anticipations of change in macro conditions that are truly pervasive such as real output growth, real interest rates, inflation, energy, etc., and (b) behavior-driven pervasive shocks in confidence or risk perceptions such as panics, liquidity crises, etc.

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\(^5\) For instance, see Bodie, Kane and Marcus [2011, pp. 933-937.] The “beta” discussed there is proportional to the correlation between the individual asset and the existing portfolio.
To do a really good job of optimal diversification, we must be able to identify and measure the pervasive factor perceptions and then to estimate factor sensitivities for any asset or portfolio of interest. The first job is to identify and measure the factors. Existing literature has studied several alternative approaches.

One approach relies on an entirely statistical method such as principle components or factor analysis, (e.g., Connor and Korajczyk [1988], Roll and Ross [1980].) A second approach prespecifies macro-economic variables that seem likely to be pervasive and then pre-whitens the official numbers pertaining to such low frequency constructs as industrial production, inflation, and so on, (e.g., Chen, Roll and Ross [1986] for equities, Litterman and Scheinkman [1991] for bonds.) Then there is the approach of relying on asset pricing theory to develop proxies that are empirically related to average returns (e.g., Fama/French [1992], Carhart [1997].) Finally, a lesser known but simpler approach is to employ a handful of rather heterogeneous indexes or tradable portfolios.

Each of the above approaches has particular limitations. Purely statistical methods are theoretically sound but everything has to be stationary. Pre-specified macro-economic variables are the most theoretically solid but are observed with excruciatingly low frequency. Factor proxies suggested by asset pricing are weak theoretically and are not necessarily even related to risk. A group of heterogeneous diversified portfolios can have non-stationary compositions and be observed at high frequency - but heterogeneity must be sufficient to span all relevant and pervasive underlying risk drivers.

Heterogeneous portfolios work well for spanning global factors. Pukthuanthong and Roll (2009) went to a lot of trouble to extract ten global principal components. They employed the extracted global principal components as factor proxies and demonstrated a substantial increase in global market integration for many countries. Then, as a robustness check for their purely statistical procedures, they replaced the principal components with broad indexes from ten large countries and found virtually identical results. Country indexes are evidently sufficiently heterogeneous to span the same underlying macro perceptions as principal components.
Using a set group of portfolios is arguably the easiest and best approach to factor estimation if heterogeneity can be assured, which suggests that a well-chosen set of exchange traded funds (ETFs) might serve the purpose quite well. ETFs are often diversified portfolios or derivatives-based equivalents. As such, their returns must be driven mainly by underlying factors; i.e., by high-frequency changes in market perceptions of macro-economic conditions. Their idiosyncratic volatility should be relatively small. Moreover, they are generally liquid, transparent, and cheap to trade. Their variety across several asset classes suggests a healthy degree of heterogeneity.

V. The ETF Marketplace

According to the NYSE web site, there are now more than 20,000 ETPs (exchange-traded products) listed on exchanges around the world. The NYSE ARCA, the electronic network where many ETFs are traded domestically, has almost 2,000 ETFs listings. NYSE Euronext has additional listings and so do other exchanges.

ETF trading volume has been accelerating recently, particularly in relation to trading in related cash assets and in other derivatives such as options and futures. For example, Roll, Schwartz, and Subrahmanyam [2011] document an increase in trading volume for the S&P 500 ETF “Spider” of roughly 10,000% from 1996 through 2009. This compares with increased volume of around 100% in the underlying S&P 500 cash assets and a similar increase in options on the S&P 500 index. Meanwhile trading in the original futures contract on the index has actually declined (though this has been offset by trading in the newer “E-Mini” futures.)

Just on the NYSE, there are ETFs in many asset classes, including equities, bonds, commodities, and currencies. But as might have been anticipated, heterogeneity is not that impressive within each class. Domestic equity ETFs in particular are highly correlated with each other and with the broad market indexes. For example, Table 1 shows the correlation between 25 prominent equity-class ETFs and the S&P 500 Index. As can be seen there, all of these ETFs are very highly correlated with the Index (and, of course, with each other.) There is probably not enough
heterogeneity across many equity ETFs to provide a sufficient spanning of the underlying factors.

Across asset classes, however, ETF heterogeneity might be acceptable. Within the class of bond ETFs, there is more heterogeneity than among equity ETFs, while commodity and currency ETFs provide still more; (See Roll [2010].) Heterogeneity is also enhanced by “bear market” ETFs. As the name suggests, these funds are designed to move inversely to the underlying cash assets. Hence, they possess a relatively large degree of diversification potential. Further enhancements are provided by the recent appearance of “spread” ETFs, funds constructed to be long in one asset class and short in another.

VI. Spread ETFs as Factors

Spread ETFs may be particularly intriguing because they seem to promise good spanning over the underlying macroeconomic risk drivers without the necessity of short positions. To illustrate this possibility, consider the “FactorShares” ETFs with tickers FOL, FSE, and FSU. FOL is long oil and short stocks. FSE is long stocks and short US Treasury Bonds. FSU is long stocks and short the US Dollar against a basket of foreign currencies.6

All three of these ETFs are leveraged 2:1 and the leverage position is re-established on a daily basis. As a consequence, each one is rather volatile even though the long/short nature of the underlying investments likely reduces risk to some extent. There is, however, an advantage to being leveraged, particularly for a small investor: it is easier to de-leverage a position than to add leverage to an under-leveraged position. De-leveraging can be accomplished simply by investing a smaller amount in the ETF.

Given their particular structures, these three spread ETFs should have sensitivities to the underlying macroeconomic factor perceptions something like the hypothetical numbers in Table 2, which I have concocted for purposes of discussion but are meant to be more or less in the

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6 Technically, these ETFs use futures instead of cash positions. The stock futures are for the S&P 500 index, while the oil, bond and dollar are in specific futures contracts. A full explanation can be found at FactorShares.com.
ballpark. For example, FOL is positively sensitive, +2, to energy and negatively sensitive, -2, to equities since it is long oil futures and short stock index futures.

These sensitivities are rather large because the FOL is leveraged 2:1. By assumption, this particular ETF is neutral with respect to investor confidence under the reasonable presumption that its long and short elements would move together by roughly the same amount when that particular variable was perceived to change in value. Similar considerations have suggested the numerical sensitivities hypothesized in Table 2 for the two other spread ETFs, FSE and FSU.

Now imagine several investors whose preferences are motivating them to construct portfolios with a variety of differing risk exposures to the underlying macroeconomic factors. Investor #1 is relatively agnostic about the factors and decides to simply become equally exposed to all three but at a fairly low level of risk. After consulting with a savvy investment counselor, he invests in the three ETFs according to the percentage shown in Table 3. The resulting sensitivities to the underlying factors are shown in the last column. Each one is a relatively low 0.375.

Investor #2 is just as agnostic about the macro factors but is willing to take on more risk. He can tolerate a beta sensitivity of 1.0 to each macro risk. Consequently, the appropriate investments in the ETFs are relatively the same as for Investor #1 but they have to be leveraged upward by borrowing. This is achieved by borrowing 1 and 2/3 dollar for every dollar of equity and then investing 2 and 2/3 dollars in the same relative proportions as Investor #1. The investment percentages in the three ETFs are, respectively, 108.333%, 33.333%, and 125% (percent of initial equity.)

Another example might be an investor who is acutely intolerant of price changes induced by unexpected shocks in investor confidence and energy. Indeed, his most desirable portfolio would be completely insensitive to these macro variables but, conversely, he is willing to tolerate an average level of sensitivity to Industrial Production, a $\beta_{ip} = 1.0$. Using the same three ETFs, table 4 presents the investments that provide the desired risk profile.

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7 This is consistent with the leveraged ETFs listed in Table 1, such as ProShares Ultra S&P 500, whose estimated empirical “beta” is approximately 2.
Finally, an investor such as an airline might set aside funds to pay for future energy costs while avoiding exposure to the risks of other macroeconomic shocks. This dictates zero sensitivity to both industrial production and investor confidence and average sensitivity to energy. Table 5 provides optimal ETF investments for this energy hedger, who must borrow 50% of the available funds and invest the entire amount as shown.

All the investment allocations discussed above and portrayed in Tables 3-5 have totally positive weights in the three ETFs. Yet the range of risk profiles, \([\beta_{\text{Ind.Prd}}, \beta_{\text{Inv.Conf}}, \beta_{\text{Energy}}]\) is quite large; the four discussed risk profiles are, respectively, \([0.375, 0.375, 0.375]\), \([1.0, 1.0, 1.0]\), \([1.0, 0., 0.]\), and \([0., 0., 1.0]\). (Some leverage is required for the second and fourth profile.)

VII. Options on Spread ETFs and Implied Correlations

The implied volatility from options on Spread ETFs depends on correlations between the underlying constituent assets. This means that market consensus anticipations about future correlations can potentially be directly observed from prices. The spread ETFs now trading offer an opportunity to observe such prospective correlations between bonds and stocks, gold and stocks, the US$ and stocks, and oil and stocks.

To make the required calculations, we need three simultaneous option prices for the long (L) and short (S) components of the ETF and for the ETF itself.

To work out this calculation algebraically, it is convenient to use the following notation:

\[ R_L: \] return on the long asset held in the spread ETF, (subscript “L”)
\[ R_S: \] return on the short asset held in the spread ETF, (subscript “S”)
\[ w_j: \] weighting (or leverage ratio) for spread ETF position \( j \), \( j=L,S \)
\[ \sigma_j: \] implied volatility, standard deviation, of asset \( j \) (from an option written on \( j \))
\[ \rho: \] implied correlation between the returns on assets held long and short in the ETF

Assuming that there is no volatility in the borrowing undertaken to finance the leveraged spread ETF, the risky part of the ETF’s daily return is

\[ R_{\text{ETF}} = w_L R_L + w_S R_S. \]
For example, FOL is long futures in light sweet crude oil and short E-mini futures on the S&P 500 equity index. Both positions are leveraged 2:1. So the weightings for FOL are $w_L = 2.0$ and $w_S = -2.0$, which are fixed daily by rebalancing at the end of each trading day.

The variance of a spread ETF’s return is

$$\text{Var}(R_{ETF}) = \sigma_{ETF}^2 = w_L^2 \sigma_L^2 + w_S^2 \sigma_S^2 + 2w_Lw_S\rho\sigma_L\sigma_S$$

where $\rho$ is the correlation, whose value is the objective of our calculations.

The implied (i.e., market consensus) value of the spread ETF’s volatility can be obtained from its option while the implied volatilities of the ETFs constituents can be obtained from separate options on those assets. Consequently, everything is known or can be obtained from marketed options in the above variance formula, except for the correlation itself.

To solve for the correlation, we invert the formula and collect term to obtain

$$\rho = \frac{\sigma_{ETF}^2 - w_L^2 \sigma_L^2 - w_S^2 \sigma_S^2}{2w_Lw_S\sigma_L\sigma_S}.$$

This is the general formula for the implied correlation between the two asset classes held in a spread ETF.

Given the FOL values of $w_L = 2.0$ and $w_S = -2.0$, the correlation above is

$$\rho = \frac{\sigma_L^2 + \sigma_S^2}{2\sigma_L\sigma_S} - \frac{\sigma_{ETF}^2}{8\sigma_L\sigma_S}.$$ 

As a numerical example, suppose that options on light crude oil futures, S&P 500 E-mini futures, and Factor Shares FOL, imply, respectively, annual percentage volatilities of 20%, 15%, and 35%; (the latter seems large but remember that the FOL is levered 2:1.) Plugging in these numbers in the expression just above, we obtain

$$\rho = \frac{.2^2 + .15^2}{2(.2)(.15)} - \frac{.35^2}{8(.2)(.15)} = 0.53125.$$

This particular correlation, slightly above 0.5 between oil and equities, might strike some as implausibly high (or low) and, if so, the ETF option is too cheap (expensive) and a profitable
trade would involve buying (selling) it and offsetting the trade with a dynamically hedged short (long) position in the ETF itself.

Since correlation is always between +1 and -1, it is not hard to prove that the implied volatility from options on FOL must lie between 10% and 70%, given the oil and equity volatilities assumed in this numerical example. Anything outside that range would offer a pure arbitrage trade.

VIII. Conclusions

When evaluating the diversification benefit from combining two portfolios or when combing a single asset with an existing portfolio, the simple correlation is misleading in a multi-factor world. This is because a mimicking portfolio can be constructed from one portfolio to match the other portfolio or the single asset. A mimicking portfolio has the same risk profile as the portfolio or asset being mimicked; the same sensitivities to the true underlying high frequency macro perception shocks that comprise the factor risk drivers. When a mimicking risk profile can be engineered, the only thing that matters for diversification is the residual volatility that is not explained by the factors. If, in the limit, there is no residual volatility, there can be no benefit from diversifying.

This altered way of thinking about diversification makes it apparent that a fundamental investment concern involves measuring the underlying factors and then using those measurements to estimate risk profiles. Several different methods for measuring factors have been developed in the literature. Each has its own peculiar difficulties. However, one of the seemingly most attractive methods is simply to rely on a set of heterogeneous existing indexes or liquid tradable portfolios. With sufficient heterogeneity and liquidity, the true underlying factor space is likely to be spanned and the observations are of sufficiently high frequency.

Exchange traded funds (ETFs) exist in a large variety of flavors, are highly liquid for the most part and are themselves individually well-diversified. ETFs from divergent asset classes might
very well be some of the best proxies for the unobservable macro perceptions that are the true underlying risk drivers.

One ETF newcomer, the “spread ETF,” which is long one asset class and short another, is a particularly promising factor proxy. A large number of highly diverse risk profiles can be engineered just from long-only positions in spread ETFs. Spread ETFs also offer a unique opportunity for directly observing the market’s consensus belief about the future correlation between constituent asset classes.
References


Table 1

Characteristics of Twenty-Five Prominent Equity ETFs

The correlation shown is between the returns of the ETF and the S&P 500 index. N is the available sample size in months, since the inception of the ETF through March 2010. The Beta is the simple slope coefficient from a regression of the ETF’s returns on the S&P 500’s returns and T(Beta) is the associated t-statistic. Volatilities are standard deviations in percent per annum. EDM(%) is a measure of diversification potential from combining the ETF with the S&P 500.8

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<tr>
<td>OEF</td>
<td>iShares S&amp;P 100</td>
<td>0.9799</td>
<td>112</td>
<td>0.987</td>
<td>51.5</td>
<td>16.01</td>
<td>15.90</td>
<td>0.225</td>
</tr>
<tr>
<td>ISI</td>
<td>iShares S&amp;P 1500</td>
<td>0.9665</td>
<td>74</td>
<td>1.024</td>
<td>102.1</td>
<td>15.34</td>
<td>14.93</td>
<td>0.000</td>
</tr>
<tr>
<td>IVV</td>
<td>iShares S&amp;P 500</td>
<td>0.9979</td>
<td>117</td>
<td>0.989</td>
<td>165.4</td>
<td>15.67</td>
<td>15.80</td>
<td>0.852</td>
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<td>DDM</td>
<td>ProShares Ultra Dow30</td>
<td>0.9779</td>
<td>45</td>
<td>1.836</td>
<td>30.7</td>
<td>34.23</td>
<td>18.23</td>
<td>0.000</td>
</tr>
<tr>
<td>UVG</td>
<td>ProShares Ultra Russell 1000 Value</td>
<td>0.9833</td>
<td>35</td>
<td>2.142</td>
<td>37.2</td>
<td>43.76</td>
<td>20.18</td>
<td>0.000</td>
</tr>
<tr>
<td>SSO</td>
<td>ProShares Ultra S&amp;P 500</td>
<td>0.9963</td>
<td>45</td>
<td>2.018</td>
<td>76.2</td>
<td>36.93</td>
<td>18.23</td>
<td>0.000</td>
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<td>RSU</td>
<td>Rydex 2x S&amp;P 500</td>
<td>0.9975</td>
<td>28</td>
<td>2.041</td>
<td>72.5</td>
<td>44.93</td>
<td>21.96</td>
<td>0.000</td>
</tr>
<tr>
<td>XLG</td>
<td>Rydex Russell 50</td>
<td>0.9781</td>
<td>58</td>
<td>0.884</td>
<td>35.2</td>
<td>14.78</td>
<td>16.37</td>
<td>9.670</td>
</tr>
<tr>
<td>ELR</td>
<td>SPDR DJ LargeCap</td>
<td>0.9889</td>
<td>52</td>
<td>0.972</td>
<td>47.0</td>
<td>16.79</td>
<td>17.08</td>
<td>1.712</td>
</tr>
<tr>
<td>TMW</td>
<td>SPDR DJ Total Market</td>
<td>0.9858</td>
<td>112</td>
<td>0.995</td>
<td>61.5</td>
<td>15.88</td>
<td>15.90</td>
<td>0.413</td>
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<tr>
<td>SPY</td>
<td>SPDR S&amp;P 500</td>
<td>0.9969</td>
<td>205</td>
<td>0.991</td>
<td>179.6</td>
<td>14.90</td>
<td>14.99</td>
<td>0.628</td>
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<tr>
<td>MGC</td>
<td>Vanguard Mega Cap 300</td>
<td>0.9989</td>
<td>27</td>
<td>0.972</td>
<td>105.0</td>
<td>21.76</td>
<td>22.37</td>
<td>2.725</td>
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<tr>
<td>MGK</td>
<td>Vanguard Mega Cap 300 Growth</td>
<td>0.9790</td>
<td>27</td>
<td>0.939</td>
<td>24.0</td>
<td>21.45</td>
<td>22.37</td>
<td>4.073</td>
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<tr>
<td>VIG</td>
<td>Vanguard Mergent Dividend</td>
<td>0.9754</td>
<td>47</td>
<td>0.810</td>
<td>29.7</td>
<td>14.86</td>
<td>17.90</td>
<td>16.997</td>
</tr>
<tr>
<td>VV</td>
<td>Vanguard MSCI LargeCap</td>
<td>0.9979</td>
<td>74</td>
<td>1.009</td>
<td>129.6</td>
<td>15.09</td>
<td>14.93</td>
<td>0.000</td>
</tr>
<tr>
<td>VTV</td>
<td>Vanguard MSCI LargeCap Value</td>
<td>0.9763</td>
<td>74</td>
<td>1.030</td>
<td>38.3</td>
<td>15.75</td>
<td>14.93</td>
<td>0.000</td>
</tr>
<tr>
<td>VTI</td>
<td>Vanguard MSCI Total Market</td>
<td>0.9938</td>
<td>104</td>
<td>1.010</td>
<td>90.0</td>
<td>15.78</td>
<td>15.52</td>
<td>0.000</td>
</tr>
<tr>
<td>EPS</td>
<td>Wisdom Tree Earnings 500</td>
<td>0.9941</td>
<td>37</td>
<td>0.976</td>
<td>54.2</td>
<td>19.46</td>
<td>19.82</td>
<td>1.839</td>
</tr>
</tbody>
</table>

---

8 EDM(%) is the maximum percentage reduction in volatility from combining the ETF with the S&P 500 Index without shorting either. A zero in this column signifies that the Index has a smaller volatility than any long-only combination of the Index and the ETF.
Table 2
Hypothetical Sensitivities (β’s) of FactorShares Spread ETFs To Underlying High Frequency Macroeconomic Perception Shocks

<table>
<thead>
<tr>
<th>Underlying Macro Shock</th>
<th>FOL</th>
<th>FSE</th>
<th>FSU</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Long Crude Oil</td>
<td>Long Equities Short T Bonds</td>
<td>Long Equities Short the US$</td>
</tr>
<tr>
<td>Industrial production</td>
<td>-2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Investor Confidence</td>
<td>0.0</td>
<td>-1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Energy</td>
<td>2.0</td>
<td>-0.5</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

Table 3
Investment Percentages in FactorShares Spread ETFs For an Agnostic Investor with a Relatively Low Tolerance for Risk

<table>
<thead>
<tr>
<th>ETF</th>
<th>ETF Investment</th>
<th>Macro Factor</th>
<th>Macro Factor β</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOL</td>
<td>40.625%</td>
<td>Industrial Production</td>
<td>.375</td>
</tr>
<tr>
<td>FSE</td>
<td>12.500%</td>
<td>Investor Confidence</td>
<td>.375</td>
</tr>
<tr>
<td>FSU</td>
<td>46.875%</td>
<td>Energy</td>
<td>.375</td>
</tr>
</tbody>
</table>

Table 4
Investment Percentages in FactorShares Spread ETFs For an Investor with Zero Tolerance for Risks Driven by Investor Confidence and Energy, but Average Tolerance to Industrial Production

<table>
<thead>
<tr>
<th>ETF</th>
<th>ETF Investment</th>
<th>Macro Factor</th>
<th>Macro Factor β</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOL</td>
<td>25.000%</td>
<td>Industrial Production</td>
<td>1.0</td>
</tr>
<tr>
<td>FSE</td>
<td>33.333%</td>
<td>Investor Confidence</td>
<td>0.0</td>
</tr>
<tr>
<td>FSU</td>
<td>41.667%</td>
<td>Energy</td>
<td>0.0</td>
</tr>
</tbody>
</table>

9 The macro factor sensitivity (β) is a weighted average of the ETF sensitivities where the weights are the investment proportions. Hence -2.0(.40625)+2.0(.12500)+2.0(.46875) = 0.375 for industrial production and, similarly, 0(.40625)-1.5(.12500)+1.2(.46875) = 0.375 for investor confidence and 2.0(.40625)-.5(.125)-.8(.46875) = 0.375 for energy.

10 The macro factor sensitivities are, respectively, -2.0(.25)+2.0(.33333)+2.0(.41667) = 1.0 for industrial production, 0(.40625)-1.5(.33333)+1.2(.41667) = 0.0 for investor confidence and 2.0(.25000)-.5(.33333)-.8(.41667) = 0.0 for energy.
Table 5
Investment Percentages in FactorShares Spread ETFs
For a Pure Energy Hedger, Unexposed to Other Macroeconomic Shocks

<table>
<thead>
<tr>
<th>ETF</th>
<th>ETF Investment % of Equity</th>
<th>Macro Factor</th>
<th>Macro Factor $\beta^{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOL</td>
<td>75.000%</td>
<td>Industrial Production</td>
<td>0.0</td>
</tr>
<tr>
<td>FSE</td>
<td>33.333%</td>
<td>Investor Confidence</td>
<td>0.0</td>
</tr>
<tr>
<td>FSU</td>
<td>41.667%</td>
<td>Energy</td>
<td>1.0</td>
</tr>
</tbody>
</table>

$^{11}$ The macro factor sensitivities are, respectively, $-2.0(.75)+2.0(.33333)+2.0(.41667) = 0.0$ for industrial production, $0(.40625)-1.5(.33333)+1.2(.41667) = 0.0$ for investor confidence and $2.0(.75000)-.5(.33333)-.8(.41667) = 1.0$ for energy.
Two portfolios (indexed by P) are both driven entirely by two underlying factors (f’s) according to a linear return generating process at time t, $R_{P_t} = E_P + \beta_{P,1} f_{1,t} + \beta_{P,2} f_{2,t}$, with $\text{Var}(f_1) = \text{Var}(f_2)$ and $\text{Cov}(f_1,f_2) = 0$. For portfolios P = A and B, factor sensitivities ($\beta$’s) are related by $\beta_{A,1} = k_1 \beta_{B,1}$ and $\beta_{A,2} = k_2 \beta_{B,2}$. The constants of proportionality ($k$’s) are plotted on the horizontal axes over the range -1 to +1 and the correlation between the portfolios is plotted on the vertical axis. Perfect positive (negative) correlation requires $k_1 = k_2 > (<) 0$. 

Figure 1
Correlation Between Perfectly Integrated Portfolios
In a Two-Factor World with Diverse Factor Sensitivities