Capital Requirements, Risk Choice, and Liquidity
Provision in a Business Cycle Model

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January 10, 2014

Abstract

This paper presents a quantitative dynamic general equilibrium model for the purpose of determining the optimal capital requirement for banks. Banks play two roles in this model: They contribute to the production of a final good and they provide liquidity in the form of bank debt, which households value. Banks also benefit from an implicit bailout guarantee from the government, which motivates them to take on excessive risk. I quantify this model using data from the national income and product accounts as well as the Federal Deposit Insurance Corporation and find that the dynamics of the model are consistent with business cycle facts. Higher capital requirements lower risk-taking and increase consumption, but they also reduce the supply of bank debt. The reduction in bank debt leads to a lower interest rate on bank debt through a general equilibrium effect. This reduces the overall funding costs of banks and allows them to grow larger, which increases the capital stock and, consequently, production as well as consumption. The optimal capital requirement weighs the reduction in economic volatility and the increase in consumption against the reduction in liquidity. Welfare is maximized at 14% equity as a share of risk-weighted assets.

*I am deeply indebted to Monika Piazzesi, Martin Schneider, and Pablo Kurlat for their invaluable guidance and patience. I also want to thank Moritz Lenel, Theresa Kuchler, Laurence Wong, Andrey Fradkin, Evan Mast, Alonso Villacorta Gonzales, Marinho Bertanha, Claudia Wolff, and the participants of the Stanford Macroeconomics Seminar and the Macroeconomics Lunch. Support by the Macro Financial Modeling Group dissertation grant from the Alfred P. Sloan Foundation and the Kohlhagen Fellowship Fund, as well as the Haley-Shaw Fellowship Fund of the Stanford Institute for Economic Policy Research (SIEPR), is gratefully acknowledged. Correspondence: Department of Economics, Stanford University. 579 Serra Mall, Stanford, CA, 94305. Email: begenau@stanford.edu. For the latest update, please go to www.stanford.edu/~begenau/jmp_begenau.pdf.
1 Introduction

The recent financial crisis has emphasized the fact that banks take substantial risks. They operate under a number of frictions and receive government subsidies that create the potential for moral hazard. Capital requirements are a regulatory tool designed to make banks safer, but the crucial question is whether higher capital requirements can reduce banks' risk-taking without substantially reducing the useful services that they provide, such as the provision of loans and liquidity.

In this paper, I develop a quantitative model for analyzing this important trade-off. The model includes a banking sector that invests in capital and creates money-like-securities in the form of bank debt such as deposits. Its key features are the preferences for bank debt holdings of households\(^1\), subsidies for banks that provide excessive risk-taking incentives, and a subset of production that depends on banks. The model captures the business cycle dynamics of the banking sector, as well as macroeconomic aggregates. I find that the capital requirement should be increased to 14%.

An optimal capital requirement weighs reduced liquidity provision by banks against lower output volatility and increased bank lending. The increase in lending is due to a general equilibrium effect: Raising the capital requirement above the current status quo lowers the amount of bank debt. Since the marginal utility of bank debt is decreasing, it becomes more desirable to households. Therefore, to reduce the demand for bank debt, its interest rate decreases. If this decrease is large enough, banks face lower funding costs for their assets, which entices them to increase their credit supply. The increase in credit supply then leads to a higher capital stock, higher output, and more consumption.

To quantify the above trade-off, I have matched the model to data from the national income and product accounts (NIPA) and banks' regulatory filings from the Federal Deposit Insurance Corporation (FDIC). The curvature parameters governing the technology of bank-dependent production and households' preferences for liquidity are especially important because they determine the response of bank lending to an increase in the capital requirement. In the model, the households' liquidity preference parameter determines the variance of the bank liability-consumption ratio. I have used this parameter to match the volatility of the ratio of FDIC bank liabilities to NIPA consumption. The curvature parameter of the bank-dependent production technology governs how much capital is transformed into output. I select the value of this parameter to match banks' income-asset ratio.

\(^1\)Households receive utility from holding money-like bank debt which reflects the value from bank debt being accepted and redeemed at par.
The parametrization implies that a reduction in the liquidity supply by banks lowers their funding costs. The quantified model matches balance sheet and income statement data from banks together with macroeconomic aggregates. Moreover, its dynamics are consistent with many business cycle moments in the data that have not been targeted. For example, it is consistent with the procyclicality and volatility of balance sheet and income statement variables. It also captures the correlations between NIPA and balance sheet variables, which makes it particularly suitable for studying the effects of capital requirements on the economy.

The model environment is a two-sector stochastic growth economy with sector-specific aggregate shocks. One sector is populated by frictionless and perfectly competitive firms that rent capital and labor from households to produce the final good. The other sector produces output with capital and a decreasing returns to scale technology. This sector consolidates the banking system with bank dependent borrowers\(^2\): banks decide how much to invest in this sector. They can also choose the riskiness of their assets, which implies that bank risk is not only determined by leverage. Using their balance sheet, banks produce liquidity services through bank debt.

Banks finance themselves with equity and debt. When households invest in bank debt, they receive an interest rate and a utility flow. The utility flow from bank liabilities implies that households are willing to accept a lower interest rate on bank debt than on any other riskless asset that does not provide utility. In other words, bank debt is the lowest return asset in this economy.

Government guarantees act effectively as an implicit subsidy by lowering the debt financing costs for banks because default risks are not included in the claims that banks issue. I captured these effects in this model by introducing an explicit subsidy to the banking sector that depends positively on leverage, risk-taking, and the size of the banking sector. This subsidy features a complementarity between risk-taking and leverage\(^3\) so that banks can increase the amount that the subsidy pays by taking on more risk when they are highly leveraged. Banks are subject to a Basel-II type capital requirement, which means that they have to hold a certain percentage of their risky assets in equity. The capital requirement is binding since households accept a discount on the interest rate on bank debt.

\(^2\) This assumption is based on a similar argument as in Brunnermeier and Sannikov (2012), which states that banks and their borrowers can be consolidated when borrowers are penniless and banks are equipped with a monitoring technology that eliminates the moral hazard of their borrowers.

\(^3\) The complementarity of the subsidy between risk-taking and leverage captures the following effect of mispriced limited liability and government guarantees: Close to the point of insolvency, banks can bet on receiving a large positive shock by choosing a high amount of risk. In case the bet goes wrong, the losses are bounded from below. The closer banks are to insolvency, the less they have to lose, which increases their risk-taking incentive.
The size of the banking sector is determined by the curvature in the bank dependent production technology as well as the benefits and costs of assets. These costs are a weighted average of equity and debt financing costs. Since the capital requirement constraint is binding, the weights are determined by the regulator. When banks choose risk they trade off the benefit from the subsidy against a loss in efficiency that occurs with high risk. The presence of the government subsidy is an incentive for banks to undertake excessively risky projects, which are inefficient. The higher the leverage of banks, the more incentive they have to take on excessive amounts of risk.

The downside of a higher capital requirement is a reduction in the supply of liquidity in the form of bank liabilities. A higher capital requirement implies that a larger share of risky assets has to be financed with relatively more expensive equity. When interest rates do not adjust, the funding costs of banks increase. In this case, banks want to reduce their assets, which means that they reduce their liabilities as well.

There are two benefits from a higher capital requirement. First, there is a reduction in volatility, which comes from the complementarity between risk-taking and leverage in the subsidy. When banks decrease their leverage, the subsidy is also decreased. In this case, banks have less incentive to choose a high amount of risk because the subsidy payment is now too small in comparison with the potential loss in efficiency. Less risk-taking by banks reduces the volatility of total output and raises the efficiency of the banking sector.

The second benefit is an increase in bank lending, which results in higher consumption levels. This finding is due to a general equilibrium effect on the interest rate for bank debt that comes from the liquidity preferences of households as well as the two-sector nature of the model. More specifically, it connects the amount of bank debt to its interest rate and therefore to the funding costs of assets. As long as there is complementarity between consumption and bank liabilities in the utility, households want to hold more bank debt the scarcer it is relative to consumption. When the amount of bank debt falls in response to a tightened capital requirement, the interest rate on bank debt must decrease to make it less attractive to households. If the decrease in the interest rate on bank debt is large enough, banks’ funding costs fall despite the fact that a larger share of assets has to be financed with more expensive equity. Both higher efficiency (due to less risk-taking) and lower financing costs for assets entice banks to increase their assets, which raises the output of the bank-dependent sector and the capital stock in the economy. As a result, consumption also increases.

In the quantitative implementation of the model, I match bank-dependent production variables to FDIC data (which include all deposit insured U.S. commercial banks and savings
institutions) and non-bank dependent production variables to NIPA data. Using these data and the model’s equilibrium conditions, I identify the parameters that determine the behavior of the model. The quantitative effects of the higher capital requirement in the model depend mainly on four parameters: the curvature parameters in the bank-dependent production technology and households’ preference for liquidity, and two parameters that govern banks’ risk choice. In the model, banks take on more risk when the subsidy (included in profits) is high. I infer the value for the sensitivity of the subsidy with regard to risk-taking by matching the volatility of banks’ income-asset ratio conditional on past profits to the data. To identify the parameter that governs how much efficiency banks lose if they increase their risk-taking, I employ the unconditional volatility of the income-asset ratio, together with the optimality condition with respect to the risk-taking of banks.

I use the model to derive the optimal capital requirement based on households’ utility. The optimal requirement is 17% when transition dynamics to the new equilibrium are ignored and 14% when they are included. A higher requirement implies a larger banking sector and a higher capital stock. Since assets accumulate slowly over time, early in the transition, banks must reduce bank liabilities by more than what would be necessary to arrive at the new steady state. Under the new requirement, total output and consumption increase by 0.10%, banks reduce their risk-taking by 1.5 percentage points, and liquidity decreases by 2%.

**Related Literature**

This paper builds upon optimal banking regulation theory and dynamic macroeconomic models with financial frictions and intermediaries. It is most closely related to other quantitative studies that have investigated the effects of capital requirements and leverage constraints.

The recent financial crisis has sparked a discussion — motivated by theoretical models — of whether banks’ capital requirements should be increased. This relates to the question of why banks are highly leveraged. One strand of the literature presents high leverage ratios as a solution to governance problems (for example Dewatripont and Tirole (1994, 1994a, 2012)), or attributes high leverage ratios to banks’ role as liquidity providers (for example Diamond and Rajan (2001), Diamond and Rajan (2000), Gorton and Winton (1995) and DeAngelo and Stulz (2013)).

The present model incorporates the role of banks as liquidity providers and thus captures the effects of higher capital regulation on liquidity creation. In contrast to the previous strand of literature, Admati, DeMarzo, Hellwig, and Pfleiderer (2010, 2013) have argued that equity is costly because of subsidies provided by government guarantees and preferential tax
treatment. They have found that higher capital requirements reduce incentives for excessive risk-taking and debt overhang problems. In the present paper, I quantify the potential costs (a lower supply of liquidity) and benefits (less risk-taking by banks) of a higher requirement that have been identified in the theoretical literature.

Macroeconomics models with financial frictions are rooted in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2010) have incorporated credit market imperfections into New Keynesian models. Recently, more models have focused on the role of financial intermediaries in the development of crises (see, for example, Gertler, Kiyotaki, and Queraltó (2012), Gertler and Kiyotaki (2013), He and Krishnamurthy (2012), Brunnermeier and Sannikov (2012) and Gertler and Karadi (2012)). In contrast, in this paper, I develop a tractable macroeconomic framework with a focus on the effects of capital requirements.

This paper is more closely related to work that quantifies the effects of capital requirements and leverage constraints, for instance Christiano and Ikeda (2013), Bigio (2012), Martinez-Miera and Suarez (2012), Van Den Heuvel (2008), and Corbae and D’Erasmo (2012). The theoretical literature has identified the main effects of a change in capital requirement, namely changes in the liquidity supply, risk choice, and lending activities of banks. Van Den Heuvel (2008) was one of the first to use a quantitative general equilibrium growth model with liquidity demand of households to assess the effects of capital requirement on welfare. He found that the main effect of the capital requirement was a reduction in deposits and that the current requirement was therefore too high.

Christiano and Ikeda (2013) studied leverage constraints in a New Keynesian model where bankers have an unobservable effort choice. Martinez-Miera and Suarez (2012) analyzed the effects of capital requirements on the systemic risk choice of banks in a small open economy. Corbae and D’Erasmo (2012) focused on the interaction between competition and capital requirements in a quantitative model of banking industry dynamics. Bigio (2012) developed a theory about risky financial intermediation under asymmetric information and analyzed how capital requirements change the risk capacity of an economy. A common fea-

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4Hanson, Stein, and Kashyap (2010) and Kashyap, Rajan, and Stein (2008) also argue for higher capital requirements referring to the tax-advantage of debt and competitive pressure over cheap funding sources as the leading source for banks’ high leverage.

5Goodhart, Vardoulakis, Kashyap, and Tsomocos (2012) have non-quantitatively assessed different regulatory tools. They found that capital requirements give banks incentives to move activity to a shadow bank and that they alone are not sufficient for preventing a crisis.

6Keohoe and Chari (2013) have shown that limits to debt-to-value ratios can improve welfare by eliminating the incentives for a government bailout.

7There is also a larger strand of quantitative literature that focuses on the procyclical effects of capital requirement (see, for example, Covas and Fujita (1998) and Suarez and Repullo (2012)).
ture in these papers is that a tightening of the constraint reduces the riskiness of the banking system but that it also reduces the amount of lending, which results in a lower GDP.

In the present model, the effects on risk-taking and lending activities from a change in the capital requirement are still present, but I have also incorporated the consequences of a change in the liquidity supply. With preferences for liquidity, the trade-off of a higher capital requirement with regard to banks’ lending activities (in general equilibrium) is reversed: Since households value bank debt more when it is relatively scarce, they are willing to accept an even higher discount on the interest rate on bank debt. This lowers the overall funding costs of bank assets, leading to more — not less — lending in the economy.

In this paper, I assume that mispriced government guarantees are the fundamental reason for excessive risk-taking of banks. Pennacchi (2006) used a stylized static model to show that actuarially fair deposit insurance premiums can create a moral hazard that leads banks to concentrate their loan portfolio on systematic risk. Schneider and Tornell (2004) were the first to study the effects of government bailout guarantees in a dynamic setting. They found that a bailout guarantee gives individual banks incentives to take correlated risks and thus increase volatility. Beigenu, Piazzesi, and Schneider (2013) demonstrated empirically that banks take on sizable amounts of risk, even in securities like derivatives that are traditionally supposed to hedge balance sheet risks.

The paper proceeds as follows. Section 2 presents the model. Section 3 describes the mechanism that drives the trade-off from higher capital requirements. Section 4 describes how the model is matched to the data and how well it captures moments that have not been targeted. Section 6 discusses the welfare implication of the model.

2 Model

The model incorporates banks into a business cycle model with capital accumulation where the consumption good is produced in two sectors. For one of these sectors, banks operate a production technology and determine its risk. They also produce deposits which households value. Banks receive a subsidy that depends positively on leverage, risk taking, and bank size.

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8Similarly, Gollier, Koehl, and Rochet (1997) have shown that firms have excessive risk-taking incentives when there is limited liability.

9Ljungqvist (2002) used a dynamic general equilibrium model to show how government guarantees may increase asset price volatility.
2.1 Technology

Consider a single-good economy that produces good \( c \) in two different sectors. These two sectors are a bank-independent sector (sector \( f \)) and a bank-dependent sector (sector \( h \)). The firms in the bank-independent sector rent labor and capital from household to form output with a Cobb-Douglas technology

\[
y^f_t = Z^f_t \left( k^f_{t-1} \right)^\alpha \left( N^f_t \right)^{1-\alpha},
\]

(1)

where \( Z^f_t \) is the productivity level at time \( t \), \( k^f_{t-1} \) is the capital stock installed in \( t-1 \), \( \alpha \) is the share of capital, and \( N^f_t \) is the quantity of labor input. Productivity is stochastic

\[
\log Z^f_t = \rho^f \log Z^f_{t-1} + \sigma^f \epsilon^f_t,
\]

(2)

where \( \epsilon^f_t \) is drawn from a multivariate normal distribution.

The bank-dependent production sector is owned and run by banks. Using capital \( k^h_{t-1} \), they produce output \( y^h_t \) with a decreasing returns to scale technology

\[
y^h_t = Z^h_t \left( k^h_{t-1} \right)^v.
\]

(3)

The productivity level \( Z^h_t \) follows

\[
\log Z^h_t = \rho^h \log Z^h_{t-1} + (\phi_1 - \phi_2 \sigma^h_{t-1}) \sigma^h_{t-1} + \sigma^h_{t-1} \epsilon^h_t,
\]

(4)

where \( \epsilon^h_t \) is drawn jointly with \( \epsilon^f_t \) from \( \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \sigma^{fh} \\ \sigma^{fh} & 1 \end{bmatrix}\right) \), where \( \sigma^{fh} \) is the covariance between \( \epsilon^f_t \) and \( \epsilon^h_t \).

The process of \( \log Z^h_t \) is persistent: Its autocorrelation is \( \rho^h \). The term \((\phi_1 - \phi_2 \sigma^h_{t-1}) \sigma^h_{t-1}\) affects the conditional mean. In period \( t \), banks can choose the amount of risk \( \sigma^h_t \) (i.e. exposure to the aggregate shock \( \epsilon^h_t \)) at which they want to operate in \( t+1 \). The choice of \( \sigma^h_t \) also determines the expected productivity level in \( t+1 \). The parameters \( \phi_1 \) and \( \phi_2 \) govern the shape of the risk-productivity frontier.

*Capital Accumulation*

Capital is not sector-specific. That is, the capital stock of banks and firms can be transformed from and into each other one for one. Moreover, both capital types depreciate
at the same rate $\delta$. Capital in sector $j \in \{f, h\}$ accumulates according to
\[ i^j_t = k^j_t - (1 - \delta) k^j_{t-1}. \]

Adjustments to the stock of capital are costly. When investment exceeds the replacement of depreciated capital, investors incur a proportional capital adjustment cost of
\[ \varphi_j \left( \frac{i^j_t}{k^j_t} - \delta \right)^2 k^j_t, \]
where $\varphi_j$ is the sector-specific adjustment cost parameter.

### 2.2 Bank

Banks make up the financial system in this economy\(^{10}\). They play two roles: First, they produce a good that households consume. Second, they provide liquidity to households who value holding deposits. Banks are owned by households and maximize shareholder value by generating cash flow that is discounted with the stochastic discount factor from households.

Banks enter the period with capital $k^h_{t-1}$, government security holdings $b_{t-1}$, bank debt $s_{t-1}$, equity $e_{t-1}$, and a risk level $\sigma^h_{t-1}$. The balance sheet equates risky assets $k^h_{t-1}$ and riskless assets $b_{t-1}$ to bank debt $s_{t-1}$ and equity $e_{t-1}$:
\[ k^h_{t-1} + b_{t-1} = s_{t-1} + e_{t-1}. \]

At the beginning of the period $t$, the economy’s states $\epsilon^h_t$ and $\epsilon^f_t$ are realized. Banks generate income from operating their production technology and investing in riskless assets. Their expenses are interest payments on bank debt. Therefore, profits are defined as
\[ \pi_t = \underbrace{y^h_t - \delta k^h_{t-1}}_{\text{production income}} + \underbrace{r^B_{t-1} b_{t-1}}_{\text{interest inc.}} - \underbrace{r_{t-1} s_{t-1}}_{\text{interest exp.}}. \]

In period $t$, they choose investment $i^h_t$ as well as risk taking $\sigma^h_t$ in order to operate the production sector. Additionally, banks have a leverage and a portfolio choice. The leverage choice determines with how much debt $s_t$ and with how much equity $e_t$ banks finance their assets. The portfolio choice determines the amount of risky assets $k^h_t$ and the amount

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\(^{10}\)Since all banks are identical and the shock to the bank-dependent sector is an aggregate shock, banks’ risk choices are perfectly correlated and we can speak of a representative bank that takes prices as given.
of riskless assets $b_t$. Finally, banks decide how much dividends $d_t$ to distribute to households.

**Market Imperfections in the Banking Sector**

Banks face a regulator who stipulates a constraint on the amount of debt with which banks can finance risky assets:

$$e_t \geq \xi k^h_t,$$

where $\xi$ determines the amount of equity $e_t$ needed to finance risky assets $k^h_{t-1}$.

Banks receive a subsidy from the government:

$$TR \left(k^h_{t-1}, \frac{e_{t-1} + \pi_t}{k^h_{t-1}}, \sigma^h_t\right) = \omega_3 k^h_{t-1} \exp \left(-\omega_1 \left(\frac{e_{t-1} + \pi_t}{k^h_{t-1}}\right) + \omega_2 \sigma^h_t\right), \quad (5)$$

where $\omega_1$, $\omega_2$, and $\omega_3$ are positive constants. The scalar $\omega_1$ is the sensitivity of the transfer with respect to leverage after profits have been realized $\left(k^h_{t-1}/(e_{t-1} + \pi_t)\right)$. The scalar $\omega_2$ is the sensitivity of the transfer with respect to current risk taking $\sigma^h_t$, and $\omega_3$ determines the average transfer per unit of physical capital. Moreover, since $\sigma^h_t$ affects the conditional mean of banks’ profits in $t+1$, there is an additional benefit of risk taking when banks are highly leveraged.

The adjustment of dividends is costly: Banks incur a cost if their dividend payout deviates from the target level $\bar{d}$. The dividend payout cost introduces intertemporal rigidities into the balance sheet. Following Jermann and Quadrini (2012), the payout cost has the following form:

$$f(d_t) = \frac{\kappa}{2} (d_t - \bar{d})^2,$$

where $\kappa$ governs the size of this cost.

**Problem of Banks**

Banks use equity, profits and the cash flow from government transfers $TR(\cdot)$ to finance next period’s equity $e_t$, the capital adjustment costs, and the dividend payout to households. Due to the equity payout costs, the necessary cash flow to payout $d_t$ is $d_t + f(d_t)$. Therefore, dividends are defined as:

$$d_t = e_{t-1} + \pi_t - f(d_t) + TR \left(k^h_{t-1}, \frac{e_{t-1} + \pi_t}{k^h_{t-1}}, \sigma^h_t\right) - e_t - \varphi_h \left(k^h_t - (1 - \delta) k^h_{t-1} - \delta\right)^2 k^h_t,$$

where $\varphi_h$ is the capital adjustment costs.
The bank problem is written recursively. For the statement of the problem, it is useful to define \( \tilde{e} = e + \pi \) as equity after profits. The state of the economy \( \varepsilon \) is determined by the realizations of the shocks \( \epsilon^f \) and \( \epsilon^h \). Thus, the state variables of banks are the aggregate state vector \( X \) (to be described later), the state of the economy \( \varepsilon \), banks’ equity after profits \( \tilde{e}(\varepsilon, X) \), as well as \( k^h \) due to the adjustment costs of capital.

Banks discount the future with the pricing kernel \( M(X', \varepsilon') \) from households. They choose capital, government securities, bank debt, the amount of risk-taking, equity after profits (and therefore book equity), as well as dividends to solve

\[
V^B(\tilde{e}, k_h, X, \varepsilon) = \max_{k^h, y', s', \sigma^h, \tilde{e}(\varepsilon', X')} \left[ d + E_{\varepsilon'|\varepsilon} \left[ M(X', \varepsilon') \left( V^B(\tilde{e}', X', k^h, X', \varepsilon') \right) \right] \right] \tag{6}
\]

\[\text{s.t.}\]

\[
d = \tilde{e} - e' - f(d) + TR \left( k^h, \frac{\tilde{e}}{k^h}, \sigma^h \right) - \varphi_h \left( \frac{k^h - (1 - \delta) k^h}{k^h} - \delta \right)^2 k^h
\]

\[
\tilde{e}'(\varepsilon', X') = e' + \pi \left( k^h, \sigma^h, b', s', X', \varepsilon' \right)
\]

\[
e' + k^h = b' + s'
\]

\[
e' \geq \xi k^h.
\]

Banks have unlimited liability: if \( \tilde{e} < 0 \) they set \( d < 0 \).

### 2.3 Households

Households are all identical and live indefinitely. They own capital \( k^f \) for firm production and supply labor \( N^f \) to firms inelastically. They are also the owners of banks and as such receive dividends \( d \).

Households care about consumption \( c \) and holding liquidity in the form of bank debt \( s \). Bank debt gives utility in the period it is acquired and pays interest in the following period. Their felicity function is thus defined over consumption and bank liabilities \( (s') \) in a money-in-the-utility specification

\[
U(c, s') = \log c + \theta \frac{s'^{1-\eta}}{c^{1-\eta}},
\]

where \( \theta \) is the utility weight on deposits and \( \eta \) governs the curvature of the deposit-consumption ratio in the utility. This utility specification ensures that more consumption raises the marginal utility of liquidity. At the beginning of the period after the shocks have been realized (realizations of \( e^h \) and \( e^f \) are summarized in vector \( \varepsilon \)), the state variable of the household
- its net worth $n$ - is

$$n (\varepsilon) = \text{Financial Wealth} + \text{Capital} - \text{Taxes}.$$  

Financial wealth consists in dividends $d$ and share value $p$ from owning $\Theta$ shares of the banking sector and $s$ bank debt\textsuperscript{11}:

$$\text{Financial Wealth} = (d (X, \varepsilon) + p (X, \varepsilon)) \Theta + (1 + r (X)) s.$$  

That is, households do not hold bonds. Later, I will verify that in equilibrium they also do not want to hold bonds. Households own capital $k^f$ which they rent out to firms

$$\text{Capital} = (r^f (X, \varepsilon) + 1 - \delta) k^f.$$  

Lump sum taxes are denoted as $T$. Additionally, households receive labor income from supplying $N_k$ hours inelastically to firms, earning wage $w^f$. Thus labor income is

$$\text{Labor Income} = w^f (X, \varepsilon) N^f.$$  

Households’ value function is determined by $n$, $k^f$, the aggregate state vector $X$, and the realization of shocks $\varepsilon$. They maximize their value function by choosing consumption $c$, new deposit balance $s'$, capital $k'^f$, labor supply $N^f$, and bank shares $\Theta'$ subject to a budget constraint. Thus, their problem is to solve

$$V^H (n, k_k, X, \varepsilon) = \max_{\{c, s', k'^f, N^f, \Theta, n'(X', \varepsilon')\}} U (c, s') + E_{\varepsilon'} [M (X', \varepsilon') V^H (n' (X', \varepsilon'), k'^f, X', \varepsilon')],$$  

subject to the budget constraint

$$c + s' + 1 + \varphi_f \left( \frac{k'^f - (1 - \delta) k^f}{k'^f} - \delta \right)^2  k'^f + p (X, \varepsilon) \Theta' = n (\varepsilon) + w^f (X, \varepsilon) N^f, \quad (8)$$

\textsuperscript{11}The model captures the effects of government guarantees though the government guarantee itself is not formally in the model. A consequence of a government guarantee is that depositors regard bank debt as perfectly riskless.
and net worth tomorrow
\[ n'(X', \varepsilon') = (d(X', \varepsilon') + p(X', \varepsilon')) \Theta' + (1 + r(X')) s' + (r^f(X', \varepsilon') + 1 - \delta) k^{lf}. \tag{9} \]

When installing new capital in excess of depreciation, the household incurs the cost 
\[ \varphi_f \left( \left( k^{lf} - (1 - \delta) k^f \right) / k^{lf} - \delta \right)^2 \]
per unit of capital \( k^{lf} \). The stochastic discount factor in the economy is given by
\[ M(X', \varepsilon|\varepsilon) = \beta \left( \frac{U_c(c(X', \varepsilon'), s')}{U_c(c(X, \varepsilon), s)} \right). \]

2.4 Government

The government follows a balanced budget rule where it maintains debt levels at \( B' = B \) so that:
\[ TR(\cdot) + r^B B = T. \tag{10} \]

2.5 Recursive Competitive Equilibrium

The timing in the model is as follows: shocks occur and decisions are made subsequently. Then a new period starts again. The state vector \( X \) contains the aggregate net worth of banks \( \tilde{E} \), the aggregate net worth of households \( N \), the aggregate capital stock of households \( K^f \), the aggregate capital stock of banks \( K^h \), and the productivity levels of firms and banks \( Z^f \) and \( Z^h \) respectively.

**Definition.** Given an exogenous\(^{12}\) government debt policy \( B \), a recursive competitive equilibrium is defined by a pricing kernel \( M(X, \varepsilon) \) and prices: \( w^f(X, \varepsilon), r^f(X, \varepsilon), r^h(X, \varepsilon), p(X, \varepsilon), r(X), \) and \( r^B(X) \), value functions for households \( V^H \) and banks \( V^B \), and policy functions of households for consumption \( P^c_H \), bank debt \( P^d_H \), capital \( P^k_H \), bank equity shares \( P^e_H \), labor supply \( P^l_H \), as well as policy functions of banks for their capital stock \( P^k_B \), bonds \( P^b_B \), bank debt \( P^d_B \), equity \( P^e_B \), dividends \( P^d_B \), and \( P^\sigma_B \), and \( H \) the function governing the law of motion for \( X \) such that

1. Given the price system and a law of motion for \( X \):

\(^{12}\)Government securities are not a choice variable in this model because otherwise the government could optimally set \( B = \infty \), financed with non distortionary taxes. It would be optimal to do so, because households receive utility from deposits which can be produced with government debt.
(a) the policy function \( P_{B}^{B}, P_{B}^{B}, P_{B}^{'B}, P_{B}^{'B}, P_{B}^{r^{h}}, \) and the value function for banks \( V^{B} \) solve the Bellman equation, defined in equation 6.

(b) the policy function \( P_{H}^{H}, P_{H}^{H}, P_{H}^{H}, P_{H}^{H}, P_{H}^{N}, P_{H}^{N} \), and the value function for households \( V^{H} \) solve the Bellman equation, defined in equation 7.

2. \( w^{f}(X, \varepsilon) \) and \( r^{f}(X, \varepsilon) \) satisfy the optimality conditions of firms.

3. For all realization of shocks, the policy functions imply

(a) market clearing for

i. government bonds: \( P_{B}^{B} = B \)

ii. bank debt: \( P_{B}^{B} = P_{H}^{B} \)

iii. capital: \( P_{H}^{h} + P_{B}^{B} = k^{H} + k^{h} \)

iv. labor \( P_{H}^{N} = N^{f} \)

v. bank shares: \( \Theta = 1 \)

vi. consumption:

\[
c = y^{h} + y^{f} + (1 - \delta) k^{f} + (1 - \delta) k^{h} - \frac{k}{2}(d - \bar{d})^{2} - k^{f} \left(1 + \varphi_{f} \left(\frac{i^{f}}{k^{f} - \delta}\right)^{2}\right) - k^{h} \left(1 + \varphi_{h} \left(\frac{i^{h}}{k^{h} - \delta}\right)^{2}\right)
\]

(b) consistency with aggregation: \( n = N, \bar{e} = \bar{E}, k^{f} = K^{f} \) and \( k^{h} = K^{h} \).

4. The government budget constraint in equation 10 is satisfied.

5. The law of motion for \( X \) is consistent with the policy functions, rational expectations, and \( X'=H(X) \).

2.6 Discussions of Assumptions

This section discusses the assumptions that underlie the set-up.

**Bank-Owned Production Sector**

The final good is produced by two production sectors: bank-dependent and non-bank-dependent. This assumption assigns banks an important role in the provision of a good that households value. The idea that some agents need lenders (banks) to realize production
projects underlies Bernanke and Gertler (1989) as well as Kiyotaki and Moore (1997). This assumption makes a part of production dependent on the ability to obtain funds from lenders.

The bank-dependence of one production sector reflects the fact that banks generally provide funds to borrowers who do not have access to capital from elsewhere due to informational asymmetries. Those borrowers are usually small businesses and households who want to buy property. To fix ideas, the bank dependent production sector can be thought of the construction sector that depends on households’ access to mortgages. In the literature (see Freixas and Rochet (1998) on the role of banks), banks emerge as a solution to the asymmetric information problem between borrowers and lenders by gathering information (for instance through long term relationships as in Sharpe (1990)) and by screening and monitoring (as emphasized by Diamond (1984) and Tirole and Holmstrom (1997)). These practices allow banks to choose the riskiness and investment scale of their borrowers.

The present model goes a step further: banks own the capital stock used in the bank dependent sector and operate the production technology. This idea has been used by Brunnermeier and Sannikov (2012). It can be shown that this set up is isomorphic to a model in which bank borrowers have zero net worth and banks own a monitoring technology that allows them to effectively eliminate the asymmetric information. By allowing banks to own a production sector, I can study the behavior of banks in a tractable set-up. This abstraction serves the purpose to focus on the market imperfections that affect banks’ investment, leverage, and risk choices the most.

Decreasing Returns to Scale in Bank Dependent Sector

The bank dependent sector operates a decreasing returns to scale technology in capital. This captures the idea that not all projects in the world are suitable to be carried out by the banking sector. In other words, it is a stand-in for the degree to which banks can profitably eliminate the asymmetric information between them and their borrowers. This assumption also allows me to analyze the size of the banking sector in a meaningful way.

Banks can profitably lend to bank dependent borrowers because their monitoring and long term relationship building mitigates the asymmetric information problems that hinder these borrowers to access capital markets. These borrowers, however, are not homogeneous: There are top borrowers that are very productive with low default risk and other borrowers that are not. Also, monitoring is costly. It is only profitable for banks to lend to borrowers as long as the benefit of lending matches or exceeds its costs. This is particularly true for capital intensive projects where it is easier to monitor the investment process. When banks start lending to the bank-dependent sector they first lend to the profitable borrowers which
makes these investments attractive. After that there are only less profitable investments left because the remaining borrower pool requires more monitoring, defaults more, or is less productive. This idea is captured with the assumption of diminishing returns to banks’ risky assets.

Bank’s Risk and Return Menu

The Cobb-Douglas technology of banks has a multiplicative stochastic productivity term which is described in equation (4). Productivity is persistent and has a mean component which is affected by banks’ risk choice $\sigma^h$. The choice of $\sigma^h$ is equivalent to picking a project from a risk-return menu (i.e. the particular combination of mean and risk exposure). This specification postulates a trade-off between mean and exposure. The menu of projects $Z^h$ is set to have an interior maximum in $\sigma^h$. As a consequence, there exists a $\sigma^h$ that is optimal in the sense of maximizing mean productivity $Z^h$. This is a new feature of this model.

The concavity of $Z^h$ is meant to capture a decline in returns for high amounts of risk: risky assets $k^h$ can be thought of as the loan portfolio of banks. They could choose to invest in projects with high or low idiosyncratic risk, e.g. default probability. The choice of investing into high and low idiosyncratic risk projects is akin to a choice of the mean return of the loan portfolio. At the same time, some of the high idiosyncratic risk projects involve higher exposure to aggregate risk. So the bank can choose projects from a menu: projects with low mean - low exposure, high mean - medium exposure, and low mean - high exposure\textsuperscript{13}.

Generally, when investing in the stock market, mean returns can be increased with higher risk. Another way to think about the concavity in the trade-off between return and exposure is the following. Banks have incentives to take on systematic risk (i.e. exposure to $\epsilon^h$) because it increases their chances of being saved by the government. Farhi and Tirole (2011) show how the bailout promise leads banks to take on correlated risks. Regulators do not want banks to take on too much systematic risk. Indeed, regulators assess how diversified banks are and subject them to a more stringent capital requirement if banks are insufficiently diversified. Also, regulators discourage banks from investing into the stock market. If banks nevertheless want to increase their exposure, they have to do this in ways that escape regulators. These evasive investment strategies can compromise mean returns since they involve the inefficient use of resources to avoid regulatory scrutiny.

\textsuperscript{13}There are not strong theoretical restrictions on the shape of the risk-return trade-off. For example, Backus and Gregory (1993) show how in the Mehra-Prescott (1985) framework the shape of the risk return trade-off depends on the process (in particular on the autocorrelation) governing the evolution of the state.
Adjustment Costs to the Banking Capital Stock

Bank borrowers choose banks because they find it more difficult to obtain funds elsewhere. Banks build relationships with their customers to overcome the asymmetric information as for instance described by Sharpe (1990). It is costly to build up these customer relationships. A sudden reduction in the loan portfolio may also be costly because other market participants lack the information that the selling bank has acquired over time.

Dividend Adjustment Costs

Corporations, including banks, smooth dividends. Lintner (1956) showed that managers smooth dividends over time. In the case of banks, Dickens, Casey, and Newman (2002) used Morningsart’s Stocktools/Principia Pro data from 1998-2000 to show that past dividends strongly predict future dividends of banks.

Easterbrook (1984) proposed a theory under which dividends arise as a tool to reduce the agency costs between managers and shareholders because dividends can lead to more monitoring of managers. When firms are expected to continuously pay dividends, they may have higher needs for external funds. To access capital markets, banks must subject themselves to their scrutiny. Therefore, dividends can lead to more monitoring by capital markets.

As in Jermann and Quadrini (2012), I capture the smoothness of dividends through a quadratic dividend adjustment costs function. Costs arise when the payout deviates from the steady state target level:

\[ f(d) = \frac{\kappa}{2} (d - \bar{d})^2. \]

In this model, dividend adjustment costs introduce intertemporal rigidities into the balance sheet, which make banks’ choice of equity dependent on the current level of equity. This is consistent with the observation of Adrian and Shin (2011), who found that bank equity is sticky. The stickiness of equity can derive from debt overhang problems (see Admati et al. (2012)) and equity issuance costs. The latter can arise from underwriting fees and adverse selection premiums (see Myers and Majluf (1984)). Altinkilic and Hansen (2000) found empirical evidence for quadratic issuance costs: Initially, scale economies lower average costs, but with larger offers agency costs worsen as it becomes harder to find buyers willing to purchase the stock.

Paying out too much dividends can also be costly because of an increasing marginal tax rate on equity distributions, (see Hennessy and Whited (2007)).
The Subsidy Function

In the model, banks receive a subsidy from the government. These payments are increasing in (i) size, (ii) leverage, and (iii) risk taking of banks. I parametrize the subsidy function in the following way:

\[ TR\left(k^h, \frac{e + \pi}{k^h}, \sigma'^h\right) = \omega_3 k^h \exp\left(-\omega_1 \left(\frac{e + \pi}{k^h}\right) + \omega_2 \sigma'^h\right), \]

where \(k^h\) are the risky assets that banks hold, \(e + \pi\) represents equity after profits, and \(\sigma'^h\) denotes the risk choice of banks for the next period.

In the data, banks have limited liability and are beneficiaries of explicit (FDIC insurance) or implicit government guarantees (i.e. bailout of Bear Stearns). Without government protection, the risk of default is reflected in the cost of borrowing. If instead governments act as a backstop to banks\(^{14}\), debt holders do not require compensation for default risk. This lowers the cost of debt financing and helps explain high leverage ratios of banks in the data.

In the model, banks have unlimited liability but receive a subsidy that depends on leverage and risk taking. The subsidy in the form of the transfer function captures the effects of a banking system that is considered too big to fail. This assumption has two consequences. First, default does not occur in equilibrium. Second, government guarantees act effectively as subsidies by lowering the debt financing costs for banks because default risks are not priced into the claims that banks issue. The value of government guarantees is reflected in the transfer function’s positive dependence on leverage, risk-taking, as well as the size of the bank. Moreover, the transfer function captures the value of tax rules that benefit debt over equity financing. One of the first papers to model the effect of bailout guarantees over the business cycle is by Schneider and Tornell (2004). Government subsidies are also the core friction in Admati, DeMarzo, Hellwig, and Pfeiderer (2010, 2013).

Bank owners may have incentives to take on excessive risks when they have limited liability. In fact, equity claims are call-options on bank assets, an analogy that has first been discussed by Black and Scholes (1973a). More risk increases the value of the call option. Gollier, Koehl, and Rochet (1997) show that the risk exposure of firms with limited liability is always larger than that of firms with unlimited liability. Pennacchi (2006) presents a model in which deposit insurance subsidizes banks and that banks can increase the subsidy by concentrating their loan portfolio in systematic risk. Begenau, Piazzesi, and Schneider (2013) demonstrate empirically that commercial banks’ use of derivatives increases the risk

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\(^{14}\)Deposit insurance by the FDIC is a particular feature of commercial banks. If deposit insurance is mispriced, it distorts banks’ debt financing costs similar to implicit government guarantees.
exposure of banks’ balance sheet instead of hedging that exposure.

In the present model, risk-taking incentives are captured through the subsidy’s dependence on $\sigma^h$ and through the functional form of the transfers which captures complementarity between risk taking and leverage. That is, risk taking incentives are particularly strong when banks are highly leveraged. The transfer is increasing in $\sigma^h$ because banks effectively save the risk premium which they would need to pay without a guarantee.

There is a subsidy on debt for all firms in the US through its function as a tax shield. In the financial industry, where competition is on small net interest margins, it matters that debt is cheaper even though the tax advantage is quantitatively small. This argument has been advanced by Hanson, Stein, and Kashyap (2010). The scalar $\omega_3$ in the transfer function captures what would be the tax-advantage in this model if banks were purely financed with debt, had no profits, and were taking no risks. Then $\omega_3$ reflects the tax-advantage per dollar of debt.

The appendix shows how a model with an explicit default choice by banks and government bailout implies a bailout payoff function that resembles the reduced form subsidy function considered here (see appendix E). In the model with default choice and bailout, higher levels of leverage increase the probability of a default in which the government bails out the bank. Likewise, higher risk-taking by banks increases the probability of a default. Moreover, banks that are highly leveraged and take on more risks are especially likely to hit the default line. And finally, the size of the bailout increases with the size of the bank. The payments by the government occurring with some probability in the future as well as the savings that occur when banks are backed by the government can also be expressed as a stream of payments each period that depend on the same variables that increase the probability of a default and bailout amount, namely on after-profit leverage, risky assets, and the risk choice by banks.

**Household’s Liquidity Demand**

Diamond and Dybvig (1983)\textsuperscript{15} were the first who explicitly analyzed the idea of households’ liquidity demand and the role of banks as liquidity providers. In their model, liquidity demand arises because it improves households risk sharing possibilities. In this model, households have preference for liquidity in the form of bank liabilities. These preferences capture the benefits from money-like-securities such as their liquidity and transaction service. As in Diamond and Dybvig (1983), banks are important because they provide liquidity. This

\textsuperscript{15}Other papers have build upon this idea, see Diamond and Rajan (2000), Gorton and Pennacchi (1990), and Holmström and Tirole (2011).
role makes banks and their debt special compared to other firms. The fact that bank debt consists mainly in deposits is also behind the rationalization of deposit insurance. Due to households’ liquidity demand, it is optimal for banks to be highly leveraged aside of government subsidies. DeAngelo and Stulz (2013) show this mechanism in a stylized model.

Since Sidrauski (1967) money-in-the-utility specifications have been used to capture the benefits from money-like-securities for households in macroeconomic models. Feenstra (1986) showed the functional equivalence of models with money-in-the-utility and models with transaction or liquidity costs. The utility function in this paper additionally reflects that more consumption raises the marginal utility of bank debt holdings for households. These preferences are a version of those used in Christiano, Motto, and Rostagno (2010)\textsuperscript{16}. There are other ways of eliciting liquidity demand of households. For instance, Chari, Christiano, and Eichenbaum (1995) use a shopping time technology in which deposits help to reduce the time spent on purchasing good. Schneider and Doepke (2013) rationalize the existence of money through its use as a dominant unit of account.

*Bank Capital Requirements*

Banks are subject to a Basel-II type of capital requirement. The Basel-II accords stipulate that banks must hold a certain percentage of risk-weighted assets in terms of equity. Under these rules, assets that are considered safe such as government securities receive a 0% risk weight. In the model banks have to hold $\xi$ dollars of equity $e$ for each dollar of risky assets $k^h$.

3 Steady State Characterization

I define a deterministic steady state equilibrium as an equilibrium in which $Z^h$ and $Z^f$ are constants.

**Definition.** Given an exogenous government debt policy $B$, a steady state equilibrium is defined by a constant level of $Z^h$ and $Z^f$, a pricing kernel $M(X)$ and prices: $w^f(X)$, $r^f(X)$, $r^h(X)$, $p(X)$, $r(X)$, and $r^B(X)$, value functions for households $V_H$ and banks $V_B$, and policy functions of households for consumption $P_{cH}$, bank debt $P_{sH}^e$, capital $P_{kH}^f$, bank equity shares $P_{eH}^e$, labor supply $P_{NfH}$, as well as policy functions of banks for their capital stock $P_{kB}^{kH}$, bonds $P_{bB}^{bH}$, bank debt $P_{sB}^e$, equity $P_{eB}^e$, dividends $P_{dB}$, and $P_{sB}^{dH}$ and a law of motion for $X$ such that the equilibrium definition in section 2.5 is satisfied.

\textsuperscript{16}Their money and deposit utility parameter relates to the bank debt-consumption elasticity $\eta$ in the following way: $\sigma_q = 2 - \eta$. 

20
In this section, I characterize this non-stochastic steady state to illustrate some properties of the equilibrium and in particular the forces that generate the new trade-off when the capital requirement is increased. In this model, households’ preferences for bank debt lower the equilibrium interest rate on bank debt. This is a new feature of this model, which drives the effect of capital requirements on lending. It also implies that banks’ capital constraint is binding even without government subsidies. Second, the benefit from the bailout guarantee to banks drives a wedge between the costs and benefits of risky assets. This wedge gives banks incentives to take on excessive risks.

*Interest Rate Discount on Bank Liabilities*

In this model, bank debt is special because agents receive an utility flow from them. The preference for bank debt implies the existence of a discount on its interest rate which is the amount households are willing to give up in exchange for holding a riskless assets which gives a flow of utility compared to a riskless asset that does not give a flow of utility. This amount equals the marginal increase in utility from increasing the holdings of bank debt by one dollar keeping the marginal utility of consumption constant. This is spelled out in the first order condition of households with respect to bank debt holdings in the non-stochastic steady state:

\[
\frac{\partial U(c,s)}{\partial s} \times \left( 1/ \frac{\partial U(c,s)}{\partial c} \right) = \left( \frac{r^e - r}{1 + r^e} \right),
\]

where \(1/M(X) \equiv 1 + r^e\) and \(\partial U(c,s)/\partial s^l = \theta s^{-\eta}c^{\eta-1}\) is the marginal utility of bank debt holdings and \(\partial U(c,s)/\partial c = 1/c - \theta s^1 - \eta c^{\eta-2}\) is the marginal utility of consumption, which are both positive. The left hand side of equation (11) is the ratio of the marginal utility of bank debt holdings to the marginal utility of consumption. That is, the marginal increase in utility from one unit of bank debt normalized by the marginal utility of consumption. The right hand side (11) is the spread between equity and bank debt financing in the steady state.

Equation (11) says that as long households derive utility from bank debt and as long as the economy is not saturated with liquidity, the interest rate on bank debt is always less than the interest rate on any other riskless asset.

The marginal utility of bank debt is increasing in consumption whenever \(\eta > 1\). When agents become richer and want to consume more, they also want to hold more bank debt. Likewise, the marginal utility of consumption is also increasing in deposits as long as \(\eta > 1\). The interest rate discount on bank liabilities is high whenever agents hold less bank debt (high marginal utility of bank debt) relative to consumption (low marginal utility of consumption).
In this case, the equilibrium interest rate on bank debt \( r \) is low to reduce the demand of deposits.

In the households’ problem, households were not given the option to invest in government bonds. In fact, households do not want to hold government bonds because they have the same risk characteristics as bank liabilities without providing utility. Moreover, government bonds earn the same interest rate because government bonds are risk free and receive a risk weight of zero in the capital constraints of banks \( (e \geq (\xi k^h + 0 \times b)) \). If returns were not equated, there would be an arbitrage opportunity: banks could issue more debt to buy bonds driving down the interest rate. Or if bonds are more expensive than bank debt (low interest rate), banks would not want to hold bonds.

**Banks’ Capital Constraint**

In the non-stochastic steady state and for every combination of parameters, the capital constraint of banks is binding if either households have preference for bank debt holdings or banks receive transfers from the government that imply a debt benefit. To see this, consider the first order condition of banks with respect to equity in the non-stochastic steady state:

\[
\mu = \Lambda - M \Lambda (1 + r) \left( 1 - \omega_1 TR \left( 1, \frac{\bar{e}}{k^h}, \sigma^h \right) \right),
\]

where \( \mu \) is the Lagrange multiplier on the capital constraint and \( \Lambda \) is the Lagrange multiplier on banks’ budget constraint in equation (6). In the non-stochastic steady state dividends are at their long run level \( d = \bar{d} \) and therefore \( \Lambda = 1 \). Moreover, the stochastic discount factor of households is \( M = \beta \Gamma^{-1} = 1/(1 + \bar{r}^e) \) where \( \Gamma \) is the growth rate of the economy. This first order condition tells us that when banks increase equity by one unit, the capital constraint is relaxed by \( \mu \). At the same time, banks do not take advantage of the interest rate discount on debt that they would receive if they had used the other source of funding (debt) instead. Moreover, banks receive less transfers due to an increase in profits which increases their equity after profits \( \bar{e} \). I rewrite (12) into

\[
\mu = \frac{r^e - r}{1 + r^e} + \omega_1 TR \left( 1, \frac{\bar{e}}{k^h}, \sigma^h \right) \left( \frac{1 + r}{1 + r^e} \right).
\]

The capital constraint is binding for any parametrization because it is cheaper for banks to finance themselves with debt rather than with equity: one dollar of debt raised today results in a positive net profit \( r^e - r > 0 \) tomorrow. The government subsidy is an additional reason for a binding capital constraint. Without households’ preference for liquidity or a subsidy for banks the rate on deposits would equal the interest rate on capital (Friedman rule) in
the non-stochastic steady state and so \( r = r^e \). In this case, banks are indifferent between debt and equity to finance their assets and the capital requirement would have no effect on the equilibrium.

**Optimal Size of the Banking Sector**

The combined first order condition of risky assets and equity determines the costs and benefits of risky assets. To see that, I first define

\[
\frac{TR}{k^h} \equiv TR \left(1, \frac{\bar{e}}{k^h}, \sigma^h\right) \\
r^h \equiv v \frac{y^h}{k^h} \\
1 + r^e \equiv 1/M \\
\bar{M} \equiv M \frac{\Lambda'}{\Lambda},
\]

and the equity after profits of banks as a fraction of risky assets

\[
\frac{\bar{e}}{k^h} = y^h \frac{k^h}{k^h} - (r + \delta) + (1 + r) \frac{e}{k^h}.
\]

The first order condition of banks’ risky assets in the steady state is

\[
\mu \xi = M \Lambda \left( \left(v \frac{y^h}{k^h} - r - \delta \right) \left(1 - \omega_1 TR \left(1, \frac{\bar{e}}{k^h}, \sigma^h\right)\right) + TR \left(1, \frac{\bar{e}}{k^h}, \sigma^h\right) \left(1 + \omega_1 \frac{\bar{e}}{k^h}\right) \right),
\]

which together with the first order condition of banks with respect to equity (equation (12)) is combined to:

\[
\frac{TR}{k^h} \left(1 + \omega_1 (1 - v) \frac{y^h}{k^h}\right) + (1 + r^h - \delta) = \xi (1 + r^e) + (1 - \xi) (1 + r). \tag{13}
\]

This equation tells us what an additional unit of capital is worth, keeping leverage constant. Together with the first order condition of households (equation (11)) governing the interest rate on bank debt, this equation determines the size of banks. The right hand side represents the funding costs of one unit of risky assets while the left hand side is the benefit of risky assets. The subsidy from the government is higher for highly leveraged banks (small \( y^h/k^h \)). The marginal benefit of capital consists in the net marginal product on capital and the
The marginal transfer is

\[
\frac{\partial TR}{\partial k^h} = \omega_3 \exp \left( -\omega_1 \frac{\bar{e}}{k^h} + \omega_2 \sigma^h \right) - \omega_1 TR \left( \frac{k^h \frac{\partial \bar{e}}{\partial k^h} - \bar{e}}{(k^h)^2} \right)
\]

\[
= \frac{TR}{k^h} + \omega_1 \frac{TR}{k^h} \left( \frac{\bar{e}}{k^h} - \frac{\partial \bar{e}}{\partial k^h} \right),
\]

where \((\bar{e}/k^h - \partial \bar{e}/\partial k^h) = (1 - v) y^h/k^h + (1 + r) \xi\) is the difference between the average and the marginal product on capital plus the return on equity. That is, an increase in capital gives first one additional \(TR/k^h\). Second, since the transfer is assessed for the end-of-period equity of banks (\(\bar{e}\) instead of \(e\)), more capital raises end of period leverage while at the same time it reduces profits, lowering \(\bar{e}\).

The funding cost of \(k^h\) (right hand side of equation (13)) is a weighted average between two interest rates: \(1 + r^e\) is the interest rate that needs to be paid to the shareholder and \(1 + r\) is the interest rate that needs to be paid to depositors. In the steady state \(r^e = r^f - \delta\) because households first order condition with respect to capital \(k^f\) in the steady state is \(1/M = (1 + r^f - \delta)\). This means that bank equity holders must be paid the same return as they would obtain from investing one dollar into the firm sector and receiving the return \((1 + r^f - \delta)\). The government subsidy drives a wedge between the funding costs and the return on risky assets. The higher the value of the subsidy the lower the marginal product of risky assets. This implies that banks are larger in a world with subsidies than in a world without subsidies. This is consistent with a finding by Gandhi and Lustig (2010). They find a size factor in the stock return of banks that helps explain why larger bank stocks have lower risk-adjusted returns than small and medium-sized bank stocks. They attribute the presence of the size factor to the higher likelihood of recovery of larger banks.

**Risk-Taking Incentives**

The risk-return trade-off embedded in the technology of banks allows me to define an interior choice of risk-taking. The features of the model allow me to infer banks’ risk choice even in the steady state. The variable \(\sigma^h\) determines not only the exposure to the aggregate shock (which is not defined in the non-stochastic steady state) but also the mean productivity in banks’ technology and the flow payment from the government subsidy which are both defined in the steady state. In the stochastic model, the first order condition of
banks with respect to the amount of risk is

\[ 0 = \mathbb{E}_{\varepsilon' | \varepsilon} \left\{ M (X', \varepsilon') \Lambda (X', \varepsilon') \left( \frac{y^h(X', \varepsilon')}{k^h} \left( \phi_1 - 2\phi_2 \sigma^h + \varepsilon'_h \right) \right) \right\} \times \left( 1 - \omega_1 TR \left( 1, \bar{\varepsilon}(X', \varepsilon'), \sigma^h \right) \right) \left( 1 - \omega_1 TR \left( 1, \bar{\varepsilon}(X', \varepsilon'), \sigma^h \right) \right) + \omega_2 TR \left( k^h, \bar{\varepsilon}(X, \varepsilon), \sigma^h \right) \frac{1}{\Gamma k^h} \Lambda (X, \varepsilon), \right. \]

(14)

The first line is the expectation of tomorrow’s discounted marginal return of choosing today a project with the risk and return characteristics implied by \( \sigma^h \). Since the subsidy is decreasing in profits, the marginal return is reduced by \( \omega_1 TR (1, \bar{e}(X', \varepsilon') / k^h, \sigma^h) \), the factor by which the transfer is reduced for one unit increase in profits. The last term on the second line of equation (14) is the marginal subsidy from risk taking.

The optimal choice of \( \sigma_h \) in the steady state can be derived from (14)

\[ \sigma^h = \max Z^h \frac{\phi_1}{2\phi_2} + \frac{(1 + r^e) k^h}{2\phi_2} \frac{\omega_2 TR / k^h}{y^h 1 - \omega_1 TR / k^h}. \]

Without the transfer, the optimal risk choice is equal to the first term of equation (15) which maximizes the productivity level of the production technology of banks in the steady state. When banks receive a subsidy which is sensitive to risk taking, that is \( \omega_2 > 0 \), banks take on more risk than the amount that maximizes the productivity of banks’ technology. The scalar \( \omega_2 \) governs how much more risk banks want to choose over the level that maximizes productivity \( Z^h \).

Effect of an Increase in the Capital Requirement

An increase in the capital requirement reduces the supply of liquidity in the form of bank liabilities. But it also decreases risk-taking by banks and increases production as well as consumption. Banks can respond in three ways to comply with a higher requirement. They can increase equity holding assets constant. They can decrease assets holding equity constant. Or they can increase equity and assets which may or may not hold bank liabilities constant.

Holding interest rates constant, a higher capital requirement increases the funding costs of risky assets in equation (13) because a larger share of assets has to be financed with more expensive equity. Indeed, this is a familiar result from partial equilibrium models. The decreasing returns to risky assets in the bank dependent production technology implies that a reduction in risky assets leads to a higher return. Thus to increase the return on assets to
match higher funding costs, banks have to decrease their holdings of assets and consequently bank debt.

In general equilibrium, however, the interest rates on equity and bank debt may change with an increase in the capital requirement, which has consequences for the funding costs of assets. In the non-stochastic steady state, the interest rate on equity is equal to the interest rate on firm capital and is therefore not affected by the capital requirement. The interest rate on bank liabilities is determined by the marginal utility of bank debt holdings relative to the marginal utility of consumption in households’ Euler equation (11). How the interest rate is going to adjust in response to a fall in the supply of bank debt depends on how the ratio of marginal utilities responds to a fall in bank debt holdings. The derivative of the marginal utility ratios is

$$\partial \left( \frac{\partial U(c,s)}{\partial s} \times \left( 1/ \frac{\partial U(c,s)}{\partial c} \right) \right) \frac{\partial s}{\partial s} = \left( \frac{c}{s} \right)^{\eta} s^{-1} \frac{(1 - \eta) \theta (\xi)^{1-\eta} - \eta \theta (1 - \theta (\xi)^{1-\eta})}{(1 - \theta (\xi)^{1-\eta})^2} < 0$$

if $\eta > 1$ since $(1 - \theta (\xi)^{1-\eta}) > 0$ as this is the product of consumption with the marginal utility of consumption. This implies that a reduction in bank debt holdings increases the ratio of marginal utilities and therefore increases the interest rate discount on bank debt. When the fall in the interest rate is large enough, banks’ funding costs of risky assets fall, even though a higher fraction of risky assets has to be financed with relatively more expensive equity. The larger $\eta$, the larger the sensitivity of the interest rate $r$ to changes in $c/s$.

Since the capital constraint of banks is binding, an increase in $\xi$ leads to a reduction in leverage. Because of the complementarity of the subsidy between banks’ risk choice and leverage, less leverage lowers the marginal subsidy from risk-taking which causes banks to choose lower levels of $\sigma^h$. A reduction in $\sigma^h$ leads to an increase in the mean productivity level of banks’ investment technology. This raises the marginal product of risky assets. Outside the steady state, lower levels of $\sigma^h$ imply a lower variance of bank dependent output and thus total output.

Both, the reduction in funding costs and the increase in the marginal product make risky assets more desirable. For this reason banks want to increase their risky asset holdings. Higher productivity and a higher stock of risky assets increase the capital stock of the economy and the output from the bank dependent part of production. As a consequence, overall output and consumption increase. The optimal capital requirement trades off the fall in liquidity in the form of bank debt against the rise in consumption. When shocks are included in the analysis, the optimal capital requirement also takes into account the reduction in output volatility.
The magnitude of the fall in $r$ depends on the curvature parameter $\eta$ in the utility function of the households. Larger values for $\eta$ imply that households do not like changes in the bank debt-consumption ratio. In this case, the interest rate responds more to changes in the amount of bank debt holdings in order to keep $s/c$ constant. In addition to $\eta$, the curvature parameter in the banking sector technology $v$ matters for how much banks increase their assets in response to tighter requirements. The larger $v$ the easier assets are transformed into bank dependent output and therefore the larger the effect on consumption will be. The other two parameter that matter are $\omega_2$ and $\phi_2$ which together affect the optimal risk choice of banks. A high value for $\omega_2$ implies a larger sensitivity of the subsidy with respect to risk-taking. A high value of $\phi_2$ means that the productivity level $Z^h$ decreases faster in risk taking. In the next section, I present how the model is matched to the data.

4 Calibration

The full set of equilibrium describing equations is listed in section A.

The model is calibrated for the United States at quarterly frequency. The data covers the first quarter of 1999 to the last quarter of 2012. This period reflects a deregulated banking system which arguably started with the passing of the Gramm-Leach-Bliley Act\textsuperscript{17}. This bill completely repealed the Glass-Steagall Act that had restricted the activities of commercial banks. The firm parameters are calibrated using NIPA data while bank parameters are calibrated using data from commercial banks and savings institutions. This data stems from aggregated regulatory filings, so called call reports, which comprise balance sheet and income statement data\textsuperscript{18}. The dollar quantities are converted to trillion dollars and normalized by the St. Louis Fed population numbers measured in billions.

The calibrated parameters can be divided into three groups. The first group (summarized in table 2) are parameters that are directly set to their data counterpart. These parameters are the growth rate of GDP $\Gamma$, the depreciation rate $\delta$, the parameters governing the productivity process of firms $\rho^f$ and $\sigma^f$, and the correlation $\sigma^fh$ between $\epsilon^f$ and $\epsilon^h$ average hours worked $N^f$, level of riskless securities on the balance sheet of banks $B$, the persistence of banks’ productivity $\rho^h$, and the transfer parameter $\omega_3$. The second group of parameters (summarized in table 3) uses moments in the data together with the steady state conditions of the model to identify the parameters. In this group are the capital share in firm

\textsuperscript{17}In 1996 the Federal Reserve reinterpreted the Glass-Steagall Act several times, eventually allowing bank holding companies to earn up to 25 percent of their revenues in investment banking. But it was not before 1999 when the Glass-Steagall Act was completely repealed. This bill is called the Gramm-Leach-Bliley Act.

\textsuperscript{18}http://www.fdic.gov/bank/statistical/guide/index.html
Table 1: Mapping the Model to the Data

<table>
<thead>
<tr>
<th>Model</th>
<th>NIPA and FDIC balance sheet &amp; income statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^h$: bank output</td>
<td>income – securities interest income</td>
</tr>
<tr>
<td>$k^h$: bank capital</td>
<td>assets – sec – cash – fixed assets</td>
</tr>
<tr>
<td>$y^f$: firm output</td>
<td>NIPA total GDP – bank output</td>
</tr>
<tr>
<td>$k^f$: firm capital</td>
<td>NIPA $K - k^h$</td>
</tr>
<tr>
<td>c: consumption</td>
<td>NIPA consumption</td>
</tr>
<tr>
<td>s: bank debt</td>
<td>net income + non interest expense</td>
</tr>
<tr>
<td>$\pi$: profits</td>
<td>bank liabilities</td>
</tr>
<tr>
<td>r: rate on bank debt</td>
<td>interest expenses / bank liabilities</td>
</tr>
<tr>
<td>$\sigma^h$: risk choice</td>
<td>STD of HP-filtered log ($y^h/k^h$)</td>
</tr>
<tr>
<td>e: equity</td>
<td>Tier-1 equity</td>
</tr>
</tbody>
</table>

This table presents the model objects in the left and their data analogue in the right column.

Table 2: Parameters selected without Steady State Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function</th>
<th>Target Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma = 1.0075$</td>
<td>average growth rate p.c.</td>
<td>p.c. quarterly GDP growth</td>
</tr>
<tr>
<td>$\delta = 0.025$</td>
<td>capital depreciation</td>
<td>NIPA capital consumption</td>
</tr>
<tr>
<td>$\rho^f = 0.95$</td>
<td>$z^f$ productivity process</td>
<td>firm TFP persistence - from literature</td>
</tr>
<tr>
<td>$\sigma^f = 0.0075$</td>
<td>firm TFP volatility</td>
<td></td>
</tr>
<tr>
<td>$N^f = 1.43$</td>
<td>average hours (1/1000)</td>
<td>hours: Simona Cociuba</td>
</tr>
<tr>
<td>$B = 14.231$</td>
<td>riskless securities</td>
<td>bank balance sheet riskless assets</td>
</tr>
<tr>
<td>$\rho^h = 0.75$</td>
<td>persistence of $Z^h$</td>
<td>persistence of HP-filtered log ($y^h/k^h$)</td>
</tr>
<tr>
<td>$\sigma^{th} = 0.41$</td>
<td>corr: $\epsilon^h$ and $\epsilon^f$</td>
<td>corr: TFP and HP-filtered log ($y^h/k^h$)</td>
</tr>
<tr>
<td>$\omega_3 = 0.0043$</td>
<td>transfer parameter</td>
<td>tax benefit of debt 4.3 % - Graham (2000)</td>
</tr>
</tbody>
</table>

This table presents the parameter values that have been selected to match the moments in the right column.
Table 3: Parameters Selected Using Steady State Condition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function</th>
<th>Target Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.33$</td>
<td>firm production</td>
<td>firm labor share in firm GDP</td>
</tr>
<tr>
<td>$\beta = 0.9767$</td>
<td>discount rate</td>
<td>capital output ratio in firm sector</td>
</tr>
<tr>
<td>$E^f = 18$</td>
<td>effective hours</td>
<td>matches Cobb Douglas $y^f$</td>
</tr>
<tr>
<td>$\nu = 0.3735$</td>
<td>bank production</td>
<td>income-asset</td>
</tr>
<tr>
<td>$\xi = 10.75$</td>
<td>capital constraint</td>
<td>averages Tier 1 capital over risk based assets</td>
</tr>
<tr>
<td>$\theta = 0.0256$</td>
<td>deposit utility weight</td>
<td>interest rate spread on deposits</td>
</tr>
<tr>
<td>$\omega_1 = 2.11$</td>
<td>transfer parameter</td>
<td>bank profits</td>
</tr>
<tr>
<td>$\phi_1 = 0.13$</td>
<td>$Z_h$ productivity process</td>
<td>normalize mean productivity level = 1</td>
</tr>
<tr>
<td>$\phi_2 = 1.07$</td>
<td></td>
<td>$\text{std}(\text{HP-filtered } \log(y^h/k^h)) = 0.147$</td>
</tr>
</tbody>
</table>

This table contains the parameter values that have been selected to satisfy steady state conditions of the model together with the target moments in the right column.

Table 4: 2nd Moment Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function</th>
<th>Target Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = 1.2$</td>
<td>s/c elasticity</td>
<td>$\frac{\text{std}(s/c)}{\text{std}(GDP)} = 1.61$</td>
</tr>
<tr>
<td>$\omega_2 = 2.91$</td>
<td>transfer parameter</td>
<td>conditional variance of income-asset ratio</td>
</tr>
<tr>
<td>$\varphi_f = 0.0012$</td>
<td>Adjustment cost of $k^f$</td>
<td>$\frac{\text{std}(f)}{\text{std}(GDP)} = 5.6$</td>
</tr>
<tr>
<td>$\varphi_h = 0.265$</td>
<td>Adjustment cost of $k^h$</td>
<td>$\frac{\text{std}(h)}{\text{std}(GDP)} = 56$</td>
</tr>
<tr>
<td>$\kappa = 0.002$</td>
<td>Dividend payout costs</td>
<td>$\frac{\text{std}(d)}{\text{std}(GDP)} = 26$</td>
</tr>
</tbody>
</table>

This table contains parameter values that govern second moments in the model. They have been selected to match the target moments in the right column.

productiono, the discount factor of households $\beta$, effective hours $E^f$ in firm production, the capital share in banks $\nu$, the capital requirement $\xi$, the weight on bank liability holdings in the utility $\theta$, the transfer parameter $\omega_1$, and the parameters governing the risk-return trade-off of banks $\phi_1$ and $\phi_2$. The remaining parameters $\eta$ (elasticity of the deposit-consumption ratio in the utility), $\omega_2$ (the sensitivity of the transfer with respect to risk-taking), $\varphi_h$ and $\varphi_f$ (adjustment costs of capital parameters in sector $h$ and $k$ respectively), and $\kappa$ (dividend payout cost parameter) determine second moments of the model (summarized in table 4) which are jointly calibrated with the other parameters in table 3. This leaves several other second moments which can be used to check the model. For instance, I can check the model with business cycle moments and cross-correlations of balance sheet and income statement variables that have not been targeted. I will now explain in more detail how each parameter is calibrated.
One important question for the quantification of this model is, what is the data counterpart of bank output. GDP can be measured with the value added-, expenditure-, or income approach. Bank income is thus part of GDP and can be viewed as the value added from the banking sector. In the model, bank dependent output is produced with capital and banks are able to extract all its rents. In this case, rents to capital in the bank dependent part of production equal the income of banks. The calibration uses this analogy and measures bank dependent production output as the sum over interest and non-interest income net of interest income from securities using the aggregated income statements of commercial banks and savings institutions. The value added by firms is measured as the difference between total GDP from NIPA tables and banks’ value added. According to this measure banks account for 5% of GDP. An overview on how model objects are mapped to the data is given in table 1.

For capital used in bank production, I use banks’ risky assets from the balance sheet: total assets net of government securities, fixed assets, and cash. This capital measure of banks implies a capital-output ratio of roughly 12. In the model, the amount of government debt $B$ is exogenous. I set $B$ to the average level of riskless assets on banks’ balance sheets which consist mainly in government securities. The average of riskless assets amounts to $14.232$ ($14,231$ per capita). Government securities alone are $10.3$.

For simplicity, I assume that capital depreciates at the same rate in both sectors. I compute the depreciation rate using gross investment data and data on capital consumption from NIPA to obtain $\delta = 0.025$. The average capital-output ratio is 6.25 which implies a consumption-capital ratio of about 0.12.

The capital-output ratio of firms is roughly 7. The firm Cobb-Douglas function parameter $\alpha$ is chosen to match the share of salaries and wages in GDP which gives $\alpha = 0.33$. In the model, households supply labor inelastically. I use this fact to normalize hours worked to a constant, using the hours series which has been constructed and kept updated by Cociuba, Ueberfeldt, and Prescott (2012)\textsuperscript{19}. The number of average hours worked in the firm sector is around 1433 hours (at an annual rate), so $N_f = 1.43$. For the parameters and firm sector size to match the restriction of the Cobb-Douglas function, I convert hours into effective hours, so that one worker works more efficiently. In the model, I call this parameter $E_f$ which takes the value 18. In order to parametrize the productivity process $Z_f$ of firms I decompose GDP into its factor components. Then I apply the HP filter to the series and calculate its standard deviation which gives $\sigma_f = 0.0074$. I take the persistence parameter

\textsuperscript{19}The data can be downloaded from Simona Cociuba’s website: https://sites.google.com/site/simonacociuba/research.
from the literature which typically sets a value of $\rho^f = 0.95$.

The growth rate $\Gamma$ is computed using real GDP which results in an annualized growth rate of 3%. The time preference rate $\beta = 0.977$. It is picked such that $\beta$ is consistent with the steady state investment optimality condition as well as the marginal product of firm capital.

The parameter $\xi$ denotes the Tier-1 capital requirement in the model. According to the FDIC rules banks are deemed adequately capitalized if they hold 6 percent Tier-1 capital (common stock, noncumulative perpetual preferred stock, and minority interests in consolidated subsidiaries) to risk-weighted asset ratio. On average, banks hold 10.75% of risky assets (measured as assets net of fixed assets, government securities, and cash) in terms of Tier-1 equity (common stock, noncumulative perpetual preferred stock, and minority interests in consolidated subsidiaries) so that $\xi = 10.75\%$. That is the regulatory capital requirement is not binding for the average bank. Note though that regulators employ a more sophisticated risk-weighting technology than the one employed here. For instance credit and market risks are not considered here, which usually increase the amount of risk-weighted assets. Nevertheless the ratio of Tier-1 capital to risk-weighted assets as assessed by regulators is 9.5%, thus not far from 10.75%. Moreover, the FDIC data includes savings institutions that may hold more equity than the typical commercial bank and therefore increase the average. Still, the market seems to impose tougher limits on equity-asset ratios than regulators. Hanson, Stein, and Kashyap (2010) show that banks kept a Tier-1 capital to risk-weighted asset ratio well above the regulatory minimum, even after the crisis (8 percent). They cite the IMF (2010) estimate of 7 percent cumulative credit losses at US banks during the recession showing that losses were severe and capital buffers well above 6 percent are needed. The conclusion therefore should not be that capital requirements don’t matter or that they have been too high, but rather that capital requirements have been too low to effectively do the things they were designed to do. That is not to say that the regulatory capital requirement does not matter. Van Den Heuvel (2008) shows that most banks hold the ratio of total equity to risk-weighted asset relatively close to the regulatory limit.

Using the data on risky assets of banks as well as interest and non-interest income, the decreasing return to scale parameter $v = 0.37$ matches the income-risky asset ratio in the data. This parameter governs how much bank dependent output can be generated with one unit of risky assets.

Now I describe the calibration of the parameters governing the government subsidy and the risk-return frontier in banks’ productivity process as well as the preference parameters for bank debt holdings. These parameters are essential for the behavior of banks and specific
to this model. The scalar $\omega_3$ in the subsidy function is calibrated so that the subsidy without the benefit for leverage and risk-taking equals the tax-benefit on debt per unit of risky assets:

$$\text{Tax Advantage} = TR \left(1, \frac{e}{k^h}, \sigma^h | \omega_1 = 0, \omega_2 = 0 \right)$$

$$= \omega_3 \exp \left(-0 \times \frac{e}{k^h} + 0 \times \sigma^h \right).$$

Graham (2000) estimated the tax benefit of debt to be 4.3% (net of personal taxes) of firm market value. The ratio of banks’ market value to risky assets is approximately 0.10. Thus the tax benefit of debt per dollar of risky assets is on the magnitude of 0.43 cents, giving $\omega_3 = 0.0043$.

I obtain an estimate for $TR \left(1, \frac{e}{k^h}, \sigma^h \right)$ from the model by finding $TR \left(1, \frac{e}{k^h}, \sigma^h \right)$ as the difference from profits in the model to profits in the data which results in $TR \left(1, \frac{e}{k^h}, \sigma^h \right) = 42$ basis point. As a robustness check I use an bailout estimate from Veronesi and Zingales (2009) to derive $TR \left(1, \frac{e}{k^h}, \sigma^h \right)$. To this end, I translate the bailout amount into a stream of income:

$$\text{bailout-stream} = \frac{\text{Bailout Amount}}{k^h} \cdot \frac{r^{FDIC}(1+r^{FDIC})^{NB}}{(1+r^{FDIC})^{NB}-1},$$

where $r^{FDIC}$ is the interest rate at which the bailout is translated into quarterly payments and $NB$ is the number of quarters between bailouts. I set $NB = 58$ corresponding to two bailouts between 1984 and 2012 (the 1987 savings and loan crisis and 2008)\textsuperscript{20}. I set $r^{FDIC}$ to the average quarterly assessment rates of the FDIC which is around 20 basis point on each dollar of the assessment base (assets-equity), that is around 25 basis point for each dollar of $k^h$. Veronesi and Zingales (2009) estimate the bailout value to be $130$ billion which amounts to 1.63% per dollar of risky assets or around 0.9% of GDP in 2008. Scaling the bailout stream value to the size of banks’ risky assets implies that the bailout benefit amounts to approximately 7 basis point of GDP per year. The transfer value is approximated as:

$$TR \left(1, \frac{e}{k^h}, \sigma^h \right) = \text{Tax-Advantage of Debt +bailout -stream},$$

which is 46 basis point. If instead I use the bailout number of 175 billion dollars of the New York Times for the bank TARP recipients, the quarterly transfer is 47 basis point.

I find the sensitivity of the transfer function with respect to $\omega_1$ by using the condition for the marginal benefit of risky bank assets keeping leverage constant. Rewriting equation

\textsuperscript{20}Using only the number of bailouts of the sample period $NB$ can be alternatively set to 56. The results are not sensitive to either value.
(13), the sensitivity of the bailout benefit to leverage is
\[
\omega_1 = \frac{(\xi (1 + r^e) + (1 - \xi) (1 + r) - (1 + r^h - \delta))}{1 - \nu} \frac{1}{TR(1, \tilde{\xi}, \sigma^h)} - 1.
\] (16)

The scalar \(\omega_1\) takes on large values for banks that operate at a low marginal product and high capital intensity levels, and that are additionally highly leveraged. Using the data analogues of the interest rates, the transfer per unit of risky assets, and the decreasing returns to scale parameter \(\nu\) the sensitivity of the transfer with respect to leverage is \(\omega_1 = 2.1\).

The parameters \(\omega_2\) and \(\phi_2\) determine the risk choice of banks. The parameter \(\omega_2\) governs how much risk banks want to take because of the subsidy. The parameter \(\phi_2\) governs how much risk reduces the productivity of banks. The model pins one of these parameters down through banks’ optimality condition with respect to risk taking. I identify \(\phi_2\) using this optimality condition where \(\sigma^h\) in the model is set to the the unconditional variance of the HP filtered income-asset ratio in the data. I choose \(\omega_2\) such that the conditional variance of the HP filtered income-asset ratio given past profits is matched. In the data, the subsidy to banks is included in profits. I infer a high value for \(\omega_2\) if the income-asset volatility is high conditional on the profit-asset ratio being high a period ago. That is, I regress the demeaned business cycle component of the income-asset ratio on the lagged profit-asset ratio
\[
\left(\log \left(\frac{y^h}{k^h}\right)\right)^2 = \text{const} + \text{coefficient} \frac{\pi}{k^h} + \text{error},
\]
in the data and obtain an estimate for the coefficient on \(\pi/k^h\). Then I solve the model given \(\omega_2\) and simulate data to find the model implied regression coefficient. I find \(\omega_2\) such that the model coefficient matches the one in the data.

The scalar \(\phi_1\) matches the unconditional mean of the stationary process in \(Z^h\). To calculate this process, I use again the HP filtered business cycle component of \(\log y^h/k^h\). The parameter \(\rho^h\) equals the autocorrelation of this series. Since this is a demeaned, stationary process its unconditional mean is
\[
E(Z^h) = 0 = \exp \left(\frac{\phi_1 \sigma^h - \phi_2 (\sigma^h)^2}{1 - \rho^h} + \frac{(\sigma^h)^2}{2}\right).
\] (17)

Given an observed volatility of banks, \(\phi_2\) controls where the unconditional mean \(Z^h\) reaches its maximum. This parameter is chosen to satisfy the first order condition of banks with respect to the risk choice \(\sigma^h\) while at the same time satisfying the restriction of \(Z^h\)’s unconditional mean. A higher number of \(\phi_2\) implies a lower productivity maximizing amount of
risk $\sigma^h$ as

$$\sigma^h = \frac{\phi_1}{2\phi_2}.$$  

The government guarantee distorts the risk choice and drives a wedge between the productivity maximizing amount of risk $\sigma^h$ and the observable amount of risk:

$$\sigma^h = \frac{\phi_1 + \omega_2 \Gamma M y^h \frac{TR/k^h}{1-\omega_1 TR/k^h}}{2\phi_2}.$$  

(18)

For the calibration, $\sigma^h$ is set to the volatility of the income to risky assets ratio. The scalar $\phi_2$ is identified from the level of the government transfer per unit of capita. This transfer per unit of capital is high, when the return on assets is high but the marginal product of risky banking assets low. High transfers imply larger risk taking incentives for banks. Therefore, $\phi_2$ is identified by the transfer to banks. Figure 1 shows the conditional mean of $Z^h$ with the calibrated parameters $\phi_1 = 0.13$ and $\phi_2 = 1.07$.

Households have log utility with respect to consumption for simplicity. The preferences for bank liabilities are governed by parameters $\eta$ and $\theta$. The parameter $\theta$ determines the interest rate discount on bank liabilities. I select $\theta$ to match the interest rate discount using the first order conditions of the model. More concretely given $\eta$, the scalar $\theta$ is identified by the following first order condition of households with respect to bank debt holdings:

$$\frac{\partial U (c, s)}{\partial s} / \frac{\partial U (c, s)}{\partial c} = \frac{r^e - r}{1 + r^e}.$$  

(19)
Table 5: Second Moment Calibration

<table>
<thead>
<tr>
<th>1999q1 - 2011q4</th>
<th>STD - D</th>
<th>STD - M</th>
<th>Rel. STD - D</th>
<th>Rel. STD - M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend</td>
<td>34.40</td>
<td>26.03</td>
<td>26.83</td>
<td>20.67</td>
</tr>
<tr>
<td>Investment</td>
<td>7.21</td>
<td>4.80</td>
<td>5.62</td>
<td>3.81</td>
</tr>
<tr>
<td>Investment Bank</td>
<td>72.88</td>
<td>69.01</td>
<td>56.84</td>
<td>55.00</td>
</tr>
<tr>
<td>Deposit/Consumption</td>
<td>2.02</td>
<td>2.02</td>
<td>1.61</td>
<td>1.61</td>
</tr>
</tbody>
</table>

This table contains the standard and the targeted relative standard deviations of the data (D) and compares those to the model (M).

Variables: HP-Cycle component of logged variable expressed in percent
†Variables: HP-Cycle component of variable

The interest rate on bank debt is on average 0.27% (annualized). The return on equity in the steady state is $1 + r^e = 1/\beta \Gamma^{-1}$, which implies a $\theta$ value of 0.0256.

The parameter $\eta$ determines the curvature in the utility of the bank debt holdings to consumption ratio. As such, it determines how much this ratio varies. Naturally, the target moment in the data to calibrate $\eta$ is the volatility of the bank debt holdings to consumption ratio. I choose $\eta$ jointly with the other second moment parameters (government subsidy sensititivty to risk-taking $\omega_2$ and the adjustment costs parameters $\varphi_f$, $\varphi_h$, and $\kappa$) to minimize the average distance (relative to GDP) between the volatility of bank investment as well as aggregate investment, the volatility of dividend, the volatility of the bank debt holdings to consumption ratio, and the conditional income-asset ratio volatility and their data counterparts.

The adjustment cost parameter $\kappa$ and $\varphi_h$ interact with each other in an interesting way. When the adjustment of dividends is costly relative to the adjustment of risky assets, the volatility of dividends is low whereas the volatility of balance sheet variables is high. The reason is that banks respond to shocks with varying the balance sheet instead of adjusting dividends. If instead banks find it more expensive to adjust the balance sheet instead of dividends, they respond to shocks with large adjustments in dividends. The model does not generate enough volatility in bank dividends. But without $\kappa > 0$ the model does not generate enough volatility on the balance sheet. The parameter values that bring investment, dividend, and $s/c$ volatilities closest to the data are $\omega_2 = 2.91$, $\kappa = 0.002$, $\varphi_h = 0.265$, $\varphi_f = 0.0012$ and $\eta = 1.2$. Table 5 shows the result from the calibration. The model does not produce enough volatility in particular in dividend and total investment.
Table 6: Volatilities

<table>
<thead>
<tr>
<th>1999q1 - 2011q4</th>
<th>STD - D</th>
<th>STD - M</th>
<th>Rel. STD - D</th>
<th>Rel. STD - M</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.28</td>
<td>1.26</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bank GDP</td>
<td>7.26</td>
<td>21.21</td>
<td>5.66</td>
<td>16.85</td>
</tr>
<tr>
<td>Assets</td>
<td>1.76</td>
<td>2.36</td>
<td>1.37</td>
<td>1.87</td>
</tr>
<tr>
<td>Bank Debt</td>
<td>1.98</td>
<td>2.29</td>
<td>1.55</td>
<td>1.81</td>
</tr>
<tr>
<td>Risky Assets</td>
<td>3.15</td>
<td>3.21</td>
<td>2.45</td>
<td>2.55</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.04</td>
<td>0.49</td>
<td>0.81</td>
<td>0.39</td>
</tr>
<tr>
<td>Profits</td>
<td>33.10</td>
<td>36.50</td>
<td>25.81</td>
<td>28.31</td>
</tr>
</tbody>
</table>

This table contains the standard and the relative standard deviations of the model (M) and compares those to the data (D).

Variables: HP-Cycle component of logged variable expressed in percent

†Variables: HP-Cycle component of variable

5 Business Cycle Statistics

In this section, I discuss the business cycle implications of the model. The model is solved as described in section D using the benchmark calibration with \( \xi = 10.75\% \). I simulate the model 1000 times for double as many periods as are in my sample (roughly 120). Half of the observations are discarded. Then I apply the HP filter with a smoothing parameter of 1600 to the remaining simulated data points. I use this data to compute volatilities, autocorrelations, and cross-correlations which are reported in table 6 and 7.

Table 6 reports the volatilities of key variables in the model. In the model, GDP is almost as volatile as in the data which is largely driven by the volatility of the banking sector. The model captures the volatility of assets, bank debt, and bank profits. It overstates the volatility of bank income and understates the volatility of consumption. The low consumption volatility is a familiar feature of many business cycle models.

Table 7 summarizes the business cycle - and cross-correlations of the model and compares it to the data. Overall, it produces reasonable correlations, in particular regarding the business cycle correlations which can be seen in the first column of table 7. Consumption and total investment comove with GDP because the marginal product on capital is higher during booms, leading to better investment opportunities and higher output during booms. Banking output and balance sheet variables are procyclical as in the data. Banks expand their assets (risky assets) during booms and contract them during recessions. The positive correlation between banking and firm sector productivity, the curvature on the deposit-consumption ratio in the utility \( \eta \), as well as the adjustment costs of capital in both sectors
are important for producing the procyclicality of balance sheet variables and output. When $\eta$ takes on a large value, households are less flexible with regard to changes in the bank debt holdings-consumption ratio. When only the firm sector is hit by a positive shock, the marginal product of firm capital $k^f$ is higher. This raises the opportunity costs for $k^h$ and the rate at which shareholders want to be compensated. Without adjustment costs to capital or relatively strong curvature in the preferences for the bank debt holdings to consumption ratio, risky assets of banks flow immediately to firms at times when the productivity of capital in the firm sector is higher than in the banking sectors, producing a negative correlation between balance sheet variables and total GDP. Adjustment costs make it expensive to change the current stock of capital in either sector and therefore slow down the response to shocks. Since banks are at the capital constraint, movements in bank capital stock $k^h$ are perfectly correlated with movements in bank debt stocks. Relatively inelastic preferences for the bank debt holdings to consumption ratio therefore represent another reason for a slow response of $k^h$ to shocks. The marginal utility of bank debt holdings is increasing in consumption which is also governed by $\eta$. With $\eta > 1$, during booms agents like to consume more of both, consumption goods and bank debt holdings. As a result, the model is able to generate a positive correlation between bank investment and aggregate investment.

Since bank debt holdings are procyclical, the interest rate on bank debt is procyclical, too. The model captures these procyclicalities. Movements in the interest rate come from movements in the ratio of marginal utilities of bank debt and consumption, which is governed the optimality condition of households with respect to bank debt holdings. Since bank debt is procyclical the marginal utility of bank debt is countercyclical, which induces comovement of the interest rate on bank debt rates with GDP.

The model produces similar business cycle correlations of bank output, return over risky assets, profits, and dividends as in the data. Good times mean higher profits for banks because the marginal product of risky assets increases. The higher profitability of banks during booms implies lower payoffs of the government subsidy whose value is higher during bad times. When the payments from the subsidy are lower, banks have less incentive to take on excessive risks so that they choose less risky and more efficient projects, which increases profits and lowers the subsidy further.

The return over risky assets depends on the marginal product of risky assets and the government transfer payments. The increase of the marginal product outweighs the decrease of the transfer payment during booms resulting in a positive correlation between return over risky assets and GDP. Dividends are a function of banks’ equity after profits, the government subsidy, adjustment costs to dividend payout and capital, as well as new book equity. Banks
Table 7: Business Cycle Correlations (D=data, M=model) 1999Q1 - 2011Q4

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Bank Output</th>
<th>Investment</th>
<th>Assets</th>
<th>Bank Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D</td>
<td>M</td>
<td>D</td>
<td>M</td>
<td>D</td>
</tr>
<tr>
<td>GDP</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y^h$ Bank Output</td>
<td>0.63</td>
<td>0.63</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>0.95</td>
<td>0.97</td>
<td>0.52</td>
<td>0.69</td>
<td>1</td>
</tr>
<tr>
<td>Assets</td>
<td>0.27</td>
<td>0.39</td>
<td>0.31</td>
<td>0.69</td>
<td>0.16 †</td>
</tr>
<tr>
<td>Bank Debt</td>
<td>0.21†</td>
<td>0.39</td>
<td>0.25</td>
<td>0.69</td>
<td>0.11†</td>
</tr>
<tr>
<td>$k^h$ (Bank Risky Assets)</td>
<td>0.49</td>
<td>0.39</td>
<td>0.57</td>
<td>0.69</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tilde{e}$ Equity</td>
<td>0.38</td>
<td>0.62</td>
<td>0.36</td>
<td>0.97</td>
<td>0.34</td>
</tr>
<tr>
<td>$d$ Dividend</td>
<td>0.44</td>
<td>0.61</td>
<td>0.28</td>
<td>0.99</td>
<td>0.44</td>
</tr>
<tr>
<td>$r$</td>
<td>0.62</td>
<td>0.64</td>
<td>0.92</td>
<td>0.64</td>
<td>0.52</td>
</tr>
<tr>
<td>$c$ Consumption</td>
<td>0.94</td>
<td>0.86</td>
<td>0.68</td>
<td>0.37</td>
<td>0.89</td>
</tr>
<tr>
<td>$ROK^h = y^h/k^h$</td>
<td>0.65</td>
<td>0.62</td>
<td>0.67</td>
<td>0.96</td>
<td>0.68</td>
</tr>
<tr>
<td>$\pi$ Profit</td>
<td>0.41</td>
<td>0.58</td>
<td>0.32</td>
<td>0.94</td>
<td>0.49</td>
</tr>
<tr>
<td>$i^h$ Bank Investment</td>
<td>0.58</td>
<td>0.48</td>
<td>0.33</td>
<td>0.75</td>
<td>0.70</td>
</tr>
</tbody>
</table>

This table displays the business cycle correlations of model object (M) and compares those to their data counterpart (D).

Variables: HP-Cycle component of logged variable / GDP trend, HP smoothing = 1600

†: p-value ≥ .10 ; †† Variables: HP-Cycle component variable / GDP trend

can pay more dividends during booms because their equity after profits increases more thanks to their higher profitability.

Overall, the model is able to produce the correct sign of the correlation. In four cases (the correlations of equity, dividends, return over risky assets, and profits with balance sheet) the model appears to generate a wrong correlation. Three out of four of these cases concern correlations in the data that change when being computed over a longer horizon (equity, dividends, return over risky assets). In the model, risky assets and book equity are perfectly correlated because banks are constrained by the capital requirement. This is consistent with the data where the ratio of equity to assets is acyclical implying that equity expands along with assets during booms. When computing the correlation between equity and assets in the data for the longer period from 1984 to 2011, the correlation is significantly positive.

The model also appears to not generate the negative correlation between dividends as well as the return over risky assets and balance sheet variables. Again, the negative correlation of $ROK^h$ and dividends disappears in the data when the correlations are computed for the period between 1984 and 2011. Moreover, the model produces a stronger correlation of $\tilde{e}$ and balance sheet variables as in the data, even when computed for a longer horizon where
<table>
<thead>
<tr>
<th></th>
<th>$k^h$</th>
<th>$\bar{\hat{\epsilon}}$</th>
<th>$\bar{\hat{d}}$</th>
<th>$\bar{\hat{r}}$</th>
<th>$\bar{\hat{c}}$</th>
<th>$\bar{\hat{ROK}_h}$</th>
<th>$\bar{\hat{\pi}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D M</td>
<td>D M</td>
<td>D M</td>
<td>D M</td>
<td>D M</td>
<td>D M</td>
<td>D M</td>
</tr>
<tr>
<td>$k^h$</td>
<td>1 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\hat{\epsilon}}$</td>
<td>0.11†</td>
<td>0.79</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\hat{d}}$</td>
<td>-0.07†</td>
<td>0.66</td>
<td>0.18†</td>
<td>0.96</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\bar{\hat{r}}$</td>
<td>0.68</td>
<td>0.86</td>
<td>0.25</td>
<td>0.72</td>
<td>0.34</td>
<td>0.61</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{\hat{c}}$</td>
<td>0.45</td>
<td>0.38</td>
<td>0.41</td>
<td>0.39</td>
<td>0.53</td>
<td>0.35</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\hat{ROK}_h}$</td>
<td>-0.43</td>
<td>0.53</td>
<td>0.47</td>
<td>0.92</td>
<td>0.54</td>
<td>0.96</td>
<td>0.54</td>
</tr>
<tr>
<td>$\bar{\hat{\pi}}$</td>
<td>-0.35</td>
<td>0.58</td>
<td>0.47</td>
<td>0.90</td>
<td>0.46</td>
<td>0.95</td>
<td>0.12†</td>
</tr>
<tr>
<td>$\bar{\hat{i}}$</td>
<td>0.27</td>
<td>0.25</td>
<td>0.14†</td>
<td>0.69</td>
<td>0.03†</td>
<td>0.75</td>
<td>0.33</td>
</tr>
</tbody>
</table>

This table displays the business cycle correlations of model object (M) and compares those to their data counterpart (D).

Variables: HP-Cycle component of logged variable / GDP trend, HP smoothing = 1600
†: p-value $\geq .10$  †† Variables: HP-Cycle component variable / GDP trend

The correlation between equity and assets is 0.27 and significant. The model explains most of the movements in dividends with movements in the equity after profits of banks. This link is less strong in the data. The model fails to capture the negative correlation between assets and profits. This correlation does not disappear when looking at a longer horizon albeit it loses in strength, being only -0.29. The reason for this counterfactual correlation is a strong comovement of balance sheet variables and profits with GDP in the model. Since assets and profits increase during booms, they also exhibit a positive cross-correlation. Furthermore, banks in the model generate profits from balance sheet variables only. In the data, banks derive their profits also from off-balance sheet items such as derivatives and structured investment vehicles.

An increase in consumption raises the marginal utility of bank debt holdings as long as there is enough curvature on the bank debt holdings to consumption ratio. The model consequently produces a relatively strong correlation between bank debt holdings and consumption. In the data, consumption and bank debt holdings are positively correlated albeit not significantly. However, for the sample period of 1984 and 2011 the correlation is 0.43 which is closer to the correlation predicted by the model.
6 Welfare

First, I present the solution to the first best which gives intuition for the mechanics of the model. Next, I discuss how a regulator should optimally set the capital requirement of banks. To solve the model, I use local perturbation methods that are discussed in the appendix D.

6.1 First Best: Complementarity between Bank Debt and Risky Assets

In this section, I discuss the first best solution as one in which government debt is taken as given and a social planner uses the two production technologies and the balance sheet technology to decide over the optimal allocation. For simplicity assume further that there are no adjustment costs (it does not matter in the steady state anyways) and that the growth rate is $\Gamma = 1$. As in the full model, each sector’s technology is stochastic and varies with shocks. The social planner maximizes the present value of households’ lifetime utility and takes into account that bank debt must be produced with $k^h$. The problem is

$$V^{FB} (X, \varepsilon) = \max_{\sigma^h, k^h, k^f, c} \log c + \theta \frac{(k^h \cdot c)^{1-\eta}}{1-\eta} + E_{\varepsilon|\varepsilon} \left[ M (X', \varepsilon'|\varepsilon) V^{FB} (X', \varepsilon') \right],$$

s.t.

$$c + k^f + k^h = y^f + y^h + (1 - \delta) (k^h + k^f),$$

using $s = k^h$. The state vector $X$ contains the productivity levels $Z^h$ and $Z^k$. The arranged first order conditions are

$$M (X', \varepsilon', \varepsilon) = \beta \left( \frac{c (X', \varepsilon')^{-1} - \theta k^h (X, \varepsilon)^{1-\eta} c (X', \varepsilon')^{\eta-2}}{c (X, \varepsilon)^{-1} - \theta (k^h)^{1-\eta} c (X, \varepsilon)^{\eta-2}} \right),$$

$$1 = E_{\varepsilon'|\varepsilon} \left[ M (X', \varepsilon', \varepsilon) \left( \frac{\partial y^f (X', \varepsilon')}{\partial k^f (X, \varepsilon)} + (1 - \delta) \right) \right],$$

$$1 = \frac{\theta (k^h \cdot c (X, \varepsilon)^{-1})^{-\eta}}{1 - \theta (k^h \cdot c (X, \varepsilon)^{-1})^{1-\eta}} + E_{\varepsilon'|\varepsilon} \left[ M (X', \varepsilon', \varepsilon) \left( \frac{\partial y^h (X', \varepsilon')}{\partial k^h (X, \varepsilon)} + (1 - \delta) \right) \right],$$

$$0 = E_{\varepsilon'|\varepsilon} [M (X', \varepsilon', \varepsilon) y^h (X', \varepsilon') (\phi_1 - 2\phi_2 \sigma^h + \epsilon^i)].$$
In the steady state

\[
\frac{c}{k} = \frac{y}{k} + (1 - \delta)
\]

\[
\frac{\partial y^f}{\partial k^f} - \delta = \frac{1}{\beta} - 1
\]

\[
\frac{\partial y^h}{\partial k^h} - \delta = \frac{1}{\beta} - 1 - \frac{\theta (k^h/c)^{-\eta}}{\beta (1 - \theta (k^h/c))^{1-\eta}}
\]

\[
\sigma^h = \frac{\phi_1}{2\phi_2},
\]

where \( y = y^h + y^f \). The planner picks the project that maximizes the productivity of the banking sector: \( \sigma^h = \frac{\phi_1}{2\phi_2} \), that is \( Z^h = \exp \left( \frac{(\phi_1 - \phi_2 \sigma^h)}{1 - \rho^h} \right) > 1 \). From equation (20) and (21) it is clear that the marginal product of capital in the firm sector is higher than the marginal product in bank sector. Banks have a higher capital output ratio in their sector in order to satisfy the demand for bank debt holdings. The marginal product of capital across these two production sectors does not equalize because capital in the banking sector produces (partially when banks hold government securities) liquidity and the final good. The marginal product of capital in the banking sector is equal to the marginal product of firm capital less the interest rate discount on bank debt. With preference for liquidity, the capital stock is higher than the modified golden rule level (also discussed in Van Den Heuvel (2008)). This has implications for the behavior of consumption in the welfare analysis. Without liquidity demand

\[
r^h - \delta = r^f - \delta,
\]

and investment occurs at the modified golden rule. In the steady state, \( Z_k = 1 \) and the optimal amount of \( k^f \) is

\[
k^{f*} = \left( \frac{1}{\beta + \delta - 1} \right)^{\frac{1}{\alpha - \tau}} N^f.
\]

The optimal amount of \( k^h \) is the solution to this pair of equations

\[
c = y + (1 - \delta) (k^f + k^h)
\]

\[
vZ_h k^h^{\nu-1} + \frac{1}{\beta} \frac{\theta (k^h/c)^{-\eta}}{1 - \theta (k^h/c)^{-\eta}} - \delta = \frac{1}{\beta} - 1.
\]
The right hand side of equation 23 describes the opportunity cost of investing one unit of the final good in the banking sector instead of in the non-banking sector and the left hand side describes its benefit.

Figure (2) depicts the equilibrium in the first best. It plots equation (23)’s left hand side as a dashed line and the ride hand side as a solid line. The optimal amount of $k^* h$ is found at the intersection of the dashed and solid line. Without the deposit premium, the equilibrium would be found outside to the left on this picture where the marginal product of $k^h$ (dot-dash line in figure (2)) equates the marginal product of $k^f$. The interest rate discount introduces a wedge between the marginal products of capital in the two sectors, leading to a higher equilibrium level of $k^h$. Any $k^h > k^* h$ implies too much investment into the banking sector. Any $k^h < k^* h$ implies too little investment into the banking sector, not satisfying households’ liquidity demand. The optimal amount of risky banking assets depends on the utility parameters of the households. Figure (3) shows how the supply of $k^h$ (bank debt actually) depends $\theta$ and the shape of the benefit of capital (the left(2)) on $\eta$. The higher the weight on bank debt in the utility (high $\theta$) the more capital must be allocated to the banking sector, lowering the marginal product there. The less curvature on the bank debt holdings to consumption ratio, that is the lower $\eta$, the flatter the slope of the $k^h$ benefit curve.

What would happen if a regulator imposed a capital constraint: Social Planner Solution

The social planner maximizes the present value of household’s lifetime utility given a
level of government debt and imposing a capital constraint where \( s = (1 - \xi) k^h \). Assume that the level of government debt equals \( 0 \). The social planner takes into account that bank debt is produced with \( k^h \) thus the problem becomes:

\[
V^{FB}(X, \varepsilon) = \max_{\sigma^h, k^h, k^f, c} \log c + \theta \left( \frac{(1-\xi)k^h}{c} \right)^{1-\eta} + E_{\varepsilon' | \varepsilon} \left[ M(X', \varepsilon' | \varepsilon) V^{FB}(X', \varepsilon') \right],
\]

s.t.

\[
c + k^f + k^h = y^f + y^h + (1 - \delta) (k^h + k^f),
\]

using \( s = k_h \). From the first order conditions above, only the condition with respect to \( k_h \) has changed to:

\[
1 = \frac{\theta (1 - \xi)^{1-\eta} \left( \frac{k^h}{c(X, \varepsilon)} \right)^{-\eta}}{1 - \theta (1 - \xi)^{1-\eta} \left( \frac{k^h}{c(X, \varepsilon)} \right)^{-\eta}} + E_{\varepsilon' | \varepsilon} \left[ M(X', \varepsilon', \varepsilon) \left( \frac{\partial y^h(X', \varepsilon')}{\partial k^h} + (1 - \delta) \right) \right].
\]

The steady state allocations look identical to before except for the deposit premium and the
therefore the marginal product on banking assets:

\[ vZ^h \left( k^h \right)^{v-1} - \delta + \frac{1}{\beta} \frac{\theta (1 - \xi)^{1-\eta} \left( \frac{k^h}{c} \right)^{1-\eta}}{1 - \theta (1 - \xi)^{1-\eta} \left( \frac{k^h}{c} \right)^{1-\eta}} = \frac{1}{\beta} - 1 \]

\[ \frac{c}{k^h + k^f} = \frac{y^h + y^f}{k^h + k^f} + (1 - \delta) \]

In the first best, welfare is always maximized by choosing a capital requirement of \( \xi = 0 \) since any \( \xi > 0 \) reduces the amount of bank debt. When the demand for bank debt is not very elastic, more capital compared to the first best is needed in order to keep bank debt holdings at a similar level as in the first best. This case is shown in figure 4. In contrast when \( \eta < 1 \), the social planner chooses a smaller level of \( k^h \) compared to the first best.

An increase in \( \xi \) (from 0 as in the first best to 10.75%) changes how much of \( k_h \) can serve as liquidity in the utility. By reducing the amount of bank debt holdings \( (k^h (1 - \xi)) \), an increase in \( \xi \) reduces utility. The agent can increase utility again if she increases the banking capital stock in equilibrium which is akin to an income effect. At the same time an increase in the capital stock also decreases the marginal utility of bank debt which leads the agent to hold less bank debt i.e. substitution effect. Whether forced capital requirements result in more or less \( k^h \) depends on the value of \( \eta \) which determines whether the income or the substitution effect dominates. When \( \eta > 1 \), the income effect dominates the substitution effect.
effect and forced capital requirement leads to more banking capital as shown in figure 4
where welfare for the first best is shown as a function of $k^h$. When $\eta = 1$ the income and
substitution effect cancel. The term $(1 - \xi)$ drops out of the utility and we are back to the
first best. When $\eta < 1$, the substitution effect dominates the income effect and households
prefer to install less banking capital in equilibrium.

6.2 The Return on Risky Assets

The key equations that govern banks’ behavior are the first order condition with respect to
bank liabilities

$$E_t (M_{t+1}) (1 + r_t) = \left(1 - \frac{\theta}{\Gamma} s_t^{-\eta} c_t^{\eta-1} \left(\frac{1}{\xi_t} - \theta (s_t)^{1-\eta} c_t^{\eta-2}\right)\right),$$

the optimality condition of banks with respect to risk-taking

$$0 = E_t \left\{ \tilde{M}_{t+1} \left(\frac{y_{t+1}^h}{k_t^h} \left(\phi_1 - 2\phi_2 \sigma_{t+1}^h + \epsilon_{t+1}^h\right) \right) \times \left(1 - \omega_1 TR \left(1, \frac{\tilde{e}_t + 1}{k_t^h}, \sigma_{t+1}^h\right)\right) \right\} + \frac{1}{\Gamma k_t^h} TR \left(1, \frac{\tilde{e}_t}{k_t^h}, \sigma_t^h\right) \omega_2,$$

and the optimality condition of banks with respect to the physical capital of banks, keeping
leverage fixed:

$$\xi \left(1 + E_t r_t e_{t+1}^e \right) + (1 - \xi) \left(1 + r_t\right) = (1 + E_t r_t e_{t+1}^e) E_t \left[ \tilde{M}_{t+1} R_{t+1}^k \right],$$

where $1/(1 + r_t e_{t+1}^e) = \tilde{M}_{t+1} = M_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t}$ is the stochastic discount factor of households,
augmented by the ratio of banks dividend payout Lagrange multipliers. The return on
capital $k^h_{t+1}$ is defined as:

$$R_{t+1}^k = 1 + r_{t+1}^h - \delta + TR \left(1, \frac{\tilde{e}_{t+1}}{k_t^h}, \sigma_{t+1}^h\right) \left(1 + \omega_1 \frac{y_{t+1}^h}{k_t^h} (1 - v)\right).$$

Changes in $\xi$ affect the expected return on banking capital in equation (26) and therefore
how much $k^h$ banks are going to invest. This in turn affects how much bank debt holdings
banks offer to households and how much risk banks want to take. The covariance between
the augmented stochastic discount factor and the return on risky bank assets matters for a
risk premium on risky assets:
\[ E_t R^k_{t+1} - (\xi (1 + E_t r^k_{t+1}) + (1 - \xi)(1 + r_t)) = -(1 + E_t r^k_{t+1}) \left( COV_t \left( \tilde{M}_{t+1} R^k_{t+1} \right) \right) \]

where \( COV_t \left( \tilde{M}_{t+1} R^k_{t+1} \right) < 0 \) as returns move with the business cycle but the augmented stochastic discount factor moves against it.

### 6.3 Optimal Capital Requirement

In this section, I present the welfare implications of this model.

**Optimal Capital Requirement**

I use the model to compute the optimal capital requirement. To this end, I solve the model for different levels of capital requirement and obtain its decision rules. Then I simulate the model under the benchmark capital requirement of \( \xi = 10.75\% \). To compute the optimal requirement taking into account the transition effects, I use the decision rules for each value of \( \xi \) to simulate time paths for consumption and bank debt, which start at a random point on the time path of the benchmark capital requirement. This procedure is repeated over the number of simulations, starting the new regime each time at a different point on the old regime’s time path. Then - for each \( \xi \) - I evaluate the realized utility and compute the value function for the period before the new regime is introduced by discounting the time path of utility with households’ pricing kernel. Finally, I average over simulations.

Figure 5 depicts the result expressed in consumption equivalents percentage units, that is, the percentage permanent change in consumption if the economy moves from the current regime (\( \xi = 10.75\% \)) to any other capital requirement on the x-axis. The value function reaches its maximum at about \( \xi = 14\% \), which is above the level that commercial banks and savings institutions currently hold on their balance sheet.

When the capital requirement is increased banks reduce the supply of bank debt and increase equity as shown in figure 6. This figure presents the time path of bank debt (left panel) and equity (right panel) for different levels of capital requirement over the transition periods. Banks can comply with the higher level of capital requirement by either keeping equity constant and reducing bank debt, or by increasing equity and expanding assets. But even in the latter case, banks need to reduce bank debt because a balance sheet expansion must go through the decreasing returns to scale on banks’ asset side. That is, banks only want to become larger when the funding costs of assets decrease. Without reducing the
Figure 5: Optimal Level of Risked Based Capital Ratio

Figure 6: Transition of Banks’ Funding Sources

Bank Debt

Equity
The reduction in the supply of bank debt affects the interest rate as discussed in section 3. Figure 7 shows how the interest rate on bank liabilities responds to changes in the capital requirement. The marginal utility of bank debt holdings depends negatively on the amount. That is, the fall in the supply increases the demand for bank debt by households. To reduce the demand, the interest rate has to decrease. Intuitively, the reduction in the interest rate makes bank debt holdings less attractive to households.

The fall in the interest rate on bank liabilities is large enough to lower the funding costs of banks. This entices banks to increase their size financed by more equity (as seen in the right panel of figure 6). The increase in size also prevents the supply of bank debt from falling too much. The higher the levels of $\xi$, the larger the fall in bank debt holdings and their rate. That is, for high values of $\xi$, say 22%, households accept a larger reduction in the interest rate, which prevents the supply of bank debt from falling too much.

The increase in bank assets and equity occurs relatively quickly because the adjustment costs to capital and dividends are low. However, it takes time to expand the balance sheet because capital (necessary for the balance sheet expansion) accumulates slowly over time.
Figure 8 shows the time path of capital in the banking sector (left panel) and non-banking sector (right panel). Only for a high value of the capital requirement, say $\xi = 22\%$, banks need to initially lower their assets. To increase assets in the banking sector, banks must invest more. For capital accumulates slowly over time, part of the initial increase in the banking sector’s physical capital stock $k^h$ originates from the firm sector. Consequently, the outflow of capital from the firm sector is higher the higher the capital requirement. Once the new equilibrium has been reached, the firm sector capital stock returns back to its original level. Figure 9 depicts the effect on the total capital stock of the economy and output.

In the beginning of the transition, capital falls. This is due to the banking sector’s need for capital that drains the firm sector’s capital stock. Even though the capital stock falls initially, output immediately rises.

The reason for the initial increase in output (despite the lower initial capital stock) is the higher efficiency of the banking sector. Banks use not only more assets, but they also use them more efficiently. The reason for the increase in efficiency is a reduction in risk-taking (see right panel of figure 10). The government subsidy depends positively on the interaction between leverage and risk-taking. That is, the higher the leverage of banks the higher the incentive to take on risk. Since banks have to reduce their leverage to comply with the capital requirement, the payoff from the subsidy (given a level of risk-taking) is lower. As discussed in section 3, the choice of $\sigma^h$ implies a trade-off between the benefit from the subsidy and the loss in efficiency (reduction in the expected mean $Z^h$). The capital requirement lowers the benefit from the subsidy which entices banks to take on more efficient levels of risk.
Figure 9: Transition Dynamics

Table 9: Percentage Change in Comparison to Old Steady State

<table>
<thead>
<tr>
<th>% Change</th>
<th>Output</th>
<th>Cons.</th>
<th>Bank Output</th>
<th>Assets</th>
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<td>Std</td>
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<td>-1.5</td>
<td>-6</td>
<td>-6</td>
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<td>$\pi/k^h$</td>
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<td>Risk Taking</td>
<td>$Z^h$</td>
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<tr>
<td>Levels</td>
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<td>Std</td>
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<td>-10</td>
<td>-11</td>
<td>-5.5</td>
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<tr>
<td>Equity</td>
<td>$k^h$</td>
<td>Bank Debt</td>
<td>$r$</td>
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<tr>
<td>Levels</td>
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<td>-40</td>
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<tr>
<td>Std</td>
<td>-6</td>
<td>-6</td>
<td>-6</td>
<td>-3.7</td>
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An increase in overall output increases consumption (see the left panel of figure 10). The optimal capital requirement trades off the fall in utility due to the reduction in deposits against the rise in utility through a reduction in economic volatility and higher consumption levels.

How does the Economy behaves under the Optimal Capital Requirement

How does the economy under the optimal capital requirement of $\xi = 14\%$ compares to the economy under the current regime? Table 9 shows how (averaged over simulations and time paths) the economies differ from each other. The first row of each block in table 9 presents the percentage difference in average levels between the new and the old regime.
The second row presents the average difference in the standard deviations.

Households prefer the higher capital requirement regime over the current one because it leads to higher consumption (+0.1%) and lower volatility (−2.5%). To reach this new level of capital requirement, they accept a 2% reduction in the holding of bank debt. Banks profit also from an increase in the capital requirement. The fall in the funding costs, driven by a 40% reduction in the rate on bank debt, increases profits per unit of capital by 4%. Table 9 makes the case that higher capital requirements do not necessarily cause a fall in output, bank activity, and profits. The reason for this is the general equilibrium effect that changes the funding costs of assets.

The reduction in banks’ risk-taking by 6% means that risk-taking is decreased by 1 percentage point. This drives the fall in the standard deviation of output. It also increases the productivity of banks by 1.5%. Banks’ output increases because they are more productive and employ more capital in the production process. To comply with the higher requirement, banks need to increase equity by 30%.

Figure 11 depicts the simulation bounds of welfare as a function of the capital requirement. The blue dashed line denotes the 95% percentile, the red dashed line the 5%, and the black solid line depicts the mean.

Welfare Gain of Optimal Capital Requirement

In the spirit of Lucas (1987), I compute the welfare cost of current capital requirement
as the percentage change in consumption needed to make households indifferent between the current regime and the optimal regime in case of an immediate implementation. That is, I find the scalar $\lambda_0$ that keeps households indifferent between

$$E_0 \left( \sum_{t=0}^{\infty} M_{t+1} U((c_t, s_t) | \xi = 10.75\%) \right) = E_0 \left( \sum_{t=0}^{\infty} M_{t+1} U((\lambda_0 c_t, s_t) | \xi = 17\%) \right),$$

starting from their respective steady states (that is not taking into account the transition dynamics). The $\lambda_0$ that makes households indifferent between the two regimes is 0.999. In other words, households are indifferent between the old regime and the new regime, if they accept a permanent reduction of 0.1% (quarterly) in consumption. This is a small improvement in welfare and familiar in the literature (see Lucas (1987) and Van Den Heuvel (2008)).

**Optimal Capital Requirement without transition dynamics**

When transition dynamics are ignored the optimal capital requirement is $\xi = 17\%$, as shown in figure 12. When transition dynamics are included, deposits fall by more than what is necessary to reach the new equilibrium. The reason for that is that capital accumulates slowly over time. When transition dynamics are not included, households do not suffer the
initial reduction in deposits to reach the new steady state. This reduces the costs of an increase in the capital requirement, which results in a higher optimal level.

**Optimal Dynamic Policy**

The above section has shown that the transition path matters for the optimal capital requirement. A natural question is then how to design a dynamic policy which has the potential to mitigate the effects of a sudden change in the capital requirement. In particular, I design a policy that gradually increases the capital requirement, giving banks time to slowly increase their size. This avoids the large initial reduction in deposits to comply with a new requirement. I postulate the following dynamic policy:

\[ \xi_t = \xi_{\text{new policy}} + \left( \xi_{\text{old policy}} - \xi_{\text{new policy}} \right) \nu^{-t}, \]

where \( \nu \) is the speed at which the transition to the new regime occurs. The old policy starts at \( \xi = 10.75\% \) and then increases exponentially until it reaches the new level of capital requirement. Banks take into account the path of the transition in their optimal choices.

Figure 13 presents four welfare plots for a different value of the speed parameter \( \nu \), which governs how gradual banks need to comply with the new requirement. When the
policy becomes effective within less than three years, the optimal requirement continues to be 14%. The capital requirement can be even higher if policy makers can commit to a longer implementation phase. The top-left panel of figure 13 shows that it would be optimal to implement $\xi = 17\%$ over 24 years. If policy-makers set even 29 years of transition time, $\xi$ should be set to 19%.

Welfare is exponentially increasing in the phase-in time. Figure 14 presents four plots where welfare for different levels of $\xi$ is depicted as a function of the speed parameter $\nu$. For each level of capital requirement, welfare can be improved if its implemented gradually over time.

Non-Stochastic Steady State Optimal Capital Requirement

Non-stochastic steady state welfare depends on the capital requirement because $\xi$ determines how much deposits and consumption are produced. As discussed in section 3, the optimal amount of capital requirement in the steady state trades off the increase (decrease) in consumption against the decrease (increase) in bank debt holdings from higher (lower) capital requirement. Figure 15 depicts welfare as a function of $\xi$. The optimal capital requirement in the steady state is about $\xi = 4.4\%$ lower than 14% when transition dynamics and shocks
Figure 14: Welfare as a Function of the Transition’s Speed

\[
\xi = 0.12 \\
\xi = 0.15 \\
\xi = 0.18 \\
\xi = 0.23
\]

Figure 15: Steady State Welfare maximizing \( \xi = 4.4\% \)
are included. Since welfare overall changes only modestly with $\xi$, the reduction in volatility matters. This shows that it is important to take into account the effects of shocks as well as the effects from the transition when determining the optimal level of a capital requirement.

**When to Increase the Capital Requirement**

In this exercise, the transition to the new capital requirement is started during a recession (solid line in figure 16) or in a boom (dashed line in figure 16). A crisis is defined as a below average shock in both sectors over at least three quarters and a boom is defined as an above average shock in both sectors over at least three quarters. The model implies that welfare is maximized at a higher capital requirement when the policy is introduced during a recession than during a boom. Banks increase loan supply when funding costs decrease. Higher capital requirements trigger a larger fall in bank debt holdings and therefore a larger fall in the funding costs of banks. During a recession, banks should be enticed to lend more even though current profits are low. The way to convince banks to increase their assets despite low returns is by reducing the funding costs of banks through a higher level of the capital requirement.
7 Conclusion

This paper has developed a quantitative dynamic general equilibrium model to study the effects of capital requirements on the economy and to determine the optimal requirement. I quantify the model with data from banks’ regulatory filings and NIPA. The model is consistent with business cycle dynamics of US macroeconomic aggregates and the banking sector. I use the model to compute the optimal level of capital requirement. Taking into account the transition dynamics, the capital requirement should be increased to 14%. The model proposes a new trade-off from higher capital requirement: the reduction in bank debt holdings from a higher capital requirement leads to an increase in the demand of bank debt holding. To reduce this demand, the interest rate on bank liabilities adjusts downwards leading to overall lower funding costs of banks. This entices banks to increase their balance sheet, which leads to more output and consumption.
References


Cociuba, S. E., A. Ueberfeldt, and M. Shukayev (2013). Interest Rate Policy and Financial Regulation: How to Control Excessive Risk Taking?


A Equilibrium Conditions

This part shows the equilibrium conditions for the detrended model. The original variables, say $Y_t$, are decomposed into the part that is purely driven by the deterministic balanced growth path trend and into the part that is the stochastic variation around that trend. That is, one can write $Y_t = \bar{Y} X_t \exp(\hat{y}_t)$ where $\hat{y}_t$ is the log deviation from that trend. Eventually, I want to express the model in terms of the log deviation from its steady state. As a first step, I need to transform the original model by taking out the trend $X_t$. Also, I want to express the model in per capita terms by dividing all variables by $N_t$ total economy hours. Since labor is measured in efficiency units $N_k$ generally represent effective hours per worker of non-bankers and bankers respectively. The lower case $y$ denotes the trend stationary equivalent of $Y$ (except for $T$, taxes).

The household utility is

$$U(c_t, s_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + \theta \frac{s_t^{1-\eta}}{1-\eta}$$

and the resource constraint:

$$c_t + \Gamma k_t^f \left( 1 + \varphi_f \left( \frac{i_t^f}{k_t^f} - \delta^* \right)^2 \right) + \Gamma k_t^h \left( 1 + \varphi_h \left( \frac{i_t^h}{k_t^h} - \delta^* \right)^2 \right) = y_t^h + y_t^f + (1 - \delta) k_{t-1}^f + (1 - \delta) k_{t-1}^h - \frac{\kappa}{2} (d_t - \bar{d})^2$$

where $\delta^* = (\Gamma - 1 + \delta)$. The equilibrium conditions are

$$M_{t+1} = \beta^{-1} \left( \frac{1}{c_{t+1}} - \theta (s_{t+1})^{1-\eta} (c_{t+1})^{\eta-2} \right)$$

$$n_t = (d_t + p_t) \Theta_{t-1} + (1 + r_{t-1}) s_{t-1} + \ldots$$

$$w_t^f N_f + (r_t^f + 1 - \delta) k_{t-1}^f - T_t$$

$$n_t = c_t + \Gamma s_t + \left( 1 + \varphi_f \left( \frac{i_t^f}{k_t^f} - \delta \right)^2 \right) \Gamma k_t^f + p_t \Theta_t$$

$$i_t^f = k_t^f - (1 - \delta) k_{t-1}^f$$

$$E_t (M_{t+1}) (1 + r_t) = \left( 1 - \frac{\theta}{\Gamma} s_t^{-\eta} c_t^{\eta-1} \left( \frac{1}{c_t} - \theta (s_t)^{-\eta} c_t^{-2} \right) \right)$$

$$1 + \varphi_f \left( \frac{i_t^f}{k_t^f} - \delta^* \right)^2 + 2 \varphi_f \left( \frac{i_t^f}{k_{t-1}^f} - \delta^* \right) (1 - \delta) \frac{k_{t-1}^f}{\Gamma k_t^f} = E_t \left[ M_{t+1} \left( r_{t+1}^f + \left( 1 + 2 \varphi_f \left( \frac{i_{t+1}^f}{k_t^f} - \delta^* \right) \right) (1 - \delta) \right) \right]$$

$$1 = E_t \left[ M_{t+1} \frac{(d_{t+1} + p_{t+1})}{p_t} \right]$$
\begin{align*}
w_t^f &= (1 - \alpha) Z^f \left( k_{t-1}^f \right)^{\alpha} (N^f)^{\alpha} \\
r_t^f &= \alpha Z_t^f \left( k_{t-1}^f \right)^{\alpha-1} (N^f)^{1-\alpha} \\
y_t^f &= Z_t^f \left( k_{t-1}^f \right)^{\alpha} (N^f)^{1-\alpha} \\
y_t^r &= Z_t^r \left( k_{t-1}^r \right)^{\alpha} \\
\log Z_t^f &= \rho^f \log Z_{t-1}^f + \sigma^f e_t^f \\
\log Z_t^h &= \rho^h \log Z_{t-1}^h + \left( \phi_1 - \phi_2 \sigma_{t-1}^h \right) \sigma_{t-1}^h + \sigma_{t-1}^h e_t^h \\
\pi_t &= y_t^h - \delta k_{t-1}^h + r_{t-1}^B b_{t-1} - r_{t-1} s_{t-1} \\
r_t^h &= \frac{y_t^h}{k_{t-1}^h} \\
d_t &= \tilde{e}_t - \frac{\kappa}{2} \left( d_t - \bar{d} \right)^2 + TR \left( k_{t-1}^h, \frac{e_{t-1} + \pi_t}{k_{t-1}^h}, \sigma_t^h \right) \\
&- \varphi_h \left( \frac{i_t^h}{k_t^h} - \delta \right)^2 \Gamma k_t^h - \Gamma e_t \\
\tilde{e}_t &= \pi_t + e_{t-1} \\
e_t &= k_t^h + b_t - s_t \\
e_t &= \xi k_t^h \\
A_t &= s_t + e_t \\
TR \left( k_{t-1}^h, \frac{e_{t-1} + \pi_t}{k_{t-1}^h}, \sigma_t^h \right) &= \omega_3 k_{t-1}^h \exp \left( -\omega_1 \left( \frac{e_{t-1} + \pi_t}{k_{t-1}^h} \right) + \omega_2 \sigma_t^h \right) \\
TR \left( k_{t-1}^h, \frac{e_{t-1} + \pi_t}{k_{t-1}^h}, \sigma_t^h \right) + \left( 1 + i_{t-1}^b \right) b_{t-1} &= \Gamma b_t + T_t \\
1 &= \Lambda_t \left( 1 + \kappa \left( d_t - \bar{d} \right) \right) \\
w_t^h &= (1 - v) Z_t^h \left( k_{t-1}^h \right)^{\nu} \\
i_t^h &= \Gamma k_t^h - (1 - \delta) k_{t-1}^h \\
0 &= E_t \left[ M_{t+1} \Lambda_{t+1} \left( r_t^B - r_t \right) \right] \\
\mu_t &= \Lambda_t - E_t \left[ M_{t+1} \Lambda_{t+1} \left( 1 + r_t \right) \left( 1 - \omega_1 TR \left( 1, \frac{\tilde{e}_t}{k_t^h}, \sigma_{t+1}^h \right) \right) \right]
\end{align*}
\[ \mu_t \xi = -\Lambda_t \left( \varphi_h \left( \frac{i_t^h}{k_t^h} - \delta^* \right)^2 + 2\varphi_h \left( \frac{i_t^h}{k_t^h} - \delta^* \right) (1 - \delta) \frac{k_t^h-1}{\Gamma k_t^h} \right) \ldots \\
+ E_t \left\{ M_{t+1} \Lambda_{t+1} \left( \left( \frac{y_{t+1}^h}{k_t^h} - r_t - \delta \right) \left( 1 - \omega_1 TR \left( 1, \frac{e_{t+1}}{k_t^h}, \sigma_{t+1}^h \right) \right) \right) \\
+ 2\varphi_h \left( \frac{i_{t+1}^h}{k_{t+1}^h} - \delta^* \right) (1 - \delta) \\
+ TR \left( 1, \frac{e_{t+1}}{k_t^h}, \sigma_{t+1}^h \right) \left( 1 + \omega_1 \frac{e_{t+1}}{k_t^h} \right) \right\} \right\} \\
= \frac{\left( \xi 1/M + (1 - \xi) (1 + r) - (1 + \nu y^h - \delta) )}{(1 + \omega_1 \left( \frac{y^h}{k^h} (1 - \nu) \right))} = TR/k^h \\
0 = E_t \left\{ M_{t+1} \Lambda_{t+1} \left( \frac{y_{t+1}^h}{k_t^h} (\phi_1 - 2\phi_2 \sigma_{t+1}^h + e_{t+1}^h) \right) \\
\times \left( 1 - \omega_1 TR \left( 1, \frac{e_{t+1}}{k_t^h}, \sigma_{t+1}^h \right) \right) \right\} + \frac{1}{\Gamma k_t^h} \Lambda_t \Lambda_t \left( 1, \frac{\tilde{e}_t}{k_{t+1}^h}, \sigma_t^h \right) \omega_2 \right\} \\
\]
### Business Cycle Correlations (D=data, M=model)

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<th>1984q1 - 2011q4</th>
<th>GDP</th>
<th>Yₜ</th>
<th>Investment</th>
<th>Assets</th>
<th>Deposits</th>
<th>kₜ</th>
<th>Equity</th>
<th>Dividend</th>
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<th>c</th>
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<td>0.183</td>
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</table>

Variables: HP-Cycle component of logged variable / GDP trend

†: p-value $\geq .10$; †† Variables: HP-Cycle component variable / GDP trend
C Regulatory Environment

The banking system is regulated by the following entities: Office of the Comptroller of the Currency (OCC), the Board of Governors of the Federal Reserve System (FRB), the Federal Deposit Insurance Corporation (FDIC), the National Credit Union Administration (NCUA) and the Consumer Financial Protection Board (CFPB). Banks can select their regulator agency by deciding which charter to obtain: national bank, state bank, credit union, etc.

The OCC is an independent bureau within the Treasury Department. It regulates and supervises national banks and federal savings associations and federal branches of foreign banks. The FDIC is a government corporation that provides deposit insurance and supervises and regulates banks as well. They are responsible for overseeing insured financial institution adherence to FFIEC (federal financial institutions examinations council) reporting requirements. These call reports are required by all insured national and state non-member\(^{21}\) commercial banks as well as state-chartered savings banks on a quarterly basis. It is their data that is used in this paper. The FRB regulates bank and financial holding companies. It also regulates state chartered banks that are members of the FRB. The NCUA supervises credit unions and the CFPB supervises many different financial institution with the focus on consumer financial protection.

D Computation

The model is solved with a higher order perturbation method using Dynare. See the manual and Dynare guides. The following exposition closely follows Tommaso Mancini Griffoli Dynare user guide.

Generally, a dynamic stochastic model is a collection of first order and equilibrium conditions that take the general form:

\[
E_t \{ f (y_{t+1}, y_t, y_{t-1}, u_t) \} = 0
\]

\[
E (u_t) = 0
\]

\[
E (u_t u_t') = \Sigma_u
\]

\(^{21}\) Banks that are not members of the FRB can only be state chartered because all nationally chartered banks must be member of the FRB. They are also subject to reserve requirements but are generally less regulated than member banks. Examples of a FRB non-member banks are GE Capital Bank and the Bank of the West.
where \( y \) is a vector of endogenous variables of any dimension, \( u \) is a vector of exogenous stochastic shocks of any dimension. The solution to this system is a set of equations relating variables in the current period to the past state of the system and current shocks, that satisfy the original system. This is what we call the policy function. Sticking to the above notation, we can write this function as:

\[
y_t = g(y_{t-1}, u_t).
\]

It is then straightforward to write \( y_{t+1} \) as a function of \( y_t \) and \( u_{t+1} \). We exploit here the recursive nature of the solution. Proceeding in this way, the above system can be rewritten with the help of a new function \( F \), such that

\[
F(y_{t-1}, u_t, u_{t+1}) = f(g(g(y_{t-1}, u_{t}), u_{t+1}), g(y_{t-1}, u_t), y_{t-1}, u_t).
\]

\( F \) helps then to rewrite \( E_t \{ f(y_{t+1}, y, y_{t-1}, u_t) \} \) in the following way:

\[
E_t \{ F(y_{t-1}, u_t, u_{t+1}) \} = 0.
\]

We can then hope to find the system’s solution as the solution to the first order approximation around a steady state which is defined as

\[
f(\bar{y}, \bar{y}, \bar{y}, 0) = 0
\]

with the property \( \bar{y} = g(\bar{y}, 0) \). The first order Taylor expansion around \( \bar{y} \) yields:

\[
E_t \{ F^{(1)}(y_{t-1}, u_t, u_{t+1}) \} = E_t \left\{ f(\bar{y}, \bar{y}, \bar{y}, 0) + f_{y_{t+1}}(g_{y} (g_{y} \hat{y} + g_{u} u) + g_{u} u') + f_{y_{t}}(g_{y} \hat{y} + g_{u} u) + f_{y_{t-1}}(g_{y} \hat{y}) + f_{u} u \right\} = 0
\]

where \( \hat{y} = y_{t-1} - \bar{y} \). Taking expectations

\[
E_t \{ F^{(1)}(y_{t-1}, u_t, u_{t+1}) \} = (f_{y_{t+1}} g_{y} g_{y} + f_{y_{t}} g_{y} + f_{y_{t-1}} g_{y}) \hat{y} + (f_{y_{t+1}} g_{y} g_{u} + f_{y_{t}} g_{u} + f_{u}) u = 0
\]

One can see that future shocks only enter with their first moments (which are zero in expectations). That is why they drop out when taking expectations of the linearized equations. This is technically why certainty equivalence holds in a system linearized to its first order. The second thing to note is that we have two unknown variables in the above
equation: $g_y$ and $g_u$ each of which will help us recover the policy function $g$.

Since the above equation holds for any $\dot{y}$ and any $u$, each parenthesis must be null and we can solve each at a time. The first, yields a quadratic equation in $g_y$, which we can solve with a series of algebraic tricks that are not all immediately apparent (but detailed in Michel Juillard’s presentation). Incidentally, one of the conditions that comes out of the solution of this equation is the Blanchard-Kahn condition: there must be as many roots larger than one in modulus as there are forward-looking variables in the model. Having recovered $g_y$, recovering $g_u$ is then straightforward from the second parenthesis. Finally, notice that a first order linearization of the function $g$ yields:

$$y_t = \bar{y} + g_y \dot{y} + g_u u.$$

Any higher order solution uses the same “perturbation methods” as above (the notion of starting from a function you can solve - like a steady state - and iterating forward), yet applies more complex algebraic techniques to recover the various partial derivatives of the policy function. But the general approach is perfectly isomorphic. Note that in the case of a second order approximation of a model, the variance of future shocks remains after taking expectations of the linearized equations and therefore affects the level of the resulting policy function.

**E Motivation for Transfer Function**

I motivate the transfer function through a two period model with bailout and limited liability by banks. Banks choose first risk $\sigma^h$ given a starting value of equity $e$ and assets $k^h$. From the balance sheet identity $e + s = k^h$, profits $\Pi(\sigma^h, \epsilon) = Z^h(k^h)\nu - \delta k^h - rs$ are only a function of the shocks and the risk choice through $Z^h$. The technology variable $Z^h$ depends on the risk choice variable $\sigma^h$ and the shock in the following way: $\log Z^h(\sigma^h) = (\phi_1 - \phi_2 \sigma^h) \sigma^h + \sigma^h \epsilon$ where $\epsilon \sim N(0, 1)$. There exists a government that pursues the following bailout policy:

$$\text{Bailout} = \max \{0, -\{e + \Pi\}\}.$$

Banks have the following objective:

$$\max_{\sigma^h} E \left[ (e + \Pi(\sigma^h, \epsilon) + \text{Bailout}) \right].$$
This objective is thus the expectation of the sum of the bank business part $e + \Pi (\sigma^h, e)$ and the \emph{Bailout} part. The value of the expected \emph{Bailout} depends on $\sigma^h, k^h$, and $e$.

The following plot shows the value of the three-dimensional expected bailout function over different values of leverage $k^h/e$ and $\sigma^h$, keeping fix the scale of the bank $k^h$. The value of the expected bailout increases in leverage and $\sigma^h$. Given the size of the bank and when leverage is given by the regulator through a binding capital constraint, the expected bailout depends only on risk-taking. When banks could choose leverage freely, they could maximize the value from the expected bailout by being highly leveraged and taking on a lot of risk at the same time.

![Graph showing the value of the expected bailout function.](image)

This graph suggests that the bailout is an increasing function of leverage, risk-taking, and assets with complementarities in leverage and risk-taking. In a model with limited liability of banks and government bailout, higher levels of leverage increase the probability of a default in which the government would need to step in and bailout the bank. Likewise, higher risk-taking by banks increases the probability of a default. And finally, the absolute amounts of transfers are greater for larger banks as measured by assets. The payments by the government occurring in the future can be also expressed as a constant stream of subsidies that depends on equity after the realization of profits, risky assets, and risk-taking by banks. These variables would otherwise affect the probability of a default. The subsidy function thus becomes:

\[
TR \left( k^h, \frac{\bar{e}}{k^h}, \sigma^h \right) = \omega_3 k^h \exp \left( -\omega_1 \left( \frac{\bar{e}}{k^h} \right) + \omega_2 \sigma^h \right).
\]

The $\omega$ parameters are found by minimizing the difference between the expected subsidy function and the expected bailout. With the parameters values of $\omega_1 = 4$, $\omega_2 = 2$, and $\omega_3 = .001$ the transfer function takes on the following shape.
That is, the reduced form subsidy function captures the shape of the bailout function that was derived from first principles.