Incentivizing Impact Investing

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Abstract

We consider a project which produces both a public social good and a private good — an impact investment. When the project is financed with external capital, the owner may have an incentive to under or over invest in social good. Under investment arises when the owner does not fully internalize the social value of the public good. Over investment arises because repayment uses up only the private good, making the social good relatively more attractive. The model provides a theoretical foundation for funding impact investments through Social Impact Bonds — to discourage over investment — or Social Impact Guarantees — to discourage under investment. When social investors have sufficient capital, socially responsible investment strategies such as equity investments in socially responsible firms are also optimal.

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1 Introduction

Investors increasingly rely on environmental, societal, and governance factors when evaluating investment opportunities and when choosing assets for their investment portfolios. Over the past two decades, the universe of U.S. sustainable, responsible, and impact (SRI) investments has increased tenfold from $639 billion in 1995 to $6.57 trillion in 2014.\(^1\)\(^2\) These investment strategies, which seek to achieve long-term financial returns along with positive societal impact, now account for more than 15% of total assets under management for U.S. managers.\(^3\) Despite the growing importance of SRI investments, it is not well understood how socially conscious investors influence the level of social good nor how firm managers are incentivized to balance financial and social returns.

This paper derives optimal contracts for impact investing when (i) social capital is limited in comparison to funds available for commercial investments\(^4\) and (ii) project owners/managers allocate scarce resources between the competing goals of profit maximization and improved social outcomes. In doing so, we relate impact investing to a multitask principal-agent problem (Holmstrom and Milgrom, 1991) in which some tasks provide greater financial return while others provide greater social benefit. We show that the optimal security design critically depends (i) on the preferences for financial versus social returns for the firm’s various stakeholders (owner/manager and external investors) and (ii) on available and contractible metrics of cash flow and social output on which security repayment can be conditioned. In these contexts, optimal securities serve the dual role of leveraging limited social capital in pursuit of social impact and rewarding project owners for effectively balancing financial versus social output.

Our approach provides three important insights into the design of SRI investments. First, it rationalizes the use of Social Impact Bonds (SIB), or pay-[it-back]-for-success bonds, when funding impact investments in the public sector. While SIBs have been hailed in the popular press as a financial engineering triumph, our analysis, to the best of our knowledge, provides the first theory of the SIB design. Second, when impact investments are in the private sector, the model prescribes new securities, Social Impact Guarantees (SIG), which compensate SIG investors with a greater financial return when desired social goals are not attained. Finally, in the absence of first-best SIG securities, the model provides theoretical evidence that standard equity investments in sustainable

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\(^2\)Throughout this manuscript we use the terms “SRI investment,” “social investment,” and “impact investment” interchangeably.


\(^4\)While estimates of the total size of social investor capital exceed $6.5 trillion (US SIF, 2014), the pool of dollars directed towards social projects is still limited in comparison to the amount of funds commercially available for investments that generate market returns.
and responsible firms are often sufficient SRI strategies. Furthermore, equity contracts may be optimal when social output is difficult to measure and/or when socially conscious investors have sufficient investment capital.

On January 29, 2014, the Commonwealth of Massachusetts announced the largest Social Impact Bond in history. The contract raised $12 million from private investors to provide life skills and employment training to young, at-risk men in the Boston area with the objectives of reducing recidivism and generating taxpayer savings. If the project is successful in reducing re-conviction rates, the contract requires up to $16 million in success payments to investors. Social Impact Bonds are an increasingly common tool for funding impact investments in the public sector. In these contracts, private investors provide upfront investment for social programs that may also generate cost savings. In time, if the project succeeds at generating the desired outcomes, an outcome payer (e.g., the Commonwealth of Massachusetts) returns capital to private investors. The more successful the project along social dimensions, the higher the investment return. In our multitask setting with limited social capital, we show that the optimal security for funding public works projects exhibits a pay-[it-back]-for-success feature, which is the hallmark of the SIB design. By penalizing public officials — who in equilibrium care relatively more about their projects’ social benefit than financial return — with a large security repayment when social output is high, the optimal security limits the tendencies of public officials to overemphasize social goals.

While SIBs have been used extensively to fund SRI investments in the public sector, securities that contract directly on social output are less common in private sector investments. As discussed by Besley and Ghatak (2007), the private sector is often the more natural provider of public goods when social output is inherently bundled with the production of a private good — as is the case in impact investments. When an impact investment is in the private sector, the first-best contract encourages pro-social investment within for-profit firms by rewarding the firm owner following strong social performance and penalizing him otherwise. That is, the security promises a smaller repayment from the firm owner to social investors when social output metrics are high and a greater repayment from the firm owner to social investors when social output metrics are low. We coin this new security a Social Impact Guarantee, since it compensates SIG investors — who value social outcomes — with a greater financial return when their desired social goals are not attained. As such, our analysis recommends the adoption of new financial securities that contract directly on observed social output (when available) for funding impact investments in the private sector.

In the absence of first-best SIG securities, we show that standard equity contracts increase social output in socially responsible firms. When a firm owner sells a larger equity stake to outside investors, he retains a smaller claim on the firm’s financial profit. As a result, the firm owner’s
incentive to balance for-profit and social output is distorted, leading to greater investment in the social technology. When social investors can afford to purchase a sufficiently large equity stake in socially responsible firms, equity investments can be just as effective as contracting explicitly on social output. The analysis therefore provides theoretical support for the most common SRI investment strategy — targeted equity investment in socially responsible firms.

Turning now to the details of the model, we consider a project that produces both cash profit (equivalently cost savings) and social good, and we assume that the economy is populated with two investors: a social investor and a for-profit investor. While both investors value the consumption of the cash output equally, only the social investor places value on the social good. The project is endowed to one of the investors.\(^5\) We describe an impact investment as a public works opportunity if it is endowed to the social investor (i.e., a public works project managed by a service provider), and as a private sector opportunity if it is endowed to the for-profit investor (i.e., a firm managed by a firm owner). A distinguishing feature of these projects is that the balance between generating social good and maximizing profit is chosen by the manager. Namely, the project owner allocates a scarce resource — effort, attention, or investment — over the cash production and the social good technologies separately.\(^6\) While the owner’s choices are unobservable, outputs are observable and contractible.

In the case of the Massachusetts pay-for-success initiative, the public works project to reduce recidivism generates two outputs: (i) the social good associated with reformed citizens and (ii) cost savings to taxpayers. While a reduction in re-conviction rates may generate both the desired social good and taxpayer savings through a decrease in the number of resources expended in the criminal justice system, these cost savings may not be genuine if individuals kept out of prison end up instead in the welfare system. Thus, it is not obvious that a reduction in recidivism leads to an overall reduction in government spending, despite the social benefits. In administering the project, there is therefore a balance between focusing on morality training which has been shown to be effective at reducing recidivism versus employment training which may lead to greater long-term cost savings to taxpayers.

In the private sector, consider an impact investment to improve employee health through more expansive health insurance and greater emphasis on preventive care and healthy living. The two outputs from the project are (i) cost savings to the firm via lower turnover and a reduction in sick

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\(^5\) In Besley and Ghatak (2001), under investment in a public good is mitigated by granting a project’s control rights to the agent with the greatest valuation for the project’s output. Conversely, we consider a setting in which control rights are a project characteristic and cannot be transferred.

\(^6\) See Baron (2001), Bagnoli and Watts (2003), Kotchen (2006), and Besley and Ghatak (2007) for settings in which profitability and social good production are complements.
leave and (ii) the social good which stems from improvements in long-term health outcomes. A firm may stress greater sick patient care in order to reduce employee leave, but do little to affect overall health outcomes or life expectancy. Conversely, the firm could provide nutrition counseling and subsidized gym access to reduce the likelihood of diabetes and heart disease but do little to reduce employee sick days due to common illnesses. Finally, consider an impact investment to provide grocery services in the inner city. While the storefront produces revenue from the sale of food, it may also provide urban families with increased access to fresh produce, promoting healthy diets. Again, there is a balance between the provision of high margin foods such as cereals and canned goods versus nutrient rich fruits and vegetables which provide the greater nutritional value.

We first consider the optimal contract when the project is a public works opportunity. If the impact investment can be self-financed, the service provider efficiently allocates investment between the for-profit and social outputs. In reality, social capital is relatively scarce and the ability of governments to impose taxes on citizens is limited and inefficient. In these cases, the service provider must raise additional capital from the commercial investor in order to help fund the impact investment. However, doing so distorts the service provider’s incentive to effectively balance cost savings and social goals. In particular, the service provider over emphasizes the social output because he is forced to split the project’s cash output with an external investor. With this tension in mind, we derive the optimal security that can be sold to the for-profit investor to fund an impact investment when social output metrics are available and contractible. The security assigns a greater repayment to the investor when social output metrics are high (a pay-for-success contract). When the constrained social investor operates the project, the optimal contract tapers the service provider’s incentive to overweight social value by forcing a greater security repayment when social output is high.\footnote{Our design shows that, in a principal-agent setting, social output and costly effort are related. Under the optimal contract, the social owner surrenders greater profits when outputs are indicative of high social investment and keeps greater profits when outputs are indicative of low social investment. This result is, most notably, related to Innes (1990), which shows that the optimal contract between manager and investor requires the manager to surrender greater profits when output is a signal of low effort and keep greater profits when output is a signal of high effort.}

Thus, a pay-for-success contract (or SIB) is optimal whenever a public works opportunity relies on external financing.

Next, we consider an impact investment in the private sector — the project is administered by the for-profit firm owner and partially funded by the social investor. Because the firm owner places no value on social good, all else equal, he under emphasizes social output in favor of greater profit opportunities. In this case, the security features a greater repayment when social output metrics are low. By punishing the firm owner with a high repayment when social output is low and rewarding him with a low repayment when social output is high, the security encourages pro-social
behavior by the for-profit firm owner.

We then extend our analysis to settings in which the commercial investor is not solely profit motivated and is willing to pay for his expected social value.\(^8\) In these cases, both the commercial investor and social investor initially under invest in the social technology because neither fully internalizes the aggregate value of the social good. However, funding the impact investment with external capital pushes the project owner towards greater social investment, since repaying financiers uses up only the private output. When the firm is owned by the unconstrained commercial investor, the firm owner under invests in the social technology, and the optimal security, as in the base model, encourages greater social investment by punishing the firm owner with a high repayment when social output metrics are low (a SIG). When the impact investment is instead owned by the social investor, the service provider tends to over invest in the social technology, and the optimal security is a SIB or pay-for-success security.\(^9\)

Finally, for many impact investments, social output metrics may be unobservable or difficult to measure, implying that SIBs and SIGs are not possible. In these cases, the commercial investor and social investor are restricted to simple revenue sharing contracts such as standard equity claims. When the project owner maintains only a small fraction of the project’s future revenue, his incentive to balance for-profit and social output is distorted, leading to greater investment in the social technology. That is, if the project owner places some positive value on social output, an equity investment leads him to favor greater social investment. Our analysis prescribes an optimal equity contract that, similar to first-best contracts, serves a dual role of leveraging limited social capital and effectively balancing the project owner’s financial and social pursuits. When the social investor’s capital budget is sufficiently large, equity investment can be just as effective in this dual role as contracting explicitly on social output. The analysis thus supports the common strategy of using environmental and societal factors in social investors’ portfolio selection problem.

This paper contributes to an emerging literature which studies the effects of SRI investment on firm outcomes. Heinkel, Kraus, and Zechner (2001) argues that, by lowering the cost of capital for socially responsible firms, SRI investment strategies lead to an increase in the number of impact investments that may be undertaken. In our model, this effect is captured by the fact that the social investor overpays for his claims to the firm’s cash output, since he pays for his consumption

\(^8\)Besley and Ghatak (2005) considers a setting in which principals and agents esteem the public good similarly. Their analysis shows that greater productivity is achieved via selection (matching principals to “motivated” agents with similar “missions”) rather than providing explicit incentives.

\(^9\)The result that the service provider over invests in the social output contrasts standard results concerning the under provision of public goods. While the service provider under provides the social good if the impact investment is self-financed, he (weakly) over emphasizes social output whenever the project is partially funded by the commercial investor.
of social good. Consistent with this result, evidence suggests that social investors pay a premium for SRI investment. Hong and Kacperczyk (2009) examines the financial cost in avoiding stocks that are not SRI acceptable, i.e., “sin stocks.” Sin stocks have lower price-to-book ratios and higher risk-adjusted returns than comparable stocks. Geczy, Stambaugh, and Levin (2005) provides similar evidence; investors that adhere to socially responsible investments add an additional constraint to their portfolio selection problem and under-perform relative to an unconstrained portfolio by as much as 30 basis points per month. Moreover, we show that SRI also increases social investment by making social output relatively more attractive for the firm owner. The simplicity of the mechanism is its strength: we do not require socially responsible investors to induce pro-social preferences through corporate governance as in Baron (2008). Instead, social investors may achieve greater social impact by passively investing in socially responsible firms. Our result is also agnostic regarding the organizational design of the firm. Besley and Ghatak (2015) consider a tradeoff between profitability and social good and focus on the optimal organizational structure (e.g., a for-profit or a non-profit) as a means to balance this tension. The authors show that a hybrid structure, i.e., an organization in which the manager is granted discretion over which output to emphasize, is optimal when the manager’s incentives are naturally aligned with the founder’s. Our analysis shows that the explicit tradeoff between profitability and social good makes equity investment a powerful tool for balancing cash and social outputs within many existing for-profit firms.

2 Model

Consider a project which may produce both cash profit (cost savings) and social good — an “impact investment.” The project requires an upfront investment equal to $I$. The project manager then allocates an unobservable scarce resource between two production technologies, one which aids in the production of social good and another that aids in the production of cash output. The unobservable scarce resource could be unobservable investment, effort, or attention. Without loss of generality, we refer to the unobservable resource as investment hereafter. Denote by $i_s$ the level of investment in the social technology and by $i_x$ the level of investment in the traditional for-profit technology, where $I = i_s + i_x$.\(^{10}\)

The cash profit generated by the project is a random variable $x \in \{0, 1\}$. When $x = 1$, the project succeeds in producing cash output, and when $x = 0$ the project fails to generate cash output. The probability of success is given by $Pr(x = 1 | i_x, i_s) = f(i_x)$, which is an increasing, concave,

\(^{10}\)While the upfront investment $I$ is necessary for our model, the bifurcation of $I$ across the two technologies is not. The qualitative implications would be the same if the project required an upfront investment $I$, and the manager allocated some attention or effort budget $B$ across the two technologies.
twice continuously differentiable function. The conditions on \( f(\cdot) \) are natural: greater investment in the for-profit technology increases the probability of successfully producing cash output, but at a diminishing rate. In addition, the project may either succeed \((s = 1)\) or fail \((s = 0)\) in its production of social output \(s\). The probability of success is given by \( Pr(s = 1 \mid i_x, i_s) = g(i_s) \), which is also an increasing, concave, twice continuously differentiable function. For simplicity we assume that, conditional on the chosen levels of investment, the probabilities of successfully producing cash output and social output are independent. The distinguishing feature between \(x\) and \(s\) is that \(s\) is non-excludable. Consequently, \(s\) cannot be sold after it is produced.

In the Massachusetts example, the public works project produces cost savings to taxpayers only if lower incarceration rates lead to a reduction in total government expenditures. The present value of these savings is the cash component \(x\) in the model. The impact investment also educates an at-risk group of individuals in order to reduce criminal activity. This represents the social good component \(s\) in the model. While undertaking the impact investment positively affects both taxpayer savings and social good, there is tradeoff between the two objectives. The service provider must choose how much to invest in each output; some interventions may lead to greater taxpayer savings (i.e., employment training) while other interventions may provide a greater reduction in criminal behavior and re-conviction rates (i.e., morality training).\(^{11}\) In our private sector example, providing greater health care benefits promotes cost savings due to lower turnover and a reduction in sick leave \(x\) as well as improved long-term health outcomes \(s\). Generous sick patient coverage may help employees return to work quickly, but do nothing to promote healthy living or to increase life expectancy. Conversely, efforts to increase preventive care and healthy diets may provide greater improvement in overall well-being and long-term health, but be less effective at reducing sick days.

The economy is populated with two risk-neutral investors: a commercial investor and a social investor. While both investors place equal value on cash profit, the social investor places greater value on the social output relative to the commercial investor. We use the superscript \(\pi\) to denote the regular commercial investor and the superscript \(\psi\) to denote the social investor. The utility function for the commercial investor and social investor, respectively, are

\[
\begin{align*}
    u^\pi(x^\pi, s) &= x^\pi + \alpha s, \quad (1) \\
    u^\psi(x^\psi, s) &= x^\psi + s, \quad (2)
\end{align*}
\]

where \(x^\pi\) denotes the cash profit consumed by the commercial investor, \(x^\psi\) denotes the cash profit consumed by the social investor, and \(\alpha \in [0, 1]\) captures the relative value that the commercial

\(^{11}\)While we focus on the example of recidivism, there are numerous pay-for-success contracts globally that fund projects to increase childhood literacy, reduce homelessness, and provide services to the mentally ill.
investor places on the social output. In the main analysis we set \( \alpha = 0 \) to stress the distinction between for-profit investors and socially minded investors.\(^{12}\) In an extension, we consider non-zero values of \( \alpha \). Investment occurs in period 1 and the payoffs occur in period 2. The gross market rate of return on alternative investments is \( \rho \geq 1 \). The for-profit investor is endowed with initial wealth \( \beta^\pi > I \) and is therefore able to fully fund the impact investment. The social investor, however, is financially constrained and has a maximum capital budget \( \beta^\psi < I \). Thus, the social investor must raise additional funds from the for-profit investor in order to undertake the impact investment. Furthermore, we assume that the for-profit investor does not find the impact investment to be profitable for individual investment,

\[
f(I - i_s) \leq \rho I \text{ for all } i_s,
\]

where we have made the substitution \( i_x = I - i_s \). These assumptions are made to capture an important feature of impact investing — the funds available from socially-minded investors are limited so that many social projects are forgone because they cannot be solely funded by investors who are willing to accept a smaller rate of cash return on their invested capital. In addition, we assume \( g(0) > \rho \beta^\psi \) and \( f(0) > \rho (I - \beta^\psi) \). The first assumption guarantees that expected social output covers the social investor’s maximum contribution to the upfront investment cost \( I \), while the latter assumption guarantees that expected cash output covers the regular investor’s minimum contribution.\(^{13}\) Furthermore, these assumptions highlight the distinguishing feature of impact investments as projects capable of producing both social value and cash output. However, within this context, there is a tradeoff regarding which output to emphasize.

We begin by considering the levels of investment which maximize the joint surplus across investors in a frictionless economy, that is an economy without agency conflicts or financial constraints. Assuming \( \alpha = 0 \), the surplus maximizing share of investment directed towards the social technology solves,

\[
\max_{i_s \in [0,I]} f(I - i_s) + g(i_s), \tag{4}
\]

where we have again made the substitution \( i_x = I - i_s \). We make the following assumptions to ensure that the solution to the first-best choice problem involves investment in both the for-profit

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\(^{12}\)Although we focus on the case in which \( \alpha = 0 \) in the base model, the analytic proofs in the Appendix, whenever possible, consider the more general setting in which \( \alpha \in [0,1] \).

\(^{13}\)These assumptions provide sufficient conditions for the existence of an optimal security in sections 2.1 and 2.2.
and social components,

\[
\lim_{i_s \to 0} \frac{g'(i_s)}{f'(I - i_s)} = \infty, \quad (5)
\]

\[
\lim_{i_s \to I} \frac{f'(I - i_s)}{g'(i_s)} = \infty. \quad (6)
\]

The solution \(i^{FB}_s\) which maximizes joint surplus is therefore characterized by the first-order condition,

\[
f'(I - i^{FB}_s) = g'(i^{FB}_s). \quad (7)
\]

The expression in (7) implies that surplus is maximized when the marginal benefits of cost savings and social good are equated. What we have in mind is that there is a wide (and in the model continuous) menu of choices a manager could make; while each choice generates both profit and social good with positive probability, some tilt more heavily towards cash production and others more heavily towards social good production.

2.1 Social Impact Bond

Consider first the setting in which the social investor owns and operates the project. We term the project a public works opportunity and the social investor a service provider. The service provider’s wealth is limited and additional funds must be raised from the for-profit investor in order to undertake the impact investment. Again, the term for-profit investor is used to characterize a commercial investor who does not value social output, i.e., \(\alpha = 0\).\(^{14}\) The service provider’s investment choice is not observable, however, outputs \(x\) and \(s\) are observable and contractible.\(^{15,16}\) In the next section, we consider a setting in which social output is non-contractible.

Consider a traded security offered by the service provider to the for-profit investor. The security has price \(p^\pi\) and pays \(y^\pi(x, s)\) in period 2. The superscript \(\pi\) is used to denote a security which

\(^{14}\)While our model focuses on the bilateral contract between two agents, it is equivalent to a setting in which the service provider contracts with many investors who do not value social output. In these cases, the service provider’s value of the social output represents the aggregate value to society. For example, the Bill and Melinda Gates Foundation makes contributions to AIDS research that are well beyond Mr. and Mrs. Gates’ private values of the research’s social output.

\(^{15}\)While social output is valued by at least one agent in our model, such a condition is unnecessary for the optimal security to contract on social output. As noted in Holmstrom (1979), any signal that reveals information about the project manager’s choices should be included in the contract.

\(^{16}\)The feasibility of contracting on this set of impact investments relies crucially on well-defined measurable metrics of social value. This concern has been somewhat addressed for public works opportunities funded by SIBs through the use of third-party verification and often times randomized experiments to measure program outcomes. While additional steps are necessary to penetrate private sector opportunities, several recent developments make social-output-contingent contracts possible in the private sector. For example, Impact Reporting & Investment Standards (IRIS: http://iris.thegiin.org/) and GIIRS Ratings and Analytics for Impact Investing (http://giirs.org/) provide independent evaluations of social value similar to the role credit rating agencies play in providing default information on corporate bonds.
is purchased by the for-profit investor. When considering the design of the security, we assume standard limited liability constraints; (i) \( y^s(x, s) \leq x \): the service provider cannot be required to repay more than the cash profit produced by the project,\(^{17}\) and (ii) \( y^s(x, s) \geq 0 \): the investor’s liability is limited to the initial level of investment \( p^s \).\(^{18}\) Since no repayment is possible when the project fails to produce cash output, limited liability requires \( y^s(0, s) = 0 \). For ease of notation, we therefore express the repayment \( y^s(1, s) \) as \( y^s(s) \).

In the case of the Massachusetts initiative, the project is administered by Rocca, a non-profit organization which focuses on reducing incarceration and poverty among high-risk youth, while success payments are made by the Commonwealth of Massachusetts. In the model, we consider both the service provider and outcome payer to be the same agent, i.e., the social investor who controls the project. The primary for-profit investor is Goldman Sachs in the Massachusetts initiative. The contracting problem is to design a security that can be sold by the Commonwealth of Massachusetts to Goldman Sachs in order to help fund the upfront program costs while providing a fair market return to the for-profit investor.

Because the service provider values both cash output and social good, the expected future value of investment to the service provider is equal to the sum of cash and social outputs minus the expected security repayment to the for-profit investor minus the service provider’s contribution to the upfront investment cost,

\[
\text{Expected output} - f(I - i_s) + g(i_s) - \left( I - i_s \right) \left[ g(i_s) y^s(1) + (1 - g(i_s)) y^s(0) \right] - \rho \left( I - p^s \right). \tag{8}
\]

The investment decision is made after the security has been sold to the for-profit investor. Since the share of investment allocated to the social technology is unobservable, the service provider does not internalize the impact of this investment choice on the price of the security. Consequently, the level of social investment chosen by the service provider is the solution to the following optimization,

\[
\max_{i_s \in [0,I]} f(I - i_s) + g(i_s) - \left( I - i_s \right) \left[ g(i_s) y^s(1) + (1 - g(i_s)) y^s(0) \right]. \tag{9}
\]

For tractability, we assume that (9) is concave in \( i_s \) for all values of \( y^s(1) \) and \( y^s(0) \) in \([0, 1]\). When

\(^{17}\) A recidivism impact investment in New York City was abandoned in 2015 after it was deemed to not be cost effective, implying that limited liability with respect to cash production is reasonable (Barron’s 2015). Furthermore, many local governments face large budget deficits with as many as 8 general-purpose local governments filing for bankruptcy protection between 2010 and 2014.

\(^{18}\) See also Laux (2001) and Bond and Gomes (2009) for the analysis of multitasking problems with limited liability. In our setting, the limited liability constraints only apply to the impact investment’s private good as the public good is non-excludable.
$i_s \in (0, I)$, the chosen level of social investment is defined implicitly by

$$
0 = -f'(I - i^*_s) + g'(i^*_s) + \left[ f'(I - i^*_s)g(i^*_s) - g'(i^*_s)f(I - i^*_s) \right] y(1) \\
+ \left[ f'(I - i^*_s)(1 - g(i^*_s)) + g'(i^*_s)f(I - i^*_s) \right] y(0).
$$

(10)

The equation in (10) describes the effect of $y(1)$ and $y(0)$ on the service provider’s incentive to invest in the social technology. Define,

$$
\kappa_0(i_s) \equiv f'(I - i_s)(1 - g(i_s)) + g'(i_s)f(I - i_s),
$$

(11)

and,

$$
\kappa_1(i_s) \equiv f'(I - i_s)g(i_s) - g'(i_s)f(I - i_s).
$$

(12)

Using the preceding notation, (10) is rewritten as,

$$
0 = -f'(I - i^*_s) + g'(i^*_s) + \kappa_1 y(1) + \kappa_0 y(0).
$$

(13)

The functions $\kappa_0(i_s)$ and $\kappa_1(i_s)$ represent incentive weights associated with the security repayment schedule. Note that $\kappa_0(i_s) > 0$ for all $i_s \in [0, I]$. Thus, an increase in $y(0)$ compels the service provider to increases the share of investment directed toward the social technology. The intuition is straightforward; increasing $y(0)$ penalizes the service provider with a high security repayment when the project successfully produces cash output but fails to achieve its social objectives. This increases the ex ante incentive for socially directed investment. Alternatively, decreasing $y(0)$ dulls the incentive to invest in the social technology. The sign on $\kappa_1(i_s)$ is ambiguous. When the project succeeds along both the cash production and social value dimensions, it is not generally clear whether investment tilted in favor of the cash or social technologies. Thus, the effect of an increase in $y(1)$ on the service providers incentive for social investment is ambiguous. When $\kappa_1(i_s)$ is positive, an increase in $y(1)$ increases the share of investment in the social good. The opposite is the case when $\kappa_1(i_s)$ is negative.

In what follows, we consider the security design which maximizes joint welfare when investment is unobservable. Without loss of generality, we assume that all surplus from investment accrues to the service provider.$^{19,20}$ As such, maximizing the service provider’s payoff is equivalent to maximizing joint surplus. Given this assumption, the equilibrium security price is,

$$
p^\pi = \frac{f(I - i^*_s) \left[ g(i^*_s)y(1) + (1 - g(i^*_s))y(0) \right]}{\rho},
$$

(14)

$^{19}$In the analysis that follows, if the security achieves the first-best investment level, the surplus may be divided between the service provider and commercial investor via a constant term in the security’s price. If the service provider’s budget constraint binds and first-best is not achieved, accruing all surplus to the service provider is equivalent to maximizing the joint surplus.

$^{20}$One interpretation is that commercial investors are competitive and therefore earn zero profits in expectation.
implying that the for-profit investor is willing to pay a price exactly equal to the expected security payment, discounted by the gross market rate of return. At this price, the portion of the project cost coming from the service provider is minimized. Since social capital is limited, this price provides the greatest (weakly) opportunity for the funding of valuable social projects. The contract that maximizes joint surplus solves,

\[
\max_{\{i_s^*, y^{\pi}(0), y^{\pi}(1)\}} f(I - i_s^*) + g(i_s^*) \tag{15}
\]

\[
\text{s.t. } i_s^* \in \arg \max_{i'_s \in [0, I]} f(I - i'_s) + g(i'_s) - f(I - i_s^*) \left[ g(i_s^*) y^{\pi}(1) + (1 - g(i_s^*)) y^{\pi}(0) \right] \tag{15.1}
\]

\[
0 \leq y^{\pi}(0) \leq 1 \tag{15.2}
\]

\[
0 \leq y^{\pi}(1) \leq 1 \tag{15.3}
\]

\[
I - f(I - i_s^*) \left[ g(i_s^*) y^{\pi}(1) + (1 - g(i_s^*)) y^{\pi}(0) \right] \rho \leq \beta \psi. \tag{15.4}
\]

Inequality (15.4) is the service provider's capital budget constraint, where we have substituted for \(p^{\pi}\) from (14).

At first blush, it may seem reasonable to sell the project’s entire cash flow claim to the for-profit investor to satisfy the service provider’s budget constraint, i.e., \(y^{\pi}(0) = y^{\pi}(1) = 1\). By repaying all realized cash flow to the for-profit investor in period 2 the security’s price is maximized, fixing investment. However, under the full repayment contract, constraint (15.1) simplifies to,

\[
i_s^* \in \arg \max_{i'_s \in [0, I]} g(i'_s). \tag{16}
\]

In this case, the service provider overinvests in the social technology and chooses the corner solution \(i_s^* = I\). The full repayment contract highlights a tension in the contracting problem outlined: when external financing is raised from the for-profit investor, the service provider must repay the for-profit investor part of the cash profits generated by the project. While this diminishes the service provider’s own benefit from cash output, the service provider’s utility from social output is unaffected by the existence of the security (because social output is non-excludable). As such, this contracting tension leads the service provider to over invest in the social technology.

Under the full repayment contract, the service provider retains no cash in period 2 and therefore benefits only from the social value produced by the project. Thus, to reduce the service provider’s incentive to over invest in the social technology, the service provider must partially internalize his investment decision’s impact on cash production and either \(y^{\pi}(0)\), \(y^{\pi}(1)\), or both must be reduced below one. Consider an alternative contract to the full repayment design, \(\{\hat{y}^{\pi}(0), \hat{y}^{\pi}(1)\}\), which induces a level of investment \(\hat{i}_s^* = \hat{i}_s > \hat{i}_s^{FB}\) and satisfies the service provider’s budget constraint. If both \(\hat{y}^{\pi}(0)\) and \(\hat{y}^{\pi}(1)\) are interior valued, there must exist an alternative contract
that is more efficient. To see this, recall that the incentive effect of increasing (decreasing) \( \hat{y}^{\pi}(0) \) and \( \hat{y}^{\pi}(1) \) is captured by incentive weights \( \kappa_0(\hat{i}) \) and \( \kappa_1(\hat{i}) \) described with the service provider’s incentive compatibility constraint in (13). The cost of reducing \( \hat{y}^{\pi}(0) \) and \( \hat{y}^{\pi}(1) \) is reflected in the service provider’s budget constraint in (15.4). Since the security repayments enter linearly into each constraint, we can, roughly speaking, capture the benefit-to-cost ratio of adjusting \( \hat{y}^{\pi}(0) \) and \( \hat{y}^{\pi}(1) \) by scaling the marginal effect on incentives by the marginal cost of a smaller expected repayment.

For \( \hat{y}^{\pi}(0) \) this ratio is,

\[
\frac{\kappa_0(\hat{i})}{f(I - \hat{i})(1 - g(\hat{i}))} = f'(I - \hat{i}) + \frac{g'(\hat{i})}{f(I - \hat{i})},
\]

and for \( \hat{y}^{\pi}(1) \) this ratio is,

\[
\frac{\kappa_1(\hat{i})}{f(I - i)g(\hat{i})} = f'(I - \hat{i}) - \frac{g'(\hat{i})}{g(\hat{i})}.
\]

Clearly, the preceding equations imply,

\[
\frac{\kappa_0(\hat{i})}{f(I - \hat{i})(1 - g(\hat{i}))} \geq \frac{\kappa_1(\hat{i})}{f(I - i)g(\hat{i})},
\]

because \( \frac{g'(\hat{i})}{1 - g(\hat{i})} \geq 0 \) and \( \frac{g'(\hat{i})}{g(\hat{i})} \geq 0 \). Using the preceding intuition, one may construct an alternative contract to \( \hat{y}^{\pi}(s) \) which is more efficient. When \( \kappa_1(\hat{i}) \) is positive, an incentive neutral deviation from the contract \( \hat{y}^{\pi}(s) \) is accomplished by decreasing \( \hat{y}^{\pi}(0) \) to a smaller value \( \hat{y}^{\pi}(0) \) and also increasing \( \hat{y}^{\pi}(1) \) to \( y^{\pi}(1) \). By continuation, the most affordable contract which induces a level of investment \( i_s = \hat{i} \) is characterized by the feature \( \hat{y}^{\pi}(1) > \hat{y}^{\pi}(0) \). By relaxing the service provider’s budget constraint under this more affordable contract, it is intuitive that there exists a more efficient contract that elicits investment \( i \in [i^{FB}, \hat{i}) \) and is affordable, e.g., \( \hat{y}^{\pi}(0) \) could be lowered further while fixing \( \hat{y}^{\pi}(1) \). In the case in which \( \kappa_1(\hat{i}) \) is negative, increasing \( \hat{y}^{\pi}(1) \) is optimal because it both dulls the incentive to over invest and most easily satisfies the service provider’s budget constraint. Hence, a more efficient and affordable security will tend to penalize the service provider when social output is high. The following proposition formalizes the result.

**Proposition 1.** If the impact investment is a public works project, there exists a pay-for-success contract featuring \( y^{\pi}(1) > y^{\pi}(0) \) which maximizes the joint surplus of the public service provider and the for-profit investor. When the optimal contract is not unique (i.e., when first-best investment is obtained), a pay-for-success contract has the highest price (among the set of optimal contracts) and is therefore most affordable for the service provider.

**Corollary 1.1.** Under the optimal pay-for-success contract, the service provider over invests (weakly) in the social output relative to the first-best level of social investment, \( i_s^* \geq i^{FB} \).
From Proposition 1, there exists an optimal security which features greater repayment from the service provider to the for-profit investor following a successful social outcome. The result reconciles what may seem a particularly puzzling contract design — pay-for-success contracts, including the Massachusetts initiative, appear to penalize governments for doing the “right” thing. Our analysis provides justification for the design of Social Impact Bonds (SIBs). Since the social benefit that accrues to the service provider cannot be captured by the for-profit investor, the service provider has an incentive to pursue social goals at the expense of cash output when the service provider relies on external financing. The pay-for-performance feature arises for two reasons. First, in order to dull the tendency to over-invest in the social technology, the pay-for-performance feature penalizes the social foundation when social output is high. Second, the contract most easily satisfies the service provider’s budget constraint, that is, in any set of contracts that induce the same level of equilibrium investment, the most affordable security in the set is a pay-for-success contract. Pay-for-success bonds are therefore optimal when social projects are operated by financially constrained agents with pro-social preferences and funded in-part by outside investors.

The result in Corollary 1.1 also implies that the optimal contract leads the service provider to over-invest (weakly) in the social technology. The result contrasts with the standard tragedy-of-the-commons’ intuition. Here, rather than under providing the public good, the service provider produces the public good excessively. The result may be appreciated by considering the features of the environment that lead to the result. The key elements of the environment are (i) the public good is naturally bundled with a private good, (ii) the producer of the bundle is financially constrained, and (iii) investment is unobservable. Because the producer must repay financiers, he does not fully internalize the profits generated from the produced bundle. He does, however, fully internalize his own value of the public good under any repayment design. Therefore, his relative valuation of the public good to his share of profits increases when he retains a smaller fraction of the cash output. This leads him to over-emphasize the public good relative to what is socially optimal. In the base model, the service provider fully internalizes the aggregate social value of the public good. Thus, when the service provider over provides the public good relative to his owner personal value, he also over provides the public good relative to what is socially optimal. In Section 2.3, we relax this assumption and allow both the commercial investor and social investor to jointly benefit from the production of the social good. Even in this setting, the social investor tends to over-emphasize the public good whenever the project is financed in part with external capital.
2.2 Social Impact Guarantee

In the previous section, we analyze the case in which the social investor owns and operates the project and argue that this setting is most reflective of public sector projects. We now turn our attention to impact investments in the private sector in which the for-profit investor owns and operates the project. We interpret this setting as a private sector opportunity. We term the project a firm and the for-profit investor the firm owner. For clarity, we term the social investor a social foundation.

In order to increase the firm owner’s attention to social value, we consider a traded security offered by the firm owner to the social foundation. The superscript $\psi$ is used to denote the security which is purchased by the social foundation. The security has price $p^\psi$ and pays $y^\psi(1)$ in period 2 when the project succeeds in producing both cash output and social output and pays $y^\psi(0)$ when the project succeeds in producing cash output but fails to achieve its social objective. We again restrict attention to securities in which the firm owner cannot be required to repay more than the cash profit produced by the project and the social foundation’s liability is limited to the initial level of investment.

Given security repayments $y^\psi(0)$ and $y^\psi(1)$, the level of social investment chosen by the firm owner is the solution to the following optimization,

$$\max_{i_s \in [0, I]} f(I - i_s) - f(I - i_s) \left[ g(i_s)y^\psi(1) + (1 - g(i_s))y^\psi(0) \right].$$

(20)

When $i_s \in (0, I)$, the chosen level of social investment is defined implicitly by,

$$0 = -f'(I - i_s) + \left[ f'(I - i^*_s)g(i^*_s) - g(i^*_s)f(I - i^*_s) \right] y^\psi(1)$$

$$+ \left[ f'(I - i^*_s)(1 - g(i^*_s)) + g(i^*_s)f(I - i^*_s) \right] y^\psi(0).$$

(21)

Again, we interpret $\kappa_0(i_s)$ and $\kappa_1(i_s)$ as the incentive weights associated with the security repayment schedule and rewrite (21) as,

$$0 = -f'(I - i^*_s) + \kappa_1(i^*_s)y^\psi(1) + \kappa_0(i^*_s)y^\psi(0).$$

(22)

This implies, as in the previous section, that penalizing the firm owner with a high security repayment when the project fails to produce social output increases the incentive for socially directed investment. We maintain the assumption that all surplus accrues to the social foundation. Consequently, the price of the security is,

$$p^\psi = I - \frac{f(I - i^*_s) - f(I - i^*_s) \left[ g(i^*_s)y^\psi(1) + (1 - g(i^*_s))y^\psi(0) \right]}{\rho}. $$

(23)
As discussed in the previous section on SIBs, since the project’s maximum expected cash return is less than the market rate, \( p^\psi > 0 \), implying that the project must be jointly funded by the firm owner and social foundation. As a result, the social foundation earns a below market financial return in expectation and therefore subsidizes the financial return for the firm owner.\(^{21}\)

The security design which maximizes joint welfare when investment by the firm owner is unobservable solves,

\[
\max_{\{i^*_s, y^\psi(0), y^\psi(1)\}} f(I - i^*_s) + g(i^*_s)
\]

\[
\text{s.t. } i^*_s \in \arg \max_{i'_s \in [0,I]} f(I - i'_s) - f(I - i^*_s) \left[ g(i^*_s)y^\psi(1) + (1 - g(i^*_s))y^\psi(0) \right]
\]

\[
0 \leq y^\psi(0) \leq 1
\]

\[
0 \leq y^\psi(1) \leq 1
\]

\[
I - \frac{f(I - i^*_s) - f(I - i^*_s) \left[ g(i^*_s)y^\psi(1) + (1 - g(i^*_s))y^\psi(0) \right]}{\rho} \leq \beta^\psi.
\]

The inequality in (24.4) is the social foundation’s capital budget constraint, where we substitute the explicit form of the security’s price into the constraint. Holding fixed the levels of investment, the security which is least costly to the social foundation and which most easily satisfies the social foundation’s budget constraint sets \( y^\psi(0) = y^\psi(1) = 0 \). However, under a no repayment contract, constraint (24.1) simplifies to,

\[
i^*_s \in \arg \max_{i'_s \in [0,I]} f(I - i'_s).
\]

When all cash output remains with the firm owner, the owner places no value on social output. The no repayment design highlights the for-profit firm owner’s tendency to under invest in the social technology. To increase the firm owner’s incentive to invest in the social technology, either \( y^\psi(0) \), \( y^\psi(1) \), or both must be raised above zero. In the previous section with SIBs, the most efficient (i.e., affordable) means to taper the service provider’s tendency to over invest was accomplished with a contract that penalized high realizations of \( s \). Here, the most efficient means to induce pro-social investment is by rewarding high realizations of \( s \). This leads to the following result.

**Proposition 2.** If the impact investment is a private sector opportunity, there exists a pay-for-failure contract featuring \( y^\psi(0) > y^\psi(1) \) which maximizes the joint surplus of the firm owner and the social investor. When the optimal contract is not unique (i.e., when first-best investment is obtained), a pay-for-failure contract has the lowest price (among the set of optimal contracts) and is therefore most affordable for the social investor.

\(^{21}\)The finding is consistent with empirical investigations which suggest that socially responsible investing leads to lower expected returns. See Hong and Kacperczyk (2009) and Geczy, Stambough, and Levin (2005).
**Corollary 2.1.** Under the optimal pay-for-failure contract, the firm owner **under** invests (weakly) in the social output relative to the first-best level of social investment, \( i^*_s \leq i^{FB} \).

From Proposition 2, there exists an optimal security featuring greater repayment to the social foundation when the project fails to achieve its social objective. We coin this security a Social Impact Guarantee (SIG), since it promises the social foundation a greater security repayment when social value is low. Like Social Impact Bonds, SIGs assign a greater fraction of project cash to profit-maximizing agents when social value is high, in this case the firm owner. By rewarding the firm owner with a low security repayment when the project succeeds in producing social output, the security helps align the incentives of the firm owner and the social foundation in pursuit of social value. However, according to Corollary 2.1, the firm owner still exhibits a tendency to under supply the public good relative to first-best.

The results of Proposition 1 in Section 2.1 and Proposition 2 provide a clear contrast. While there has been much focus on the use of Social Impact Bonds to fund impact investments in social services, pay-for-success bonds in their current form have little use in the private sector. When the unconstrained agent operates the impact investment, the relevant concern is under investment in the public good, and a Social Impact Guarantee most effectively provides pro-social investment incentives.

### 2.3 Motivated Commercial Investors

Instead of the purely profit motivated commercial investor in the base model, we now consider a commercial investor who places some positive weight \( \alpha \in (0, 1] \) on the social output.\(^{22,23}\) When both investors value the social good, the first-best level of investment in the social technology is implicitly defined by,

\[
f'(I - i^{FB}_s) = (1 + \alpha)g'(i^{FB}_s).\tag{26}
\]

In this setting, both the commercial investor and the social investor under invest in the social technology absent financial contracting, because neither investor fully internalizes the aggregate value of social output.

First, consider the case in which the impact investment is operated by the social investor. Because the commercial investor now enjoys the social output, he requires less cash repayment on

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\(^{22}\) Similar to our base model, the focus on a single outside investor is arbitrary. Here, the outside investor could instead be a group of investors who are willing and able to pay for their own value of social good. The interpretation of the parameter \( \alpha \) would then be the aggregate social value across the group investors.

\(^{23}\) Besley and Ghatak (2005) uses the term “motivated agent” to label an agent that is naturally aligned with a principal’s preferences. The commercial investor we consider in this section is akin to a “motivated agent” because he is partially aligned with the social investor ex ante.
his upfront investment than in the base model. In the context of the Massachusetts initiative to reduce recidivism, other investors, the Kresge Foundation and Living Cities, participated behind Goldman Sachs. While these investors expected some repayment, they took a subordinated position to Goldman Sachs and expected a lower return. The Kresge Foundation and Living Cities are willing to earn lower returns because they value the project’s social output as a substitute for financial returns.

The contracting problem between the service provider and the motivated commercial investor is similar to that explored in Section 2.1 with two differences. First, joint surplus now accounts for both investors’ values of the social output (captured by the first-best level of investment defined above). Second, the motivated commercial investor pays a greater upfront price for the security. Because all surplus accrues to the service provider, the price the motivated commercial investor pays equals,

\[
\frac{\alpha g(i_s)}{\rho} + \frac{f(I - i_s)[g(i_s)y^a(1) + (1 - g(i_s))y^a(0)]}{\rho},
\]

which exceeds the price the for-profit investor is willing to pay outlined in (14), all else equal. While service provider under invests in the social technology absent external financing, he over invests in social output once when he relies on external funding. As in the base model, the optimal contract punishes the service provider with a large repayment when social value is high in order to minimize over investment in the social technology.

**Remark 1.** If the impact investment is a public works project, there exists a pay-for-success contract featuring \(y^a(1) > y^a(0)\) which maximizes the joint surplus of the public service provider and the motivated commercial investor. When the optimal contract is not unique (i.e., when first-best investment is obtained), a pay-for-success contract has the highest price (among the set of optimal contracts) and is therefore most affordable for the service provider.

According to Remark 1, pay-for-success contracts are optimal whenever an impact investment is operated by a service provider and funded in part by an outside investor.\(^{24}\) Even when both investors value social output, the service provider still over emphasizes the production of social good in response to external financing. That is, the service provider not only over produces the public good relative to his own preferences, but under the optimal contract, over produces the

\(^{24}\)In a setting with \(\alpha > 0\), the service provider is initially prone to under invest in the social output. However, because the service provider is constrained, the optimal contract pushes the service provider to (weakly) over invest in the social output. Since over investment is the relevant concern, the optimal contract is a SIB.
public good relative to the aggregate socially optimal level.\textsuperscript{25}

Now, consider a setting in which the impact investment is operated by the motivated commercial investor and is funded in part by the social investor. The contracting environment is similar to the setting explored in Section 2.2, and the optimal security contract again encourages greater social investment by punishing the firm owner with a high security repayment when social value is low.

**Remark 2.** If the impact investment is a private sector opportunity, there exists a pay-for-failure contract featuring $y^\psi(0) > y^\psi(1)$ which maximizes the joint surplus of the motivated firm owner and the social investor. When the optimal contract is not unique (i.e., when first-best investment is obtained), a pay-for-failure contract has the lowest price (among the set of optimal contracts) and is therefore most affordable for the social investor.

Remark 2 echoes the findings of Section 2.2: while a pay-for-failure contract provides the firm owner with a greater incentive to invest in the social technology, in many cases, the firm owner still under provides the social output even if he values the social output himself.

### 3 Non-contractible Social Output

In many settings, social output is not easily measured or contractible. Consider the example of an impact investment to open a storefront offering affordable fresh produce in the inner city. While the storefront provides urban families with greater access to nutrient rich fruits and vegetables, screening the treated population to measure the social value of a healthy diet is likely to be difficult or prohibitively costly. When social output is not contractible, the social investor and commercial investor are limited to standard revenue sharing contracts such as debt or equity. Because repayment is only possible when $x = 1$, we interpret the security as an equity claim.

First, consider a public works project operated by the service provider. To maintain consistency with prior notation, the superscript $\pi_E$ is used to denote the equity contract which is sold to the commercial investor to overcome the service provider’s capital shortfall. The equity contract has price $p^{\pi_E}$ and pays a fraction $y^{\pi_E}$ of the cash output in period 2 to the commercial investor. Since all surplus accrues to the service provider, the equilibrium equity security price is,

$$p^{\pi_E} = \frac{\alpha_g(i^*_s)}{\rho} + \frac{f(I - i^*_s)y^{\pi_E}}{\rho}.$$ \hspace{1cm} (28)

\textsuperscript{25}While we identify over investment as an inefficiency in the model, it need not be in all cases. If the motivated commercial investor were not willing to pay for his value of the public good, the contract which maximizes the service provider’s expected utility would no longer be the contract which maximizes joint surplus. In this case, the level of social investment may be closer to the first-best level when the service provider’s budget constraint binds as opposed to when the service provider is free to choose his most preferred contract and investment choice. Under this alternative setting, financial constraints on service providers may be welfare enhancing, particularly in settings in which agents are unlikely to internalize society’s full value of the public good.
Under an equity contract, the level of social investment chosen by the service provider is the solution to the following optimization,

$$
\max_{i_s \in [0,I]} f(I - i_s) + g(i_s) - f(I - i_s)y^{PE}.
$$

(29)

When \( i_s \in (0,I) \), the chosen level of social investment is defined implicitly by,

$$
0 = -f'(I - i^*_s) (1 - y^{PE}) + g'(i^*_s).
$$

(30)

Similar to the setting with contractible social output, a full repayment contract \((y^{PE} = 1)\) leads the service provider to ignore cash profit altogether. In the setting of the fresh food storefront, a full repayment contract leads the service provider to supply the most nutrient rich foods available, regardless of profit margin. While this maximizes social value, it produces the smallest amount of profit. Joint surplus is therefore increased by compelling the service provider to retain a share of the storefront’s profits. The equity contract that maximizes joint surplus solves,

$$
\max \{i^*_s, y^{PE}(0), y^{PE}(1)\} f(I - i^*_s) + (1 + \alpha)g(i^*_s)
$$

s.t. \( i^*_s \in \arg \max_{i'_s \in [0,I]} f(I - i'_s) + g(i'_s) - f(I - i'_s)y^{\pi} \)

(31)

$$
0 \leq y^{\pi} \leq 1
$$

(31.2)

$$
I - \frac{\alpha g(i^*_s) + f(I - i^*_s)y^{\pi}}{\rho} \leq \beta^{\psi}.
$$

(31.3)

Inequality (31.3) is the service provider’s capital budget constraint, where we have substituted the explicit form for \( p^{\pi E} \). Beginning with a full repayment contract, decreasing \( y^{PE} \) from 1 reduces the commercial investor’s share of profit, all else equal. However, as \( y^{PE} \) decreases, the service provider chooses smaller values of \( i^*_s \) which increases the joint surplus between the service provider and commercial investor. As such, the most efficient contract decreases \( y^{PE} \) until either joint surplus is maximized or the service provider’s budget constraint binds. At that point, no further reduction in \( y^{PE} \) is possible. The following proposition formalizes this point.

**Proposition 3.** If

$$
\frac{\alpha}{1 + \alpha} f \left( I - i^{FB}_s \right) + \alpha g \left( i^{FB}_s \right) \geq \rho \left( I - \beta^{\psi} \right),
$$

(32)

joint welfare is maximized with an equity share,

$$
y^{PE} = \frac{\alpha}{1 + \alpha}
$$

(33)

sold to the commercial investor. The service provider’s investment choice is \( i^*_s = i^{FB}_s \).
If the condition in (32) is not satisfied, joint welfare is maximized by selling an equity share,

\[ y^{\pi E} = \frac{\rho (I - \beta^{\psi}) - \alpha g(i^*_s)}{f(I - i^*_s)} \tag{34} \]

to the commercial investor, where \( i^*_s \) is the equilibrium level of investment given by,

\[ \frac{f'(I - i^*_s)}{g'(i^*_s)} = \frac{f(I - i^*_s)}{f(I - i^*_s) + \alpha g(i^*_s) - \rho(I - \beta^{\psi})}, \tag{35} \]

and the service provider over invests in the impact investment relative to first-best, \( i^*_s > i^{FB}_s \).

When \( \alpha > 0 \), the service provider initially under weights social output relative to its aggregate social value; however under an optimal equity contract, the service provider (weakly) over emphasizes social investment. That is, when the service provider’s budget constraint is small, the service provider sells a large equity stake in order to fund the project. As a result, the service provider internalizes only a fraction of the cash output produced by the impact investment and subsequently over weights production of the social output. Likewise, when \( \alpha = 0 \), the social investor always over invests in the social technology, as a result of external financing.

Now, suppose that the impact investment is operated by the commercial investor. For example, rather than the service provider opening a fresh food storefront in the inner city, a commercial grocery store operates the storefront and acquires funding from a social foundation. The commercial grocer sells a fraction \( y^{\psi E} \) of realized cash flows to the social foundation to subsidize the project’s upfront cost. With the equity contract \( y^{\psi E} \), the level of social investment chosen by the firm owner is the solution to the following optimization,

\[ \max_{i_s \in [0,I]} f(I - i_s)(1 - y^{\psi}) + \alpha g(i_s). \tag{36} \]

If \( \alpha = 0 \), the firm owner’s optimal choice of investment for any equity contract is \( i^*_s = 0 \). As such, an equity contract has no effect on the firm owner’s incentives, and he will choose \( i^*_s = 0 \) for all \( y^{\psi E} \in [0,1] \). This implies that the most affordable contract is a no repayment contract where the social investor simply subsidizes the upfront cost of the project.

**Lemma 1.** If \( \alpha = 0 \) and the impact investment is in the private sector, the equilibrium level of investment under any equity contract is \( i^*_s = 0 \). The equity contract which is least costly for the social investor has no repayment. The amount of the subsidy is

\[ p^{\psi E} = I - \frac{f(I)}{\rho} < \beta^{\psi}. \tag{37} \]
When \( \alpha = 0 \) and social output is non-contractible, the optimal contract involves direct subsidization of the project’s upfront cost and no repayment. Since the contract provides no incentive for social investment, the firm owner maximally emphasizes cash production at the expense of social good.

In contrast, when \( \alpha > 0 \), the motivated firm owner may sell equity to offset the impact investment’s upfront cost and improve incentives. That is, while equity contracts provide no incentive for social investment with a for-profit firm owner, equity contracts lead to strictly greater social investment with a motivated firm owner.

**Proposition 4.** If \( \alpha > 0 \), the level of social investment chosen by the firm owner is increasing in the size of the equity repayment. If

\[
\frac{\alpha}{1 + \alpha} f(I - i_s^{FB}) + \alpha g(i_s^{FB}) \geq \rho(I - \beta^\psi),
\]

(38)

joint welfare is maximized with an equity share,

\[
y^{\psi E} = \frac{1}{1 + \alpha}
\]

(39)

sold to the social foundation. The firm owner’s investment choice is \( i_s^* = i_s^{FB} \).

If \( \alpha > 0 \) and the condition in (38) is not satisfied, joint welfare is maximized by selling an equity share,

\[
y^{\psi E} = \frac{f(I - i_s^*) + \alpha g(i_s^*) - \rho(I - \beta^\psi)}{f(I - i_s^*)},
\]

(40)

to the social foundation, where \( i_s^* \) is the equilibrium level of social investment given by,

\[
\frac{f'(I - i_s^*)}{g'(i_s^*)} = \frac{\alpha f(I - i_s^*)}{\rho(I - \beta^\psi) - \alpha g(i_s^*)},
\]

(41)

and the firm owner under invests in the impact investment relative to first-best, \( i_s^* < i_s^{FB} \).

Proposition 4 provides both a normative recommendation and a positive insight. Our analysis recommends social investors take equity stakes in socially responsible firms. By doing so, firm owners’ profit motives are dulled and they are more prone to emphasize social outputs which they value. For example, when the owner of the grocery store retains a smaller equity stake, he is likely to substitute towards perishable fruits and vegetables which provide lower margins but greater nutritional value. Similarly, when a firm owner retains a small equity stake, his profit motives are dulled and he is likely to provide better employee benefits like comprehensive health care and employer subsidized meals. Our analysis also explains why social investors tend to invest in socially responsible firms (SRI) as opposed to firms who are agnostic towards social output. When firm
owner’s place a positive value on social output, equity investment by social investors leads directly to an increase in social investment and greater expected social output. In firms which are agnostic towards social output, direct equity investment has no effect on the level of social output.

Furthermore, from Propositions 3 and 4, the total expected surplus produced by the impact investment is unaffected by the agent who controls the investment technologies if the condition in (38) is satisfied. Importantly, the irrelevance of control rights is only possible under an equity contract if \( \alpha > 0 \). When social good is naturally bundled with a private good, socially responsible entrepreneurs may be just as effective as governments at providing public goods.

4 Concluding Discussion

Our analysis of impact investments shows that financial contracting between social investors and commercial investors accomplishes a dual goal. First, financial contracting leverages limited social capital, enabling socially-minded investors to pursue impactive investments they might otherwise not be able to afford. Second, optimal partitioning of ex post cash flows in the security contract leads to stronger ex ante incentives to finance social investment more efficiently. Since impact investments produce public social good, free-riding potentially impedes individual investors from participating in impact investments. In our model, free riding is limited because there is always at least one agent who is willing and able to pay for his value of the social good. Our analysis suggests that securities that finance impact investments must involve a party who is both large and willing to pay for his or her value of the public good. Thus, for public sector opportunities, the service provider must be willing to undertake a negative net present valued project in order to subsidize the returns of outside investors. In the private sector, securities contracts must be sold to large social block holders who internalize (and pay for) their direct impact on social investment, e.g., the Bill and Melinda Gates Foundation and the J. Paul Getty Trust.
References


Appendix A

Proof of Proposition 1: Before constructing a proof of Proposition 1, we provide a preliminary result.

Lemma A1. If the impact investment is a public works project, the service provider never under invests in the social technology under an optimal contract.

Proof of Lemma A1: The proof is constructed as a proof by contradiction: There exists a security \( \{y^\pi(0) = \hat{y}_0, y^\pi(1) = \hat{y}_1\} \) and a corresponding level of social investment \( \hat{i}_s < i^FB_s \) which maximizes joint surplus subject to constraints (15.1)-(15.4).

Now consider the alternative contract \( y^\pi(0) = y^\pi(1) = 1 \). In this case, the service provider has no profit motive and chooses \( i_s = I > i^FB_s \). Because the service provider’s choice problem is continuous with respect to \( y^\pi(0) \) and \( y^\pi(1) \), by the intermediate value theorem, there exists a contract \( \{\tilde{y}_0, \tilde{y}_1\} \) with \( \tilde{y}_0 \geq \hat{y}_0 \) and \( \tilde{y}_1 \geq \hat{y}_1 \) which provides the first-best level of social investment, \( \hat{i}_s = i^FB_s \).

We now show that there exists a security \( \{\tilde{y}_0, \tilde{y}_1\} \) which provides the first-best level of social investment and satisfies the service provider’s budget constraint. The price of any security is given by

\[
p^\pi = \frac{f(I - i_s) [g(i_s)y^\pi(1) + (1 - g(i_s)) y^\pi(0)] + \alpha g(i_s)}{\rho}, \tag{A1}
\]

and the derivative of the security’s price with respect to \( y^\pi(0) \) and \( y^\pi(1) \), respectively, are

\[
\frac{dp^\pi}{dy^\pi(0)} = \frac{1}{\rho} \left[ \Upsilon(i_s) + \alpha g(i_s) \right], \tag{A2}
\]

\[
\frac{dp^\pi}{dy^\pi(1)} = \frac{1}{\rho} \left[ \Upsilon(i_s) + \alpha g(i_s) \right], \tag{A3}
\]

where

\[
\Upsilon(i_s) = (-f'(I - i_s)g(i_s) + f(I - i_s)g'(i_s))y^\pi(1) + (-f'(I - i_s)(1 - g(i_s)) - f(I - i_s)g'(i_s))y^\pi(0).
\]

When \( i_s \in (0, I) \), as is the case here with under investment in the social technology, the service provider’s choice of social investment is defined implicitly by

\[-f(I - i_s) + g(i_s) = \Upsilon(i_s).
\]

Subbing into the derivatives above gives,

\[
\frac{dp^\pi}{dy^\pi(0)} = \frac{1}{\rho} \left[ [-f(I - i_s) + (1 + \alpha)g(i_s)] \frac{d}{dy^\pi(0)} + f(I - i_s)(1 - g(i_s)) \right], \tag{A2}
\]

\[
\frac{dp^\pi}{dy^\pi(1)} = \frac{1}{\rho} \left[ [-f(I - i_s) + (1 + \alpha)g(i_s)] \frac{d}{dy^\pi(1)} + f(I - i_s)g(i_s) \right]. \tag{A3}
\]
From the concavity of the production functions, \(-f(I - i_s) + (1 + \alpha)g(i_s) > 0\) for \(i_s < i_s^{FB}\). If, starting from the contract \(\{\hat{y}_0, \hat{y}_1\}\) and the level of investment \(\hat{i}_s < i_s^{FB}\), an increase in \(y^\pi(0)\) or \(y^\pi(1)\) increases the equilibrium level of social investment, then the security price also increases. Since \(\frac{\partial i_s}{\partial y^\pi(0)} > 0\) for all \(i_s\), increasing \(y^\pi(0)\) both increases joint welfare and makes the contract more affordable whenever \(i_s < i_s^{FB}\). As a result, no contract with \(y^\pi(0) < 1\) and \(i_s < i_s^{FB}\) can ever be optimal. It remains to be shown that a contract with \(y^\pi(0) = 1\) and \(i_s < i_s^{FB}\) is also never optimal. When increasing \(y^\pi(1)\) also raises the level of social investment (i.e., when \(\frac{\partial i_s}{\partial y^\pi(1)} > 0\) for \(i_s < i_s^{FB}\)), then increasing \(y^\pi(1)\) also increases joint welfare and makes the contract more affordable. As a result, if \(\frac{\partial i_s}{\partial y^\pi(1)} > 0\) for \(i_s < i_s^{FB}\), there exists a contract \(\{\hat{y}_0, \hat{y}_1\}\) which provides the corresponding level of investment \(\hat{i}_s = i_s^{FB}\) and which is affordable since \(\hat{y}_0 \geq \hat{y}_0\) and \(\hat{y}_1 \geq \hat{y}_1\).

Finally, we show that a contract with \(y^\pi(0) = 1\), \(i_s < i_s^{FB}\), and \(\frac{\partial i_s}{\partial y^\pi(1)} < 0\) for \(i_s < i_s^{FB}\) is not consistent with the service provider’s investment choice problem. First note that \(\frac{\partial i_s}{\partial y^\pi(1)} < 0\) if and only if \(f'(I - i_s)g(i_s) - f(I - i_s)g'(i_s) < 0\). Now consider the service provider’s choice of investment under the contract \(\{\hat{y}_0 = 1, \hat{y}_1\}\). If \(f'(I - i_s)g(i_s) - f(I - i_s)g'(i_s) < 0\), then

\[-\left[f'(I - i_s)g(i_s) - f(I - i_s)g'(i_s)\right](1 - \hat{y}_1) + g(i_s) > 0,

which implies that the service provider chooses \(i_s = I\) contradicting our assumption that \(\hat{i}_s < i_s^{FB}\). Thus, we have shown that if there exists a security \(\{y^\pi(0) = \hat{y}_0, y^\pi(1) = \hat{y}_1\}\) which is affordable and induces a corresponding level of social investment \(\hat{i}_s < i_s^{FB}\) there necessarily exists an alternative security \(\{\bar{y}_0, \bar{y}_1\}\) which is also affordable and induces the first-best level of social investment \(\bar{i}_s = i_s^{FB}\). As such, when the impact investment is a public works project, the optimal contract never involves under investment in the social technology.

We now proceed with the proof of Proposition 1. From Lemma A1, the optimal contract cannot involve under investment in the social technology. As such, we separate the proof into two parts. First, we consider the case in which the optimal contract provides over investment in the social good and show that the optimal contract is a pay-for-success security. Then, we consider the case in which the optimal contract provides the first-best level of social investment and show that the most affordable contract is a pay-for-success security.

**Case 1 \((\hat{i}_s > i_s^{FB})\):** The proof in this case is constructed as a proof by contradiction: There exists a security \(\{y^\pi(0) = \bar{y}_0, y^\pi(1) = \bar{y}_1\}\) with \(\bar{y}_1 \leq \bar{y}_0 \leq 1\) and a corresponding level of investment \(\hat{i}_s > i_s^{FB}\) which maximizes joint surplus subject to constraints (15.1)-(15.4)

We start by assuming that \(\hat{i}_s < I\) and then consider the case in which \(\hat{i}_s = I\). With an internal solution for \(\hat{i}_s\), the service provider’s incentive compatible first-order condition in (15.1) is satisfied.
with equality,
\[
0 = -f'(I - \hat{i}_s) + g'(\hat{i}_s) + \left[f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s)\right] \hat{y}_1 \\
+ \left[f'(I - \hat{i}_s)(1 - g(\hat{i}_s)) + g'(\hat{i}_s)f(I - \hat{i}_s)\right] \hat{y}_0.
\] (A4)

**Case 1(a):** \(f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s) < 0\)

In case 1(a), it is straightforward that there exists \(\epsilon > 0\) such that the alternative contract
\[
\tilde{y}_0 = \hat{y}_0, \\
\tilde{y}_1 = \hat{y}_1 + \epsilon,
\] (A5) (A6)
and corresponding level of social investment \(\tilde{i}_s\) (with \(\tilde{i}_s > \hat{i}_s > i_s^{FB}\)) provide greater joint welfare and are affordable. When \(f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s) < 0\), an increase in \(y^\pi(1)\) reduces the degree of over investment in the social technology and thus increases the joint welfare between the service provider and commercial investor. Furthermore, since \(\tilde{i}_s > i_s^{FB}\), we see, from the equation in (A3), that an increase in \(y^\pi(1)\) also raises the price that the commercial investor pays for the security. Since the contract \(\{\tilde{y}_0, \tilde{y}_1\}\) satisfies the social investor’s budget constraint, the contract \(\{\tilde{y}_0, \tilde{y}_1\}\), which has a higher price, must also satisfy the budget constraint. Thus, the contract \(\{\tilde{y}_0, \tilde{y}_1\}\) provides greater joint welfare and is affordable, contradicting the optimality of \(\{\tilde{y}_0, \tilde{y}_1\}\).

**Case 1(b):** \(f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s) \geq 0\)

Consider an alternative contract of the form,
\[
\tilde{y}_0 = \hat{y}_0 - \delta \left( \frac{f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s)}{f'(I - i_s)(1 - g(i_s)) + g'(i_s)f(I - i_s)} \right), \\
\tilde{y}_1 = \hat{y}_1 + \delta,
\] (A7) (A8)
for some small \(\delta > 0\). The alternative contract \(\{\tilde{y}_0, \tilde{y}_1\}\) maintains the same incentives as the contract \(\{\hat{y}_0, \hat{y}_1\}\), and thus, \(\tilde{i}_s = \hat{i}_s\). However, the alternative contract is more affordable:
\[
f(I - \hat{i}_s)\left(g(\hat{i}_s)\hat{y}_1 + (1 - g(\hat{i}_s))\hat{y}_0\right) + \alpha g(\hat{i}_s) \\
= f(I - \hat{i}_s)\left(g(\hat{i}_s)\hat{y}_1 + (1 - g(\hat{i}_s))\hat{y}_0\right) + \alpha g(\hat{i}_s) + \frac{\delta f(I - \hat{i}_s)^2 g'(\hat{i}_s)}{f'(I - i_s)(1 - g(i_s)) + g'(i_s)f(I - i_s)} \\
> f(I - \hat{i}_s)\left(g(\hat{i}_s)\hat{y}_1 + (1 - g(\hat{i}_s))\hat{y}_0\right) + \alpha g(\hat{i}_s) \\
\geq \rho \left(I - \beta^\psi\right)
\]
Now, there exists $\epsilon > 0$ such that the contract,
\begin{align*}
\hat{y}_0 &= \tilde{y}_0 - \epsilon, \\
\hat{y}_1 &= \tilde{y}_1,
\end{align*}
and corresponding level of social investment $\hat{i}_s$ (with $\hat{i}_s > \hat{i}_s > i_s^{FB}$) provide greater joint welfare and are affordable. By continuity, the contract $\{\hat{y}_0, \hat{y}_1\}$ is affordable, and by concavity, delivers $\hat{i}_s > \hat{i}_s > i_s^{FB}$. Thus $\hat{i}_s$ delivers greater surplus which contradicts the optimality of $\{\tilde{y}_0, \tilde{y}_1\}$.

Thus far we assumed that $\{\tilde{y}_0, \tilde{y}_1\}$ delivers an internal choice $\hat{i}_s \in (i_s^{FB}, I)$. Consider now the possibility, $\hat{i}_s = I$. In this case, there exists $\epsilon > 0$ such that the alternative security,
\begin{align*}
\tilde{y}_0 &= 1 - \epsilon, \\
\tilde{y}_1 &= 1,
\end{align*}
leads to $\hat{i}_s < I$ and is affordable. Since $f(0) > \rho (I - \beta^\omega)$, the contract $y^\pi(0) = y^\pi(1) = 1$ satisfies the social investor’s budget constraint. Thus by continuity, there exists $\epsilon > 0$ such that the alternative security $\{\tilde{y}_0, \tilde{y}_1\}$ is also affordable. Furthermore, any $\epsilon > 0$ provides sufficient profit motives for the service provider to choose a level of social investment which is less than $I$. Therefore, the contract $\{\tilde{y}_0, \tilde{y}_1\}$ is affordable and, by concavity, provides greater joint surplus, contradicting the optimality of $\{\tilde{y}_0, \tilde{y}_1\}$.

We now consider the case in which the optimal contract provides the first-best level of social investment and show that the most affordable contract is a pay-for-success security.

**Case 2 ($\hat{i}_s = i_s^{FB}$):** The proof in this case is constructed as a proof by contradiction: There exists a security $\{y^\pi(0) = \tilde{y}_0, y^\pi(1) = \tilde{y}_1\}$ with $\tilde{y}_1 \leq \tilde{y}_0 \leq 1$ and a corresponding level of investment $\hat{i}_s = i_s^{FB}$ which maximizes joint surplus subject to constraints (15.1)-(15.4) and which maximizes (among the set of optimal contract) the security price $p^\pi$.

**Case 2(a):** $\tilde{y}_0 < 1$

There exists $\delta > 0$, such that the alternative contract
\begin{align*}
\tilde{y}_0 &= \tilde{y}_0 - \delta \\
\tilde{y}_1 &= \tilde{y}_1 + \delta,
\end{align*}
maintains the same incentives for social investment as the contract \{\hat{y}_0, \hat{y}_1\}. As such, \(\tilde{i}_s = \hat{i}_s = i^{FB}_s\).

In addition, the alternative contract is more affordable:

\[
f(I - \tilde{i}_s) \left( g(\tilde{i}_s)\hat{y}_1 + (1 - g(\tilde{i}_s))\hat{y}_0 \right) + \alpha g(\tilde{i}_s) \\
= f(I - \tilde{i}_s) \left( g(\tilde{i}_s)\hat{y}_1 + (1 - g(\tilde{i}_s))\hat{y}_0 \right) + \alpha g(\tilde{i}_s) + \frac{\delta f(I - \tilde{i}_s) g'(\tilde{i}_s)}{f'(I - \tilde{i}_s)(1 - g(\tilde{i}_s)) + g'(\tilde{i}_s)f(I - \tilde{i}_s)} \\
> f(I - \tilde{i}_s) \left( g(\tilde{i}_s)\hat{y}_1 + (1 - g(\tilde{i}_s))\hat{y}_0 \right) + \alpha g(\tilde{i}_s) \\
\geq \rho \left( I - \beta^\psi \right),
\]

which contradicts the assertion that \(\hat{y}_1 \leq \hat{y}_0 < 1\) is the most affordable contract among the set of optimal contracts.

**Case 2(b): \(\hat{y}_0 = 1\)**

If \(f'(I - \tilde{i}_s)g(\tilde{i}_s) - g'(\tilde{i}_s)f(I - \tilde{i}_s) \geq 0\), an alternative security analogous to the one constructed in Case 2(a) provides the first-best level of investment and is more affordable. However, if \(f'(I - \tilde{i}_s)g(\tilde{i}_s) - g'(\tilde{i}_s)f(I - \tilde{i}_s) < 0\), the assertion that \(\tilde{i}_s = i^{FB}_s\) is inconsistent with the service provider’s choice of social investment under the contract \{\hat{y}_0 = 1, \hat{y}_1\}. A detailed description of this argument can be found in the proof of Lemma A1.

**Proof of Corollary 1.1:** See proof of Lemma A1.

**Proof of Proposition 2:** Before constructing a proof of Proposition 2, we provide a preliminary result.

**Lemma A2.** If the impact investment is in the private sector, the firm owner never over invests in the social technology under an optimal contract.

**Proof of Lemma A2:** The proof is constructed as a proof by contradiction: There exists a security \{\(y^\psi(0) = \hat{y}_0, y^\psi(1) = \hat{y}_1\)\} and a corresponding level of social investment \(\tilde{i}_s > i^{FB}_s\) which maximizes joint surplus subject to constraints (24.1)-(24.4).

Now consider the alternative contract \(y^\psi(0) = y^\psi(1) = 0\). In this case, the firm owner’s chosen level of investment solves

\[
\begin{cases} 
-f'(I - i_s) + \alpha g'(i_s) = 0 & \text{if } \alpha > 0 \\
i_s = 0 & \text{if } \alpha = 0
\end{cases}
\]

Thus, from the concavity of the probability functions, the firm owner chooses \(i_s < i^{FB}_s\) under the contract \(y^\psi(0) = y^\psi(1) = 0\). Because the firm owner’s choice problem is continuous with respect
to \( y^\psi (0) \) and \( y^\psi (1) \), we can therefore apply the intermediate value theorem: there exists a contract \( \{ \bar{y}_0, \bar{y}_1 \} \) with \( \bar{y}_0 \leq \bar{y}_0 \) and \( \bar{y}_1 \leq \bar{y}_1 \) which provides the first-best level of social investment, \( \hat{i}_s = i^FB_s \).

We now show that there exists a security \( \{ \bar{y}_0, \bar{y}_1 \} \) which provides the first-best level of social investment and that satisfies the social investor’s budget constraint. The price of any security is given by

\[
p^\psi = \frac{f(I - i_s) [1 - g(i_s)y^\nu (1) - (1 - g(i_s)) y^\psi (0)] + \alpha g(i_s)}{\rho}, \tag{A16}
\]

and the derivative of the security’s price with respect to \( y^\psi (0) \) and \( y^\psi (1) \), respectively, are

\[
\frac{dp^\psi}{dy^\psi (0)} = -\frac{1}{\rho} \left[ \Omega(i_s) \frac{di_s}{dy^\psi (0)} - f(I - i_s)(1 - g(i_s)) \right],
\]

\[
\frac{dp^\psi}{dy^\psi (1)} = -\frac{1}{\rho} \left[ \Omega(i_s) \frac{di_s}{dy^\psi (1)} - f(I - i_s)g(i_s) \right],
\]

where

\[
\Omega(i_s) = -f'(I - i_s)(1 - g(i_s)y^\nu (1) - (1 - g(i_s)) y^\psi (0)) + g'(i_s)(\alpha - f(I - i_s)(y^\psi (1) - y^\psi (0))).
\]

When \( i_s \in [i^FB_s, I) \), the firm owner’s choice of social investment is defined implicitly by

\[
\Omega(i_s) = 0.
\]

Alternatively, when \( \Omega(i_s) < 0 \) for all \( i_s \), then \( i_s = I \) and \( \frac{di_s}{dy^\psi (0)} = \frac{di_s}{dy^\psi (1)} = 0 \). Substituting into the derivatives above yields,

\[
\frac{dp^\psi}{dy^\psi (0)} = \frac{f(I - i_s)(1 - g(i_s))}{\rho} \geq 0, \tag{A17}
\]

\[
\frac{dp^\psi}{dy^\psi (1)} = \frac{f(I - i_s)g(i_s)}{\rho} > 0. \tag{A18}
\]

As a result, lowering either \( y^\psi (0) \) or \( y^\psi (1) \) decreases the price the social investor must pay for the security. Since \( \bar{y}_0 \leq \bar{y}_0 \) and \( \bar{y}_1 \leq \bar{y}_1 \), we have

\[
f \left( I - i^FB_s \right) [1 - g \left( i^FB_s \right) \bar{y}_1 - (1 - g \left( i^FB_s \right)) \bar{y}_0] + \alpha g \left( i^FB_s \right)
\]
\[
< f \left( I - \hat{i}_s \right) [1 - g \left( \hat{i}_s \right) \hat{y}_1 - (1 - g \left( \hat{i}_s \right)) \hat{y}_0] + \alpha g \left( \hat{i}_s \right)
\]
\[
\leq \rho \left( I - \beta^\psi \right)
\]

Thus, the security \( \{ \bar{y}_0, \bar{y}_1 \} \) is affordable and provides the first-best level of social investment, contradicting the optimality of the contract \( \{ \bar{y}_0, \bar{y}_1 \} \). That is, when the commercial investor operates the impact investment, the contract which maximizes joint welfare never involves over investment in the social technology. \( \blacksquare \)
We now proceed with the proof of Proposition 2. From Lemma A2, the optimal contract cannot involve over investment in the social technology. As such, we separate the proof into two parts. First, we consider the case in which the optimal contract provides under investment in the social good and show that the optimal contract is a pay-for-failure security. Then, we consider the case in which the optimal contract provides the first-best level of social investment and show that the most affordable contract is a pay-for-failure security.

**Case 1 (\(i_s < i_s^{FB}\)):** The proof in this case is constructed as a proof by contradiction: There exists a security \(\{y^\psi(0) = \hat{y}_0, y^\psi(1) = \hat{y}_1\}\) with \(\hat{y}_1 \geq \hat{y}_0\) and a corresponding level of investment \(\hat{i}_s < i_s^{FB}\) which maximizes joint surplus subject to constraints (24.1)-(24.4)

We start by assuming that \(\hat{i}_s > 0\) and then consider the case in which \(\hat{i}_s = 0\). With an internal solution for \(\hat{i}_s\), the firm owner’s incentive compatible first-order condition in (24.1) is satisfied with equality,

\[
0 = -f'(I - \hat{i}_s) + \alpha g'(\hat{i}_s) + \left[ f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s) \right] \hat{y}_1 + \left[ f'(I - \hat{i}_s)(1 - g(\hat{i}_s)) + g'(\hat{i}_s)f(I - \hat{i}_s) \right] \hat{y}_0.
\]

\(\text{(A19)}\)

**Case 1(a):** \(f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s) < 0\)

In case 1(a), it is straightforward that there exists \(\epsilon > 0\) such that the alternative contract

\[
\hat{y}_0 = \bar{y}_0,
\]

\(\text{(A20)}\)

\[
\hat{y}_1 = \bar{y}_1 - \epsilon,
\]

\(\text{(A21)}\)

and corresponding level of social investment \(\bar{i}_s\) (with \(\bar{i}_s < \hat{i}_s < i_s^{FB}\)) provide greater joint welfare and are affordable. When \(f'(I - \bar{i}_s)g(\bar{i}_s) - g'(\bar{i}_s)f(I - \bar{i}_s) < 0\), a decrease in \(y^\psi(1)\) reduces the degree of under investment in the social technology and thus increases the joint welfare between the firm owner and social investor. Furthermore, a decrease in \(y^\psi(1)\) reduces the price that the social investor pays for the security. Since the contract \(\{\bar{y}_0, \bar{y}_1\}\) satisfies the social investor’s budget constraint, the contract \(\{\bar{y}_0, \bar{y}_1\}\), which has a lower price, must also satisfy the budget constraint. Thus, the contract \(\{\bar{y}_0, \bar{y}_1\}\) provides greater joint welfare and is affordable, contradicting the optimality of \(\{\hat{y}_0, \hat{y}_1\}\).

**Case 1(b):** \(f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s) \geq 0\)

Consider an alternative contract of the form,

\[
\hat{y}_0 = \bar{y}_0 + \delta \left( \frac{f'(I - \hat{i}_s)g(\hat{i}_s) - g'(\hat{i}_s)f(I - \hat{i}_s)}{f'(I - \hat{i}_s)(1 - g(\hat{i}_s)) + g'(\hat{i}_s)f(I - \hat{i}_s)} \right)
\]

\(\text{(A22)}\)

\[
\hat{y}_1 = \bar{y}_1 - \delta
\]

\(\text{(A23)}\)
for some small $\delta > 0$. The alternative contract $\{\tilde{y}_0, \tilde{y}_1\}$ maintains the same incentives as the contract $\{\hat{y}_0, \hat{y}_1\}$, and thus, $\tilde{i}_s = \hat{i}_s$. However, the alternative contract is more affordable:

$$f(I - \hat{i}_s) \left(1 - g(\hat{i}_s)\hat{y}_1 - (1 - g(\hat{i}_s))\hat{y}_0\right) + \alpha g(\hat{i}_s)$$

$$= f(I - \hat{i}_s) \left(1 - g(\hat{i}_s)\hat{y}_1 - (1 - g(\hat{i}_s))\hat{y}_0\right) + \alpha g(\hat{i}_s) + \delta f(I - \hat{i}_s)^2 g'(\hat{i}_s) \frac{\delta f(I - \hat{i}_s)^2 g'(\hat{i}_s)}{f'(I - \hat{i}_s)(1 - g(\hat{i}_s)) + g'(\hat{i}_s)f(I - \hat{i}_s)}$$

$$> f(I - \hat{i}_s) \left(1 - g(\hat{i}_s)\hat{y}_1 - (1 - g(\hat{i}_s))\hat{y}_0\right) + \alpha g(\hat{i}_s)$$

$$\geq p \left(I - \beta_\psi\right)$$

Because the social investor's budget constraint does not bind under the contract $\{\hat{y}_0, \hat{y}_1\}$, there exists $\epsilon > 0$ such that the contract,

$$\hat{y}_0 = \tilde{y}_0 + \epsilon, \quad (A24)$$

$$\hat{y}_1 = \tilde{y}_1, \quad (A25)$$

and corresponding level of social investment $\hat{i}_s$ (with $\hat{i}_s > \tilde{i}_s > i_s^{FB}$) provide greater joint welfare and are affordable. By continuity, the contract $\{\hat{y}_0, \hat{y}_1\}$ is affordable, and by concavity, delivers $\hat{i}_s < \hat{i}_s < i_s^{FB}$. Thus $\hat{i}_s$ delivers greater joint surplus which contradicts the optimality of $\{\hat{y}_0, \hat{y}_1\}$.

Thus far we assumed that $\{\hat{y}_0, \hat{y}_1\}$ delivers an internal choice $\hat{i}_s \in (0, i_s^{FB})$. Consider now the possibility, $\hat{i}_s = 0$. In this case, there exists $\epsilon > 0$ such that the alternative security,

$$\tilde{y}_0 = \epsilon, \quad (A26)$$

$$\tilde{y}_1 = 0, \quad (A27)$$

leads to $\hat{i}_s > 0$ and is affordable. That is, any $\epsilon > 0$ provides sufficient incentives for the firm owner to choose a level of social investment which is greater than zero. Therefore, the contract $\{\tilde{y}_0, \tilde{y}_1\}$ is affordable and, by concavity, provides greater joint surplus, contradicting the optimality of $\{\hat{y}_0, \hat{y}_1\}$.

We now consider the case in which the optimal contract provides the first-best level of social investment and show that the most affordable contract is a pay-for-failure security.

**Case 2** ($\hat{i}_s = i_s^{FB}$): The proof in this case is constructed as a proof by contradiction: There exists a security $\{y^\psi(0) = \hat{y}_0, y^\psi(1) = \hat{y}_1\}$ with $\hat{y}_1 \geq \hat{y}_0$ and a corresponding level of investment $\hat{i}_s = i_s^{FB}$ which maximizes joint surplus subject to constraints (24.1)-(24.4) and which minimizes (among the set of optimal contract) the security price $p^\psi$. 

33
Case 2(a): $\hat{y}_0 > 0$

There exists $\delta > 0$, such that the alternative contract

\[
\tilde{y}_0 = \hat{y}_0 + \delta \left( \frac{f'(I - \hat{i}_s) g(\hat{i}_s) - g'(\hat{i}_s) f(I - \hat{i}_s)}{f'(I - \hat{i}_s)(1 - g(\hat{i}_s)) + g'(\hat{i}_s) f(I - \hat{i}_s)} \right)
\]

(A28)

\[
\tilde{y}_1 = \hat{y}_1 - \delta,
\]

(A29)

maintains the same incentives for social investment as the contract $\{\hat{y}_0, \hat{y}_1\}$. As such, $\hat{i}_s = \tilde{i}_s = i_{FB}^s$.

In addition, the alternative contract is more affordable:

\[
f(I - \hat{i}_s) \left( 1 - g(\hat{i}_s) \hat{y}_1 - (1 - g(\hat{i}_s))\hat{y}_0 \right) + \alpha g(\hat{i}_s) \\
= f(I - \hat{i}_s) \left( 1 - g(\hat{i}_s) \hat{y}_1 - (1 - g(\hat{i}_s))\hat{y}_0 \right) + \alpha g(\hat{i}_s) + \frac{\delta f(I - \hat{i}_s)^2 g'(\hat{i}_s)}{f'(I - \hat{i}_s)(1 - g(\hat{i}_s)) + g'(\hat{i}_s) f(I - \hat{i}_s)} \\
> f(I - \hat{i}_s) \left( 1 - g(\hat{i}_s) \hat{y}_1 - (1 - g(\hat{i}_s))\hat{y}_0 \right) + \alpha g(\hat{i}_s) \\
\geq \rho \left( I - \beta^\psi \right),
\]

which contradicts the assertion that $0 < \hat{y}_0 \leq \hat{y}_1$ is the most affordable contract among the set of optimal contracts.

Case 2(b): $\hat{y}_0 = 0$

If $f'(I - \hat{i}_s) g(\hat{i}_s) - g'(\hat{i}_s) f(I - \hat{i}_s) \geq 0$, an alternative security analogous to the one constructed in Case 2(a) provides the first-best level of investment and is more affordable. However, if $f'(I - \hat{i}_s) g(\hat{i}_s) - g'(\hat{i}_s) f(I - \hat{i}_s) < 0$, the assertion that $\hat{i}_s = i_{FB}^s$ is inconsistent with the service provider’s choice of social investment under the contract $\{\hat{y}_0 = 0, \hat{y}_1\}$.

■

Proof of Corollary 2.1: See proof of Lemma A2.

■

Proof of Remark 1: See proof of Proposition 1.

■

Proof of Remark 2: See proof of Proposition 2.

■

Proof of Proposition 3: When the service provider operates the impact investment and financing is provided by a motivated commercial investor, the equity contract that implements the first-best
level of social investment solves,
\[ \frac{1}{1 - y_{\pi E}} = \frac{f'(I - i_{FB}^s)}{g'(i_{FB}^s)} = 1 + \alpha, \]

\[ \implies y_{\pi E} = \frac{\alpha}{1 + \alpha}. \]

Thus, the first-best level of social investment is obtainable when the equity contract \( y_{\pi E} = \frac{\alpha}{1 + \alpha} \) satisfies the service provider’s budget constraint,
\[ \frac{\alpha}{1 + \alpha} f(I - i_{FB}^s) + \alpha g(i_{FB}^s) \geq \rho(I - \beta^\psi). \]

When this condition is not satisfied, the first-best level of social investment is not possible. In this case, the optimal equity contract, \( y_{\pi E} \), and the equilibrium level of social investment, \( i_{s}^* \), are pinned down by the service provider’s incentive compatible first-order condition and by the service provider’s budget constraint,

\[ f'(I - i_{s}^*) (1 - y_{\pi E}) + g'(i_{s}^*) = 0, \]
\[ f(I - i_{s}^*) y_{\pi E} + \alpha g(i_{s}^*) = \rho(I - \beta^\psi). \]

In this case, the service provider over invests in the social technology. An argument mimicking that in Lemma A1 shows that under investment in the social technology cannot be optimal when the social investor operates the impact investment and is thus omitted for brevity.

\[ \blacksquare \]

Proof of Lemma 1:

For non-trivial securities \( y^{\psi E} \in [0, 1) \) the firm owner’s investment choice is the solution to,
\[ \max_{i_s \in [0,I]} f(I - i_s)(1 - y^{\psi E}). \] (A30)

Because the firm owner does not internalize any value from social output, his equilibrium investment choice is,
\[ i_{s}^* = 0, \] (A31)

for all \( y^{\psi E} \in [0, 1) \). The cheapest contract solves,
\[ \min_{y^{\psi E}} I - \frac{f(I)(1 - y^{\psi E})}{\rho}. \] (A32)

Therefore, the most affordable contract is \( y^{\psi E} = 0 \) with price,
\[ p^{\psi E} = \frac{\rho I - f(I)}{\rho} < \beta^\psi. \] (A33)

\[ \blacksquare \]
**Proof of Proposition 4**: When the motivated firm owner operates the impact investment, the equity contract that implements the first-best level of social investment solves,

\[
\frac{1}{1 - y^{\psi E}} = \frac{f'(I - i_s^{FB})}{\alpha g'(i_s^{FB})} = \frac{1 + \alpha}{\alpha},
\]

\[\implies y^{\psi E} = \frac{1}{1 + \alpha}.\]

Thus, the first-best level of social investment is obtainable when the equity contract \(y^{\psi E} = \frac{1}{1 + \alpha}\) satisfies the service provider’s budget constraint,

\[
\frac{\alpha}{1 + \alpha} f(I - i_s^{FB}) + \alpha g(i_s^{FB}) \geq \rho(I - \beta^\psi).
\]

When this condition is not satisfied, the first-best level of social investment is not possible. In this case, the optimal equity contract, \(y^{\psi E}\), and the equilibrium level of social investment, \(i_s^*\), are pinned down by the firm owner’s incentive compatible first-order condition and by the service provider’s budget constraint,

\[
f'(I - i_s^*)(1 - y^{\psi E}) + \alpha g(i_s^*) = 0,
\]

\[
f(I - i_s^*)(1 - y^{\psi E}) + \alpha g(i_s^*) = \rho(I - \beta^\psi).
\]

In this case, the firm owner under invests in the social technology. An argument mimicking that in Lemma A2 shows that over investment in the social technology cannot be optimal when the commercial investor operates the impact investment and is thus omitted for brevity.

\[\blacksquare\]