Mark-to-Market: Real Effects and Social Welfare

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Current revision: August 2016

ABSTRACT

Mark-to-market (MTM) information improves welfare by facilitating external decisions. But due to feedback effects, MTM also stimulates pro-cyclical forces that make the firm’s wealth highly volatile. To counteract this increased volatility firm’s have a strong incentive to reduce their exposure to MTM adjustments by changing the asset portfolios they hold. In this paper, we assess the desirability of MTM in a setting where all of the above forces are endogenously determined. We derive the essential tradeoffs that determine the net welfare effects of MTM accounting. We show that, beyond a certain threshold, greater precision in the information provided by MTM would cause the welfare of all affected agents to decline.

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We thank Nahum Melumad, Amir Ziv, Haresh Sapra, Ron Dye, Shiva Sivaramakrishnan, Sunil Dutta, and Greg Clinch for many helpful comments on earlier versions of this paper. We have also benefited from numerous comments by seminar participants at Baruch City College, Columbia University, the University of Chicago, Northwestern University, Rice University, Southern Methodist University, University of California at Irvine, University of Toronto, Washington University in St. Louis, Rutgers Business School, the 2014 Accounting Theory Conference, the 2014 LAEF Conference at Santa Barbara, the Indian School of Business 2015 Accounting Conference and the 2016 UTS Australian Summer Accounting Conference.
1. Introduction

The financial crises of 2007-09 ignited an important debate among professional managers, regulators, the US Congress and academics about the economic consequences of mark-to-market (MTM) accounting (aka, fair value accounting). Many in the professional community argued that MTM was contributing significantly to the downward spiral in the economy.\(^1\) Many in the academic community argued that MTM measurements were mere messengers providing timely information that could only facilitate corrective actions.\(^2\) The debate between these two points of view is but one example of a larger issue. Do accounting measurements and reports to external parties merely mirror the events occurring in a firm, or could they actually contribute to those events? If the former is true then accounting reports could never have adverse consequences; they could only facilitate welfare improving actions by external parties. More information would always be better than less. If the latter is true then accounting regulators like the Financial Accounting Standards Board (FASB) must carefully balance the benefits of facilitating external decisions against the possibly adverse transformative consequences inside the firm.

In this paper, we describe and analyze a setting which illustrates how MTM accounting transforms the financial wealth of the firm while conveying information to external agents that is used to assess the firm’s financial condition. We explicitly derive and analyze the tradeoff between causing such wealth transformations and providing information to facilitate external decisions. Our analysis is indicative of how standard setting could go awry and produce many unintended consequences when standard setters assume (as the FASB apparently does) that firms’ operations are independent of how they are measured and reported to outsiders.

\(^1\) For examples, see Forbes (2009), Wallisison (2008a, 2008b) and Whalen (2008). Similar concerns were expressed by the American Bankers Association and in US Congressional debates.

\(^2\) See Barth, Beaver and Landsman (2001 and Barth (2006). Laux and Leuz (2009) argue that: “FVA is neither responsible for the crisis nor is it merely a measurement system that reports asset values without having economic effects of its own.”
As claimed by its supporters, MTM accounting certainly provides a much more accurate picture of a firm’s current financial situation. This greater transparency enables better decisions and actions by external agents in all situations where the payoff to those decisions is affected by the firm’s wealth. But, such decisions and actions are not necessarily “corrective” of a failing situation. For example, potential employees may be concerned with a firm’s financial health when deciding where to seek employment. Customers may want to know that they are buying from a firm that will remain financially viable in the future. Suppliers of raw materials and services to a firm are concerned with the financial strength of the firm, and investors demand such information when called upon to finance new debt or equity. In each of these situations the interaction between the firm and external agents is not entirely one sided. The financial condition of the firm certainly affects the payoff to external agents, but also, collectively, the actions taken by these external agents have significant effects on the firm’s wealth. The change in the firm’s wealth is likely to be procyclical, in the sense that when the situation looks good to external agents they take actions that make it better and a situation that doesn’t look good could trigger a flight of financial capital, human capital, and abandonment by suppliers and customers that make the financial condition of the firm much worse. Faced with the possibility of such increased volatility, managers are likely to take anticipatory actions to diminish the effect of MTM on external agents’ actions.

The interactive effects described above could be quite large. In their presence, the task of assessing the desirability of MTM is not as simple as asking whether the information provided by MTM is sufficiently reliable and whether outside stakeholders would want it. In most situations, individual external agents are so small that each individual’s own actions have no measurable effect on the firm’s financial condition. Additionally, any anticipatory actions taken by managers will be perceived as sunk. Therefore to each individual external agent, the firm’s financial situation at the

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3 This claim follows from Blackwell’s theorem.
time she needs to act, would indeed be a state of Nature, in which case only the decision facilitating role of MTM would be of concern. Thus every external agent would rationally demand the kind of information provided by MTM. But when the information is publicly provided, the actions of all agents are affected in the same direction, in which case the collective effect on the firm’s wealth could be highly significant. If the change in the firm’s wealth, or the anticipatory actions taken by corporate managers, adversely affects the payoff to external agents, then providing the information may be socially undesirable, even though each individual user of the information would demand it. In such situations, making accounting policy by merely acceding to the demands of external users is not the appropriate response by regulators. The social costs and benefits of increased disclosure could be quite subtle, but can be endogenously characterized, and regulators need to carefully assess them before arriving at an appropriate accounting policy.

The results we derive, when the firm’s wealth is allowed to endogenously adjust to accounting measurement, are starkly different from traditional wisdom. (i) The traditional wisdom is that information decreases uncertainty (which is always true in one sided interactions with Nature). In contrast, we find that information provided to aid in the assessment of a firm’s wealth increases the uncertainty in that wealth. (ii) The traditional wisdom is that, regardless of whether we mark-to-market the assets of a firm or don’t mark them to market, the firm’s assets remain exactly the same; only the accounting numbers change. In contrast, we find that the assets that are marked to market in an MTM regime would be quite different from the assets that the firm would hold in historical cost regime. (iii) Traditional wisdom suggests that the more relevant the firm’s wealth is to the decisions of external stakeholders the more precise should be the information supplied to them. However, we find that the greater the need of outside stakeholders to assess the firm’s wealth, the less precise should be the fair value information that is provided to them. In terms of \textit{ex ante} welfare, we find that
the firm (i.e. its shareholders) would unambiguously prefer historical cost to fair value accounting. We identify plausible conditions under which even outside stakeholders are worse off from fair value accounting. We find that there is a time inconsistency problem in determining disclosure policy that is analogous to the well-known time inconsistency of optimal monetary and government policy (Kydland and Prescott, 1977). At the time that outside stakeholders need to make their decisions they would demand the most precise fair value information that is feasible, but from an ex ante perspective such a disclosure policy could actually make them worse off.

Our results cast doubt on the desirability of fair value accounting and, more generally, on the wisdom of making accounting policy under the assumption that accounting information is analogous to information about the state of Nature. Plantin, Sapra and Shin (2008) and Allen and Carletti (2008) have also raised concerns about fair value accounting and have identified some of its negative consequences. However, in this previous research the concerns originate from measurement difficulties when there is a lack of liquidity in the market for the firm’s assets. Our analysis does not depend on measurement difficulties and liquidity issues and challenges a pervasive assumption underlying the determination of accounting standards in general.

The wealth transformation effect of MTM accounting is a special case of the “real effects” of financial markets that have been studied in many different contexts, both theoretically and empirically. One strand of this literature argues that real effects arise due to the information transmission role of prices in financial markets when traders and speculators in the market are collectively better informed than corporate managers. Bond, Edmans and Goldstein (2012) provide a comprehensive survey of this research. A second strand, concerned with the real effects of accounting measurement, assumes that a priori corporate managers possess information that markets do not have. In this second strand, a manager’s incentive to choose among various courses of action depends upon how prices in financial markets respond to these actions. Price response, in turn, depends upon the information in the capital
market as augmented by the information contained in accounting measurements. For example, in Kanodia, Sapra and Venugopalan (2004), a firm’s incentive to invest in intangible assets depends upon the observability of such investments by investors in the capital market which, in turn, depends at least partially upon the accounting treatment of intangible assets. A general discussion and survey of this literature is contained in Kanodia (2006) and Kanodia and Sapra (2016). In the current paper, we do not model capital markets, so the real effects we study here are not price mediated. Instead, we model a setting where the payoff to actions taken by external agents, like the firm’s customers or the firm’s suppliers, depends directly upon the financial strength of the firm. These external agents, therefore, demand and use accounting reports based on MTM measurements to assess the financial state of the firm and these assessments guide their actions. Collectively, these actions feed into the firm’s financial performance and wealth. This approach allows us to succinctly capture both the decision facilitating role and the wealth transformation effects of accounting information.

The remainder of this paper is organized as follows. Section 2 describes the economic setting that we analyze and sections 3 and 4 derive the equilibrium for the setting by working backwards from later actions to earlier actions. Section 3 derives the equilibrium behavior of external agents who use the information provided by MTM accounting and act after this information is released. Section 4 characterizes the equilibrium actions of the firm taken in anticipation of the equilibrium response of external agents to the release of MTM information. Section 5 contains a welfare analysis of the equilibrium and develops the tradeoffs, suggested by our model, that regulators should consider in assessing the desirability of MTM accounting. Section 6 concludes.

2. The Economic Setting

There are three dates, 0,1 and 2. Date 0 is an initial date, date 1 is an interim date, and date 2 is the final date. At the initial date, the firm chooses a portfolio of assets to hold. These assets change
value over time and finally payoff at the terminal date becoming part of the firm’s wealth at that date.

In a fair value accounting regime, the market value of the firm’s assets are estimated (or observed) at the interim date, and this estimated market value is the accounting signal that is conveyed to the firm’s external stakeholders. These stakeholders use the information to assess the firm’s terminal wealth and then choose their individual actions. The aggregate of these stakeholder actions feed into the firm’s terminal wealth.

We use the firm’s customers as its external stakeholders. The firm produces a single good, and there are a continuum of customers, uniformly distributed over the unit interval, who individually decide how much of the firm’s good to buy. Let:

\[ q_i = \text{purchase order placed by customer } i. \]

\[ Q = \int_0^1 q_i \, di = \text{the aggregate of customer orders.} \]

Each customer is so small that her individual purchase has no measurable effect on the firm’s terminal wealth, but the aggregate of customer orders has a very significant effect.

We model the firm’s asset portfolio choice in a simple way. The firm begins at date 0 with an endowment of \( m \) units of a riskless asset. One unit of the riskless asset held until the terminal date produces one unit of wealth at the terminal date. However, the firm has the opportunity to convert some or all of its endowment into a risky illiquid asset whose prior expected return is greater than that of the riskless asset. Let \( z \) be the amount that the firm chooses to invest in the risky asset at date 0 and let \( z\tilde{\theta} \) be the return at date 2. The firm’s ex post terminal wealth is:

\[ w = m - z + z\tilde{\theta} + Q \]  \hspace{1cm} (1)
Thus, the firm’s terminal wealth depends partly upon a decision made by the firm’s inside manager and partly upon the aggregate of decisions made by a continuum of outside stakeholders. These decisions are made sequentially. The inside decision is made prior to the realization of the fair value signal, while external decisions are made after receipt of the fair value signal.

We assume that except for informational differences, all customers are identical. The payoff to a customer for ordering from our incumbent firm depends partly upon a known parameter $\eta$ that describes how well the characteristics of the good produced by the firm match the needs of its customers, and partly upon the financial strength of the firm. The ex post payoff to a customer who places an order of size $q_i$ is:

$$ u_i = Aq_i - \frac{1}{2}q_i^2 $$

(2)

where $\frac{1}{2}q_i^2$ can be interpreted as the cost of using the good in whatever manner the customer uses it. The marginal benefit to a customer from purchasing its needs from the incumbent firm is described by:

$$ A \equiv \tau \eta + (1 - \tau)w, $$

where $0 < (1 - \tau) < 1$ describes the relative extent to which customers are affected by the financial strength of the firm that supplies them. Customers make their purchase decisions

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4 The model of customers used here is a variation on the model of individual investment decisions with strategic complementarities in Angeletos and Pavan (2004).

5 That customers are reluctant to place orders with suppliers who are perceived to be financially weak, is a well known empirical phenomenon. General Motors was faced with this predicament during the recent financial crisis. A possible reason for this phenomenon is that the benefit to a customer from today’s purchase depends partially upon future supplies of goods and services by the incumbent supplier, and the supplier’s ability to perform in the future is affected by his current financial strength. Another possibility is that firms that are
after receiving the fair value signal provided by the accounting system. The fair value of the risky asset at this interim date provides noisy information about its terminal value and therefore the fair value signal is incrementally informative about the terminal wealth of the firm.

3. Customers' Purchase Decisions

Let $E_i(\bar{A})$ be customer $i$'s expectation of $\bar{A}$ conditional on the information she receives at date 1. Then, the order placed by customer $i$ is the unique solution to:

$$\max_{q_i} \ E_i(\bar{A})q_i - \frac{1}{2}q_i^2$$

(3)

The first order condition to (3) yields:

$$q_i = E_i(\bar{A}) = \tau \eta + (1 - \tau)[m - z + zE_i(\hat{\theta}) + E_i(\hat{Q})]$$

(4)

The presence of $E_i(\hat{Q})$ in the first order condition for $q_i$ implies the presence of strategic complementarities in customer purchase decisions. The marginal benefit to purchasing from the firm is higher the higher the quantity that other customers purchase from the firm.

Expectations of $\hat{\theta}$ are determined by standard Bayesian updating of the prior distribution of $\hat{\theta}$. But expectations of the aggregate order $\hat{Q}$ are a much more complex object. These latter expectations depend upon what customer $i$ thinks other customers will do, and therefore on $i$’s beliefs of the beliefs of other customers and $i$’s beliefs of other customers’ beliefs of other customers beliefs, and so on. As is standard in the literature on higher order beliefs,
$E_i(\tilde{Q})$ is calculated iteratively, and is described by an infinite hierarchy of higher order beliefs of $\tilde{\theta}$. We construct this hierarchy below.

Since $Q = \int_0^1 q_i \, di$, it follows from the first order condition (4) that:

$$Q = \tau \eta + (1 - \tau)(m - z) + (1 - \tau)z \int_0^1 E_i(\tilde{\theta}) \, di + (1 - \tau) \int_0^1 E_i(\tilde{Q}) \, di$$  \hspace{1cm} (5)

We refer to $E_i(\tilde{\theta})$ as the first order belief of customer $i$, and $\int_0^1 E_i(\tilde{\theta}) \, di$ as the average first order belief about $\theta$ in the population of customers. No customer knows what this average belief is, but each customer can form a belief of this average belief which I denote by $\int_0^1 E_i(\tilde{Q}) \, di$. From (5),

$$E_i(\tilde{Q}) = \tau \eta + (1 - \tau)(m - z) + (1 - \tau)z E_i(\tilde{\theta}) + (1 - \tau) E_i \int_0^1 E_j(\tilde{Q}) \, dj$$  \hspace{1cm} (6)

In (6) the expression $\int_0^1 E_j(\tilde{Q})d\eta$ is conceptually well defined since it is customer $i$'s belief of the average belief of $Q$ in the customer population, but we don't yet know how to calculate it.

Inserting (6) into the customer's first order condition yields:

$$q_i = \tau \eta + (1 - \tau)(m - z) + (1 - \tau)z E_i(\tilde{\theta}) + (1 - \tau) \tau \eta + (1 - \tau)^2 (m - z) + (1 - \tau)^2 z E_i \int_0^1 E_j(\tilde{\theta}) \, dj + (1 - \tau)^2 E_i \int_0^1 E_j(\tilde{Q}) \, dj$$  \hspace{1cm} (7)
Integrating the expression in (7) over the customer population yields:

\[
Q = \tau \eta + (1 - \tau)(m - z) + (1 - \tau)z \int_0^1 E_i(\tilde{\theta})di + (1 - \tau)\tau \eta + (1 - \tau)^2(m - z) + (1 - \tau)^3(z - m) + \int_0^1 (1 - \tau)^2z \int_0^1 E_i(\tilde{\theta})di + (1 - \tau)^2 \int_0^1 E_i(\tilde{\theta})djdi + (1 - \tau)^3 \int_0^1 E_i(\tilde{\theta})dijdj \int_0^1 E_j(\tilde{Q})djidj
\]

In (8) the expression \(\int_0^1 E_i(\tilde{\theta})djdi\) is the average expectation of the average expectation of \(\theta\) in the customer population. We refer to it as the average second order expectation of \(\theta\). Now, (8) can be used to obtain an updated calculation of \(E_i(Q)\) and this updated expression for \(E_i(Q)\) can be inserted into the customer's first order condition (4) to yield an updated expression for \(q_i\). Integrating this updated expression for \(q_i\) yields the following updated expression for the aggregate order quantity \(Q\).

\[
Q = \tau \eta + (1 - \tau)\tau \eta + (1 - \tau)^2\tau \eta + (1 - \tau)(m - z) + (1 - \tau)^2(m - z) + (1 - \tau)^3(m - z) + (1 - \tau)^2z \int_0^1 E_i(\tilde{\theta})di + (1 - \tau)^2 \int_0^1 E_i(\tilde{\theta})djdi + (1 - \tau)^3 \int_0^1 E_i(\tilde{\theta})dijdj + \int_0^1 \int_0^1 E_i(\tilde{\theta})djdkdi + (1 - \tau)^3 \int_0^1 \int_0^1 E_i(\tilde{\theta})djdkdi
\]

Comparing (5), (8), and (9) it is clear that repeated iteration yields:

\[
Q = \tau \eta[1 + (1 - \tau) + (1 - \tau)^2 + ....... ] + (1 - \tau)(m - z)[1 + (1 - \tau) + (1 - \tau)^2 + ....... ] + \int_0^1 (1 - \tau)z[\theta^{(1)} + (1 - \tau)\theta^{(2)} + (1 - \tau)^2 \theta^{(3)} + ....... ]
\]
where, $\theta^{(t)}$, $t = 1, 2, 3, \ldots$ denotes the average $t$ th order expectation of $\theta$. Since $0 < (1 - \tau) < 1$, each of the infinite series contained in (10) is convergent and well defined. Carrying out the summation yields the final expression:

$$Q = \frac{\tau \eta + (1 - \tau)(m - z)}{\tau} + (1 - \tau)z \sum_{t=0}^{\infty} (1 - \tau)^t \theta^{(t+1)}$$  \hspace{1cm} (11)

Notice from (11) that the undefined expectations of $Q$ have vanished and have been replaced by well defined higher order expectations of $\theta$.

We assume that the information structure in the economy is as follows. The commonly known prior distribution of $\tilde{\theta}$ is Normal with mean $\mu$ and variance $\frac{1}{\alpha}$. Equivalently,

$$\tilde{\theta} = \mu + \tilde{\xi}, \quad \tilde{\xi} \simeq N(0, \frac{1}{\alpha}).$$ We assume $\mu > 1$ so that investment in the risky asset is a priori desirable. Both historical cost accounting and fair value accounting reveal the amount $z$ of investment in the risky asset but, at date 1, fair value accounting provides an additional signal that is not provided by historical cost accounting. Fair value accounting provides an estimate of the date 1 value of the risky asset. We assume that the fair value estimate at the interim date is a noisy version of its terminal value, which is $z\theta$. Since $z$ is known, we can model the fair value signal as:

$$\tilde{y} = \theta + \tilde{e}, \quad \tilde{e} \simeq N(0, \frac{1}{\beta})$$
Higher values of $\beta$ represent more precise measurement, and the lowest value of $\beta$, i.e. $\beta = 0$ is equivalent to not providing any fair value information. Therefore $\beta = 0$ is representative of historical cost accounting. In addition to the public fair value signal, customers may have private sources of information about the return to the risky asset. We model this as private unbiased signals, $x_i$, with some common precision $\gamma$:

$$x_i = \theta + \tilde{\omega}_i, \quad \tilde{\omega}_i \sim N(0, \frac{1}{\gamma})$$

We think that a setting with both public and private information is interesting in its own right, as it captures the realistic idea that in the absence of a public source of information, individual agents will have idiosyncratic beliefs regarding uncertain variables. We assume that all of the noise terms, $\xi, \tilde{\epsilon},$ and $\tilde{\omega}_i$ are independent of each other and independent of $\tilde{\theta}$.

We now proceed to derive the equilibrium aggregate order quantity $Q$ and individual beliefs $E_i(Q)$ for the specific information structure described above. The first order belief of customer $i$ is:

$$E_i(\tilde{\theta}) = \frac{\alpha \mu + \beta y + \gamma x_i}{\alpha + \beta + \gamma} \quad (12)$$

It is convenient to rewrite the above expression in a different way. Let

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6 There is also a technical motivation for introducing private signals. A game with strategic complementarity and homogenous beliefs generally has multiple equilibria. As shown by Carlsson and Van Damme (1993), the introduction of private information gets rid of the multiplicity and produces a unique equilibrium. Additionally, the limiting unique equilibrium as the precision of the private signal goes to zero, is commonly used as predictive of what would be observed in settings with only public information. Heinemann, Nagel and Ockenfels (2004) provide empirical support for this latter claim.
\[ \delta \equiv \frac{\gamma}{\alpha + \beta + \gamma}, \text{ and} \]
\[ p \equiv \frac{\alpha \mu + \beta \gamma}{\alpha + \beta} \]

Then (12) is equivalent to:
\[ E_i(\hat{\theta}) = \delta x_i + (1 - \delta)P, \quad (13) \]

where \( P \) can be thought of as the public information in the economy and \( x_i \) as the private information of customer \( i \). The average first order belief of \( \hat{\theta} \) is:

\[ \theta^{(1)} \equiv \int E_i(\theta)di = \int (\delta x_i + (1 - \delta)P)di = \delta \theta + (1 - \delta)P, \]

from which it follows that \( i \)'s belief of the average first order belief is:

\[ E_i \int E_j(\theta) dj = \delta E_i(\theta) + (1 - \delta)P \]
\[ = \delta(\delta x_i + (1 - \delta)P) + (1 - \delta)P \]
\[ = \delta^2 x_i + (1 - \delta^2)P \]

Therefore the average second order belief of \( \theta \) is:

\[ \theta^{(2)} \equiv \int E_i \int E_j(\theta) dj di = \delta^2 \theta + (1 - \delta^2)P \]
Iterating in this way gives the average \( t \)th. order expectation of \( \theta \):

\[
\theta^{(t)} = \delta^t \theta + (1 - \delta^t)P
\]  

(14)

Notice that the weight on the fundamental \( \theta \) decreases and the weight on the public signal \( P \) increases in successively higher order beliefs. This implies that the aggregate order received by the firm over-weights the public signal and is insufficiently sensitive to the fundamental \( \theta \).

This phenomenon is common to settings with higher order beliefs (see Morris and Shin (2002), and Angeletos and Pavan (2004)). In the specific context we are modeling, what this implies is that the error contained in the accounting estimate of fair value has a magnified influence on the firm’s terminal wealth.

Inserting (14) into the general expression for \( Q \) that was derived in (11), determines the aggregate demand for the firm’s good, under the specific information structure that we have modeled. Also, from this expression for \( Q \) the individual values of \( E_i(Q) \) and \( q_i \) can be calculated. These calculations yield:

**Proposition 1:**

*The equilibrium response of the firm’s customers to the fair value accounting signal is:*\[
q_i = \frac{1}{\tau} \left\{ \tau \eta + (1 - \tau)(m - z) + (1 - \tau)z \left( \lambda x_i + (1 - \lambda)P \right) \right\}
\]  

(15)

and,
\[ Q = \frac{1}{\tau} \left\{ \tau \eta + (1-\tau)(m-z) + (1-\tau)z \left( \lambda \theta + (1-\lambda)P \right) \right\} \] 

(16)

where,

\[ \lambda = \frac{\tau \delta}{1-(1-\tau)\delta} \]

**Proof:** See the Appendix.

Because \( \left( \frac{\tau}{1-(1-\tau)\delta} \right) < 1, \forall \delta > 0, \tau < 1 \), the equilibrium weight on \( x_i \) in (15) and on \( \theta \) in (16) is strictly less than \( \delta \), which is the weight that would be used in Bayesian updating. This confirms the earlier intuition that, because of the need to assess the beliefs of others, each individual customer under-weights his private information about \( \theta \) and over-weights the public information in deciding how much to order from the firm. In turn, this results in the equilibrium aggregate order quantity \( Q \) becoming less sensitive to fluctuations in the fundamentals \( \theta \) and overly sensitive to the public information provided by fair value accounting. The effect of this distortion on social welfare will be developed in a later section.

4. The Firm’s Asset Allocation Decision:

We now turn to the firm’s asset portfolio choice to be made at date 0. As specified earlier, the firm’s terminal wealth is \( w = m-z+z\theta+Q \). We assume the firm is risk averse with constant absolute risk aversion \( \rho > 0 \). If \( \tilde{w} \) is distributed Normal, as will be the case, the firm’s objective function is:
\[ \text{Max}_z \left\{ E(\tilde{w}) - \frac{1}{2} \rho \text{Var}(\tilde{w}) \right\} \]  

(17)

From the perspective of date 0, the aggregate order received by the firm at date 1 is a random variable because individual customer orders depend upon the public fair value signal and the private signals that customers receive at date 1. At date 0 these signals are random variables.

The equilibrium value of \( \tilde{Q} \), as determined in (16), is distributed Normal since it depends linearly on the Normally distributed return \( \tilde{\theta} \) as well as on the Normally distributed fair value report \( \tilde{y} \) that is released later at date 1.

Using the facts that \( E(\tilde{P}) = E(\tilde{\theta}) = \mu \), we obtain from (16):

\[ E(\tilde{Q}) = \frac{\tau \eta + (1 - \tau)[m + z(\mu - 1)]}{\tau} \]  

(18)

and,

\[ E(\tilde{w}) = m + z(\mu - 1) + E(\tilde{Q}) = \frac{\tau \eta + m + z(\mu - 1)}{\tau} \]  

(19)

The firm’s investment in the risky asset affects its expected terminal wealth in two ways. There is a direct effect and an indirect effect. Since \( \mu > 1 \), the direct effect of increased investment in the risky asset is to increase the expected net return on this asset. The indirect effect operates through the firm’s customers. A customer’s response to the firm’s investment in the risky asset depends upon both the private signal and the public fair value signal she receives at date 1. Unfavorable signals decrease a customer’s demand and favorable signals increase her
demand. The private signals received by customers affect individual demands but not the aggregate demand that feeds into the firm’s wealth. This is because variations in private signals are independent over the customer population and so cancel out in the aggregate. However, the public fair value signal is perfectly correlated across customers, so its effect on customer demand is like a systematic shock that persists over the customer population. If the fair value signal is favorable, then the firm’s investment in the risky asset increases aggregate customer demand for the firm’s good, but if the fair value signal turns out to be unfavorable then aggregate demand is decreasing in the risky investment. Since the fair value signal is unbiased, the news is expected to be neutral, and neutral news is good news because $\mu > 1$. So, in expectation, aggregate customer demand for the firm’s good is increasing in the amount invested in the risky asset.

But, the fair value signal also induces volatility in the aggregate demand for the firm’s good and this volatility is increasing in the amount invested in the risky asset. Below, we characterize the risk in the firm’s terminal wealth from a date 0 perspective.

$$var(\tilde{w}) = z^2 var(\tilde{\theta}) + var(\tilde{Q}) + 2z cov(\tilde{\theta}, \tilde{Q})$$  \hspace{1cm} (20)

The first term in (20), $z^2 var(\tilde{\theta})$, captures the direct effect of higher investment in the risky asset on the risk in the firm’s wealth. The second and third terms derive from customer assessments of the firm’s wealth and the consequent change in their order decisions due to observation of the fair value signal. We assess (20) term by term.

$$var(\tilde{\theta}) = var(\tilde{\xi}) = \frac{1}{\alpha}$$

From (16):
\[ \text{var}(\tilde{Q}) = \left(\frac{1-\tau}{\tau}\right)^2 z^2 \text{var}[\lambda \tilde{\theta} + (1-\lambda)\tilde{P}] \]

\[ = \left(\frac{1-\tau}{\tau}\right)^2 z^2 [\lambda^2 \text{var}(\tilde{\theta}) + (1-\lambda)^2 \text{var}(\tilde{P}) + 2\lambda(1-\lambda)\text{cov}(\tilde{\theta}, \tilde{P})] \quad (21) \]

where,

\[ \text{var}(\tilde{P}) = \left(\frac{\beta}{\alpha + \beta}\right)^2 \text{var}(\tilde{y}) \]

\[ = \left(\frac{\beta}{\alpha + \beta}\right)^2 \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \left(\frac{\beta}{\alpha + \beta}\right) \frac{1}{\alpha} \]

and,

\[ \text{cov}(\tilde{\theta}, \tilde{P}) = \text{cov}\left(\tilde{\theta}, \left(\frac{\beta}{\alpha + \beta}\right)\tilde{y}\right) \]

\[ = \left(\frac{\beta}{\alpha + \beta}\right) \text{var}(\tilde{\theta}) = \left(\frac{\beta}{\alpha + \beta}\right) \frac{1}{\alpha} \]

Inserting these last two calculations into (21) gives:

\[ \text{var}(\tilde{Q}) = \left(\frac{1-\tau}{\tau}\right)^2 z^2 \frac{1}{\alpha} \left[\lambda^2 + (1-\lambda)^2 \left(\frac{\beta}{\alpha + \beta}\right) + 2\lambda(1-\lambda)\left(\frac{\beta}{\alpha + \beta}\right)\right] \]

which simplifies to:

\[ \text{var}(\tilde{Q}) = \left(\frac{1-\tau}{\tau}\right)^2 z^2 \frac{1}{\alpha} \left[\lambda^2 + (1-\lambda)^2 \left(\frac{\beta}{\alpha + \beta}\right)\right] \quad (22) \]

Lemma 1:
i. Given any positive investment in the risky asset, the more precise is the fair value signal the greater is the uncertainty in the aggregate demand for the firm’s good, and

ii. Given any fixed precision in the fair value signal, the uncertainty in the aggregate demand for the firm’s good is strictly increasing in the amount invested in the risky asset.

Proof: See the Appendix.

The result in Lemma 1 is a special case of a quite general phenomenon. Information provided to a decision maker allows her to vary her decision to better fit the circumstances that exist at the time. From an ex ante perspective, such variability in the decision makes the world look more uncertain. The more precise is the information provided to the decision maker the more sensitive will be the decision to that information causing greater uncertainty from an ex ante perspective. Ex ante a decision maker’s action is most predictable if no subsequent information can possibly arrive prior to making that decision. In the context of our model, no information arrival (public or private) prior to the decisions made by customers, is equivalent to \( \beta = \gamma = 0 \). But in this case,

\[
\delta = \frac{\gamma}{\alpha + \beta + \gamma} = 0,
\]

implying that

\[
\lambda = \frac{\tau \delta}{1-(1-\tau)\delta} = 0.
\]

Then, it is immediate from (22) that

\[
\text{var}(\tilde{Q}) \to 0 \quad \text{as} \quad (\beta, \gamma) \to 0.
\]

The remaining term in (20), is:

\[
cov(\tilde{\theta}, \tilde{Q}) = \text{cov} \left( \tilde{\theta}, \left( \frac{1-\tau}{\tau} \right) \frac{z(\lambda \theta + (1-\lambda)\tilde{P})}{z[\lambda \text{var}(\tilde{\theta}) + (1-\lambda)\text{cov}(\tilde{\theta}, \tilde{P})]} \right)
\]

\[
= \left( \frac{1-\tau}{\tau} \right) \frac{z[\lambda \text{var}(\tilde{\theta}) + (1-\lambda)\text{cov}(\tilde{\theta}, \tilde{P})]}{z[\lambda \text{var}(\tilde{\theta}) + (1-\lambda)\text{cov}(\tilde{\theta}, \tilde{P})]}
\]
\[
\frac{1}{\alpha} \left[ \lambda + (1 - \lambda) \left( \frac{\beta}{\alpha + \beta} \right) \right] \quad (23)
\]

which is also strictly increasing in \( \beta \).

Inserting (22) and (23) into (20) gives:

\[
\text{var}(\tilde{w}) = \frac{z^2}{\alpha} \left( \frac{1 - \tau}{\tau} \right)^2 \left[ \lambda^2 + \left( 1 - \lambda^2 \right) \frac{\beta}{\alpha + \beta} \right] + 2z^2 \left( \frac{1 - \tau}{\tau} \right) \left[ \lambda + (1 - \lambda) \frac{\beta}{\alpha + \beta} \right] \quad (24)
\]

**Proposition 2:**

*Keeping fixed the firm’s investment in the risky asset, the more precise is the fair value information provided to outside stakeholders to assist in assessing the firm’s wealth the more uncertain the wealth of the firm becomes from an ex ante perspective.*

**Proof:** See the Appendix.

Proposition 2 is a stark departure from traditional wisdom, since in most economic models the arrival of information decreases uncertainty. But, it is important to ask from whose perspective are we assessing the uncertainty in the environment. It is certainly true that information provided at date 1 decreases the uncertainty faced by economic agents who make decisions at date 1. But, what happens to the uncertainty faced by economic agents who must move earlier, before the information is provided? Given that the decision made by later economic agents is sensitive to the information
provided to them, the earlier economic agents must perceive the decisions made by the later economic agents as random variables and therefore the information actually *increases* the uncertainty they face.

Proposition 2 is also indicative of how misleading accounting disclosure studies could be when the variable of interest is assigned an exogenously specified distribution. In such studies information *always* reduces uncertainty, since statistically a conditional variance is smaller than an unconditional variance. In our study too, if the firm’s wealth is an exogenously given random variable then information used to assess the firm’s wealth can only decrease the uncertainty in wealth. But such a scenario is an over-simplification of the real world. Realistically, a firm’s wealth depends not just upon the state of Nature, but also upon decisions made by both insiders and outsiders. If the disclosure of information alters the decisions of outsiders and if these decisions affect the distribution of the firm’s wealth then it is not necessarily true that information is uncertainty reducing.

We can now characterize the firm’s date 0 investment in the risky asset. Inserting (19) and (24) into the firm’s objective function, as described in (17), and differentiating with respect to \( z \) gives the first order condition:

\[
\tau \rho \frac{1}{\alpha} \left[ 1 + \left( \frac{1 - \tau}{\tau} \right) \left( \lambda^2 + (1 - \lambda^2) \frac{\beta}{\alpha + \beta} \right) + 2 \left( \frac{1 - \tau}{\tau} \right) \left( \lambda + (1 - \lambda) \frac{\beta}{\alpha + \beta} \right) \right] \frac{\mu - 1}{\beta} = 0
\]

Equation (25) indicates that the firm is not passive to the provision of fair value information.

The firm chooses its asset portfolio anticipating the revaluation that occurs at subsequent dates and the effect such revaluations have on the decisions of outsiders. To see the effect of fair value information on the firm’s asset choices let us examine how variations in \( \beta \) affect the firm’s choice of \( z \). From
(25) it is clear that the effect of $\beta$ on $z$ is through the two factors $\lambda^2 + (1 - \lambda^2)\left(\frac{\beta}{\alpha + \beta}\right)$ and 

$$\lambda + (1 - \lambda)\left(\frac{\beta}{\alpha + \beta}\right)$$

contained in the denominator of (25). In the proofs of Lemma 1 and Proposition 2, we established that both factors are strictly increasing in $\beta$. Therefore, the denominator in (25) is strictly increasing in $\beta$, implying that $\frac{\partial z}{\partial \beta} < 0$. Noting that $\beta = 0$ is representative of historical cost, we have:

**Proposition 3**

i. The firm holds a different portfolio of assets in a fair value accounting regime than it would hold in a historical cost regime.

ii. Fair value accounting causes the firm to shift away from assets whose fair values are more volatile and invest more in assets whose fair values are stable.

iii. The more precise is the fair value information the greater the shift in the firm's asset portfolio.

The result described in Proposition 3 is due to the fact that the precision of the fair value information increases not only the prior uncertainty in the aggregate order $Q$ but also increases the marginal effect of $z$ on this uncertainty. The firm decreases its holdings of risky assets in order to decrease the uncertainty in customer orders.

Proposition 3 is an important testable real effect of fair value accounting. It is noteworthy that corporate managers argued against fair value accounting on the grounds that such accounting treatment would increase the volatility of reported income. In order to understand preferences over accounting reports one must derive them from more primitive preferences over real objects. We have
shown here how volatility in reports translates into volatility in the firm’s real income. Disliking volatility in real income, the firm takes defensive measures to counteract the volatility. Such predictions are absent when it is assumed that a firm’s assets are fixed and given and independent of how they are measured and reported to outsiders.

5. Welfare Analysis

Having characterized the equilibrium decisions of both insiders and outsiders, we now turn to the main question of interest: In equilibrium, who benefits and who loses by moving from a historical cost regime to a fair value accounting regime? This question can be answered by examining how variation in the precision of the fair value information affects aggregate welfare. If the welfare of both parties (the firm and its customers) is uniformly declining in $\beta$, then fair value accounting unambiguously decreases social welfare. If the equilibrium payoff to the firm is declining in $\beta$, but the welfare of the firm’s customers is increasing in $\beta$ then there is a conflict of interest, and so on.

Welfare from the Firm’s Perspective:

The firm’s welfare is simply the maximized value of its objective function at the equilibrium value of $\tilde{Q}$. Therefore, the effect of fair value accounting on the firm’s welfare is described by:

$$\frac{\partial}{\partial \beta} \left\{ \text{Max}_z \left( E(\tilde{w}) - \frac{1}{2} \rho \text{var}(\tilde{w}) \right) \right\}$$

where $E(\tilde{w})$ is as described in (19) and var($\tilde{w}$) is as described in (24). Using the envelope theorem, this derivative is:
We have previously established in the proofs of Lemma 1 and Proposition 2 that both

\[
\frac{1}{2} \rho \left[ z^2 \frac{1}{\alpha} \left(1 - \tau \right)^2 \frac{\partial}{\partial \beta} \left( \lambda^2 + (1 - \lambda^2) \frac{\beta}{\alpha + \beta} \right) + 2 z^2 \frac{1}{\alpha} \left(1 - \tau \right) \frac{\partial}{\partial \beta} \left( \lambda + (1 - \lambda) \frac{\beta}{\alpha + \beta} \right) \right]
\]

are strictly increasing in \( \beta \). Therefore the firm’s welfare is strictly decreasing in \( \beta \), which establishes the result:

**Proposition 4**

Firms strictly prefer historical cost to fair value accounting. In a fair value accounting regime the firm’s welfare is strictly decreasing in the precision of the fair value signal.

The decrease in welfare described in Proposition 4 is entirely due to the increase in volatility of the firm’s real income and wealth. Many firms lobbied strongly against the introduction of fair value accounting standards on the grounds that such accounting would increase the volatility of reported income. Proposition 4 parallels such lobbying behavior. FASB, however, dismissed these claims arguing that accounting does not create volatility; it only makes the volatility that is already present more transparent to outsiders. This argument, together with a reading of FASB’s Conceptual Framework for Financial Reporting (2006), indicates that FASB believes that accounting reports only mirror the events that occur in a firm and has no role in shaping those events. Our analysis indicates that FASB’s argument has merit only when the actions taken by a firm’s stakeholders in response to accounting information impacts merely their own payoffs but has no effect on the wealth of the firm. We feel that such a setting is unrealistic. When the actions of outsiders does affect the firm’s wealth, in at least a collective sense, fair value accounting does create additional volatility in the firm’s true
income (not merely reported income), and this increased volatility does have negative economic consequences.

**Welfare from Customers’ Perspective:**

We now examine the effect of fair value accounting on the *ex ante* social welfare of the firm’s customers. As in Angeletos and Pavan (2004), we define the *ex post* social welfare $\Omega$ of the customer population as the aggregate of their individual *ex post* payoffs, i.e.,

$$\Omega \equiv \int u_i(di) = A \int q_i(di) - \frac{1}{2} \int q_i^2(di)$$

We decompose this aggregate welfare in the same manner as Angeletos and Pavan (2004).

Substituting $\int q_i(di) = \bar{Q}$, $\int q_i^2(di) = \left[ \int (q_i - \bar{Q})^2 di + \bar{Q}^2 \right]$, and inserting the expression for $A$ gives:

$$\Omega = \left( \tau \eta + (1 - \tau)(m - z + z \theta) + (1 - \tau)\bar{Q} \right)\bar{Q} - \frac{1}{2} \bar{Q}^2 - \frac{1}{2} \int (q_i - \bar{Q})^2 di$$

or, equivalently,

$$\Omega = \left( \tau \eta + (1 - \tau)(m - z + z \theta) \right)\bar{Q} - \frac{1}{2} (2\tau - 1)\bar{Q}^2 - \frac{1}{2} \int (q_i - \bar{Q})^2 di \quad (26)$$

The expression $\int (q_i - \bar{Q})^2 di$ in (26) indicates that social welfare is enhanced if individual customer purchases are coordinated so that each customer orders exactly the same amount, i.e. if $q_i = \bar{Q}$, $\forall i$. This social benefit to coordination is due to the convexity in the cost function of individual customers.

If the public fair value signal is the only information available to the firm’s customers then customers would be perfectly coordinated and the last term in (26) would disappear. However, the presence of private information prevents such perfect coordination. In what follows, we assume, as in Angeletos and Pavan (2004), that the weight that customers put on the financial wealth of the firm is not too
large, specifically, \((2\tau - 1) > 0\), i.e., \((1-\tau) < \frac{1}{2}\). This ensures that when individual orders are perfectly coordinated, aggregate customer welfare is not increasing in unbounded fashion with \(Q\).

In order to facilitate interpretation, it is useful to first calculate customer welfare if the information that customers receive perfectly reveals the value of \(\theta\) to all of them. This is equivalent to having \(\beta \to \infty\). In this case:

\[
q_i = A = \tau \eta + (1-\tau)(m-z+z\theta) + (1-\tau)Q, \forall i
\]

implying:

\[
q_i = Q = \frac{1}{\tau} \left[ \tau \eta + (1-\tau)(m-z+z\theta) \right], \forall i
\]

Therefore, from (26):

\[
\Omega(z | \text{perfect information}) = \frac{1}{\tau} \left[ \tau \eta + (1-\tau)(m-z+z\theta) \right]^2 - \frac{1}{2} (2\tau - 1) \frac{1}{\tau^2} \left[ \tau \eta + (1-\tau)(m-z+z\theta) \right]^2
\]

\[
= \frac{1}{2\tau^2} \left[ \tau \eta + (1-\tau)(m-z+z\theta) \right]^2
\]

The \textit{ex ante} welfare of the customer group conditional on perfect information is the expectation of the above expression with respect to \(\theta\):

\[
E(\Omega | z, \text{perfect information}) = \frac{1}{2\tau^2} \left( E[\tau \eta + (1-\tau)(m-z+z\theta)] \right)^2 + \frac{1}{2\tau^2} \text{var} (\tau \eta + (1-\tau)(m-z+z\theta))
\]

\[
= \frac{1}{2\tau^2} \left[ \tau \eta + (1-\tau)(m-z+z\theta) \right]^2 + \frac{1}{2\tau^2} (1-\tau)^2 \frac{1}{\alpha} \quad (27)
\]

The first term in (27) is the expected welfare of customers in a regime where there is no incremental information (private or public) about the return to the risky asset, and the second term is the gain in
expected welfare due to perfect information. The gain is due to the fact that information allows
customers to better fit their real decisions to the true wealth of the firm. Because of the quadratic
nature of aggregate payoffs to customers, the amount of the gain is proportional to the extent of
uncertainty reduction caused by the information.

Now, consider the case of noisy public and private information. Then, working with
$q_i$ and $Q$ as described in (15) and (16), we obtain:

**Proposition 5:**

$$E(\Omega | z, \text{noisy public and private information}) = \frac{1}{2\tau^2} \left[ \tau \eta + (1 - \tau) (m - z + z \mu) \right]^2$$

$$+ \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left[ \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \tau \gamma} - \frac{\tau \gamma (1 - \tau)}{(\alpha + \beta + \tau \gamma)^2} \right]$$

(28)

**Proof:** See the Appendix

As before, the first term in (28) is expected customer welfare in the absence of any
incremental information, while the second term is the change in customer welfare brought about by
noisy public and private information. Holding $z$ fixed, the change in customer welfare is strictly
positive, as shown below, even though the information is noisy and even though the private
component of information results in coordination losses.

$$\frac{1}{\alpha} - \frac{1}{\alpha + \beta + \tau \gamma} - \frac{\tau \gamma (1 - \tau)}{(\alpha + \beta + \tau \gamma)^2} > \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \tau \gamma} - \frac{\tau \gamma (1 - \tau)}{\alpha (\alpha + \beta + \tau \gamma)}$$

$$= \left( \frac{\beta + \tau^2 \gamma}{\alpha + \beta + \tau \gamma} \right) \frac{1}{\alpha} > 0$$

(29)
The overweighting of public information caused by higher order beliefs makes the gain from fair value accounting smaller than it would otherwise be. To see this, compare (28) to (27). When the public fair value information is infinitely precise, as in (27), private information becomes redundant so there is no overweighting of public information. In this case the gain from fair value accounting is proportional to \( \text{var}(\theta) = \frac{1}{\alpha} \) since this is the amount of uncertainty eliminated by the information that is being provided. With noisy public and private information the residual uncertainty after providing information is \( \text{var}(\theta | x_i, y) = \frac{1}{\alpha + \beta + \gamma} \), so that the uncertainty eliminated by the information provided is \( \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \gamma} \). But, the benefit from fair value information is proportional to:

\[
\frac{1}{\alpha} - \left( \frac{1}{\alpha + \beta + \gamma} + \frac{\tau \gamma (1 - \tau)}{(\alpha + \beta + \gamma)^2} \right) < \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \gamma}
\]

because,

\[
\frac{1}{\alpha + \beta + \gamma} - \frac{1}{\alpha + \beta + \gamma} - \frac{\tau \gamma (1 - \tau)}{(\alpha + \beta + \gamma)^2} =
\]

\[
\frac{(\alpha + \beta + \gamma)^2 - (\alpha + \beta + \gamma)(\alpha + \beta + \gamma) - \tau \gamma (1 - \tau)(\alpha + \beta + \gamma)}{(\alpha + \beta + \gamma)(\alpha + \beta + \gamma)^2} < 0
\]

The full benefit from uncertainty reduction is not obtained precisely because public information is over-weighted and private information is under-weighted.

The following result is immediate from (28):

**Proposition 6:** If the firm’s asset portfolio remains fixed, customer welfare is strictly increasing in the precision of fair value information.
Proposition 6 succinctly captures FASB’s argument in support of fair value accounting, viz., fair value information is relevant to external users of accounting statements and improves their decisions. If asked at the time they choose their decisions, each of our customers would express a positive demand for fair value accounting and would want the information to be as precise as possible. This is not surprising. At the time that a customer needs to act, she would view the firm’s assets as sunk and the size of the collective order quantity $Q$ as an exogenous variable that is beyond her control. Therefore, from the perspective of each individual customer, at date 1, the wealth of the firm is like a state of Nature and Blackwell’s theorem would apply.

It is tempting for accounting regulators, charged with maximizing the welfare of external users, to accede to their sequentially rational demands. But, the larger wisdom requires regulators to take an *ex ante* perspective, and from such a perspective the firm’s assets cannot be viewed as fixed and sunk. In Proposition 3, we established that the firm anticipates the increased volatility caused by the fair valuing of its assets at a later date, and counters the increased volatility by shifting its portfolio away from the risky asset. The more precise is the fair value information the greater the shift in the firm’s asset portfolio. So the welfare result described in Proposition 6, while true in a partial equilibrium sense, may no longer be true when the decline in $z$ is taken into account. Given that $\mu > 1$, customer welfare is strictly increasing in $z$ as can be seen from visual inspection of (28) and the inequality in (29). Therefore, from an *ex ante* perspective, an increase in the precision of the fair value signal generates two *opposing* effects: It enables better decisions which increase customer welfare, but it also causes lower investment in the risky asset which decreases customer welfare. Whether or not customers are better off, in an overall sense, depends on which of these two effects dominate. Below, we investigate the net effect on customer welfare.

The *ex ante* welfare of customers, as derived in (28) consists of two additive terms. The first term in (28) depends on $\beta$ only through $z$ and is increasing in $z$. Since $z$ declines as $\beta$ is
increased, the first term in (28) is unambiguously declining in $\beta$. The second term in (28) captures the two opposing effects described earlier. The parenthetical expression

$$\left[ \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \tau \alpha} - \frac{\tau \gamma (1 - \tau)}{(\alpha + \beta + \tau \alpha)^2} \right]$$

captures the decision facilitating benefit of fair value information. It is strictly positive, as shown in (29) and is strictly increasing in $\beta$. But when $z$ is decreased the firm’s expected wealth is lower and this reduces the expected benefit to the consumer from *every* decision she makes. The factor multiplying the parenthetical expression in the second term of (28) describes this welfare reducing effect of increasing the precision of fair value information caused by the fact that $z$ declines when $\beta$ is increased. The net effect of increasing the precision of fair value information could thus go either way. To gauge the net effect, insert the equilibrium value of $z$, as derived in (25) into the second term of (28). After considerable algebraic simplification, and

using the facts that $\lambda + (1 - \lambda) \left( \frac{\beta}{\alpha + \beta} \right) = \frac{\beta + \tau \gamma}{\alpha + \beta + \tau \gamma}$ as shown in the proof of Proposition 2, and

$$\lambda^2 + (1 - \lambda^2) \left( \frac{\beta}{\alpha + \beta} \right) = \frac{(\beta + \tau \gamma)(\alpha + \beta + \tau \gamma) - \alpha \tau \gamma}{(\alpha + \beta + \tau \gamma)^2}$$

as shown in the proof of Lemma 1, this substitution for $z$ into the second term of (28) yields:

$$\frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left[ \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \tau \alpha} - \frac{\tau \gamma (1 - \tau)}{(\alpha + \beta + \tau \alpha)^2} \right] =$$

$$\left( \frac{(1 - \tau)^2}{2\tau^2} \right) \left( \frac{\mu - 1}{\tau^2 \rho^2} \right) \left[ \frac{(\alpha + \beta + \tau \gamma)(\beta + \tau \gamma) - \alpha \tau \gamma (1 - \tau)}{(\alpha + \beta + \tau \gamma)^2 \left[ 1 + \left( \frac{1 - \tau^2}{\tau^2} \right) \left( \frac{\beta + \tau \gamma}{\alpha + \beta + \tau \gamma} \right) \left( \frac{1 - \tau}{\tau} \right) \left( \frac{\alpha \tau \gamma}{(\alpha + \beta + \tau \gamma)^2} \right) \right]^2} \right]$$

(30)
We wish to study the behavior of (30) with respect to variations in $\beta$. Unfortunately the effect of $\beta$ on (30) is complex and ambiguous. This implies that the overall effect of increasing the precision of fair value information on customer welfare is parameter specific.

However, considerable insight is obtained by suppressing private information and examining consumer welfare when the only information available to consumers is the public fair value information. Algebraically, this is equivalent to letting the precision of private signals converge to zero. Letting $\gamma \to 0$ in (25) and (30) yields:

$$z \to \frac{\mu - 1}{\tau \rho \left( \frac{1}{\alpha} \left( 1 + \frac{1 - \tau^2}{\tau^2} \left( \frac{\beta}{\alpha + \beta} \right) \right) \right)}, \quad (31),$$

and

$$\frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left[ \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \gamma} - \frac{\tau \gamma (1 - \tau)}{(\alpha + \beta + \gamma)^2} \right] \to \frac{(1 - \tau)^2 z^2}{2\tau^2} \left( \frac{\beta}{\alpha + \beta} \right) \frac{1}{\alpha} \quad (32)$$

Substituting (31) into (32), the second term of (28) can be expressed as:

$$\frac{(1 - \tau)^2 (\mu - 1)^2}{2\tau^2 \rho^2} \frac{\beta}{\alpha + \beta} \left[ \frac{1}{\alpha} \left( 1 + \frac{1 - \tau^2}{\tau^2} \left( \frac{\beta}{\alpha + \beta} \right)^2 \right) \right].$$

We derive, below, the precise conditions under which this expression is decreasing in $\beta$. Let:
If $L(\beta)$ is increasing in $\beta$ then both the first and second terms of customer welfare are monotone decreasing in the precision of the fair value signal, so the provision of fair value information decreases the welfare of the customer group. Simplifying the expression for $L(\beta)$ gives:

$$L(\beta) = \frac{1}{\alpha} \left[ 1 + \left( \frac{1 - \tau^2}{\tau^2} \right) \left( \frac{\beta}{\alpha + \beta} \right) \right]^2$$

Differentiating gives:

$$\frac{\partial L}{\partial \beta} = -\frac{1}{\beta^2} + \left( \frac{1 - \tau^2}{\tau^2} \right)^2 \frac{1}{(\alpha + \beta)^2}$$

Therefore,

$$\frac{\partial L}{\partial \beta} > 0 \text{ if and only if } \frac{1 - \tau^2}{\tau^2} > \frac{\alpha + \beta}{\beta}$$

or, equivalently

$$\frac{\partial L}{\partial \beta} > 0 \text{ if and only if } \frac{\alpha}{\beta} < \frac{1}{\tau^2} - 2$$  \hspace{1cm} (33)
When (33) is satisfied, and the effect of providing fair value information on the firm’s asset portfolio is taken into account, an increase in the precision of fair value information must decrease customer welfare. Therefore, when the change in the firm’s asset portfolio is taken into account, a necessary (but, but sufficient) condition for fair value information to be welfare increasing is that (33) is not satisfied, i.e.,

$$\frac{\alpha}{\beta} > \frac{1}{\tau^2} - 2$$ \hspace{1cm} (34)

Inequality (34) implies that there is an upper bound to the precision of fair value information beyond which customer welfare is guaranteed to decrease. Now, the right hand side of (34) is strictly decreasing in $\tau$, so it is strictly increasing in $(1 - \tau)$ where $(1 - \tau)$ is the weight that customers put on the firm’s wealth in assessing the marginal benefit to buying from the firm. Thus, we have the following result:

**Proposition 7:**

The greater is the relevance of firm wealth to customer decisions (i.e. the greater the value of $(1 - \tau)$) the less precise should be the information provided by fair value accounting if the accounting channel is the only source of information to outsiders.

Proposition 7 is very counterintuitive and would make no sense at all if the firm’s wealth is independent of the actions of the external users of accounting information. It begins to make sense only if we take into account the real effects of accounting disclosure. In our setting this real effect occurs in the following way. Greater precision in the information provided by fair value accounting induces greater variability in the actions of outside stakeholders, which causes greater volatility in the
firm’s wealth, which induces the firm to become more cautious in its investment strategy which, in turn, damages the welfare of the firm’s outside stakeholders.

6. Concluding Remarks

The usual debate surrounding MTM accounting is as follows: The FASB and the SEC argue that marking-to-market a firm’s assets and liabilities improves the accuracy of reported equity. Professional managers argue that MTM unrealistically increases the volatility of reported income and reported equity.\(^7\) Both arguments are framed in the space of reports. Preferences over reports are not primitive preferences, so it is difficult to make tradeoffs and welfare comparisons. What we have done, in this paper, is to explicitly identify an important decision to be made by external agents and we model how the payoff to such decisions varies with the financial condition of the firm. This allows us to be precise about how economic agents use MTM information to assess the financial condition of the firm and adjust their decisions accordingly. In turn, this allows us to quantify the welfare consequences of providing a more accurate report on the firm’s equity. On the other side of the argument, we have shown how volatility in reported equity translates into volatility in the firm’s real wealth/equity, and we have characterized a real decision that corporate managers would change in anticipation of this increased volatility. All of this allows us to be explicit about tradeoffs and their net effect on welfare.

Two key ingredients in our model drive the negative welfare results described above. First, the sequential nature of decisions: Outside stakeholders choose their actions after the accounting information arrives, but the firm chooses its action before the accounting measurement occurs and in anticipation of that measurement. Second, both sets of actions affect the object that is of interest to both parties, viz., the firm’s wealth. We conjecture that both these ingredients are present in most

\(^7\) See Beatty, Chamberlain and Magliolo (1996) for an articulation of these arguments.
settings where external financial reporting matters in an informational sense. To the extent that this is true, the approach taken here should be useful for studying many other accounting issues besides fair value accounting. For example, it would be interesting to study accounting for firms’ hedging activities from such a perspective. Notably, our approach allows us to endogenously capture both the costs and benefits of accounting measurement and disclosure, so that there is a meaningful tradeoff that can be studied.

Such endogenous tradeoffs between costs and benefits have been missing in FASB’s approach to standard setting and in much of the research in accounting. FASB’s view, as enunciated in the Conceptual Framework for Financial Reporting (2006) is: “The objective of general purpose external financial reporting is to provide information that is useful to present and potential investors and creditors and others in making investment, credit, and similar resource allocation decisions.” There has been no recognition that firms anticipate and respond to accounting measurements and reports to external parties, and that these anticipatory actions also affect the welfare of investors and other stakeholders. The main tradeoff indicated in FASB positions is a tradeoff between “relevance and reliability” which is difficult to understand in a Bayesian world with rational agents. In such a rational world, if the only objective of financial reports is to improve external decisions, then any measurement that has incremental relevant information is valuable and should be disclosed, regardless of how noisy or unreliable that measurement is. In rational Bayesian updating, noisy measurements are not misleading; they are simply assigned less weight. In academic research, it is all too common to use exogenous compliance costs or exogenous proprietary costs to offset the benefits to disclosing more information.

It may seem that our assumption that customers are risk neutral is a key limitation of our analysis. What if customers are equally risk averse as the firm’s owners? Unfortunately, an explicit analysis of how risk aversion among users would change our welfare results cannot be undertaken in
the setting that we have studied. This is because, risk averse customers would respond to any investment in the risky asset by decreasing their demand for the good that they are purchasing from the firm. Knowing this, the firm would not invest at all in the risky asset, barring some strong and unrealistic assumptions. Thus there would be no assets that could be marked-to-market. The effect of risk aversion among users needs to be studied in a different kind of model. However, the assumption of risk neutrality is not as restrictive as it might seem. As long as the sequential decision making that we have modeled is preserved, it will always be true that the sensitivity of external decisions to the fair value signal will cause a sharply increased volatility in the firm’s real equity. This increased volatility is of concern to the firm’s owners but is of no concern to external decision makers because the uncertainty in the fair value report is completely resolved by the time external agents need to act. So the firm is inherently in a more risky position than the external decision makers that rely upon the accounting information, implying that their preferences over investment in risky assets will generally not coincide.
Appendix

**Proof of Proposition 1:**

First, we calculate the value of $Q$ from (11). From (14) it follows that:

\[
\sum_{t=0}^{\infty} (1-\tau)^t \theta^{(t+1)} = \sum_{t=0}^{\infty} (1-\tau)^t [\delta^{(t+1)} \theta + (1-\delta^{(t+1)})P] 
\]

\[
= \delta \theta \left[ \sum_{t=0}^{\infty} (1-\tau)^t \delta^t \right] + P \sum_{t=0}^{\infty} (1-\tau)^t - \delta P \left[ \sum_{t=0}^{\infty} (1-\tau)^t \delta^t \right] 
\]

\[
= \frac{\delta \theta}{1-(1-\tau)\delta} + \frac{P}{\tau} - \frac{\delta P}{1-(1-\tau)\delta} 
\]

\[
= \frac{1}{\tau} \left[ \left( \frac{\tau \delta}{1-(1-\tau)\delta} \right) \theta + \left( 1 - \frac{\tau \delta}{1-(1-\tau)\delta} \right) P \right] 
\]

Inserting this expression into (11) yields the expression described in (16).

Now, from (16) it follows that:

\[
E_i(Q) = \frac{1}{\tau} \left[ \tau \eta + (1-\tau)(m-z) + (1-\tau)z \left[ \left( \frac{\tau \delta}{1-(1-\tau)\delta} \right) E_i(\theta) + \left( 1 - \frac{\tau \delta}{1-(1-\tau)\delta} \right) P \right] \right] \quad (A1) 
\]

Substituting (A1) into (4) and using $E_i(\theta) = \delta x_i + (1-\delta)P$ gives:

\[
q_i = \left[ \tau \eta + (1-\tau)(m-z) \right] \left[ 1 + \left( \frac{1-\tau}{\tau} \right) \right] + (1-\tau)z[\delta x_i + (1-\delta)P] + 
\]

\[
(1-\tau)z \left( \frac{1-\tau}{\tau} \right) \left[ \left( \frac{\tau \delta}{1-(1-\tau)\delta} \right) \{\delta x_i + (1-\delta)P\} + \left( 1 - \frac{\tau \delta}{1-(1-\tau)\delta} \right) P \right] \quad (A2) 
\]
Collect the terms in (A2) that depend on $x_i$ and the terms that depend on $P$. The term that depends on $x_i$ is:

$$\begin{align*}
(1-\tau)z\delta x_i \left[ 1 + \frac{1-\tau}{\tau} \left( \frac{\tau\delta}{1-(1-\tau)\delta} \right) \right] &= \frac{(1-\tau)z\delta x_i}{1-(1-\tau)\delta} \\
\end{align*}$$

which is convenient to write as:

$$\begin{align*}
= \left( \frac{1-\tau}{\tau} \right) \left( \frac{\tau\delta}{1-(1-\tau)\delta} \right) x_i \\
(A3)
\end{align*}$$

Also in (A2) the terms that depend on $P$ are:

$$\begin{align*}
(1-\tau)zP \left[ (1-\delta) \left\{ 1 + \frac{1-\tau}{\tau} \left( \frac{\tau\delta}{1-(1-\tau)\delta} \right) \right\} + \frac{1-\tau}{\tau} \left( \frac{1-\delta}{1-(1-\tau)\delta} \right) \right] \\
= (1-\tau)zP \left[ \frac{1-\delta}{1-(1-\tau)\delta} + \frac{1-\tau}{\tau} \left( \frac{1-\delta}{1-(1-\tau)\delta} \right) \right] \\
= \left( \frac{1-\tau}{\tau} \right) \left( \frac{1-\delta}{1-(1-\tau)\delta} \right) zP \\
= \left( \frac{1-\tau}{\tau} \right) \left( 1 - \frac{\tau\delta}{1-(1-\tau)\delta} \right) zP \\
(A4)
\end{align*}$$

Inserting (A3) and (A4) into (A2) and simplifying gives:
\[ q_i = \frac{1}{\tau} \left\{ \tau \eta + (1 - \tau)(m - z) + (1 - \tau)z \left[ \left( \frac{\tau \delta}{1 - (1 - \tau)\delta} \right)x_i + \left( 1 - \frac{\tau \delta}{1 - (1 - \tau)\delta} \right)P \right] \right\} \]

as claimed in Proposition 1.

**Proof of Lemma 1:**

Both parts of the Lemma are true if the factor \( \lambda^2 + (1 - \lambda^2)\left( \frac{\beta}{\alpha + \beta} \right) \) is strictly increasing in \( \beta \).

Using \( \lambda = \frac{\tau \delta}{1 - (1 - \tau)\delta} \) and \( \delta = \frac{\gamma}{\alpha + \beta + \gamma} \) gives \( \lambda = \frac{\tau \gamma}{\alpha + \beta + \tau \gamma} \). Therefore the factor:

\[
\lambda^2 + (1 - \lambda^2)\left( \frac{\beta}{\alpha + \beta} \right) = \left( \frac{\tau \gamma}{\alpha + \beta + \tau \gamma} \right)^2 + \left( 1 - \frac{\tau^2 \gamma^2}{(\alpha + \beta + \tau \gamma)^2} \right)\left( \frac{\beta}{\alpha + \beta} \right)
\]

\[
= \frac{1}{(\alpha + \beta + \tau \gamma)^2} \left[ \tau^2 \gamma^2 + [(\alpha + \beta + \tau \gamma)^2 - \tau^2 \gamma^2] \left( \frac{\beta}{\alpha + \beta} \right) \right]
\]

\[
= \frac{\tau^2 \gamma^2 + \beta(\alpha + \beta) + 2\beta \tau \gamma}{(\alpha + \beta + \tau \gamma)^2}
\]

Therefore,

\[
\text{sign} \frac{\partial}{\partial \beta} \left\{ \lambda^2 + (1 - \lambda^2)\left( \frac{\beta}{\alpha + \beta} \right) \right\} =
\]

\[
\text{sign} \left\{ (\alpha + \beta + \tau \gamma)^2(\alpha + 2\beta + 2\tau \gamma) - 2(\alpha + \beta + \tau \gamma)(\tau^2 \gamma^2 + \beta(\alpha + \beta) + 2\beta \tau \gamma) \right\} =
\]

\[
\text{sign} \left\{ (\alpha + \beta + \tau \gamma)((\alpha + \beta + \tau \gamma)(\alpha + 2\beta + 2\tau \gamma) - 2(\tau^2 \gamma^2 + \beta(\alpha + \beta) + 2\beta \tau \gamma)) \right\} =
\]
Proof of Proposition 2:
We have previously argued that from the perspective of date 0,  

\[ \text{var}(\tilde{w}) = z^2 \text{var}(\tilde{\theta}) + \text{var}(\tilde{Q}) + 2z \text{cov}(\tilde{\theta}, \tilde{Q}). \]

In Proposition 2 we are holding \( z \) fixed and \( \text{var}(\tilde{\theta}) \) is a prior variance that is unaffected by the precision of accounting disclosure. We have shown in Lemma 1 that \( \text{var}(\tilde{Q}) \) is strictly increasing in the precision of public disclosure.

Therefore, it suffices to establish that \( \text{cov}(\tilde{\theta}, \tilde{Q}) \) is also strictly increasing in the precision of public disclosure. But, from (20), \( \text{cov}(\tilde{\theta}, \tilde{Q}) \) is strictly increasing in \( \beta \) if the factor

\[ \lambda + (1 - \lambda) \left( \frac{\beta}{\alpha + \beta} \right) \]

is strictly increasing in \( \beta \). Inserting \( \lambda = \frac{\tau_\gamma}{\alpha + \beta + \tau_\gamma} \) gives,

\[ \lambda + (1 - \lambda) \left( \frac{\beta}{\alpha + \beta} \right) = \frac{\tau_\gamma}{\alpha + \beta + \tau_\gamma} + \left( 1 - \frac{\tau_\gamma}{\alpha + \beta + \tau_\gamma} \right) \left( \frac{\beta}{\alpha + \beta} \right) \]

\[ = \frac{\beta + \tau_\gamma}{\alpha + \beta + \tau_\gamma} \]

which is strictly increasing in \( \beta \).

Q.E.D.

Proof of Proposition 5:
As derived in (16):
\[
Q = \frac{1}{\tau} \left\{ \tau \eta + (1 - \tau)(m - z) + (1 - \tau)z \left( \lambda \theta + (1 - \lambda)P \right) \right\}
\]

The term \( \lambda \theta + (1 - \lambda)P \) can be written as:
\[
\lambda \theta + (1 - \lambda) \left( \frac{\alpha (\theta - \theta + \mu) + \beta (\theta + \varepsilon)}{\alpha + \beta} \right) = \theta - (1 - \lambda) \left( \frac{\alpha}{\alpha + \beta} \right) (\theta - \mu) + (1 - \lambda) \left( \frac{\beta}{\alpha + \beta} \right) \varepsilon
\]
\[
= \theta - \left( \frac{\alpha}{\alpha + \beta + \gamma} \right) (\theta - \mu) + \left( \frac{\beta}{\alpha + \beta + \gamma} \right) \varepsilon
\]
where we have used \( (1 - \lambda) = \frac{\alpha + \beta}{\alpha + \beta + \gamma} \) as derived in the proof of Lemma 1. Therefore,
\[
Q = \frac{1}{\tau} \left[ \tau \eta + (1 - \tau)(m - z + z\theta) + (1 - \tau)z \left\{ -\left( \frac{\alpha}{\alpha + \beta + \gamma} \right) (\theta - \mu) + \left( \frac{\beta}{\alpha + \beta + \gamma} \right) \varepsilon \right\} \right]
\]

Now, we use this last expression for \( Q \) to evaluate (26) term by term.
\[
\left[ \tau \eta + (1 - \tau)(m - z + z\theta) \right] Q = \frac{1}{\tau} \left[ \tau \eta + (1 - \tau)(m - z + z\theta) \right]^2 + \frac{1}{\tau} \left[ \tau \eta + (1 - \tau)(m - z + z\theta) \right] \left( (1 - \tau)z \left\{ -\left( \frac{\alpha}{\alpha + \beta + \gamma} \right) (\theta - \mu) + \left( \frac{\beta}{\alpha + \beta + \gamma} \right) \varepsilon \right\} \right]
\]
and,

\[ Q^2 = \frac{1}{\tau^2} \left[ \tau \eta + (1 - \tau)(m - z + \theta) \right]^2 \]
\[ + \frac{2}{\tau} \left[ \tau \eta + (1 - \tau)(m - z + \theta) \right] \left[ (1 - \tau) z \left\{ -\frac{\alpha}{\alpha + \beta + \tau \gamma} (\theta - \mu) + \frac{\beta}{\alpha + \beta + \tau \gamma} \right\} \right] \]
\[ + \left( \frac{1 - \tau}{\tau} \right)^2 z^2 \left\{ -\frac{\alpha}{\alpha + \beta + \tau \gamma} (\theta - \mu) + \frac{\beta}{\alpha + \beta + \tau \gamma} \right\}^2 \]

Also, from (15) and (16),
\[ \int (q_i - Q)^2 \, di = \frac{1}{\tau^2} (1 - \tau)^2 z^2 \left( \frac{\tau \gamma}{\alpha + \beta + \tau \gamma} \right)^2 \int (x_i - \theta)^2 \, di \]

Therefore, (26) can be expressed as:
\[ \Omega = [\tau \eta + (1 - \tau)(m - z + \theta)] Q - \frac{1}{2} (2\tau - 1) Q^2 - \frac{1}{2} \int (q_i - Q)^2 \, di \]
\[ = \frac{1}{2\tau^2} \left[ \tau \eta + (1 - \tau)(m - z + \theta) \right]^2 + \]
\[ \left( \frac{1 - \tau}{\tau^2} \right) \left[ \tau \eta + (1 - \tau)(m - z + \theta) \right] \left[ (1 - \tau) z \left\{ -\frac{\alpha}{\alpha + \beta + \tau \gamma} (\theta - \mu) + \frac{\beta}{\alpha + \beta + \tau \gamma} \right\} \right] \]
\[ - \frac{1}{2\tau^2} (2\tau - 1)(1 - \tau)^2 z^2 \left( -\frac{\alpha}{\alpha + \beta + \tau \gamma} (\theta - \mu) + \frac{\beta}{\alpha + \beta + \tau \gamma} \right)^2 \]
\[ - \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left( \frac{\tau \gamma}{\alpha + \beta + \tau \gamma} \right)^2 \int (x_i - \theta)^2 \, di \]
Taking an expectation over the random variables \( \theta \) and \( \varepsilon \) and using \( E[(\theta - \mu)^2] = \text{var}(\theta) = \frac{1}{\alpha} \)

and \( E[(x_i - \theta)^2] = \text{var}(x_i) = \frac{1}{\gamma} \), gives:

\[
E(\Omega \mid z) = \frac{1}{2\tau^2} \left[ \tau \eta + (1 - \tau)(m - z + z\mu) \right]^2 + \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \frac{1}{\alpha} \\
- \left( \frac{1 - \tau}{\tau^2} \right)(1 - \tau)^2 z^2 \frac{\alpha}{\alpha + \beta + \tau\gamma} E[(\theta)(\theta - \mu)] \\
- \left( \frac{2\tau - 1}{2\tau^2} \right)(1 - \tau)^2 z^2 \left[ \left( \frac{\alpha}{\alpha + \beta + \tau\gamma} \right)^2 \frac{1}{\alpha} + \left( \frac{\beta}{\alpha + \beta + \tau\gamma} \right)^2 \frac{1}{\beta} \right] \\
- \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \frac{\tau^2\gamma}{(\alpha + \beta + \tau\gamma)^2}
\]

Collecting terms and using \( E[(\theta)(\theta - \mu)] = E(\theta^2) - \mu^2 = \text{var}(\theta) = \frac{1}{\alpha} \) gives,

\[
E(\Omega \mid z) = \frac{1}{2\tau^2} \left[ \tau \eta + (1 - \tau)(m - z + z\mu) \right]^2 \\
+ \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left[ \frac{1}{\alpha} - \frac{2(1 - \tau)}{(\alpha + \beta + \tau\gamma)} - (2\tau - 1) \frac{\alpha + \beta}{(\alpha + \beta + \tau\gamma)^2} - \frac{\tau^2\gamma}{(\alpha + \beta + \tau\gamma)^2} \right] \\
= \frac{1}{2\tau^2} \left[ \tau \eta + (1 - \tau)(m - z + z\mu) \right]^2 \\
+ \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left[ \frac{1}{\alpha} - \frac{1}{(\alpha + \beta + \tau\gamma)^2} (2 - 2\tau)(\alpha + \beta + \tau\gamma) + (2\tau - 1)(\alpha + \beta + \tau^2\gamma) \right]
\]
\begin{align*}
&= \frac{1}{2\tau^2} \left[ \tau \eta + (1 - \tau)(m - z + z \mu) \right]^2 \\
&\quad + \frac{1}{2\tau^2} (1 - \tau)^2 z^2 \left[ \frac{1}{\alpha} - \frac{1}{\alpha + \beta + \gamma} - \frac{\tau \gamma (1 - \tau)}{\alpha + \beta + \gamma^2} \right],
\end{align*}

which is the result that we wished to prove.

\textbf{Q.E.D.}
References


Wallison, P.J. 2008b, Judgment too Important to be Left to the Accountants, *Financial Times*, May 1, 2008.
