A Theory of Subprime Mortgage Lending*

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Abstract

We present a general equilibrium model of a subprime economy characterized by limited recourse mortgages, asymmetric borrower credit quality information, and mortgage lenders that either own or sell the loans they originate. Because portfolio lenders can acquire soft information at low cost and are capacity constrained, there is another potential funding source for consumers: the conduit loan market. Conduit lenders originate mortgages based on hard information only, but have access to the securitized investment market. This trade-off between adverse selection and secondary market liquidity determines the equilibrium size of the portfolio and conduit loan markets – in our model consumers can choose between portfolio loans and conduit loans and depending on the parameters of the economy the equilibrium regime may change. Our theory rationalizes the emergence of the subprime conduit mortgage market and subsequent collapse of the traditional lending model, and also the recent rise and fall of the subprime conduit mortgage market. In addition, the model sheds some light on the access to and fragmentation of the rental and owner-occupied segments of the housing market, and also illustrates how house prices respond to changes in the credit scoring technology and mortgage securitization rate, among other things.

Key words: subprime lending; soft information; credit scoring technology; portfolio lenders; conduit lenders; originate-to-distribute lending model; general equilibrium; mortgage market collapses.

JEL Classification numbers: R21, R3, R52, D4, D5, D53.

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1 Introduction

This paper attempts to rationalize the emergence of the subprime conduit mortgage market and its dominance over the traditional relationship lending model, and also the recent rise and fall of subprime mortgage lending. To this end, we propose a theory of the subprime mortgage market that relies on a general equilibrium model of a subprime economy characterized by limited recourse mortgages, asymmetric borrower credit quality information, and two funding sources for consumers: the portfolio mortgage market and the conduit mortgage market.

Portfolio lenders originate-to-own, and as such are subject to lending capacity constraints. They can further be thought of as traditional relationship lenders with a comparative advantage in their ability to acquire soft credit risk information at low cost. In contrast, conduit lenders primarily (but not necessarily exclusively) originate-to-distribute - they are transactions-based -, and heavily rely on observable hard credit information, such as credit history and FICO scores, to evaluate a consumer’s credit risk profile. We exploit these differences in information asymmetry and access to the securitized investment market to provide a model where portfolio and conduit loan rates, house prices, and the sizes of the portfolio and conduit loan markets are all endogenously determined in equilibrium. In addition, the household’s tenure choice (owning versus renting) is also endogenous.

Limited recourse mortgages are another feature of the subprime mortgage market that our model incorporates. The nature of the limited recourse contract protects the subprime borrower from consuming less than a subsistence rent (mortgage exemption). Under this contract a good type consumer (no default risk) can credibly commit to pay back the loan even if the loan repayment is higher than the house value, but a bad type consumer (with default risk) cannot - hence there is a potential for adverse selection. Because bad type borrowers misrepresent their type, they are not able to simultaneously honor their loan payment (designed for a good type borrower) and consume the subsistence rent. Therefore, in our model, the lemons in the conduit mortgage market end up defaulting and giving their housing asset to the lenders. Hence, the limited recourse mortgage is effectively a non-recourse mortgage for the bad type borrowers (the lemons).

Asymmetric borrower credit quality information and the different lenders’ credit scoring technologies are important to understand the pricing of mortgages and the sorting of borrowers into the different mortgage markets. Portfolio lenders have access to soft information, so they can discriminate between consumer types and lend only to good type consumers. The only hope for consumers of bad type is to borrow from conduit lenders, who cannot perfectly screen between consumer types. Also, because portfolio lenders are capacity constrained, not all good type consumers can get a portfolio loan and must go to the alternative funding source: the conduit mortgage market. While portfolio lenders

1Soft information may include listening to and analyzing the borrower’s explanation for past difficulties in making credit payments and determining whether the hard numbers for the borrower or property make sense given what a loan agent can perceive about them. For a discussion of how securitization discourages lenders from engaging in “soft” mortgage underwriting, see "Comments to the Federal Deposit Insurance Corporation" by the National Association of Consumer Advocates on February 22, 2010.

2Subprime conduit mortgages were negotiated on internet based on observable hard credit information, such as credit history and FICO scores. Soft information acquisition was at best limited.
incorporate soft information into the determination of a (borrower specific) risk-based subprime loan rate, conduit lenders recognize that their borrower-lending clientele is lower credit quality on average. Thus, the conduit mortgage rate contains an adverse selection component, captured by the lack of soft information, but also a liquidity component coming from the conduit lender’s access to the securitized investment market. These two components move the conduit loan rate in opposite directions. On the one hand, securitization allows customization (conduit loans are priced using the investors’ time discount rate), which lowers the cost of capital in the conduit loan market. On the other hand, adverse selection in the primary mortgage increases the cost of capital in the conduit loan market. This trade-off between secondary market liquidity and adverse selection is the key driver of the rise and fall of the subprime lending market in our model.

Another important feature of our general equilibrium model is that consumers can choose between portfolio loans and conduit loans. This possibility is absent in previous equilibrium models and is key to determine the equilibrium size of the portfolio and conduit loan markets. We incorporate the consumers’ mortgage market choice in a two-periods economy where mortgage payments are due in the second period. We show that a competitive equilibrium with endogenous segmented markets exists for this economy. Then, to understand the evolution of the subprime mortgage market we consider a sequence of two-period economies where parameters of the economy, and hence the equilibrium regime, may change.

Our model rationalizes the emergence of the subprime conduit mortgage market and dominance over the traditional lending model. Consider a subprime mortgage market composed only by (traditional) portfolio lenders - our model can rationalize this by considering a poor (or non-existent) hard credit scoring technology. When securitization of subprime mortgages found a niche in the secondary securities market and new and better credit scoring technologies, such as FICO scores and consumer’s credit history, became available, conduit lenders were able to attract good type consumers by offering them a better mortgage rate than before, but still at worse terms than portfolio lenders. We identify this equilibrium regime where both portfolio lenders and conduit lenders actively lend to different pools of borrowers at different mortgage rates. When investors’ appetite for subprime mortgage-backed securities further increases (conduit lenders’ mortgage distribution rate increases and investors’ time discount factor decreases), all higher quality borrowers prefer to migrate to the subprime conduit lending market leaving portfolio lenders with a smaller market share. This happens because the conduit mortgage rate decreases below what the traditional portfolio lenders charge for their mortgages and can occur even when the non-traditional conduit lenders relied on an imperfect credit scoring technology and thus some degree of adverse selection is present in the conduit loans market. This boom of subprime credit, driven by the rise and subsequence dominance of the conduit mortgage market, is accompanied in our model by a sharp increase and subsequent jump in house prices.

The subprime conduit mortgage market can also collapse in our model, in a similar way

\footnote{Incorporating the consumers’ mortgage market discrete choice into a two-periods general equilibrium economy with a continuum of agents brings new subtleties to the existence proof, which we discuss in the Appendix. Extending this setting to a fully dynamic infinite-horizon general equilibrium economy with a continuum of agents would considerably be more complicated from a technical point of view. Up to our knowledge this possibility have not been studied yet.}
than the recent bust of subprime mortgage credit. This happens in our model when lending standards seriously deteriorate, investors are more anxious, liquidity in the secondary securities market dries, and foreclosure costs increase. When this happens the subsidy paid by the higher quality borrowers to support a pooling loan rate is so high that discourages home ownership - renting is a preferred option. High credit quality consumers that were not able to borrow from portfolio lenders would then leave the conduit loan market, resulting in the collapse of this market. This lower access to credit lowers the demand for owner-occupied housing and decreases equilibrium house prices in an environment with inelastic housing supply.

Alternatively, we can also rationalize the collapse of the subprime conduit market under the lens of land use regulations - unrelated to the recent financial crisis but still interesting from an urban economic point of view. This happens when land use regulations prevent subprime borrowers with small loans from buying houses with lot size below a minimum threshold. This result illustrates how housing regulations, in the form of costs associated with minimum lot and house size constraints, which are often imposed by local land use regulators, prevent the least well-endowed subprime consumers who cannot afford from purchasing a house with a minimum lot size.

In this paper we also consider a number of extensions of the baseline model to assess its robustness. We show that soft information is endogenously acquired when the mortgage distribution rate is low, which is consistent with our assumption in the baseline model that portfolio lenders have access to soft information, while conduit lenders don’t. We then allow investors who only rely on hard information to buy mortgage-backed securities from lenders that have superior (soft) information, and show that adverse selection in secondary markets lowers the conduit lender’s mortgage rate and that sophisticated portfolio lenders are always the first choice for borrowers. Another interesting insight is that when investors are selected against by informed mortgage originators, investor’s default expectations is lower than their realized default. In another robustness check, we explain that the predictions of the baseline model do not change if we consider instead an stochastic economy with uncertainty in the second period endowment realization or the notion of separating equilibrium. Also, we examine the implications of considering non-recourse mortgage contracts instead and show that non-recourse, by eliminating adverse selection, causes the G-type to delay some consumption until the second period. This is welfare decreasing because households prefer to consume more in the first period and derive more utility from owning a house than renting.

1.1 Relationship with the literature

The literature on collateralized lending with asymmetric information is vast and has expanded rapidly in recent years in light of the subprime mortgage lending and financial crisis. In brief, and at a high level, this paper contributes to the literature that studies how both information frictions and mortgage securitization possibilities affect debt contract design, mortgage originations, securitization, and house prices.\(^4\)

\(^4\)See Jaffee and Russell (1976), Stiglitz and Weiss (1981) and Akerlof (1986) for classic papers on the effects of information frictions on screening, sorting and borrower default. For recent work that focuses
Our equilibrium analysis of the subprime mortgage market also contributes to the recent empirical literature that attempts to identify the pricing determinants of differences between portfolio loans and conduit loans, and also differences among different types of conduit loans themselves (see Keys, Mukherjee, Seru, and Vig (2010) and Krainer and Laderman (2014)). Agarwal, Amromin, Ben-David, Chomsisengphet and Evano (2011) recognized the lack of a theoretical model. To this extent, our paper provides the first theoretical framework that enables to decompose the conduit mortgage spread into a credit information component, a foreclosure recovery rate component, and a component that captures the access to liquidity in the securitized investment market. We then show how these different pricing components can drive the rise and fall of the subprime conduit mortgage market.

Our pricing results also have some analogies with Sato’s (2015) analysis of transparent versus opaque assets. Sato shows that transparent firms own transparent assets and opaque firms own opaque assets in equilibrium. This is analogous to us showing portfolio lenders hold only higher quality loans and conduit lenders own a mix. The reasons for such holdings are different in the two models, however. In our model, conduit lenders are intermediaries that transform a mix of assets into opaque subprime mortgage-backed securities (MBS). Sato also shows that opaque assets trade at a premium to transparent assets. This is primarily due to agency distortions in the opaque firm. For us a premium in opaque asset prices comes through the investors’ demand for subprime MBS.

Our paper is also related to the literature of shadow banking and subprime lending. As in Gennaioli, Shleifer, and Vishny (2012), our model shows that investors’ wealth drives up securitization, but in addition our model is able to generate the result that adverse selection in the loan origination market can be the only reason why the conduit loan market shuts down, even when there is investors’ appetite for mortgage-backed securities. This provides a different angle to the role of adverse selection on the rise and fall of subprime mortgage lending, which so far has focused on adverse selection in the secondary mortgage market. Our paper also departs from Mayer, Piskorski, and Tchistyi (2013), Makarov and Plantin (2013), and Piskorski and Tchistyi (2011) by distinguishing between shadow bank and formal bank funding models, and relating their change in market share to different equilibrium subprime mortgage configuration regimes that result from changes in the credit scoring technology, securitization, foreclosure costs, or lenders’ capacity constraints.
Importantly, our model captures the ebbs and flows of shadow bank activity, often peaking just prior to a downturn. The peak corresponds with poor access to soft information acquisition by conduit lenders and high liquidity flowing from security investors to conduit lenders (which is their largest if not exclusive source of funds). This is consistent with Purnanandam’s (2010) evidence that lack of screening incentives coupled with leverage-induced risk-taking behavior significantly contributed to the current subprime mortgage crisis. Our equilibrium mechanism links subprime mortgage lending standards to the run-up and eventually collapse in home-prices (endogenously determined in our model), and thus fills a gap in the literature that studies mortgage leverage and the foreclosure crisis (Corbae and Quintin (2015)).

Our model also differs from Ordonez’s (2014) theory that crisis appear when mortgage-backed security investors neglect systemic risks by focusing instead on the information problems that are specific to the conduit loan origination market.

We depart from the classical general equilibrium theory with collateralized lending in the following respects. First, we consider limited recourse mortgages, while most of the previous literature has focused on non-recourse mortgages. A second aspect is that we let households make endogenous rent versus own decisions by considering different housing contract durations - owner-occupied (two periods contract) versus rental (one period contract) -, and thus we bring the tenure housing decision into a general equilibrium model. A third difference is that in our model home ownership requires mortgage debt financing originating from two potential sources: portfolio lenders and conduit lenders. We then closely examine, together and separately, the effects of conduit lenders’ resolved (soft information acquisition) and unresolved (adverse selection) private information on borrower sorting outcomes. The structural details underlying mortgage contract design and market organization consequently feed back to affect the rent versus own decision.

Our model also provides a micro-founded approach to soft information acquisition and its consequences on the subprime mortgage market. Up to our knowledge, we are the first ones to show that conduit lenders do not acquire (enough) soft information in equilibrium when the mortgage distribution rate is high. This result complements Keys, Mukherjee, Seru, and Vig’s (2010) empirical evidence that existing securitization practices did adversely affect the screening incentives of subprime lenders.

Our interpretation of the credit scoring technology is similar to Chatterjee, Corbae, and Rios-Rull (2011) and Guler (2014) in that the technology dictates the fraction of borrowers

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8As Ashcraft, Adrian, Boesky and Pozsar (2012) point out, at the eve of the financial crisis, the volume of credit intermediated by the shadow banking system was close to $20 trillion, or nearly twice as large as the volume of credit intermediated by the traditional banking system at roughly $11 trillion.

9Other relevant papers that study foreclosure dynamics while taking exogenous house prices are Guler (2014) and Cambell and Cocco (2014).

10This literature on general collateral equilibrium is vast. See Geanakoplos (1997, 2003) and Geanakoplos and Zame (2014) for leading models, and Fostel and Geanakoplos (2014) for a review of the theory of leverage developed in collateral equilibrium models with incomplete markets. See also Geanakoplos (2010) for a more applied view of the role of this models in the understanding of the recent credit crisis.

11See Poblete-Cazenave and Torres-Martinez’s (2013) for a recent descriptive analysis of a model with limited liability mortgage loans.
of a given type. However, in their models the credit scoring technology is the same for all lenders and is interpreted as hard information. We distinguish between hard information and soft information. Also, importantly, we allow lenders to choose their credit scoring technologies by acquiring the optimal amounts of soft information. Also, in their models there is no loan securitization - all lenders keep their loans in portfolio - whereas in our model we allow for both portfolio and conduit lenders. Another difference is in the type of mortgage contracts. Chatterjee, Corbae, and Rios-Rull (2011) consider unsecured consumer loans, and Guler (2014) considers non-recourse contracts. We consider instead limited recourse contracts. This choice is motivated by market practice in the US subprime lending market where the majority of US states adopt recourse contracts subject to some bankruptcy exemption (see Davila (2015). Another difference with Chatterjee, Corbae, and Rios-Rull (2011) is that they allow consumers to borrow multiple times to study the role of reputation acquisition where the individual’s type score is updated every period according to some exogenous rule. These are characteristics of prime borrowers who build some credit reputation over time by borrowing in multiple occasions. In our paper we study subprime consumers whose access to credit is rather limited and in general can borrow only once. Thus, there is no reputation acquisition in our model, nor a need to update the individual’s type score.

1.2 Paper structure

The rest of this paper is as follows. In Section 2 we present the baseline model with two types of consumers, limited recourse mortgages, two types of lenders (conduit and portfolio), one state of nature in the second period, and asymmetric information in the conduit primary mortgage market. Section 3 presents the equilibrium definition, gives the result of equilibrium existence, and also presents the pricing formulas that characterize the different lenders’ optimal mortgage rates and corresponding excess premia. Section 4 focuses on the particular case of household’s tenure choice between owner-occupied housing and rental housing. Here we identify the different equilibrium regimes that our model can generate. In Section 5 we discuss how our model explains the rise and fall of the subprime mortgage conduit market, and also provide some simulations of the behavior of house prices, loan amounts and house sizes as a function of the predictive power of the credit scoring technology (hard information) and the liquidity from the secondary MBS market. Section 6 shows how to accommodate soft information acquisition into our baseline model. Section 7 discusses how the predictions of our model change if we introduce adverse selection into the secondary mortgage market. Section 8 makes additional remarks that demonstrate the robustness of our model. The Appendix is devoted to the proofs.

12 See also Arslan, Guler and Taskin (2015), Chatterjee, Corbae, Nakajima and Rios-Rull (2007) and Chatterjee, Corbae, and Rios-Rull (2008). See Chatterjee and Eyigungor (2012) for a departure from these models where long-maturity debt is issued against collateral which value may fluctuate over time.

13 In the extensions to our model we study the effect of considering instead non-recourse mortgages.
2 The baseline two-periods model

Our baseline model consists of a two-periods deterministic economy with asymmetric information in the primary mortgage market and the following main ingredients. There are households ($h$), portfolio lenders ($r$), conduit lenders ($k$), and security investors ($i$). Households are subprime households, and all have a first period subsistence endowment $\omega^{SR}$. In the second period a fraction of these households experience a positive shock in their second period endowment $\omega^2 > \omega^{SR}$, e.g., some households find a better job while remaining households remain at their current income levels. Individual households possess private information regarding their type in the first period. Mortgage contracts are recourse, but subject to some ungarishable minimum subsistence consumption ($\omega^{SR}$) by the borrower. Thus, the only source of default is the borrower’s bad income realization. Lenders cannot perfectly screen the type of borrowers using hard information only; only additional soft information can identify the type of borrower. In this first part of the model investors rely on the same credit scoring technology than those lenders without soft information, thus leaving aside the possibility of adverse selection in the secondary market of mortgage backed securities. We also ignore any agency issues regarding securitization and its implications on distressed loans.\footnote{See Cordell, Dynan, Lehnert, Liang, and Mauskopf (2009), Piskorski, Seru, and Vig (2010), Agarwal, Amromin, Ben-David, Chomsisengphet, and Evano (2011), Ghent (2011), and Adelino, Gerardi, and Willen (2013) for a discussion of the role of securitization on residential mortgages.}

We find convenient to denote an agent type by $a = h, r, k, i$, the set of agents of type $a$ by $A(a)$, and the whole set of all agents in the economy by $A$. The non-atomic measure space of agents in this economy is given by $(A, \mathcal{A}, \lambda)$, where $\mathcal{A}$ is a $\sigma$-algebra of subsets of the set of agents $A$, and $\lambda$ is the associated Lebesgue measure. The measure of the set of type $t = G, B$ consumers is set to be equal to 1, i.e., $\lambda(A(l))$. The measures of portfolio lenders, conduit lenders and investors are all set to be equal to 1, i.e., $\lambda(A(l)) = 1$ for both $l = r, k$ and $\lambda(A(i)) = 1$.

2.1 Subprime households

Let us consider a two-periods economy with a continuum of consumers that we will call subprime consumers. These consumers are also called households. The set of households is denoted by $A$. Consumption can take two forms: owner-occupied housing and a numeraire composite good (e.g., food, clothing, shelter, etc.)\footnote{The composite good represents what is given up along consumer’s budget constraint to consume more of the owner-occupied housing good.} The baseline economy is deterministic and the only source of uncertainty for the lenders is to identify the consumer type. We leave the details that characterize subprime consumers, subprime housing markets and subprime mortgage markets for the Appendix. In period 1 a household can buy a house of size $H_1$ at price $p_1$ per house size unit or buy an amount $R_1$ of the numeraire good at per unit price 1. Houses are durable goods that once purchased can be “consumed” in both periods 1 and 2 (or consumed in period 1 and sold in period 2). In particular, if the consumer buys a house in period 1, the same house enters in period 2 budget constraint as an asset endowment evaluated at market price $p_2$.\footnote{See Cordell, Dynan, Lehnert, Liang, and Mauskopf (2009), Piskorski, Seru, and Vig (2010), Agarwal, Amromin, Ben-David, Chomsisengphet, and Evano (2011), Ghent (2011), and Adelino, Gerardi, and Willen (2013) for a discussion of the role of securitization on residential mortgages.}
The numeraire good, on the other hand, is such that if purchased can only be consumed in one period.

Once the second period starts, households expect to die at the end of the period. Thus, we refer to households in period 2 as old households, and households in period 1 as young households. When households are old, they can also choose to consume owner-occupied housing $H_2$ and the numeraire good $R_2$. Household $h$’s preferences are represented by the following time separable utility function

$$u^h(R_1, H_1, R_2, H_2) = \bar{u}(R_1, H_1) + \theta^h \tilde{u}(R_2, H_2)$$

where $\theta^h < 1$ denotes the consumer’s discount factor, and $\bar{u}(\cdot)$ denotes a Bernoulli utility function that is continuous, concave and monotonic.

In our economy all subprime households fall below some subsistence poverty line and have a subsistence income in period 1 equal to $\omega^{SR}$ units of the numeraire good (e.g., government subsidy). This income is fungible in the sense that it can be used to fund a down payment on a owner-occupied house should the borrower qualify for a sub-prime mortgage. In the second period some of the subprime consumers experience a positive income shock (e.g., get a better job) $\omega^+ > \omega^{SR}$, while the rest of the pool remains at their current (poverty) income level $\omega^{SR}$. Label the consumers that experience an increase in their second period endowment as a G-type and those who don’t as a B-type. Consumers know their type in period 1, but G-type consumers are unable to verifiably convey their unrealized increase in income level to outside parties. This is another important aspect of our model with subprime consumers - as discussed below, the lenders’ credit scoring technology that screens borrower types is coarse in absence of soft information, and, in general, considerably worse that the credit scoring technology in the prime lending market. The measures of types G and B households in the economy are $\lambda_G \equiv \lambda(A(G))$ and $\lambda_B \equiv \lambda(A(B))$, respectively.

The owner-occupied housing consumption space is $[0, \bar{H}]$ where $\bar{H}$ denotes the aggregate amount of owner-occupied housing in the economy.\textsuperscript{16} For simplicity, we set $\bar{H} = 1$. In the baseline model, we take the aggregate supply of owner-occupied housing in the first period and the aggregate demand of owner-occupied housing in the second period as exogenously given.

In period 1 (impatient) households can increase their consumption by borrowing from a subprime lender. The matching between consumers and lenders is endogenous in our model and addressed later. Here, we describe the optimization problem of a consumer that has already been matched with a subprime lender. Denote the loan amount by $q > 0$, where $q < 1$ denotes the mortgage discount price (which in equilibrium will depend on whether the lender is a conduit lender or portfolio lender) and $\psi \geq 0$ is the loan repayment due at the beginning of the second period. For simplicity, we normalize the loan interest rate to 0. The budget constraint in period 1 can then be written as follows:

$$p_1 H_1 + R_1 \leq q\psi + \omega^{SR}$$

Notice that the amount of owner-occupied housing that a borrower can buy depends on

\textsuperscript{16}Below, in Section 4, we will study the impact of introducing a minimum housing consumption $H_{min}^> 0$ resulting from local land use regulation in the form of minimum quality standards for owner occupied houses.
how much of the numeraire good he wants to consume. This degree of freedom incorporates a nice feature into the model: borrowers freely choose their mortgage down payment.

Borrowers are subject to a short sale constraint that prevents them to borrow more than an exogenous upper bound $B$ (this upper bound will depend on the type of lender, as we will point out below):

$$\psi \leq B$$ (3)

Sub-prime loans are subject to a limited recourse mortgage contract that stipulates that a borrower is allowed to consume only the subsistence income $\omega^{SR}$ if default occurs. We also refer to the amount $\omega^{SR}$ as the “bankruptcy exemption” (see Davila (2015) for an analysis of bankruptcy exemptions from a welfare point of view). Accordingly, we write the second period budget constraint as follows:

$$p_2 H_2 + R_2 \leq \max \{ \omega^{SR}, \omega'_2 + p_2 H_1 - \psi \}$$ (4)

where $\omega'_2$ denotes the period 2 endowment of a consumer of type $t = G, B$ ($\omega'_2 = \omega^+$ and $\omega'_2 = \omega^{SR}$). The term $p_2 H_1$ captures the value of the house purchased in the previous period and is interpreted as a sale at market price $p_2$. The consumer can then use the proceeds of this sale to buy a house or the numeraire good, after repaying his mortgage. The maximum operator in (4) allows the household to strategically default and consume at least the minimum subsistence income $\omega^{SR}$. There is no default if $p_2$, $H_1$, and $\psi$ are such that $\omega^{SR} \leq \omega'_2 + p_2 H_1 - \psi$. In Section 6 we elaborate on the details of limited recourse mortgage contracts, their implications for adverse selection, and also explain the differences if we were to consider non-recourse mortgages instead. For now, notice that we are not requiring borrowers to constitute any particular collateral (e.g., buying a house in the first period). Loan payment is (partially) enforced by the nature of the limited recourse loan.

Households’ optimization problem is as follows: each household maximizes his utility function (1) subject to constraints (2), (3) and (4).

2.2 Lenders

Households can borrow from either portfolio lenders or conduit lenders. Both types of lenders originate mortgages in a competitive environment, although they differ in the terms of their contracts. Portfolio lenders ($r$) originate mortgages to be held in the entity’s asset portfolio (“originate-for-ownership”) and are able to acquire and analyze non-verifiable soft information (portfolio lenders know their borrowers and their communities and borrowers maintain checking and other personal accounts with them). In contrast, conduit lenders ($k$) are transactional, specializing only in originating mortgages for sale to a third

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17 Lenders cannot take everything and leave a consumer homeless when he defaults and becomes bankrupt. In fact, bankruptcy is designed to shield consumers from too much recourse on mortgage loans. See [law...]

18 A consumer with an owner-occupied house at the beginning of period 2 decides whether to sell it at market price, or to consume it. The latter is equivalent to the joint transactions of selling the house the consumer owns at the beginning of period 2 and then buying immediately after a house with same size.

19 Strategic default is simply an optimality condition in which the borrower, subject to the relevant recourse requirements, decides whether mortgage loan payoff to retain ownership of the house or default with house forfeiture generates greater utility. For a discussion of the default option, see Deng, Quigley, and Van Order (2000). See Davila (2015) for an exhaustive analysis of exemptions in recourse mortgages.
party ("originate-for-distribution"). This access to secondary mortgage markets can possibly reduce the cost of capital when secondary subprime mortgage markets are liquid and competitive. Also, conduit lenders can be characterized as technology oriented, generally working out of a small office with computers, coming and going quickly, with no established presence in a community. Conduit lenders generally have access to limited amounts of capital with which to fund mortgages, and it is their mortgage distribution business what provides them with enough capital to originate mortgages.

The relationship-transactional distinction goes to the heart of low-cost soft information acquisition associated with portfolio lenders versus the exclusive reliance on hard information generally associated with conduit lenders. Later in the paper we will show how this credit scoring technology outcome can be endogenized, and also will allow for heterogeneous conduit lenders with different possibilities credit scoring technologies and ability to distribute mortgages in the secondary market. For now, we consider a baseline model as summarized by Table 1

<table>
<thead>
<tr>
<th></th>
<th>Soft information</th>
<th>Originate-to-distribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio lender (r)</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Conduit lender (k)</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

Table 1

There is a mass 1 of each type of lender in our economy. By abuse of notation, we write \( l \) to denote a lender independently of his type (portfolio lender or conduit lender). We denote the set of lenders by letter \( L \).

Portfolio and conduit lenders are subject to the lending capacity constraint \( v(r) \) and \( v(k) \) (in terms of the number of borrowers).\(^{20}\) Notice that the portfolio lender’s capacity constraint is not necessary equivalent to his endowment, but the two can be made to coincide. Thus, our model can accommodate cases where a lender can only process a number of loans given its business infrastructure (given by the capacity constraint parameter \( v(l) \) in our model), and then, for this number of borrowers, the lender has a limited amount of resources for lending (given by the endowment parameter \( \omega' \)). In terms of our modeling, the portfolio lender’s capacity constraint plays an important role, as it prevents all G-type consumers from borrowing from getting portfolio loans. In particular, we assume \( \lambda_G > v(r) \). G-type consumers that cannot get a mortgage from portfolio lenders due to the capacity constraint have the possibility to get a mortgage from the conduit lenders, as long as the conduit lender’s credit scoring technology assigns them a G-rating.

2.2.1 The Credit Scoring Technology

Portfolio lenders and conduit lenders have different credit scoring technologies. Conduit lenders have access to hard credit information, which is always accurate, but it does not necessarily lead to a perfect assessment of consumer type. Portfolio lenders have soft information as a supplement to the available hard credit information, and by assumption

\(^{20}\)Modeling the capacity constraint as the maximum number of mortgages (households) that a bank can serve is according to common practice where banks announce at the beginning of the year the number of mortgages that can sell during the year.
this is enough to fully reveal the borrower’s type. As such conduit lenders are not capable of resolving asymmetric information over and above what is available with hard information and their credit scoring technology.

We denote the probability that a lender \( l \) gives a good rating to a G-type type borrower by \( \Pr^l(\text{rating}=G|G) \). Similarly, we denote the probability that a lender \( l \) gives a good rating to a B-type type borrower by \( \Pr^l(\text{rating}=G|B) \). We refer to these probabilities as the lender’s credit scoring technology. By assumption, portfolio lenders always assign a good signal to G-type consumers, that is \( \Pr^l(\text{rating}=G|G) = 1 \). Conduit lenders, however, have an imperfect credit scoring technology and thus \( \Pr^l(\text{rating}=G|G) < 1 \).

Denote the measure of consumers that receive a good rating coincides with the measure of consumers that receive a loan from conduit lenders. This measure is denoted by \( \mu^k(\text{rating}=G) \) and is equal to

\[
\mu^k(\text{rating}=G) = \Pr^l(\text{rating}=G|G)\mu^k_G + (1 - \Pr^l(\text{rating}=G|G))\mu^k_B
\]

where \( \mu^k_G \) and \( \mu^k_B \) denote the measure of G-type consumers and B-type consumers that try to borrow from conduit lenders.

We restrict our analysis to mortgages that are awarded to consumers that receive a good rating. This is done to simplify the analysis, but observe that the adverse selection problem in the mortgage market of good ratings between the borrower and the conduit lender would not disappear if we would also allow for a market of bad ratings, since B-type consumers would still prefer to misrepresent their type and borrow a large loan amount as G-type consumers do.

We can use Bayes’ rule and write the expected probability of lending to G-type consumers given that the conduit lender gives a good rating in the pool:

\[
\Pr^l(G|\text{rating}=G) = \frac{\Pr^l(\text{rating}=G|G)\hat{\pi}^l(G)}{\Pr^l(\text{rating}=G|G)\hat{\pi}^l(G) + \Pr^l(\text{rating}=G|B)\hat{\pi}^l(B)}
\]

where \( \hat{\pi}^k(G) \) denotes the fundamental proportion of G-type consumers available to the conduit lender. In the Appendix we show that \( \Pr^l(G|\text{rating}=G) \) can be expressed in a linear way as follows: \( \Pr(G|\text{rating}=G) = 1 - \varepsilon\hat{\pi}(B) \), where \( \varepsilon \) denotes the amount of asymmetric information between the lender and the borrower and \( \hat{\pi}^k(B) \) denotes the fundamental proportion of B-type consumers available to the conduit lender.

To simplify notation, we write the lender’s belief on the proportion of G-type consumers in the pool of borrowers as follows:

\[
\pi^l \equiv \Pr^l(G|\text{rating}=G)
\]

By assumption, a portfolio lender has \( \pi^r = 1 \). We also assume that both conduit lenders and investors only rely on hard credit information, i.e., \( \pi^k = \pi^l < 1 \). This assumption is convenient as it allows us to focus on the adverse selection problem between borrowers and conduit lenders, leaving aside potential information problems that may arise between conduit lenders and secondary mortgage investors. To further economize on notation, we will write below \( \tilde{\pi} \equiv \pi^k = \pi^l \) and \( \pi^* \equiv \pi^r = 1 \).
2.2.2 The Originate-To-Distribute Constraint

Denote by $\varphi^l \geq 0$ the total amount of mortgages originated by an $l$-lender and $z^l \geq 0$ the amount of mortgages that lender $l$ originates to distribute (sell) to the investors (homogeneous loans are pooled and securitized into one asset). A lender cannot distribute more than a fraction $d^l$ of his originated mortgages. $d^l$ is the same for all lenders of the same type. The originate-to-distribute constraint takes the following form:

$$z^l \leq d^l \varphi^l \quad (5)$$

where $d^l \in [0, 1]$ is the maximum fraction of mortgages that a lender of type $l$ can sell to security investors (a parameter in our model). The remaining part, $(1 - d^l) \varphi^l$, is kept in the bank’s portfolio.

By assumption, portfolio lenders keep all their originated mortgages in their portfolio, i.e., $d^k = 0$. Conduit lenders, on the other hand, distribute some fraction of their mortgages, i.e., $d^k > 0$. In general, $d^k$ is typically close to 1 for conduit lenders. A distribution rate smaller than 1 can be the result of a regulation or a self-imposed constraint due to reputation concerns (not modelled here).

2.2.3 Profit Function

In our model a subprime borrower defaults when he receives a bad income realization in the second period. This is the case of B-type consumers in our baseline model (later, when we introduce a stochastic economy we let both G-type and B-type consumers to default, although with different probabilities). Given the nature of the limited recourse mortgage contract, when there is default, the lender garnishes all borrower’s income above $\omega^{SR}$. This includes repossessing the house and reselling it if the borrower happened to buy one in the first period. However, the foreclosure process is costly for the lender: foreclosure cost and other indirect costs associated with foreclosure delays result in a loss of $(1 - \delta)p_2H_1$ units of the numeraire good to the lender, where $\delta \in [0, 1]$ denotes the foreclosure recovery rate.

Because the limit recourse contract allows the borrower to keep $\omega^{SR}$ for personal consumption, we have that B-type borrowers will default in the second period, and hence the lender only recovers the foreclosure value of the house that the borrower purchased in period 1, $\delta p_2H_1$. Hence, for the B-type borrower, the mortgage is effectively non-recourse in nature. A key difference with standard non-recourse contracts, however, is that under the limited recourse contract the borrower is not required to constitute housing as collateral in the first period. Later, in Section 4, we will present a setting where the borrower - independently of his type - finds optimal to buy owner-occupied housing, and thus the lender recovers a positive amount from B-type borrowers (i.e., $\delta p_2H_1 > 0$).

The mortgage contract for the G-type borrower is limited recourse, because in the second period the G-type borrower credibly commits to pay any income (endowment of the numeraire good) and wealth (housing purchased in the first period) he has on top of the subsistence rent $\omega^{SR}$ to the lender (we will see that in equilibrium the G-type borrower pays $\varphi^l = \omega^+ + p_2H_1^G - \omega^{SR}$).

The lender $l$’s revenue from mortgage lending in the second period depends on his credit scoring technology through probability $\pi^l$ and has the following expression: $\Pi^l \equiv$
\[ \pi^t \varphi^t + (1 - \pi^t) \delta p_2 H_1^G, \]
where the first component captures the payment from the fraction \( \pi^t \) of G-type borrowers who honor their promise, while the second component captures the payment from the fraction \( 1 - \pi^t \) of B-type borrowers who default.\(^{21}\) Only a fraction \( 1 - d^l \) of mortgages affect the lender’s profit function in the second profit, as he distributes a fraction \( d^l \) of the mortgage payment proceeds to investors. Accordingly, we write the lender \( l \)'s profit function as follows:

\[
\Phi(\varphi^l, z^l) = (\omega^l_1 - q^l \varphi^l + \tau z^l) + \theta^l (1 - d^l)(\pi^l \varphi^l + (1 - \pi^l) \delta p_2 H_1^G),
\]

where \( q^l \) and \( \theta^l \) denote the lender \( l \)'s mortgage discount price and discount factor, respectively, and \( \tau \) denotes the sale price of each mortgage unit in the secondary market. To simplify the expression (6), we assume that the lender’s second period endowment is equal to 0 (i.e., \( \omega^l_2 = 0 \)). The lender’s first period endowment is positive, \( \omega^l_1 > 0 \). Since \( \theta^k < \theta^l \), lenders care less about period 1 consumption than households do.

The lenders optimization problem is as follows. Each lender \( l \) chooses \( \varphi^l \) and \( z^l \) to maximize his profit function \( \Phi(\varphi^l, z^l) \) subject to the originate-to-distribute constraint (5). Lenders are risk-neutral\(^{22}\), and thus their first order conditions determine the competitive mortgage prices \( q^l \), \( l = r, k \). We will elaborate more on this later in a subsection on mortgage pricing.

Notice that the interaction between the originate-to-distribute constraint (5) and the profit function (6) determines the two possible loan origination models contemplated here. On the one hand, conduit lenders can distribute a fraction \( d^k > 0 \) of the originated mortgages, but lack soft information so \( \pi^k < 1 \). Portfolio lenders, on the other hand, have soft information (\( \pi^r = 1 \)) but don’t sell their mortgages (\( d^r = 0 \)).

We refer to the pairs \( (q^*, \varphi^*) \) and \( (q^k, \varphi^k) \) as the pooling contracts offered by portfolio lenders and conduit lenders to borrowers, respectively. For the case of conduit loans, \( q^k \) is the pooling price that captures the presence of lemons in the conduit loan market.\(^{23}\) The pair \( (q^*, \varphi^*) \) represents the optimal contract offered by portfolio lenders, who by definition keep all originated mortgages in their portfolio. Since portfolio lenders have soft information, the mortgage price \( q^* \) can be seen as the risk-free rate in this economy. On the other hand, the mortgage price \( q^k \) incorporates the cost (in terms of a lower mortgage payment) associated with the conduit lenders inability to fully resolve information asymmetries at the time of loan origination. However, this cost can be offset by high relative demand (liquidity) of originated mortgages by investors in the secondary market, which may raise the discount price accordingly.

\(^{21}\)This interpretation is similar to Chatterjee, Corbae, and Rios-Rull (2001) where the credit scoring technology is thought as the fraction of people with a given score that can repay.

\(^{22}\)Lenders are risk neutral. The assumption of lender’s risk neutrality is common in the literature. See e.g. Arslan, Guler, and Taskin (2015), Chatterjee, Corbae and Rios-Rull (2011), Chatterjee and Eyigungor (2015), and Guler (2015).

\(^{23}\)In subsection “Separating equilibrium” below, we will show that in the baseline setting there is no separating equilibrium.
2.3 Investors

We consider a mass 1 of investors that buy securitized subprime loans at market price \( \tau \). The investor’s endowment in periods 1 and 2 is denoted by \( \omega^i_1 \) and \( \omega^i_2 \), respectively. To economize on notation in the investor’s profit function, we assume that \( \omega^i_2 = 0 \), and thus omit this element. First period endowment \( \omega^i_1 \) is positive, however. Notice that if investors’s mortgage purchases were constrained by a low \( \omega^i_1 \), lenders’ conduit mortgage origination would then be constrained as well. We discuss this possibility below.

Investors assign a smaller weight to period 1 consumption than lenders do, and therefore we write \( \theta^l < \theta^i \). For now assume that investors rely on the same hard credit information than conduit lenders, and thus \( \pi^i = \pi^k \). The optimization problem of an investor \( i \) consists on choosing \( z^i \) to maximize the following profit function:

\[
\Lambda^i(z^i) \equiv \omega^i_1 - \tau z^i + \theta^i (\pi^i z^i + (1 - \pi^i) \delta p_2 H^G_1) \tag{7}
\]

The term \( \pi^i z^i + (1 - \pi^i) \delta p_2 H^G_1 \) captures the investor’s second period revenue from buying mortgages in the first period. The first term, \( \pi^i z^i \), corresponds to the payment from the fraction \( \pi^i \) of G-type borrowers. The second term, \( (1 - \pi^i) \delta p_2 H^G_1 \), corresponds to the income from lending to a fraction \( (1 - \pi^i) \delta \) of B-type borrowers. Notice that \( \delta \) here stands for the percentage of total mortgages that investors buys, hence investors are entitled to that revenue. Also notice that we are assuming that investors have the same foreclosure cost than lenders do, i.e., \( \delta \) is the same for both of them.

3 Equilibrium and Mortgage Pricing

Our notion of equilibrium assumes that agents (consumers, lenders and investors) form beliefs about the composition of the pool of borrowers for a given type of lender. These beliefs are common, degenerate and governed by the credit scoring technology associated with a particular lender. Lenders take these beliefs as given and optimize without taking into account the decision of consumers to apply for their loans.\(^{\text{24}}\) Also, consumers take those beliefs as given and choose the type of mortgage to apply for. Investors also take those beliefs as given to price the mortgage-backed security bought to conduit lenders. Thus, in our model all agents’ beliefs about the prices of mortgages and goods, as well as about the composition of each mortgage market, are common and degenerate. An equilibrium for our economy then consists of a vector of goods, mortgages and mortgage-backed security prices such that each agent optimizes given his constraints, the respective markets clear, and the matching between agents is consistent in terms of the aggregate of choices (a formal definition of equilibrium is given below).

In our model default risk is a result of the inability of conduit lenders to perfectly screen borrower types and thus can be attributed to the endogenous behavior of consumers with whom they are matched in equilibrium. We treat this risk as idiosyncratic in the sense that the matching of lenders with consumers is assumed to be independent and uniform, and assumes that the law of large numbers applies (see Zame (2007) and Duffie and Sun (2007, 2012)). For example, if the matching between a lender of type \( k \) and consumers of type \( G \)

\(^{\text{24}}\)This is similar to Zame (2007) where agents optimize without taking the supply of jobs into account.
is governed by probability $\pi^k$ (dictated by his credit scoring technology), then the fraction of the total continuum population of G-type consumers that attempt to borrow from this type of lender is $\pi^k$.

Because consumers of the same type are ex-ante identical, all consumers of the same type obtain the same ex-ante expected utility. However, given the portfolio lenders’ capacity constraint and the imperfect credit scoring technology of conduit lenders, consumers of the same type will end up with different loan types, and thus obtain different realized housing consumption and ex-post realized utility (e.g., there will be an equilibrium configuration where some G-type consumers are lucky and obtain a portfolio loan, some G-type consumers obtain a conduit loan, and the remaining G-type consumers cannot borrow and must rent). Our approach to equilibrium existence is consistent with this interpretation: a consumer randomizes over the possible consumptions in the set of different types of mortgage markets (portfolio mortgage, conduit mortgage, and no-mortgage markets).

Because the theory developed here is intended as a competitive theory of lending relationships, we require that a “group” is formed by one lender and several borrowers and thus is small relative to the population as a whole. Since there is a continuum of consumers and lenders in our economy, in equilibrium we will observe a continuum of negligible (measure zero) groups in this economy. This is similar to the notion of competitive equilibrium in theory of club formation. Groups are comparable in size to agents but are negligible with respect to the society. The notion of matching between agents then requires consistency in terms of the aggregate of choices. For example, consistency means that if a third of the population are G-type borrowers in a pool where lenders have $\pi^k = 0.5$, then a third of the population must be B-type consumers in this pool. This consistency condition must hold simultaneously for all types of groups (Ellickson et al. (1999)).

Also, notice that the continuum of consumers allows us to deal with two types of non-convexities: those associated with the maximum operator in the consumer’s second period budget constraint, and those associated with the consumer’s choice of loan type (portfolio loan, conduit loan, or no loan).

The notion of a group is also important because it permits us to control for the lender’s capacity constraint by specifying number of borrowers in the definition of a group. At the same time, notice that we can choose two additional parameters in our model: the lender’s first period endowment ($\omega^l_1$) and the measure of lenders of each type.

A “primary mortgage group” is formally defined here as the result of the matching between one lender and several borrowers (one-to-many) and is defined by the triplet $g^l = (l, v(l), \pi^l)$, where the first coordinate indicates the type of lender (there is one lender in each group), and the second and third coordinates indicate the capacity constraint and credit scoring technology of an $l$-type lender, respectively. The first coordinate can take three identities: $l = r$ (portfolio lender), $l = k$ (conduit lender) and $l = \emptyset$ (no lender). When a consumer $h$ belongs to a group where $l = \emptyset$, it means that he has no lender to borrow from and thus $v^h = 0$. Group $g^\emptyset$ contains consumers who do not get a loan, or those consumers who prefer renting to owning. In a “secondary mortgage group” lenders with $d^l > 0$ match investors to transact mortgage-backed securities given beliefs $(\pi^i, \pi^l)$. Because $\lambda(A(k)) = \lambda(A(i)) = 1$ we find convenient to assume that matching is pair-wise (one lender and one investor) for each secondary mortgage group. This group is defined by the vector $g^s = (i, l, d^l, \pi^i, \pi^l)$. The universe of available group types in this economy is
restricted to the discrete set $G = \{g^r, g^k, g^b, g^s\}$.

When a primary mortgage group forms, the borrowers enter into a relationship with the lender described by a contract. In a pooling equilibrium, this contract specifies the discount mortgage price that results from the lender’s first order optimality condition (lenders are risk neutral). Given this mortgage price, the borrowers demand a loan amount, which is then satisfied by the lender (market clearing holds). Notice that our concept of group allows for adverse selection by letting $\pi^l < 1$. When $\pi^l = 1$, the lender’s pool of borrowers is only composed by G-type consumers. However, when $\pi^l < 1$ a fraction $1 - \pi^l$ of the pool of borrowers is of type B and the lender adjust the mortgage discount price accordingly.

In our model agents choose the type of group by choosing a “membership” in $G$. A “membership” is agent-type ($a = G, B, r, k, i$) and group-type ($g \in G$) specific, and is denoted by $m = (a, g)$. The set of memberships is denoted by $M$, while the maximum number of memberships that an agent $a$ can choose is denoted by $M(a)$. The following concept is needed in our definitions below. A list is a function $\iota : M \to \{0, 1, \ldots \}$, where $\iota(t, g)$ denotes the number of memberships of type $(t, g)$. We then write $\text{Lists} = \{\iota : \iota \text{ is a list}\}$ and define the agent’s group-membership choice function by $\mu : A \to \text{Lists}$. We will need these definitions below.

### 3.1 Equilibrium definition and the existence result

The following concepts are necessary to characterize agents’ choice sets. Denote the household $h$’s consumption bundle by $x^h = (H^h_t, R^h_1, H^h_2, R^h_2) \in \mathbb{R}^4_+$. The pair $(x^h, \psi^h)$ is feasible if it satisfies constraints (2), (3) and (4). The lender and investor consumption bundles are given by $x^l = (R^l_1, R^l_2) \in \mathbb{R}^2_+$ and $x^i = (R^i_1, R^i_2) \in \mathbb{R}^2_+$, respectively. The triplet $(x^l, \varphi^l, z^l)$ is feasible if it satisfies (5). We find convenient to rewrite the consumer’s utility function as a function of the his consumption and group type, e.g., $u^h(x^h, \mu^h(m))$. Similarly, we write $\Phi^l(\varphi^l, z^l, \mu^l(m))$ and $\Lambda^l(\varphi^l, z^l, \mu^l(m))$ for the lender and investor’ profit functions, respectively.

The consumer $h$’s choice set $X^h \subset \mathbb{R}^5_+ \times \text{Lists}$ consists of the feasible set of elements $(x^h, \psi^h, \mu^h)$ that this consumer can choose. Similarly, let the lender $l$’s choice set $X^l \subset \mathbb{R}^5_+ \times \text{Lists}$ be the feasible set of elements $(x^l, \varphi^l, z^l, \mu^l)$ that that this lender can choose, whereas the investor $i$’s choice set $X^i \subset \mathbb{R}^5_+ \times \text{Lists}$ stands for the set of elements $(x^i, z^i, \mu^i)$ that that this investor can choose. Also, let us define the set $\text{Lists}(a) = \{\mu^a \in \text{Lists} : \sum_m \mu^a(m) \leq M(a), \exists x^a \text{ s.t. } (x^a, \mu^a) \in X^a\}$, which represents the agent $a$’s restricted consumption set of memberships compatible with his consumption.

We make the following assumptions:

- The utility mapping $(h, x, \mu) \to u^h(x, \mu)$ is a jointly measurable function of all its arguments.
- The consumption set correspondence $a \to X^a$ is a measurable correspondence, for $a = h, l$.
- If $(x^a, \mu^a) \in X^a$ and $x^a \geq x^a$, then $(x^a, \mu^a) \in X^a$, for $a = h, l$. 

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• Each agent \( a \neq k \) chooses at most one group, while each conduit lender \( k \) can choose at most two groups, i.e., \( M(a) = 1 \) if \( a \neq k \) and \( M(a) = 2 \) if \( a = k \).

• Lenders can only belong to their corresponding primary mortgage type (i.e., \( \mu_l(l, g^l) = 1 \) for \( l = r, k \)), and conduit lenders and investors belong to the secondary mortgage group (i.e., \( \mu^s(a, g^s) = 1 \) if \( a = k, i \)).

• The endowment mapping \((\omega_1, \omega_2) : a \mapsto (\omega_1(a), \omega_2(a))\), with \( \omega_1(a), \omega_2(a) > 0 \) for \( a \in A \), is an integrable function.

• The aggregate endowment is strictly positive, i.e., \( \int_A \omega(a) d\mu > E \), where \( E > 0 \).

Next, we proceed to define the concept of consistent matching in terms of the aggregate of choices.\(^{25}\) For this, let us denote the aggregate of type \((a, g)\)-memberships by \( \hat{\mu}(a, g) \equiv \int_{A(a)} \mu^i(a, g) d\mu \). We say that the aggregate membership vector \( \hat{\mu} \in \mathbb{R}^M \) is consistent if, for every group type \( g \in G \), there is a real number \( \gamma(g) \) such that \( \hat{\mu}(a, g) = \gamma(g)n(a, g) \), \( \forall a = G, B, r, k, i \), where \( \gamma(g) \) is the measure of type \( g \) groups, and \( n(a, g) \) is the (natural) number of type \( a = G, B, r, k \) agents in group \( g \). Then, we say that the choice function \( \mu : A \rightarrow \text{Lists} \) is consistent for \( A \subseteq A \) if the corresponding vector is consistent. We write \( \text{Cons} \equiv \{ \hat{\mu} \in \mathbb{R}^M : \hat{\mu} \text{ is consistent} \} \).

**Definition 1:** Given \((\pi^r, \pi^k, \pi^s)\), an equilibrium for this economy is a vector of memberships \( \mu \), prices \((p_1, p_2, q^r, q^k, \tau)\) and allocations \((x^h, \psi^h)_{h \in A(G) \cup A(B)}, (\varphi^l, z^l)_{l \in A(l)} ; t = r, k, (z^l)_{l \in A(l)} \) such that:

1. (2.1) Each consumer \( h \) chooses \((x^h, \psi^h, \mu^h) \in X^h\) that maximizes \( u^h(x^h, \mu^h(m)) \).
2. (2.2) Each lender \( l \) chooses \((R^1_l, R^2_l, \varphi^l, z^l, \mu^l) \in X^l\) that maximizes \( \Phi^l(\varphi^l, z^l, \mu^l(m)) \).
3. (2.3) Each investor \( i \) chooses \((R^1_i, R^2_i, z^i, \mu^i) \in X^i\) that maximizes \( \Lambda^i(\varphi^i, z^i, \mu^i(m)) \).
4. (2.4) \( \hat{\mu} \) is consistent for \( A \).

(2.5) Market clearing:

\[
\int_{A(G)} \psi^{h, r} \mu^h(t(h), g^r) dh = \int_{A(r)} \varphi^r \mu^r(r, g^r) dr, \tag{8}
\]

\[
\int_{A(G) \cup A(B)} \psi^{h, k} \mu^h(t(h), g^k) dh = \int_{A(k)} \varphi^k \mu^k(k, g^k) dk, \tag{9}
\]

\[
\int_0^1 z^l dl = \int_0^1 z^l di, \tag{10}
\]

\[
\sum_{g \in G} \int_{A(G) \cup A(B) \cup A(r) \cup A(k)} R^1_m(a, g) da + \int_{A(i)} R^1_i di = \int_A \omega^1 da, \tag{11}
\]

\[
\sum_{g \in G} \int_{A(G) \cup A(B) \cup A(r) \cup A(k)} R^2_m(a, g) da + \int_{A(i)} R^2_i di = \int_A \omega^2 da, \tag{12}
\]

\[
\sum_{g \in G} \int_{A(G) \cup A(B)} H^1_g \mu^h(t(h), g) dh = \sum_{g \in G} \int_{A(G) \cup A(B)} H^2_g \mu^h(t(h), g) dh = \tilde{H} \tag{13}
\]

\(^{25}\)See also Ellickson et al. (1999).
Theorem 1 (Existence): An equilibrium specified in Definition 1 exists.

We leave the details of the existence proof for the Appendix. Next, we derive asset pricing conditions that any equilibrium in this economy must satisfy using the lender and investor’s optimality conditions.

3.2 Mortgage Discount Prices

The inability of conduit lenders to fully resolve information asymmetries implies that they will lend to some B-type households, as their hard information-based screening technology cannot perfectly distinguish B-type from G-type households. Since B-type borrowers will (endogenously) fail to comply with mortgage payment contract terms and conditions, with the net post-foreclosure sales proceeds less than the promised payment, the conduit lender incurs losses. As a result, based on observables and expectations at the time of mortgage loan origination, the lender finds it optimal to tack on a pooling rate premium to the base loan rate to account for adverse selection risk. However, the loan rate may move indirectly with the credit risk of the borrower pool in question if the lender’s access to liquidity in the secondary market is sufficiently high. This trade-off is captured by the following conduit loan pricing equation derived using the lender and investor’s optimality first order conditions:

\[ q^k = \frac{\pi \tilde{\theta}}{1 - \delta (1 - \pi) \tilde{\theta}} \]  \hspace{1cm} (14)

where \( \tilde{\theta} \equiv d^k \theta^k + (1 - d^k) \theta^l \) is the “\( d^k \)-weighted discount factor” and \( d^k \) is the lender’s mortgage distribution rate. Letter \( \pi \) is such that \( \pi = \pi^l = \pi^l \). Since \( \theta^l > \theta^l \), a higher distribution rate increases the mortgage price through parameter \( \tilde{\theta} \). Also notice that as we increase the distribution rate \( d^k \), the negative effect of adverse selection on the mortgage discount price (captured by \( \pi^l < 1 \)) is offset more and more by the higher investor’s valuation of conduit mortgages. This trade-off is one of the key drivers of the rise and fall of subprime lending market in our model: securitization allows customization, which lowers the cost of capital (i.e., lowers the mortgage rate \( 1/q^k \)) in a conduit loan market where lemons are present.

Foreclosure costs also affect the optimal pricing of conduit loans. When \( \delta^k > 0 \), the term \( 1 - \delta (1 - \pi) \tilde{\theta} \) is interpreted as the “default losses” that a conduit lender incurs when its pool of borrowers contains an expected fraction \( 1 - \pi \) of B-type borrowers. In the extreme case, when \( \delta^k = 0 \), the mortgage price coincides with the expected \( d^k \)-weighted discount factor term \( \pi^l \tilde{\theta} \). A conduit lender with only “hard information” \( \pi^l < 1 \) would set \( q^k < \tilde{\theta} \) because \( \tilde{\theta} < 1 \).

We can rewrite expression (14) in more intuitive terms as follows:

\[ q^k = \frac{\text{hard information} \times d^k \text{-weighted discount factor}}{\text{default losses}} \]

and then loglinearize it and write instead

\[ \log q^k = \log(\text{hard information}) + \log(d^k \text{-weighted discount factor}) - \log(\text{default losses}) \]

On the other hand, a portfolio lender with \( d^r = 0 \) but \( \pi^r = 1 \) sets the mortgage price equal to the lender’s discount factor \( \theta^l \), i.e.,
\[ q^* = \theta^d \]

Since \( \pi^r = 1 \) implies no default, \( q^* \) can be thought as the \textit{risk free discount price} of this economy that does not incorporate any liquidity gains from distribution of originated loans to investors, i.e.,

\[ q^* = \text{risk free discount price} \]

The discount price that investors pay for subprime mortgage securities is

\[ \tau = \frac{\tilde{\pi}\theta^d}{1 - \delta(1 - \tilde{\pi})\theta} \] \hfill (16)

Comparing pricing conditions (14) and (15) we see that, when the lemons portion converges to zero \((1 - \tilde{\pi} \rightarrow 0)\), the conduit lender’s discount price converges to the portfolio lender’s fundamental discount price if it is not possible to distribute mortgages to investors \((d^k = 0)\) or if there are no investors that buy subprime mortgage securities (i.e., if \( \omega_i^1 = 0 \)). When investors have deep pockets and \( d^k > 0 \), the conduit lender’s discount price (rate premium) can be above (below) the portfolio lender’s discount price (rate premium). The two discount mortgage prices are related as follows:

\[ q^k < q^* \text{ if } \pi^k < \pi_2 \equiv \frac{\theta^d(1 - \delta\theta)}{\theta(1 - \delta\theta^d)}. \]

Threshold \( \pi_2 \) defined in the above expression will appear again in a subsection below that characterizes mortgage market collapses. Interesting, as the distribution rate \( d^k \) increases, threshold \( \pi_2 \) decreases, and hence more information is needed to sustain an environment where the conduit mortgage rate is below the risk free rate. Below we will elaborate more on this.

By excess premium (EP) or credit spread we mean the difference between the rate of return of conduit loans and the risk free rate of portfolio loans:

\[ EP \equiv (1/q^k) - (1/q^*) \] \hfill (17)

**Proposition 1:** The excess premium increases with default losses and decreases with the predictive power of hard credit information, a higher distribution rate of mortgages to investors, and a higher risk free rate.

Figure 1 portraits the excess premium as a function of \( \pi^k \). We set \( \theta^d = 0.7, \theta^i = 0.9, \delta = 0.5, v(r) = 1, \lambda_G = 1.5, \) and \( \lambda_B = 0.5 \). In this figure we observe two lines. The first one computes \( EP \) when \( d^k = 0.8 \), and as we can see in the figure, it changes from positive to negative at \( \pi^k = \pi_2 \equiv 0.71 \). At this point the gains from intermediation exactly offset the default losses, and the \( EP \) coincides with the risk-free rate. When \( \pi^k > 0.71 \) the portfolio lender’s optimal mortgage rate is higher than the conduit lender’s rate. The second line in Figure 1 computes \( EP \) when conduit lenders do not distribute any mortgages to investors \((d^k = 0)\). We can see that in this case the conduit mortgage rate is always above the risk free rate \((EP > 0)\), but this difference decreases when the credit scoring technology improves. As explain later, when \( \pi^k \) is above threshold \( \pi_2 \equiv 0.71 \), G-types consumers
prefer conduit loans to portfolio loans in equilibrium. When that happens, the conduit lender’s fundamental proportions of G-type consumers, denoted by \( \hat{\pi}(G) \), increases from 0.5 to 0.75,\(^{26}\) and this jump creates a discontinuity on the EP that can be appreciated in this figure at \( \pi_2 = 0.71 \). Below we will elaborate more on this discontinuity effect.

When conduit lenders are heterogeneous, e.g., in terms of the credit scoring technology through the conditional probability \( \pi^k \), distribution rates \( d^k \) and foreclosure recovery rates \( \delta^k \), we expect different excess premiums between conduit lenders.

4 Tenure choice: Owner-occupied v. rental housing

So far we have assumed that consumer preferences were described by a concave utility function where \( R \) was thought as a numeraire composite good (e.g., clothing, shelter, etc.) Equilibrium for this general model was shown to exists in Section 3. In this section we think of good \( R \) as rental housing and drop the concavity assumption of the utility function to work with a more analytically tractable setting where owner-occupied housing and rental housing are perfect substitutes.\(^{27}\) In particular, we consider the linear utility function

\[
u^b(R_1, H_1, R_2, H_2) = R_1 + \eta H_1 + \theta^b(R_2 + H_2),\]

---

\(^{26}\)The discontinuity occurs because consumers start preferring conduit loans to portfolio loans. When this happens the fundamental proportion of G-type changes because the measure of G-type consumers jumps from 0.5 to 1.5. To see this notice that the measure of G-type consumers that attempts to borrow from conduit lenders increases from 0.5 to 1.5, whereas the measure of B-type consumers is 0.5.

\(^{27}\)Endowments of the numeraire good, captured by letter \( \omega \) in our model, can be thought as the right to use for one period a given amount of land (space) in a subprime neighborhood. This land can be consumed or sublet, and is fungible: if it is in hands of a subprime consumer, it can immediately be transformed at the beginning of the period into subprime rental housing at no cost, and consumed as such; if it is acquired by a lender or an investor, it can be transformed, at no cost, into commercial real estate, from which lenders and investors derive utility.
where $\eta > 1$ denotes a preference parameter that captures that, all else equal, in the first period young households prefer to consume owner-occupied housing over rental housing. This can be possibly due to a better access to schools, for example (see Corbae and Quintin (2015) for a model with also an “ownership premium” in preferences; see Hochguertel and van Soest (2001) for empirical evidence). When households are old, the utility from consumption of owner-occupied housing $H_2$ and the utility from consumption of rental housing $R_2$ are the same.

Consumers are subject to the same constraints introduced before, namely, constraints (2), (3) and (4). Therefore, a rental housing contract expires after the first period, while owner-occupied housing can be “consumed” in both periods. Hence, owner-occupied housing and rental housing can be thought in our model as contracts with different duration.

To streamline our analysis of the tenure choice decision and get simple closed form solutions, we fix $\omega_j^T = 1$, $\omega^{SR} = 1/2$ and $v(r) = 1$. In addition, we require that portfolio lenders’ capacity constraint $v(r)$ is such that not all G-type borrowers can get a loan from them - hence, there exists a potential conduit mortgage market. For this we need to assume that $\lambda_G > v(r)$. So let $\lambda_G = 1.5$. We assume that conduit lenders don’t have a capacity constraint, so that they can lend to any amount of borrowers, provided they receive a good signal.\footnote{Conduit lenders’ capacity constraints would rationalize the presence of subprime consumers that don’t get access to a mortgage when the mass of consumers unserved by portfolio lenders exceeds the conduit lenders’ capacity constraint, i.e., when $\lambda_G + \lambda_B > v(r) + v(k)$. We omit this possibility for simplicity.}

In this section we explicitly give the closed form solution corresponding to an equilibrium where consumers either choose owner-occupied housing or rental housing in each period (corner solutions). With this simple setting in mind we then illustrate the main ingredients of this model: (i) house prices, implications of local land regulation constraints and consumers’ access to loans, (ii) mortgage market collapses, and (iii) characterization of the closed form equilibrium solutions for house purchases, mortgage originations and securitization, and the size of the owner-occupied housing and rental markets.

### 4.1 House prices and local land use regulations

This subsection first takes a look at the owner-occupied housing price to understand the implications on the consumer’s housing type choice decision in periods 1 and 2. We then examine the effect of land use regulation on the exclusion of subprime borrowers from the mortgage market.

First, recall that the aggregate demand of owner-occupied housing consumption in the \textit{first} period and the aggregate supply of owner-occupied housing consumption in the \textit{second} period are inelastic, with both equal to $\bar{H} = 1$. A constant stock of owner-occupied housing is convenient to get simple closed form equilibrium solutions because the market clearing housing prices are such that $p_1 = p_2 = p$.\footnote{The owner-occupied market clearing equations in periods 1 and 2 and the households’ optimal choice $H_2^h = 0$ (shown in the Appendix) imply that $p_1 = p_2 = p$.} Defaults occur in our model due to an imperfect credit scoring technology that is not able to perfectly screen those borrowers that receive a positive endowment shock in the second period, and not due to house price movements.\footnote{For a model where default is triggered by a fall in house prices, see Chatterjee and Eyigungor (2015).}
Secondly, the equilibrium owner-occupied house price is such that \( p > 1 \). This in turn implies that old households with a mortgage will sell their house in the second period and move to rental housing,\(^{31}\) as the benefits to owning go away as the younger household transitions to older age. In the first period, however, young consumers with a mortgage will find it optimal to buy a house (provided that the credit scoring technology parameter \( \pi^k \) exceeds a certain threshold - see below).

Thirdly, portfolio lenders can in general lend to G-type consumers or to B-type consumers. Recall that portfolio lenders know the borrower’s type, and hence can charge a risk-based mortgage loan rate. So far we have assumed that portfolio lenders only lend to G-type consumers (say, because of regulation on formal banks or stigma). Here we explore a different reason. In particular, G-type consumers may end up crowding out B-type consumers from the portfolio mortgage market if there is a local policy that requires a minimum house (lot) size \( H_{\text{min}} \) equal to\(^{32}\)

\[
H_{\text{min}} = \omega^{SR}/p(1 - \delta^l)
\]

This policy also implies that those subprime consumers that don’t get a loan have no other option but to rent in the first period as they can only afford buying a house of size \( \omega^{SR}/p \), which is certainly below \( H_{\text{min}} \) since \( p > 1 \). This illustrates how local land regulations, in the form of a minimum lot size, affects the bottom of the housing market by excluding subprime borrowers from the mortgage market.\(^{33}\) Local land use regulations can be embedded in our model by modifying the owner-occupied housing consumption space as \( \{0\} \cup [H_{\text{min}}, H] \).

4.2 Mortgage market collapses

This section identifies three thresholds, \( \pi_0, \pi_1 \) and \( \pi_2 \), for \( \pi^l \equiv \Pr^l(G|\text{rating=G}) \), all functions of the parameters of our economy (\( \theta^l, \theta^o, \delta, d^l, \eta \)), that determine different subprime mortgage market configurations.

1. The presence of land regulation constraints of the type described above might give rise to a situation where the conduit market collapses when conduit lenders’ lending

\(^{31}\)Old households consumption decision of rental housing in the second period is best understood in terms of independent living, assistance living, and nursing care. In addition, Hochguertel and van Soest (2001) provide empirical evidence that young households buy a house to accommodate the new family members and possibly to get access to better schools, but when they are old and the family size decreases, these households sell their houses and move to smaller rental houses.

\(^{32}\)The portfolio mortgage contract \( (q^{B,r},\psi^{B,r}) \) specific for B-type consumers must satisfy budget constraints \( pH_{1,B,r} = \omega^{SR} + q^{B,r}\psi^{B,r} \) and \( \omega^{SR} = \omega^{SR} - \psi^{B,r} + pH_{1,B,r} \) (the latter coming from the limited recourse requirement), which implies \( \psi^{B,r} = pH_{1,B,r} \) and \( \psi^{B,r} = \omega^{SR}/(1 - q^{B,r}) \). Portfolio lender’s optimization implies that \( q^{B,r} = \theta^l \delta^l \). Thus, \( \psi^{B,r} = \omega^{SR}/(1 - \theta^l \delta^l) \) and using again equation \( \psi^{B,r} = pH_{1,B,r} \) we get \( H_{1,B,r} = \omega^{SR}/p(1 - \theta^l \delta^l) \). Then, set \( H_{\text{min}} = \omega^{SR}/p(1 - \theta^l \delta^l) \).

\(^{33}\)Local land use regulation typically imposes minimum quality standards for owner occupied houses. This creates a fixed cost that puts a lower bound on house size (in order for the builder’s profit margin to at least pay for the cost of regulation). See Malpezzi and Green (1996) and NAHB Research Center (2007) for empirical evidence and further explanations, and also the Wharton Housing Regulation Index for measures of housing regulation.
standards, captured by $\pi^k$, deteriorate.\textsuperscript{34} When $\pi^k = \pi_0$ the G-type consumer’s house size purchased with a conduit loan is equal to the minimum house size that a B-type consumer can acquire when revealing his type ($H^{\min}$ was identified in the previous subsection). In particular, threshold $\pi_0$ solves the following equation:

$$H^{G,k}_1(\pi_0) = H^{\min}$$  \hspace{1cm} (18)

When $\pi^k < \pi_0$, conduit loans are so small that borrowers with conduit loans cannot afford to buy a house with size above $H^{\min}$.\textsuperscript{35}

2. The conduit mortgage market will exist as long as G-type consumers prefer to borrow from conduit lenders than renting in the first period. When $\pi^k$ decreases below a given threshold, say $\pi_1$, the implicit interest rate is so high that G-type consumers that are unable to borrow from portfolio lenders and thus can only borrow from conduit lenders prefer to rent in both periods ($R_1 = \omega^{SR}$ and $R_2 = \omega^{+}$). Threshold $\pi_1$, at which indifference between buying a house with a conduit loan and renting in both periods occurs, solves the following equation

$$\eta H^{G,k}_1(\pi_1) + \theta^k \omega^{SR} = \omega^{SR} + \theta^k \omega^{+}$$  \hspace{1cm} (19)

The left hand side term in equation (19) represents the G-type consumer’s utility from buying a house in the first period with a conduit loan and then renting, in the case when both portfolio loan and conduit loan markets are active (hence, the market clearing house price should be computed accordingly). The right hand side term in equation (19) represents the G-type consumer’s utility from renting in both periods. When $\pi^k < \pi_1$, conduit loans are so small that G-type consumers prefer to rent in both periods.

**Lemma 1:** The conduit lender market collapses when $\pi^k < \max\{\pi_0, \pi_1\}$.

3. Consumers may prefer to borrow from conduit lenders when the conduit loan is larger than the portfolio loan. If that is the case, the portfolio market might collapse instead. Formally, there is a threshold $\pi_2$ at which the G-type consumer is indifferent between a conduit loan and a portfolio loan. This threshold solves the following expression:

$$\eta H^{G,k}_1(\pi_2) + \theta^k \omega^{SR} = \eta H^{G,r}_1 + \theta^k \omega^{SR}$$  \hspace{1cm} (20)

The left hand side term in equation (20) represents the G-type consumer’s utility from buying a house in the first period with a conduit loan and then renting, in the case when only the conduit loan market is active. The right hand side term in equation (20) represents the G-type consumer’s utility from buying a house in the first period with a portfolio loan and then renting, in the case when both portfolio

\textsuperscript{34}Conduit lenders’ lending standards are captured by $\pi^k$, which in turn depends on the proportion of G and B types and the credit scoring technology that classifies by type, more or less accurately.

\textsuperscript{35}The housing market clearing price $p$, which increases in $\pi^k$ (as the conduit loan increases), will not decrease any further when $\pi^k$ decreases below $\pi_0$.  

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loans and conduit loans markets are active.\textsuperscript{36} Observe that when $\pi^k > \pi_2$, consumers prefer conduit loans even when conduit lenders risk-price the presence of lemons and their subsequent default into the mortgage discount price, and the retail market of portfolio loans shrinks. In this case, the conduit lender’s fundamental proportions of G-type consumers, $\hat{\pi}(G)$, improves as now conduit loans are the first best option for G-type consumers. Also interestingly, when the mortgage distribution rate increases, $\pi_2$ decreases, the conduit mortgage market expands.

**Lemma 2:** The portfolio lender market shrinks when $\pi^k > \pi_2$.

Below we summarize the different possible market configurations in terms of parameter the conduit lender’s credit scoring technology and indicate the size of the portfolio and conduit mortgage markets for each of these configuration. We find convenient to write

$$\alpha^k \equiv \Pr(\text{rating}=G|G) \text{ and } \beta^k \equiv \Pr(\text{rating}=G|B).$$

For simplicity, we assume that conduit lenders are not capacity constrained (alternatively, $v(k) > \alpha \lambda_G + \beta \lambda_B$), so whenever a G-type is not able to borrow from a portfolio lender, he can always try to borrow from a conduit lender. However, not all G-type consumers that attempt to borrow from a conduit lender end up with a loan. This is because the conduit lender’s credit scoring technology identifies a G-type consumers as a bad consumer with positive probability.

**Proposition 2** (Mortgage market configurations):

- If $\alpha^k < \max\{\alpha_0, \alpha_1\}$, the conduit mortgage market collapses and only a mass $v(r)$ of G-type consumers can borrow to buy a house. The rest of the consumers, with mass $\lambda_G - v(r) + \lambda_B$, rent in both periods.

- If $\alpha^k > \alpha_2$, G-type consumers prefer the conduit mortgage market. A mass $\alpha \lambda_G + \beta \lambda_B$ of consumers receive a good rating and are able to borrow at the conduit loan rate and buy a house. Those G-type consumers without a conduit loan, with mass $\min[(1 - \alpha)\lambda_G, 1]$, will borrow from their second best option, the portfolio loan market. The rest of consumers, with mass $(1 - \alpha)\lambda_G + (1 - \beta)\lambda_B - \min[(1 - \alpha)\lambda_G, 1]$, will rent in both periods.

- When $\alpha^k \in [\max\{\alpha_0, \alpha_1\}, \alpha_2]$, portfolio lenders lend to a mass $v(r)$ of G-type consumers, whereas conduit lenders lend to a mass $\alpha(\lambda_G - v(r)) + \beta \lambda_B$ of consumers. The rest of consumers (those who receive a bad rating by the conduit lender), with mass $(1 - \alpha)(\lambda_G - v(r)) + (1 - \beta)\lambda_B$, will rent in both periods.

The proof follows immediately from our previous analysis and is thus omitted. Next we provide a graphical example that depicts the three equilibrium thresholds $\alpha_0, \alpha_1$ and $\alpha_2$, along the 45° line, as a function of the conduit lender’s credit scoring technology parameter $\pi^k$. Parameter values for this example are $d^k = 0.8$, $\theta^k = 0.4$, $\delta^k = 0.7$, $\theta^l = 0.9$, $\eta = 4$, $\delta = 0.5$, $v^k = 1$, $\lambda_G = 1.5$, and $\lambda_B = 0.5$.

\textsuperscript{36}At $\pi_2$ the market clearing house price is the same in both cases because at $\pi_2$ the conduit loan amount and the portfolio loan amount coincide.
In Figure 2 we see that when \( \alpha^k \) is below 0.15 the local land regulation constraint \( H_{\text{min}} \) shuts down the conduit lender market \((\alpha^k < \alpha_0 = 0.15)\). Then, as we increase \( \alpha^k \) until 0.50 \((0.15 < \alpha^k < \alpha_1 = 0.50)\), the conduit lender market remains shut down but now due to the bad loan terms and conditions in the conduit mortgage market with a high presence of lemons which makes G-type consumers prefer renting in the first period than buying an owner-occupied house with a conduit loan. When \( \alpha^k \in [0.50,0.71] \), both the portfolio and the conduit markets are active. And finally, when \( \alpha^k \) is above 0.71 \((\alpha^k > \alpha_2 = 0.71)\), the portfolio loan market shrinks from \( \nu(r) = 1 \) to \((1 - \alpha^k)\lambda_G\) as now liquidity from the secondary mortgage securities market allows conduit lenders to offer a better mortgage terms than portfolio lenders. Observe that threshold \( \alpha_2 \) that solves equation (20) exactly coincides with the threshold that solves equation \( q^* = q^k \) (or equivalently, \( EP = 0 \)) and also equation \( q^* \psi^* = q^k \psi^k \).

Thresholds \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) change when the parameters of our economy vary, leading to mortgage market expansions and contractions. In particular,

- When the amount of asymmetric information between borrowers and conduit lenders increases, the credit scoring technology worsens \((\alpha^k \text{ decreases})\), and therefore, all else equal, the conduit market is closer to its collapse (or enters in the collapse region).

- When the consumer’s discount factor \( \theta^h \) increases and the owner-occupied preference parameter \( \eta \) decreases, consumers find renting in the first period relatively more attractive than borrowing-to-own, and thus the conduit loan market shrinks \((\text{as } \alpha_1 \text{ increases})\).

- When the investor’s discount factor \( \theta^i \) and/or the distribution rate \( d^k \) increase, all else equal, the conduit loan market expands \((\text{as threshold values } \alpha_0, \alpha_1 \text{ and } \alpha_2 \text{ decrease})\) because conduit mortgages become more attractive due to the higher investors’ liquidity coming from the secondary mortgage market.
• Higher foreclosure costs expand the region where both portfolio and conduit loan markets are active, as a lower $\delta^k$ decreases the value of thresholds $\alpha_0, \alpha_1$ and increases the value of $\alpha_2$.

4.3 Equilibrium house prices and mortgage origination

Next we characterize the equilibrium house prices and loan amounts. First, given the mortgage discount price $q^*$ chosen optimally by portfolio lenders, G-type consumers will borrow against all their second period revenue, provided they consume exactly the subsistence rent $\omega^{SR}$. The equilibrium portfolio loan amount is an increasing function of the lender’s discount factor. In particular, it is given by the following expression

$$q^* \psi^* = \frac{\theta^l}{1 - \theta^l}$$

(21)

Similarly, a G-type consumer with a conduit loan takes as given the mortgage discount price $q^k$ that the conduit lender optimally chooses, which incorporates a penalty due to the presence of lemons, and borrows against his future income, provided that he consumes exactly the subsistence rent $\omega^{SR}$. B-type consumers that receive a good rating by the conduit lender are lucky to misrepresent their type and will borrow under the same terms than G-type consumers. The equilibrium conduit loan amount increases with the power of the credit scoring technology $\alpha^k$, the foreclosure recovery rate $\delta$, and the $d^k$-weighted discount factor $\theta^l$, which in turn increases with the distribution rate $d^k$ and the investor’s discount factor $\theta^i$ and decreases with the lender’s discount factor $\theta^l$. In particular, the equilibrium conduit loan amount is given by the following expression:

$$q^k \psi^k = \frac{\alpha^k \theta^l}{1 - \theta^l (\alpha^k (1 - \delta) + \delta)}$$

(22)

A conduit lender that sells a fraction $d^k$ of his originated mortgages to the investors generates an income that is increasing in $d^k$, the measure of consumers $\mu^k$(rating=G) that borrow from conduit lenders, and the foreclosure recovery asset value $\delta^k \theta^l (1 - \alpha^k)$. In particular, the equilibrium value of mortgages distributed to investors is given by the following expression:

$$\tau z^k = d^k \mu^k($rating=G$)$$

$$\frac{1 - d^k \theta^l (1 - \alpha^k)}{(1 - \delta \theta^l (1 - \alpha^k))}$$

(23)

where $\mu^k($rating=G$)$ is the endogenous measure of consumers that borrow from conduit lenders, i.e., $\mu^k($rating=G$) = 0$ if $\alpha^k < \max\{\alpha_0, \alpha_1\}$, $\mu^k($rating=G$) = \alpha^k (\lambda_G - v(r)) + \beta^k \lambda_B$ if $\alpha^k \in \max\{\alpha_0, \alpha_1\}, \alpha_2\}$, and $\mu^k($rating=G$) = \alpha^k \lambda_G + \beta^k \lambda_B$ if $\alpha^k > \alpha_2$. If investors had limited wealth, conduit lenders would be constrained by the total amount of credit that can be securitized, i.e., $d^k \varphi^k \leq z^k = \omega^1$ where the first inequality obeys the originate-to-distribute constraint (5) and the second equality follows from market clearing in the mortgage-backed securities market. One can now see that the equilibrium quantity of mortgages originated by conduit lenders is constrained by the investor’s wealth because $\tau z^l \leq \omega^1$. Thus, our model is also able to capture Gennaioli, Shleifer, and Vishny (2012) result that investors’ wealth drives up securitization.
Finally, the equilibrium house price depends on the mass of consumers with access to a mortgage. It is given by the following expression:

\[ p = v(r) \left( \omega^{SR} + \frac{\theta^* \omega^+}{1-\theta} \right) \text{ if } \alpha^k < \max\{\alpha_0, \alpha_1\} \]

\[ p = \begin{cases} (v(r) + \mu^k(\text{rating=G}))\omega^{SR} + \frac{\theta^* \omega^+ + \mu^k(\text{rating=G})\omega^+ \theta^* \pi^k}{1-\theta(\pi^*(1-\theta) + \beta)} & \text{if } \alpha^k \in [\max\{\alpha_0, \alpha_1\}, \alpha_2] \\ \mu^k(\text{rating=G}) \left( \omega^{SR} + \frac{\omega^+ \theta^* \pi^k}{1-\theta(\pi^*(1-\theta) + \beta)} \right) & \text{if } \alpha^k > \alpha_2 \end{cases} \]

where, as pointed out before, \( \mu^k(\text{rating=G}) = 0 \) if \( \alpha^k < \max\{\alpha_0, \alpha_1\}, \mu^k(\text{rating=G}) = \alpha^k(\lambda_G - v(r)) + \beta^k \lambda_B \) if \( \alpha^k \in [\max\{\alpha_0, \alpha_1\}, \alpha_2] \), and \( \mu^k(\text{rating=G}) = \alpha^k \lambda_G + \beta^k \lambda_B \) if \( \alpha^k > \alpha_2 \).

5 The rise and fall of subprime mortgage lending

In this section we show how our model can generate different equilibrium regimes depending on the predictive power of the credit scoring technology (hard information) and the liquidity from the secondary MBS market. We first provide a short narrative for each of the equilibrium regimes, and then illustrate the behavior of key equilibrium variables with simulations.

- **Only portfolio lenders** \((\alpha^k < \max\{\alpha_0, \alpha_1\})\)

Consider a world (pre-middle 1990s) in which subprime loan credit scoring technology was crude and there did not exist powerful summary statistics on consumer credit quality (FICO score). This meant that it was very difficult for subprime loan originators to reliably distinguish between good and bad credit borrowers based on hard information. If transaction-based lending were to occur based on hard information only, the high likelihood to confusing good and bad types in underwriting decisions would increase loan rates substantially due to adverse selection concerns, thus potentially pricing all borrowers out of the market. But relationship lenders (local depository financial institutions) are capable to soliciting soft information to improve their underwriting decision outcomes. Potentially based on regulatory requirement (e.g., CRA), localized relationship lenders are the only available source of subprime loans, but are subject to capacity constraints that result in the rationing of credit (to good types) in subprime neighborhoods.

In addition to adverse selection concerns as related to loan pricing with transaction-based lending, in this world there was also little demand for subprime loans packaged as securities. There are not strong regulatory or tax reasons to invest in pooled-tranched securities backed by mortgage or other types of loans. Capital flows into bond markets are “normal” and are not distorted by factors such as foreign capital flows looking for dollar denominated low-risk investments. This implies that a private-label subprime MBS market is non-existent, since the high cost of loan sales is not offset by any other benefits that might be associated with subprime loan securitization.
• Conduit lenders enter into the subprime mortgage market \((\alpha^k \in [\max\{\alpha_0, \alpha_1\}, \alpha_2])\)

Now consider an evolved world (say from the middle 1990s to early 2000s) in which credit information is now available to improve credit scoring decisions (FICO is introduced and provides accurate assessments of borrower credit quality), and where credit scoring models themselves improve. This creates a foundation where it is now possible to more credibly distribute subprime loans into a secondary market. Concurrent with this is the introduction of capital reserve regulation (Basel II) that increases the attractiveness of owning low credit risk (AAA-rated) securities. There has also been shocks (the Asian and Russian financial crises) that have shifted foreign capital flows towards dollar-denominated U.S. Treasuries and close substitutes. This shift in demand has decreased yields of riskless and near riskless bonds, causing fixed-income investors to move further out the credit risk curve in search for higher yields. The search for higher yields and favorable capital treatment causes demand for AAA-rated securities to skyrocket. But these securities are not in sufficient supply to meet all of the demand. The subprime mortgage market represents a vast untapped market, where the pooling of such loans can then be converted (in part, but large part) into AAA-rated securities in large quantities to help satisfy the demand.

Improved credit scoring technology along with a high demand for manufactured AAA-rated securities sets the stage for the rise of the subprime mortgage market. A reduction in the pooling rate on subprime loans due to better (perceived if not actual) sorting of good and bad types makes it feasible for low-cost transaction-based lenders (brokers and other conduit lenders) to set up shop to apply automated underwriting based on hard information only.

• Conduit lenders dominate the subprime lending market \((\alpha^k > \alpha_2)\).

Initially rates in the conduit lending market exceed rates that can be gotten from traditional portfolio lenders, so good types migrate to portfolio lenders until capacity constraints force them into the conduit market. But as demand for manufactured AAA-rated securities continues to increase, and confidence in the credit scoring technologies builds, conduit mortgage loan rates become more competitive when compared with portfolio mortgage loan rates. Subprime home ownership rates increase to increase the overall rate of home ownership.

By the early to middle 2000s, demand for AAA-rated securities has intensified. With this intensified demand and increasing confidence in the basic conduit loan business model, conduit loans rates decline to the point where the pooled conduit loan rate falls below the portfolio loan rate, and the traditional portfolio loan market shrinks (relative to the total size of the subprime mortgage market) as good subprime borrower types migrate to the conduit loan market to take advantage of the low rates. There is a housing market boom.

• The collapse of the conduit loan market \((\alpha^k < \max\{\alpha_0, \alpha_1\})\)

Finally, starting in 2006, with the start of a sustained decline in house prices and concerns about the performance of subprime MBS, confidence in the credit scoring based conduit loan business model is shaken. This causes investors to increase the pooling loan rate as the credit scoring classification system is scrutinized, and a fall-off in demand for
credit-risky MBS occurs. This causes the conduit loan market to collapse as conduit loan rates spike. Subprime home ownership rates stall and the housing boom ends (badly).

- The conduit loan market reemerges ($\alpha^k \in [\max\{\alpha_0, \alpha_1\}, \alpha_2]$)

Lastly, as a post-script, imagine it is 2018 and the U.S. economy is now “normalized”. The “broken” securitized lending business model is declared to be “... xed” as improved scoring variables are introduced and mechanisms are put into place to improve the quality of credit model assessments. The percentage of good types in the subprime population increases due to an improved job outlook and increasing wages at the low end of the labor market. Demand for highly rated securities has persisted, and once again a conduit subprime mortgage market emerges to provide financing for the lower end of the housing market.

5.1 Simulations

Using the same parameter values than in the example of Figure 2, we illustrate how the equilibrium house price and loan amounts change as a function of the conduit lender’s credit scoring technology for the different regimes identified above. For this exercise, we consider a simpler setting where

$$\Pr_l^{G}(\text{rating}=G) = \Pr_l^{B}(\text{rating}=B) = \frac{\alpha^l(\mu_G^l + (1 - \alpha^l)\mu_B^l)}{\alpha^l(\mu_G^l + (1 - \alpha^l)(\mu_G^l + (1 - \alpha^l)\mu_B^l))}$$

As before we let $v(r) = 1$, $\lambda_G = 1.5$, and $\lambda_B = 0.5$.

Figure 3 portraits the portfolio loan, conduit loan and mortgage securitization’ equilibrium values ($q^*, q^k \psi^k$ and $\tau^k$, respectively) as a function of the conduit lender’s credit scoring technology parameter $\alpha^k$. Observe that at $\alpha_2$ the portfolio loan size and the conduit loan amounts coincide, whereas when $\alpha^k > \alpha_2$ the conduit loan is larger than the portfolio loan. Also notice that threshold $\alpha_2$ is the same one as the one we identified in Figure 1 when we compared the conduit loan rate with the portfolio loan rate. At this point, the

37In the Appendix we show that this expression is equivalent to

$$\Pr_l^{G}(\text{rating}=G) = \frac{\Pr(\text{rating}=G|G) \hat{\tau}^k(G)}{\Pr(\text{rating}=G|G) \hat{\tau}^k(G) + \Pr(\text{rating}=G|B) \hat{\tau}^k(B)}$$

where $\hat{\tau}^k(G)$ denotes the fundamental proportion of G-type consumers available to the conduit lender.

38This choice of values is convenient as they imply that $\alpha_0 = \pi_0$, $\alpha_1 = \pi_1$ and $\alpha_2 = \pi_2$. For example, $\alpha_2$ solves the following equation $\alpha_2 \left[ 1 - 2\pi_2 \right] \left( \lambda_G > v(r) - \lambda_B \right) + \alpha_2 (\mu_B + 2\pi_2 (\lambda_G - v(r) - \lambda_B)) - \pi_2 \mu_g = 0$.
conduit lender’s fundamental proportion of G-type consumers \( \tilde{\pi}^k(G) \) jumps from 0.5 to 0.75, and hence \( \pi^k \) and \( q^k\psi^k \) jump too. As \( \alpha^k \) keeps increasing above \( \alpha_2 \), the conduit loan also increases. The value of the amount of securitization, captured by expression (23), is a fraction \( d^k \) of the quantity \( \varphi^k \) of mortgages originated by conduit lenders, i.e., \( \tau z^k = \tau d^k \varphi^k \).

In Figure 3 we can also see how \( \tau z^k \) also jumps at \( \alpha_2 \) (market clearing in the conduit loan market requires that \( \varphi^k = \mu^k(\text{rating}=G)\psi^k \), and \( \psi^k \) jumps when \( \alpha^k > \alpha_2 \)).

Figure 4 illustrates the equilibrium owner-occupied housing price \( p \) as a function of the credit scoring technology \( \alpha^k \). The house price is low and constant when only the portfolio mortgage market exists (i.e., when \( \alpha^k < \max\{\alpha_0, \alpha_1\} \)). The equilibrium house price jumps when \( \alpha^k > \alpha_1 \) because new home buyers with credit coming from the conduit market enter the owner-occupied housing market. Also, within range \( \alpha_1 \leq \alpha^k \leq \alpha_2 \), we observe that \( p \) increases when the credit scoring technology improves because conduit loans increase with \( \alpha^k \). Finally, when \( \alpha^k \) jumps above \( \alpha_2 \), consumers prefer to borrow from conduit lenders who offer a larger loan amount due to their access to a liquid secondary MBS market (the conduit lender’s fundamental proportion of G-type consumers \( \tilde{\pi}^k(G) \) jumps from 0.5 to 0.75), and therefore the house price jumps at \( \alpha_2 \), and then it keeps increasing as the credit scoring technology improves.
Figure 4

Figure 5 portrays the equilibrium house sizes for different borrowers ($H^{G,r}$ for borrowers with portfolio loans and $H^{G,k}$ for borrowers with conduit loans) as a function of $\alpha^k$. There we see that the house size bought with a portfolio loan is constant until $\alpha_1 = 0.5$, and then plummets as new buyers enter in the housing market when the conduit mortgage market opens (at $\alpha_1$). The house size continues decreasing as the credit scoring improves. When $\alpha^k = \alpha_2$, the equilibrium house size plummets again as conduit loans become larger (hence a bigger pressure in the housing market) as the fundamental proportion of G-type consumers in the conduit lenders’ pool of borrowers, $\hat{\pi}^k(G)$, jumps from 0.5 to 0.75. On the other hand, the equilibrium size of houses purchased with a conduit loan is increasing in the credit scoring parameter $\alpha^k$ in the region $\alpha^k \in [\alpha_1, \alpha_2]$, and then from $\alpha_2$ onwards it starts decreasing, as the convex effect of the house price dominates the concavity of the conduit loan in that region. In Figure 5 we also see that there is no much of a discontinuity at $\alpha_2$ in the equilibrium size of houses purchased with conduit loans. This is because the jump of the conduit loan amount compensates the fall in the equilibrium house price at that point. Finally, notice also that when $\alpha^k$ goes beyond $\alpha_2$, the house size of a conduit borrower becomes larger than the house size of a portfolio borrower. This also according to our result that portfolio mortgage market is not the consumers’ first option once the credit scoring technology goes beyond $\alpha_2$.

\[39\] Figures 4 and 5 also use the same parameter values as in the example of Figure 2 and hence equilibrium threshold values remain the same.
5.2 Access to and fragmentation of the rental and owner-occupied housing markets

The size of the rental market, given by the measure of consumers that are not able to borrow from either portfolio or conduit lenders, depends on the conduit lenders’ capacity to lend and their credit scoring technology. Next we illustrate how the sorting of borrowers into the different types of mortgage lending markets determines the size of the rental market. Figure 6 depicts the measure of tenants that rent in both periods for the different regimes. First, notice that portfolio lenders exhaust their lending capacity constraint $v(r) = 1$ by lending to a mass 1 of G-type borrowers. Hence, when $\pi^k < \pi_1 = 0.50$, there are only portfolio loans issued, and therefore a mass

$$\lambda_G + \lambda_B - 1$$

of households have no other option but to rent. Second, since conduit lenders can absorb all excess demand of consumers with a good rating, we have that, when $\alpha^k \in [\alpha_1, \alpha_2]$, a mass $\mu^k(\text{rating=G}) = \alpha^k(\lambda_G - 1) + (1 - \alpha^k)\lambda_B$ of consumers are able to get a conduit loan, whereas the remaining consumers, with mass

$$\frac{(\lambda_G - 1) + \lambda_B}{\lambda_G + \lambda_B} - \frac{(\alpha^k(\lambda_G - 1) + (1 - \alpha^k)\lambda_B)}{\lambda_G + \lambda_B}$$

have no other option but to rent. Third, when $\pi^k \geq \pi_2 = 0.71$, all consumers attempt to get a conduit loan first. However, only a mass $\mu^k(\text{rating=G}) = \alpha^k\lambda_G + (1 - \alpha^k)\lambda_B$ of consumers can make it. Those G-type consumers without a conduit loan, with mass
(1 – \( \alpha^k \))\( \lambda_G \), attempt to get a portfolio loan, their second option. The remaining consumers, with mass

\[
\lambda_G + \lambda_B = \frac{(\alpha^k \lambda_G + (1 – \alpha^k) \lambda_B)}{\min[(1 – \alpha^k)\lambda_G, 1]} \text{ Mass of consumers with a conduit loan (\( \mu^k(\text{rating}=\text{C}) \)) when \( \alpha^k > \alpha_2 \)}
\]

have no other option but to rent. In Figure 6 we see equilibrium values (24), (25) and (26) when \( \lambda_G = 1.5 \) and \( \lambda_B = 0.5 \), plotted against the credit scoring parameter \( \alpha^k \). The size of the rental market is the biggest when \( \pi^k < \pi_1 \). Above \( \pi_1 \) the rental market shrinks as new consumers get (conduit) loans. Then, we see how the rental market shrinks again at \( \pi_2 \) as the conduit mortgage market absorbs a substantial larger fraction of G-type and B-type consumers, while the portfolio mortgage market also absorbs those G-type consumers without a conduit loan. At \( \pi^k = \pi_2 \) a mass \( (1 – \alpha^k)\lambda_B \) of B-type consumers are able to get a conduit loan. However, as \( \alpha^k \) gets closer to 1, the mass of B-type consumers without a conduit loan that have no other option than to rent increases and converges to \( \lambda_B \).

6 Endogenous soft information acquisition

Portfolio lenders are relationship lenders that, in effect, know their borrowers and their communities, with borrowers that maintain checking and other personal accounts with the lender that has an established presence in the community. Also, portfolio lenders are subject to a stricter regulation on asset quality that generally results in having invested in people and technologies to resolve information asymmetries at low cost. Conduit lenders as shadow banks are not subject to that strict regulation and hence do not have the same incentives. So far, we have assumed that conduit lenders only relied on hard credit information, given by parameter \( \pi^k < 1 \). In this section we make the choice of soft information acquisition an
endogenous variable for conduit lenders. We aim to explain how conduit lenders’ decision to acquire soft information varies with their discount factor, their mortgage distribution rate, and their default losses.

Let us modify the profit function $\Phi^l(\varphi^l, z^l)$ as follows:

$$\Phi^l(\varphi^l, z^l) = (\omega_1^l - s - q^l \varphi^l + \tau z^l) + \theta^l (1 - d^l) (\pi^l(s) \varphi^l + (1 - \pi^l) \delta p_1 H^G_1),$$

where $s$ denotes the cost to acquire soft information in the first period, and $\pi^l(s)$ is a continuous, increasing and concave function of $s$. Taking the partial derivative with respect to $s$, with (5) binding and writing $D^l = \varphi^l - \delta p_2 H^G_1$ to denote the lender’s default losses, we obtain in equilibrium:

$$[s] : 1 = \theta^l (1 - d^l) \frac{\partial \pi^l(s)}{\partial s} D^l$$

(FOC$[s]$)

From first order condition (FOC$[s]$) we can see that the conduit lender finds optimal to acquire more soft information the higher is his discount factor $\theta^l$, the lower is the mortgage distribution rate $d^l$, the higher are the default losses $D^l(\varphi^l)$, and the stronger is the effect of $s$ on $\pi^l(s)$.

The following figure plots the marginal cost (MC) and marginal benefit (MB) functions - corresponding to the left hand side and right hand side of (FOC$[s]$) equation, respectively - as a function of the amount of soft information acquisition when $\pi^l(s) = 0.3 + \sqrt{s}$ (where the first and second components correspond to hard and soft information, respectively) with $s \in [0, 0.49]$, $\omega_2^s = 1$, $\omega_2^SR = 1/2$, and $\delta = 0.5$. In this figure MC is constant and equal to 1, while MB is decreasing with slope $-1/(4s^{3/2})$. The intersection between MC and MB pins down the optimal amount of soft information acquired by conduit lenders. In Figure 7 we plot two marginal benefit curves, one with low distribution rate ($d^k = 0.1$) and another with high distribution rate ($d^k = 0.8$). As expected, when the mortgage distribution rate increases from 0.1 to 0.8, the amount of soft information acquired by conduit lenders decreases from 0.32 to 0.02 (and hence $\pi^l$ decreases from $\pi^l(0.32) = 0.86$ to $\pi^l(0.02) = 0.34$), as conduit lenders pass default risk to the investors. This result complements Keys, Mukherjee, Seru, and Vig’s (2010) empirical evidence that existing securitization practices did adversely affect the screening incentives of subprime lenders. In particular, Keys, Mukherjee, Seru, and Vig (2010) find that conditional on being securitized, the portfolio with greater ease of securitization defaults by around 10% to 25% more than a similar risk profile group with a lesser ease of securitization. Their results are confined to loans where intermediaries’ screening efforts may be relevant and soft information about borrowers determines their creditworthiness.

In equilibrium default losses can in turn be expressed as a function of the parameters of our economy as follows:

$$D^l(\omega_2^s, \delta, d^k, p^k, \theta^l, \theta^h) = \omega_2^s \frac{(1 - \delta \theta^l)}{1 - \theta^h (\pi^l(s) (1 - s) + 0.5 \omega_2^SR s).}$$

Bubb and Kaufman (2014), on the other hand, study the effect of the moral hazard of securitization on lenders screening, and conclude that securitization did not lead to lax screening.
To sum up, we conclude that conduit lenders acquire less soft information when they care less about future consumption, when they keep less mortgages in their portfolio and/or when default losses decrease.

7 Adverse selection in the secondary mortgage market

So far we have assumed in the baseline model that conduit lenders - the ones who originate to distribute - only rely on hard information and that investors rely on the same credit scoring technology than conduit lenders, i.e., \( \pi^i = \pi^k \). Moreover, portfolio lenders, who by assumption have access to soft information, are not allowed to sell their originated mortgages to investors. This set of assumptions eliminates the possibility of adverse selection in the secondary mortgage markets. Adverse selection in secondary markets may arise if investors who only rely on hard information buy mortgage-backed securities from lenders that have superior (soft) information. This section explores this possibility and its implications on mortgage spreads and realized defaults.

Consider a setting where the conduit loan market is dominant and the portfolio loan market is relatively small, similar to the one we characterized in our previous analysis when \( \pi^k > \pi_2 \). Now consider a change in the business model of portfolio lenders to gain market quote. In particular, assume that portfolio lenders also originate-to-distribute subject to a distribution rate \( d^l \). Thus, in this setting, portfolio lenders are similar to conduit lenders, with the only advantage that portfolio lenders can acquire soft information at no (low) cost (\( \pi^r = 1 \)). We will call them “sophisticated portfolio lenders”. Investors, on the other hand, cannot rely on soft information and thus their hard credit scoring technology is such that \( \pi^i < 1 \). Accordingly, assuming that the sophisticated portfolio lender’s capacity constraint is smaller than \( \lambda_G \) (for simplicity), we can rewrite the sophisticated portfolio lender’s profit function as follows:

\[
\Phi^r(\varphi^r, z^r) = (\omega^r - q^r \varphi^r + \tau d^r \varphi^r) + \theta^r (1 - d^r) \varphi^r
\]

whereas investors maximize \( \Lambda^i(z^i) \) function (7) defined in Section 2.

\[
\Lambda^i(z^i) = \omega^i - \tau z^i + \theta^i (\pi^i z^i + (1 - \pi^i) d^i \delta p_2 H_1^i)
\]
We can then show that the sophisticated portfolio lender’s discount price is given by the following expression:\(^{42}\)

\[ q' = \frac{\pi^i d' \theta' + (1 - d') \theta'}{1 - \delta(1 - \pi^i) d' \theta'} \]  

(27)

Discount price (27) is always higher than the corresponding discount price found for conduit lenders in the baseline model (expression (14)) under the assumption that \( \pi^k = \pi^i \) (conduit lenders in the baseline model rely on the same information as investors in the modified setting where portfolio lenders can originate-to-distribute). This is because in a competitive framework sophisticated portfolio lenders, who do not face any default risk, can also benefit from the gains from distribution, which are in turn captured by a lower mortgage rate. Thus, when sophisticated portfolio lenders enter into the secondary mortgage market, the sophisticated portfolio loan rate is always smaller than the conduit loan rate, and therefore sophisticated portfolio lenders are always the first choice for borrowers (the conduit loan market can be still active when portfolio lenders are subject to regulation and capacity constraints that limit the number of loans originated). This is an important result that shows how once sophisticated portfolio lenders replace traditional portfolio lenders, the equilibrium regimes may change.

Notice that so far we have assumed that investors believe that a fraction \( 1 - \pi^i > 0 \) of the mortgages purchased to portfolio lenders will default, even if sophisticated portfolio lenders are known to have soft information and thus able to screen between borrower types. What is behind this assumption is that investors do not trust that portfolio lenders will sell them only default-free mortgages and thus rely on their hard credit scoring technology to price mortgage-backed securities.

Another interesting result that we can generate once we introduce sophisticated portfolio lenders into the model and relatively naive secondary market investors is the possibility that investors are selected against by informed mortgage originators and, as a result, investor’s default expectations are lower than their realized default. Below, we explore this possibility under the assumption that \( \pi^i < \pi^r \), which in turn may lead to different mortgage spreads between sophisticated portfolio and conduit loans (as shown above) and also to different realized default rates. Let us start this discussion by considering the following Rule I that dictates how a sophisticated portfolio lender allocates its loan originations. We will refer to loans sold to a G-type borrower as “default-free loans” or “good loans”, and to loans sold to a B-type borrower as “bad loans”.

**Rule I**: Sophisticated portfolio lenders originate G-rating (good and bad) loans and also default-free loans that the hard credit scoring technology assigns a B-rating. The number of loans originated coincides with the lender’s capacity constraint \( d_r \). The number of loans distributed to the investors is equal to \( d_r v(r) \). In a pre-stage, lenders behave as follows. When \( \Pr(\text{rating=G}|B) > 0 \), the lender replaces G-rating default-free loans with G-rating bad mortgages. The lender benefits following this strategy because he gets rid

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\(^{42}\)From the first order condition with respect \( q' \) we obtain \( q' = \pi d' + \theta'(1 - d') \), where \( p_H^G \) is a function of \( z_i \) in equilibrium (we can write \( p_H^G = d' \omega^R + q' z_i \) using consumer’s budget constraints and market clearing \( z_i = z_i' \)). Now, taking the partial derivative of \( \Lambda'(z_i') \) with respect to \( z_i' \) we obtain \( \tau = \theta'(\pi^i + (1 - \pi^i) d q') \). We substitute \( \tau \) into the \( q' \) expression and get \( q' = \theta'(\pi^i + (1 - \pi^i) d q') d' + \theta'(1 - d') \). After some algebra we get the desired price function.
of the G-rating bad mortgages and bring an equivalent number of default-free loans to his portfolio. Also, the lender will keep in his portfolio the B-rating default-free loans (if \( \Pr(\text{rating}=B|G) > 0 \)). Investors are naïve and completely rely on the hard credit scoring model. In particular, investors believe that only a fraction \( \Pr(\text{rating}=G|G) = 1 - \pi^i \) of the pool of G-rating mortgages (with size \( d^r \nu(r) \)) will default given their credit scoring technology.

**Proposition 1:** Under Rule I, the investors’ realized default rate is higher than their expected default rate. Lenders suffer no realized default as long as \( \Pr(\text{rating}=G|G) \geq \Pr(\text{rating}=G|B) \). When this inequality does not hold, only a fraction \( \hat{\pi}(G)(\Pr(\text{rating}=G|B) - \Pr(\text{rating}=B|G)) \) of portfolio mortgages defaults.

The proof of Proposition 1 is left for the Appendix E. We are now ready to examine the implication of Rule I on mortgage pricing. Assume that \( \Pr(\text{rating}=B|G) = \Pr(\text{rating}=G|B) = 1 - \alpha \). Then, according to Proposition 1, lenders suffer no default in their portfolio of loans, and investors’ expected default rate is smaller than the realized default (see Proof of Proposition 1 in Appendix E), i.e.,

\[
\frac{1 - \Pr(G|\text{rating}=G)}{1 - \pi^i} < \frac{1 - \Pr(\text{rating}=G|G) - \Pr(\text{rating}=G|B)}{\Pr(\text{rating}=G|G)}
\]

What Proposition 1 shows is that when investors buy MBS from relatively more informed lenders there are not just G and B types as in the previous sections, there are \((G|\text{rating}=G), (G|\text{rating}=B), (B|\text{rating}=B)\) and \((B|\text{rating}=G)\) types. Of course, Proposition 1 is driven by the assumptions we made in Rule I. We leave for future research the modeling of strategic considerations between portfolio lenders and secondary market investors.

8 Further remarks about the model

8.1 Extension to an stochastic economy with uncertainty

To demonstrate the robustness of our model we have included in the Appendix the technical details of extending the baseline deterministic economy with one state in the second period to an stochastic economy with uncertainty in the consumer’s second period endowment realization. Here we summarize our results.

In the first state the consumer’s endowment, irrespective of his type, is \( \omega^g_h \), whereas in the second state his endowment is \( \omega^{SR} \) and thus defaults on the loan payment. Subprime consumers differ in their probabilities attached to each of these two states. For G-type consumers the probability of \( s_1 \) is \( \beta^G \), whereas the probability is \( \beta^B \) for B-type consumers. We assume that \( 0 < \beta^B < \beta^G \). In the Appendix we show that the baseline model is in fact a particular case of the extended model and that the predictions of the model do not change in qualitative terms.
8.2 Pooling V. Separating equilibrium

We have also examined in the Appendix the possibility of a separating equilibrium for the extended model with $0 < \beta^B < \beta^G$. Here we summarize the characteristics of the separating equilibrium, and argue that our predictions for the rise and fall of subprime mortgage lending are qualitatively similar to the described dynamics of the pooling equilibrium of our baseline model.

The separating equilibrium found in the Appendix is such that $\psi^{k,G} < \psi^{k,B}$ and $q^{k,G} > q^{k,B}$, where $q^{k,G}$ and $q^{k,B}$ satisfy the conduit lender’s optimization problem when lending exclusively to type G and type B consumers, respectively. In particular, we find the following expressions:

\[
\begin{align*}
(q^{k,G}, \psi^{k,G}) &= \left( \frac{\beta^G \bar{h}}{1 - \delta \bar{h} (1 - \beta^G)/v(k)}, \frac{\omega_2^+ \eta q^{k,B} - p \theta^B \beta^B (1 - q^{k,B})}{1 - q^{k,B} \eta q^{k,G} - p \theta^G \beta^G (1 - q^{k,G})} \right), \\
(q^{k,B}, \psi^{k,B}) &= \left( \frac{\beta^B \bar{h}}{1 - \delta \bar{h} (1 - \beta^B)/v(k)}, \frac{\omega_2^+ \eta q^{k,B} - p \theta^B \beta^B (1 - q^{k,B})}{1 - q^{k,B} \eta q^{k,G} - p \theta^G \beta^G (1 - q^{k,G})} \right)
\end{align*}
\]

To economize in space, we leave the technical detail corresponding to the computation of equilibrium for the Appendix. We move directly to the discussion.

In the separating equilibrium G-type borrowers are worse off than when their type is perfectly observed (the first best), as they cannot borrow against all their future expected disposable income, $\beta^G (\omega_2^+ + p_2 H_1 - \omega^{SR})$. The borrower’s trade-off is thus between future consumption versus present consumption. G-type borrowers have a higher probability of being at the good state in the second period, and thus they value second period consumption more than B-type borrowers. As a result, the separating equilibrium has G-type borrowers with a lower second period loan repayment ($\psi^{k,G}$) than B-type borrowers ($\psi^{k,B}$). However, the loan amount in period 1 is smaller for G-type borrowers than for B-type borrowers ($q^{k,G} < q^{k,B}$). Still, G-type borrowers are better off than if they were assigned the contract designed for B-type borrowers. Also interesting is the fact that in a separating equilibrium the G-type borrower’s impossibility to borrow against all future expected disposable income results in a smaller owner-occupied housing consumption than in a pooling equilibrium, where B-types can borrow against all their disposable income of period 2, provided that the credit scoring technology is not too bad (e.g., in the extreme case, when $\pi_k = 0$, the pooling discount price is 0, and hence $q^k \psi^k = 0$).

In general, the emergence of a pooling equilibrium or a separating equilibrium depends on the rating cost for G-type consumers. To see this, notice that the B-type consumer will prefer to mimic the G-type consumer and get a pooling rate (this is better than paying a risk-based rate). In order for separation to occur, the G-type borrower must pay a signaling cost in order to signal type (in our model this cost is in terms of a lower loan amount) than if type were known to the lender ex-ante. The closed form solution for the signaling cost is rather complicated and long and thus we omit it. But it stands to reason that when the cost of signaling is too high, G-type consumers will not signal and accept a pooling equilibrium. Thus, sometimes the pooling equilibrium unravels as a result.

What is important is that, in terms of the predictions of our model, the separating equilibrium and the pooling equilibrium share the same properties as far as the effects that
the distribution rate, the difference between investors and lenders’ time discount factors and foreclosure cost are concerned. In both equilibrium types, when \(d_k\) and/or \((\theta^i - \theta^d)\) increase, and foreclosure loss rate \((1 - \delta_k)\) decreases, the loan discount price is higher (and loan repayment promise remains constant) for both G-type and B-type borrowers, resulting in larger loan amounts for both types of borrowers (see the Appendix for the closed form solutions of the loan amount and discount price in a pooling equilibrium). Similar to our analysis of market collapses in Section 3, when \(\delta^G\) is sufficiently high, G-type consumers prefer conduit loans and the portfolio loan market shrinks (in terms of the \(\pi\)-thresholds, \(\pi_0\), \(\pi_1\) and \(\pi_2\) values diminish, giving more room to the conduit loan market). Also, notice that when \(\beta^G\) is low, the loan discount price is small and so is the loan amount, and thus it can be the case that G-type consumers prefer to rent than to borrow at a high interest rate (low discount price).43 That situation would correspond to the market configuration with only portfolio loans. Finally, observe that, also in the case of a separating equilibrium, land use regulation can exclude subprime consumers from the owner-occupied housing market if \(H_{min} > \omega^{SR} + q_{B}^{k,B}\).

8.3 Recourse v. non-recourse mortgage contracts

Here we compare (limited) recourse mortgages with non-recourse mortgages, and explain the implications for the adverse selection problem to consider instead non-recourse mortgage contracts.44 In a recourse mortgage the lender can go after the borrower’s other assets or sue to have his wages garnished. So far we have assumed that mortgage contracts are recourse but subject to limited liability (mortgage exemptions in recourse mortgages are exhaustively analyzed by Davila (2015)). This is according to common practice in the US, where in most US states, subprime mortgage loans are recourse loans - there are some exceptions, such as purchase money mortgages in California and 1-4 family residences in North Dakota. Some states also limit deficiencies if a creditor proceeds through a non-judicial foreclosure.45 It is in the subprime borrowers group where one expects most limited deficiencies judgments.

Limited recourse loans are captured in our baseline model by the second period budget constraint (4): \(p_2H_2 + R_2 \leq \max\{\omega^{SR}, \omega^h + p_2H_1 - \psi\}\). Under this contract the borrower can credibly commit46 to pay back the loan even if \(p_2H_1 < \psi\) until the point where paying the promise would involve consuming below the subsistence rent (i.e., \(\omega^h + p_2H_1 - \psi < \omega^{SR}\)), which by assumption is protected by the nature of the contract. Adverse selection then arises for this type of limited recourse contracts because subprime consumers have different probabilities of receiving a high endowment in the second period. In the good state both consumer types honor the promise, and in the bad state both types default. The probability of occurrence of each state is different between the two types of consumers though.

Next, we claim that if we modify the baseline model and consider instead non-recourse

\[43\] Also, the G-type consumer has more incentives to rent than to borrow and own when the consumer’s discount factor \(\theta^h\) is high and the owner-occupied preference parameter \(\eta\) is low.

\[44\] See Kobayashi and Osano (2012) for further insights of non-recourse financing on securitization.

\[45\] See Li and Oswald (2014) and also Ghent and Kudlyak’s (2011) table 1 for a summary of different state recourse laws.

\[46\] The way bankruptcy/foreclosure law works is that non-payment results in wage garnishment.
mortgage contracts, the adverse selection problem would disappear in both the pooling and the separating equilibria. In a non-recourse mortgage, if the house does not sell for at least what the borrower owes, the lender must absorb the difference and walk away.\footnote{Notice that in both recourse and non-recourse mortgages, the lender would be able to seize and sell the house to pay off the loan if the borrower defaults.} Accordingly, the second period budget constraint (4) should be rewritten as follows: 
\[ p_2H_2 + R_2 \leq \omega^2_2 + p_2H_1 - \min\{p_2H_1, \psi\} \] (in the Appendix we indicate how to rewrite the consumer’s optimization problem when loans are non-recourse).\footnote{See Geanakoplos and Zame (2014) for the foundations of a non-recourse collateral economy in general equilibrium.}

In the pooling equilibrium of our extended economy with two states in period 2, but with \textit{non-recourse} loans, when the loan repayment amount \( \psi \) (common to both types of consumers) is above the market house value, the borrower defaults. Since \( p_2(s) = p > 1 \) in both states \( s = s_1, s_2 \) of period 2 (as both types of consumers choose to rent and sell their houses in the second period), enforcing debt repayment requires that \( \psi \leq pH_1 \), where \( \psi \) and \( H_1 \) are common for both types of borrowers in the pooling equilibrium. Thus, adverse selection is absent from this setting as both types of consumers can always repay their debt using part or all of the proceeds from the house sale, and still consume the subsistence rent \( \omega^{SR} \).

The same reasoning applies to the separating equilibrium when mortgages are non-recourse. In particular, in this equilibrium \( \psi^{G,k} \leq pH^{G,k}_1 \) and \( \psi^{B,k} \leq pH^{B,k}_1 \), and, for the same reason as before, default never occurs and hence adverse selection is absent. Key to this result is the borrowers’ equilibrium choice to rent and sell their houses in the second period, resulting in \( p_2(s) = p > 1 \).

Notice also that in both types of equilibrium the non-recourse contract does not need to include a limited liability clause, which allows the borrower to consume at least the subsistence rent in the second period, since when \( \psi \leq pH_1 \), the borrower always have means to repay the loan by selling his house and, therefore, does not need to use his own endowment to satisfy the mortgage payment.

Finally, observe that a non-recourse contract may prevent the consumer to borrow against all the second period income that is above \( \omega^{SR} \), as the promise cannot be larger than \( pH_1 \) in the baseline model. Non-recourse, by eliminating adverse selection, causes the \( G \)-type to delay some consumption until the second period. This is welfare decreasing, since households prefer to consume more in the first period. This is both because the household is impatient and because the younger household derives more utility from owning a house than renting.

\subsection*{8.4 Future research}

There are several other interesting theoretical extensions that we leave for future work. First, we think that it would be interesting to examine whether pre-payment penalties and mortgage refinancing have any role in implementing a Pareto superior equilibrium when adverse selection is present. Second, extending our model to an infinite horizon economy would give new insights on how asymmetric information and the origination-for-distribution lending model may generate bubbles and Ponzi schemes. However, we expect that incorporating the consumers’ mortgage market discrete choice into a fully dynamic
infinite-horizon general equilibrium economy with a continuum of agents would considerably be more complicated from a technical point of view. Up to our knowledge this possibility have not been studied yet. Finally, our model can be enriched by incorporating agency issues regarding securitization and examining its implications on distressed loans.

Our model and results provide new insights for empirical work. For instance, one would like to compare the severity of the adverse selection problem in the subprime mortgage market between non-recourse US states and limited recourse US states. For that, Ghent and Kudlyak’s (2011) table 1 serves as an excellent summary of the different state recourse laws. Also, it would be interesting to examine how the severity of the adverse selection problem changed during the different securitization regimes, or when differences along time in foreclosure costs, banks’ lending capacities, or the credit scoring technology are observed. Last, but not least, it would be interesting to see the economic and statistical significance of the four components that we identify in the loan yield spread.

References


