Demand Shocks, Strategic Defaults and the Dynamics of Commercial Real Estate Markets

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Markets for income producing real estate frequently respond asymmetrically to demand shocks. Following large negative shocks, e.g., California’s regional recession of the early 1990s or the recent credit crunch, asset liquidity typically declines but transaction prices change relatively little. In contrast, following positive shocks, the same markets respond with increases in prices. This paper develops a model that provides an explanation for asymmetric responses to demand shocks. Our explanation is based upon the rational response of sellers and potential buyers to the characteristics of their actual, or potential, mortgage loan contracts. The empirical relevance of the model is evaluated through a simulation base upon data from the Los Angeles County market for multi-family dwellings. We conclude that a non-recourse feature, that is prevalent in commercial mortgages, is sufficient to explain asymmetric responses to demand shocks. In addition, the model provides insights into when strategic default is a rational response to negative demand shocks. Specifically, we find that even when an owner has negative equity and negative cash flow default may not be optimal.
I. Introduction

In the short run the supply of real estate, both income producing and owner occupied housing, is fixed and elementary micro-economics tells us that the market response to demand shocks should be symmetric with: negative shocks resulting in price decreases and positive shocks resulting in price increases. In contrast to this theory, these markets typically respond to large negative demand shocks with long periods during which asset liquidity declines but transaction prices change relatively little. For example, following the onset of California’s regional recession of the early 1990s the median “time on market” for homes that sold in California [a frequently used measure of the liquidity] increased from approximately 4 weeks during 1989 to 13 weeks during 1993, where as, the median sales price declined by only 4.1 percent. Recognizing that declines in sales volume are not the same as declines in liquidity, the response of Los Angeles County’s multi-family housing market to the recent credit crunch is illustrative of the phenomenon we are trying to understand. Specifically, between the first quarter 2007 and the first quarter 2009 the sales of multi-family properties declined by more than fifty-seven percent while asset prices declined by approximately 4.2 percent. This paper provides an explanation for this phenomenon that is based upon the rational response of sellers and potential buyers to the characteristics of their actual, or potential, mortgage loan contracts.

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1California Association of Realtors.
2Costar.com. The normalized asset price index is obtained by dividing the same NOI (i.e., $1) by the market cap rate provided by CoStar for sales of apartment complexes of twenty-five or greater unit that occurred during the relevant period. During the same period the average cap rate for properties that sold increased by only 24bp. During 2009 sales continued to decline and prices started to decline.
In the US, the acquisition of income-producing real estate is typically financed by the use of a non-recourse mortgage loan.\(^3\) When an acquisition is financed in this way, the owner’s interest is equivalent to a portfolio comprised of the property, a liability in the form of a mortgage loan, and a put option that gives the owner the right to sell the property to the lender [by defaulting] at a price equal to the loan balance.\(^4\) Consequently, a seller’s reservation prices and/or a potential buyer’s maximum bid price depend on the economic characteristics of the three assets that make up the portfolio.

For organized financial markets, the characteristics of an asset that is purchased are the same as the characteristics of the asset that is sold. This need not be true for the sale of income producing real estate. For example, following a large negative demand shock, a maximum loan to value requirement frequently results in the exercise price of the option that is being “sold” [the seller’s loan balance] being greater than the exercise price of the option that would be acquired [the maximum mortgage loan for a buyer]. This is can be true even when the parties have identical expectations about the property’s investment value and the price formation process.

In addition, if the market rate of interest is greater than the contracted rate, the market value of the loan will be less than the face value of the mortgage [the balance which would be paid upon sale] and the property owner can be said to be subject to an interest rate “locked-in.”\(^5\) As we will see, the considerations of all the assets that make up the portfolio are essential to understanding the dynamics of real estate markets.

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3 Bliss and Cauley (2009) provides a rational for this phenomenon, specifically they show that non-recourse lending is an efficient response to the moral hazard issues associated with the possibility of an investor making a sequence of real estate investments.

4 A non-recourse loan precludes a deficiency judgment following loan default. Previous research [G Kau and Kim, 1994, Cornell et. al, 1996, Genesove and Mayer, 1997 and Cauley and Pavlov, 2002] has recognizes that the option to default may delay a transaction.

5 This conclusion is conditional upon the seller’s loan not being assumable and/or the costs associated with it assumption precludes a buyer from assuming the loan.
Using data for the Los Angeles County apartment building market we estimate, through a simulation, the effect of the two features on the dollar value of the motivation that is needed to induce a transaction. The results of our simulations suggest that when both features are present the non-recourse feature is likely to be more important than the interest rate “lock-in.” In addition, we show that default may not be optimal for an owner with rational expectation who has both negative equity and cash flow.

We conclude that either a non-recourse loan or an interest rate “lock-in” can result in a motivated seller’s reservation price exceeding the maximum bid price of a motivated potential buyer following a negative demand shock. This is true even when the parties have identical expectations about the property’s investment value and the price formation process. Thus, either is sufficient to explain the asymmetric response of real estate markets to demand shocks.

The major contributions of this paper are the integration of theoretical and empirical models of a market for income producing real estate. The models provide both a qualitative and quantitative examination of the response of these markets to demand shocks. We do not contend that ours is the only plausible explanation for the observed phenomenon, but we show that traditional utility maximizing behavior is sufficient to cause asymmetric responses to demand shock. Our results suggest that analysis of the dynamics of real estate markets should start with the recognition that a levered purchase of income producing real estate is the acquisition of a portfolio of assets, not just a piece of real estate.

This paper starts by reviewing alternative explanations of the decline in asset liquidity that follows negative demand shocks. Next we develop a model of a market for
income producing properties to investigate the effect of fixed interest rates and/or non-recourse loans on the conditions under which a mutually agreeable transaction takes place. Specifically, we investigate their effect on: (1) a seller’s reservation price, (2) a potential buyer’s bid price, (3) the dollar value of the motivation needed to induce a mutually agreeable transaction, and (4) the price at which transactions will take place. Finally, we use data for the Los Angeles County apartment building market, to evaluate the empirical relevance of the effects that our model has identified.

II. Why Liquidity Declines

Explanations for the decline in asset liquidity that follows a negative demand shock include market imperfections that result from high information cost and/or the combination of unrealistic expectations on the part of owners and the inability to sell real estate short. While these are important characteristics of real estate markets, they provide a more convincing explanation of short-term market responses than they do the long-term conditions that sometimes occur, e.g., during the early 1990s and post the recent sub-prime mortgage crisis. Given the importance of markets for owner occupied housing it is not surprising that recent research has attempted to provide more convincing explanations for the long-term response of these markets to negative demand shocks. One thread of literature is based upon the observation that for a large fraction of transactions a portion of the equity used to finance the purchase of a home comes from the sale of the buyer’s existing home. Stein [1995] develops a theoretical model that illustrates how homeowners may be “locked-in,” by an equity-constraint. This occurs

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6 It should be remembered that declines in the volume of transactions are not equivalent to declines in asset liquidity. This distinction is important because declines in liquidity imply decreases in economic efficiency whereas declines in volume do not.
because the sale of their existing home, at post shock values, would result in the owner having inadequate equity to purchase another home. Stein concludes that the “lock-in” can exacerbate the effect of a negative demand shock on housing markets [i.e., price decreases beget further price increases].

Genesove and Mayer [1997] investigate the empirical relevance of the “equity lock-in” using data for the Boston condominium market during the early 1990s. The starting point of their analysis is the observation that if the “equity lock-in” was important, homeowners with high loan to value ratios [e.g., over 80 percent after the shock] should have responded differently to the demand shock than homeowners with low LTVs. They find that there was a positive relationship between a condo’s LTV and: (1) the seller’s asking price, (2) time on market [i.e., a decline in liquidity], and (3) price received if the property was sold. Genesove and Mayer conclude that these results are consistent with sellers being equity constrained following a negative demand shock.

While an “equity lock-in” may be important for owner occupied housing, it is not for income producing properties. Investors are not constrained to purchase another property after a sale. Genesove and Mayer’s sample included both owner occupied and non-owner occupied [income producing] units. They note that for investors loan default is equivalent to a put option on the property. They further conjecture that this option will trigger a similar relationship between LTV, asking price, time on market and transaction price for non-owner occupied units. Our model formalizes this idea and extends it to include the possibility of an interest rate lock-up.

Cauley and Pavlov [2002], take a different tack to understanding the dynamics of a housing market’s response to demand shocks. They start with the observation that first

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7 Their sample included approximately 1300 owner occupied and 1060 investor owned units.
home mortgages are almost always (explicitly or implicitly) non-recourse loans. In this paper they consider the implications of the put option embedded in a non-recourse mortgage loan on a homeowner’s response to a negative demand shock. Specifically, they develop a statistical model to estimate the value of delaying the sale of a home and show, through an example, that it may be optimal for a homeowner to delay the sale, even when the homeowner derives no valuable services from the home.⁸ For owner occupied housing a non-recourse loan feature and an equity lock-in are not mutually exclusive explanations for a market’s response to a demand shock. The greater the decline in home value the larger the value of the put option and the more binding the equity “lock-in.” In practice, both are likely to influence the response of housing markets to demand shocks.

In a later article Genesove and Mayer [2001] apply nominal loss aversion, to explain the empirical response of the Boston condominium market [the same data set they used in their earlier paper] to a negative demand shock. Nominal loss aversion is the observation that individuals are more sensitive to losses in asset value than they are to equal size gains.⁹ For this reason, the authors hypothesize that sellers who have experienced nominal losses in condominium values will set higher reservation prices, spend a longer time on market and receive a higher selling price than sellers who have not. Genesove and Mayer conclude that the empirical evidence for nominal loss aversion, for both owner occupied and investors owned units, is very strong. They estimate the effect for owner occupied units is approximately twice that of non-owner occupied units. Again the empirical implication of nominal loss aversion and our

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⁸ A difficulty associated with estimating the effect of the put option on the insensitive to delay the sale of a home is the unknown value of the housing services received by the homeowner.

⁹ An asymmetric response to demand shocks can be thought of as an example of the disposition effect, that is the tendency to keep losers and sell winners [Shefrin and Statman, 1985].
explanations for the response of market to demand shocks are not mutually exclusive, they work in the same direction and are observationally equivalent. In fact both are likely to be present in markets for income producing real estate.

III. A Model of a Market for Income Producing Real Estate

In this section we develop a model of a real estate market designed to abstract from all other considerations, so that we can investigate the impact of the form of the loan contract on the response of a real estate market to a demand shocks. Within the model we: (1) derive the effect on a seller’s reservation price of debt financing; (2) we then determine the conditions under which a mutually advantageous transaction will take place; and; (3) finally we show that both a non-recourse loan and an interest rate “lock-in” are sufficient to generate asymmetric responses to demand shocks.

The market we consider is comprised of $N$ identical liquidity motivated sellers, who own identical income producing properties [e.g., 25 unit apartment complexes]. There are $K$ [$K=N$] identical, liquidity motivated, potential buyers for the properties. There are no transactions costs, and information is symmetric and complete. Sellers or potential buyers receive no direct utility from owning a property. There is agreement as to the price formation and cash flow generation processes. The future is comprised of a countable infinity of periods. At the beginning of a period, sellers and potential buyers are randomly allocated. They then negotiate over the sale of the property. After the negotiation is completed, the realizations of the asset price and cash flow processes for the period are observed. If the parties have not transacted during the first period, they are reallocated and negotiations start over. The assumption that $K=N$ guarantees that if

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10 Liquidity motivated agents do not think they have an information advantage regarding a property’s value.
11 Continuous time, as reflected in the asset price and cash flow stochastic processes, is broken into a sequence of non-overlapping fixed length periods [e.g., month].
an owner does not sell their property during a period, he or she will be matched with a buyer during subsequent periods [similar results can be obtained if $K > N$].

Agreement as to the price formation and cash flow generation processes implies that at the beginning of a period, sellers and potential buyers will agree that $V$ is the investment value of the property [e.g., the present value of all future cash flows for an all equity investment]. To represent the owner’s motivation to sell, we assume that if the property is not sold a constant per-period “liquidity” cost, $\tilde{c}_s > 0$, is incurred.\(^\text{12}\)

Analogously, by transacting, potential buyers are able to avoid a constant per period “liquidity” cost $\tilde{c}_b \geq 0$. These costs provide the motivation to transact.

In the absence of debt, the seller’s reservation price, $R$, is the investment value of the property minus the liquidity costs that would be born if the property were not sold and the maximum price a potential buyer will pay, $B$, is the property’s investment value plus the costs that are avoided if a property is bought ($V + c_b$). When the sum of the liquidity costs [motivations] are non-negative, a mutually agreeable transaction will occur at a price between $V - c_s$ and $V + c_b$. Consistent with the conclusions of elementary micro-economics positive shocks ($dV > 0$) result in price increases and negative shocks ($dV < 0$) in price decreases. In contrast, as we will see below non-recourse debt, actual or prospective, can eliminate otherwise mutually agreeable transactions.

A. **Seller’s Reservation Price With Debt**

At the start of the first period, the seller is assumed to have an existing, non-amortizing, non-recourse mortgage loan with an outstanding balance [face value] of $l_i$
and a net market value of $M_i$. The face and market values may differ because: (1) the market and contracted interest rates are not the same; and/or (2) because of the value of the put option embedded in a non-recourse loan. Note, in our model the value of the put includes future liquidity costs that would be incurred if the property was not sold this period and the seller acted optimally with respect to the mortgage loan [by waiting to default or sell the property] in the future. It follows that increases in the future liquidity costs decrease the current value of the option, which in turn increases the net value of the mortgage loan [i.e., $\frac{\partial M}{\partial s} > 0$].

The net value received from a potential transaction, at a price $S$, equals the net proceeds from the sale $\left(\frac{FV}{s} - \delta_i\right)$ plus liquidity cost avoided $\left(\delta_i\right)$, minus the value of the portfolio representing the seller’s interest in the property $\left(\frac{s_M}{s}\right)$. For a sale to be acceptable the seller’s surplus must be non-negative. That is,

\begin{equation}
V_R - V_M \geq 0
\end{equation}

Thus, the seller’s reservation price, $R$, is:

\begin{equation}
R \geq \frac{V_R - V_M}{s}
\end{equation}

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\footnotesize

\textsuperscript{13} Liquidity costs are the sum of the explicit and the dollar value of the implicit costs of not transacting, for example, a negative cash flow property or the costs of having a portfolio that is out of balance.

\textsuperscript{14} If a transaction is not cumulated at the beginning of a period a liquidity costs are incurred at the beginning of the period.
The above expression suggests how a below market rate mortgage can “lock-in” a property owner with a fixed interest rate loan. Specifically, if the market rate is greater than the contracted rate then, $\frac{b}{s} < \frac{M}{s}$ and if the property is sold the owner would have to pay off a loan balance that is greater than the market value of their mortgage loan. The greater the difference between the market and contracted rate of interest, the greater the “interest rate lock-in.” The owner’s reservation price may, in fact, exceed the property’s investment value [i.e., if $(\frac{F_a}{M} < s)$].

Equation (2) also allows us to determine the effect of a non-recourse feature on a seller’s reservation price. Let subscript $n$ denote all variables related to a non-recourse loan and subscript $f$ denote the corresponding values for a full recourse loan. The effect of a non-recourse feature on a seller’s reservation price is then:

\[
RR_{\text{ANF}} = \frac{F_a - F_m}{F_{m}}.
\]

Assuming the outstanding balance is independent of loan type [i.e., $F_{nF} = F$], equation (3) reduces to:

\[
RR_{\text{ANNF}} = \frac{F_a}{F_m} - 1.
\]

This is true for both full and non-recourse loans. For a variable rate mortgage the market value is independent of changes in interest rates.

While a large fraction of loans on commercial real estate are formally non-recourse we use the term to represent any legal arrangement where a property owner’s maximum potential loss is no greater than the loan’s principal $F$. For example, when the general partners in a limited partnership is a corporation.
Thus, the effect of a non-recourse feature on the seller's reservation price is the difference between the market value of the mortgage with and without this feature. This difference is determined by the [non-negative] value of the put option incorporated in the non-recourse loan.\footnote{This may not be strictly true because the holding period may be a function of the mortgage type.} When the value of the put is positive, $M_M > 0$, a seller’s reservation price with a non-recourse loan will exceed the reservation price of an identical individuals whose ownership is financed with a full recourse mortgage. The larger the value of the put incorporated in the non-recourse loan [e.g., because $dV < 0$ ] the greater the effect of the non-recourse feature on a seller’s reservation price.

We now investigate the effect of a demand shock on a seller’s reservation price. The seller’s pre shock reservation price, $R_p$, equals:

\begin{equation}
R_{p}^M = - \delta - \ldots
\end{equation}

and their reservation price after the shock, $R_a$, is

\begin{equation}
R_{a}^M = - \delta - \ldots
\end{equation}

It follows that the change in reservation price because of the demand shock is:

\begin{equation}
\Delta R_{p}^M = \Delta_{-} - \ldots
\end{equation}

Simplifying, we obtain:
For a negative demand shock the issue is, which declines more the reservation price or the properties investment value. By the definition of a negative demand shock $\Delta R^R < 0$, consequently, the answer depends upon the relationship between $M^p$ and $M^a$.

We will consider demand shocks from two sources: (1) changes in interest rates; and (2) changes in factors other than interest rates [e.g., a regional recession]. If the seller’s mortgage is full recourse, changes in interest rates effects the present value of mortgage loan payments where as changes in property values do not. Thus when a negative demand shock is triggered by an increase in interest rates $|\Delta M| > 0$, the change in reservation price for sellers who have full recourse loans is smaller, in absolute value, than the change in the properties investment value. When the demand shock is triggered by factors other than changes in interest rates the seller’s reservation price and the properties investment value change dollar for dollar in response to a demand shock.

We now consider the relationship between demand shocks and a seller’s reservation price when ownership was financed with a non-recourse loan. To analyze the effect of a demand shock stemming from factors other than changes in interest rates we assume the market rate equals the contracted mortgage rate. In this case, changes in the market value of the mortgage loan, $M$, occur because of changes in the value of the

\[ \frac{dM}{dV} = 0. \]

18 For a full recourse loan the seller’s mortgage contract does not include a put option and $dM/dV = 0$. 

option to put the property to the lender [e.g., $dV\leq 0$ and $dM < 0$], imply the put becomes more valuable and $dM < 0$.

It follows from option pricing theory that, $\frac{\partial M}{\partial V} > 0$, and $\frac{\partial^2 M}{\partial V^2} < 0$.

Consequently, a negative shock, holding interest rates constant, results in $M' > M$ and the reservation price will decline less than the property’s investment value. In addition, the fact that $\frac{\partial^2 M}{\partial V^2} < 0$ implies the larger the negative demand shock, the greater, in absolute value, the difference between the decline in seller’s reservation price and investment value.

For positive demand shocks, holding interest rates constant, the issue is, which increases more the reservation price or investment value? Following the logic used above, we can conclude that the seller’s reservation price will increase less than the property’s investment value. The larger the positive shock the smaller the difference between the increase in investment value and reservation price. For example, when an owner has substantial equity, [i.e., the value of the put option is small] the seller’s reservation price will increase almost dollar-for-dollar with the property’s investment value.

Next we consider demand shocks that are the result of unanticipated changes in interest rates. We know that changes in interest rates are negatively related too both property values, $V$, and the present value of the payments associated with a fixed interest mortgage. If we view $V$ as a function of $r$:

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19 This result provides an explanation for Genesove and Mayer’s [2001] finding that condominium owners who are subject to greater nominal losses set higher asking prices. Their explanation of this phenomenon is loss aversion.
This relationship implies that after a negative demand shocks, either exogenously determined or resulting from an increase in interest rates, the seller’s reservation price will decline less than the property’s investment value. As was previously the case, the reservation price would increase by less than the investment value following a positive demand shock.

Summarizing, we have shown in the context of our model that:

1. Regardless of loan type, if a demand shock is caused by a change in interest rates, a potential seller’s reservation price will change by less than the property’s investment value.

2. For a non-recourse mortgage loan, when a demand shock is caused by factors other than an increase in interest rates, a potential seller’s reservation price will change by less than the property’s investment value.

3. For a full-recourse mortgage loan, when a demand shock is caused by factors other than an increase in interest rates, a potential seller’s reservation price will change dollar per dollar with the property’s investment value.
B. **Buyer’s Maximum Bid Price With Debt**

If a transaction occurs, the buyer will finance the purchase with a non-amortizing, non-recourse loan with face value $F_i$ and market value $M_B$.\(^1\) The buyer’s surplus from a transaction is the value of the portfolio representing their interest in the property acquired minus the price \([S]\) paid for the property. For a transaction to be agreeable from the perspective of a potential buyer it must result in a non-negative surplus. That is,

\[
V_{MFSC_{BBB}} - S \geq 0
\]

This implies that the maximum price the potential purchaser will pay, $B$, is:

\[
B_{VF} = M_B - F_i
\]

We can now explore the effect of a demand shock on a potential buyer’s maximum bid price. Assuming the face and market value of loans used to purchase a property are equal, \([F_M = M_B]\). The maximum price a buyer will pay for a property becomes\(^2\):

\[
B_{VF} = V_{MF_{BBB}} + S
\]

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\(^1\) As noted above, $F_M$ will typically equal $M_B$ at the time of purchase. Reasons why $F_M \neq M_B$ include seller financing at a below market interest rate.

\(^2\) The buyer would be paying for the put option through a higher interest rate at loan origination.
Equation (12) shows that a negative [positive] demand shock will lower [increase], dollar for dollar, the maximum price a purchaser will pay for a property [i.e., $D_{BY}$].

C. Conditions for a Transaction

If a mutually agreeable transaction is to take place, the seller’s reservation price must be less than or equal to the buyer’s maximum bid price:

\begin{equation}
\frac{1}{2} \left( \frac{D_{BK}}{D_{MC}} \right) - \frac{1}{2} \geq 0
\end{equation}

Upon simplification, Equation (13) reduces to the following condition for a transaction:

\begin{equation}
\frac{1}{2} \left( \frac{D_{BK}}{D_{MC}} \right) - \frac{1}{2} \geq 0
\end{equation}

That is, the sum of the liquidity costs that would be born if a transaction does not occur must be at least as large as the difference between the net values of the potential transactors’ mortgage loans. If, as would be expected, mortgage interest rates are set so that at origination the borrower pays for the put option inherent in a non-recourse loan, then at origination the loan’s face value will equal its market value i.e., $F_{MV} = 0$. Thus, for a mutually agreeable transaction to take place, the sum of the liquidity costs [motivation to transact] must exceed the difference between the face value [principal] and the market value of the seller’s mortgage loan:
where $\Delta M_s$ denotes the change in the market value of the seller’s non-amortizing mortgage from origination to date.

Conceptually, we can decompose the market value of the mortgage, $M_s$, into the present value of the mortgage loan payments evaluated at a default-free \textit{but not risk} free rate of interest and the value of a put option whose exercise price is $\tilde{p}_s$. If property values and interest rates were independent, then holding the loan balance constant, declines [increases] in property values and increases [reduction] in interest rates reduce [increase] the market value of the mortgage, $M$. This in turn, increases [reduces] the liquidity costs needed to induce a mutually agreeable transaction. Changes in property values $V$ are, however, negatively related to interest rate changes. As was shown in equation (9) unanticipated increases [decreases] in interest rates, will increase [decrease] the liquidity costs needed to induce a mutually agreeable transaction.

\section*{D. Demand Shocks and Mutually Agreeable Transaction}

Within our model, the effect of demand shocks on asset liquidity is revealed by our analysis of the effect of demand shocks on the maximum bid price of potential buyers and sellers’ reservation price. We concluded that, a demand shock will lower [increase], dollar for dollar, the maximum price a purchaser will pay for a property [i.e., $D\delta V$]. In contrast, a seller’s reservation price will change less, in absolute

\footnotesize
\begin{itemize}
  \item The value of the mortgage is \textit{not} strictly additive because exercising the put results in extinguishing the loan.
  \item For a full recourse mortgage its value is independent of changes in the properties investment value.
\end{itemize}
value, than the change in a property’s investment value \( [\text{i.e., } \Delta \text{RVB} = \Delta] \). Consequently, negative demand shocks will tend to eliminate otherwise mutually agreeable transactions and positive shocks will tend to facilitate transactions. In terms of our model, where sellers are identical and buyers are identical, the outcome of each negotiation will be the same and we can conclude a sufficiently large negative demand shock will preclude any mutually agreeable transactions. In practice, at any given time, there will be distributions of loan balances and liquidity costs. Consequently, only a portion of potential transactions will be eliminated by a negative demand shock. The larger the shock the greater the fraction of potential transactions that will be eliminated and the greater the decline in asset liquidity \( [\text{i.e., increase in expected time to favorable match}] \) that will follow. In contrast, a positive demand shock will make it easier to achieve a transaction. Thus, at least theoretically, the form of the mortgage loan contract, non-recourse and/or fixed interest rate, is sufficient \[\text{but not necessary}\] to generate asymmetric response to large demand shocks.

E. Demand Shocks and Transaction Prices

Now we investigate the implications of non-recourse financing and fixed rate loans on sales prices. Specifically we show that our model implies the empirical observation that transaction prices fall less than the properties investment value following a negative demand shock.

We start by assuming that if a transaction is to take place \( B > R \) the parties “split” the difference between the seller’s reservation price \( R \) and the potential buyer’s

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25 Excepting a full recourse loan, where holding interest rates constant, \( \Delta \text{RVB} = \Delta \).
maximum bid price \([B]\) according to a proportion \(\alpha\), \(0 < \alpha < 1\). That is, the transaction price \(p\) equals:

\[
(16) \quad p = F_B(M_b) - B - \alpha B \sqrt{F_{MB}}.
\]

Simplifying, and assuming \(F_{MB} = 0\), we obtain:

\[
(17) \quad p = F_B(M_b) - \alpha B \sqrt{F_{MB}}.
\]

Taking the derivative of the transaction price with respect to the property’s investment value, \(V\), we obtain:

\[
(18) \quad \frac{dp}{dV} = \frac{1}{V} dV + \alpha - \frac{S}{V}.
\]

Because \(\frac{2M_{L}}{\partial V} > 0\) and \(0 < \alpha < 1\), \textit{when a transaction is possible, the transaction price will fall less than the properties investment value.} Note, this conclusion holds as long as the proportional split is independent of the change in \(V\).\(^{26}\)

IV. Empirical Model of a Real Estate Market

In this section we describe the model and techniques we use to assess the empirical relevance of the theoretical results presented in the previous section.\(^{27}\) Specifically, we use the least-squares Monte-Carlo (LSMC) approach developed by Longstaff and Schwartz [2001] to value the portfolio of assets that is represented by an owner’s interest in an apartment building financed by a non-recourse loan. The LSMC

\(^{26}\) In practice the parties bargaining power, as represented by \(\alpha\) may be effected by demand shocks.
approach combines the principals of risk-neutral valuation with least-squares regression to estimate the value of complex American options.

A. Joint Stochastic Processes

Without loss of generality, our analysis will be couched in terms of a single apartment unit. Underlying our implementation of the LSMC approach is the assumption that the per unit price of an apartment building, $V$, the per unit before tax cash flow yield, $\delta$, and the risk free rate of interest, $r$, are described by the following system of differential equations:\footnote{The approach used here builds upon techniques developed by Cauley and Pavlov [2002] to analyze housing markets.}

$$\frac{dv}{v} = \mu_v dt + \sigma_v dz_v$$
$$\frac{d\delta}{\delta} = \mu_\delta dt + \sigma_\delta dz_\delta$$
$$\frac{dr}{r} = \mu_r dt + \sigma_r dz_r$$

where the parameters $\mu_i, \sigma_i, [i = v, \delta, r]$ are at most functions of the state variables $v, \delta,$ and $r$ with $dz_i$ being increments to Wiener processes. The correlation coefficients between the increments to the Wiener processes are denoted by $\rho_{ij}, [i,j = v, \delta, r]$. In our model, as described below, the risk free rate of interest uniquely determines the equilibrium mortgage interest rate.

In order to estimate the joint stochastic process (19) we need to specify the functional form of the drift and diffusion coefficients. The basic assumption we make is that the expected rate of property appreciation, the drift of the cash flow yield, and the
changes in the risk free interest rate are linear functions of the state variables. This specification implies that the joint stochastic process may be written as:

\[
\begin{align*}
\frac{dy}{\nu} &= \sigma_y \sum_i \rho_{yi} dz_i \\
\delta \sum_i \rho_{yi} dz_i &= \sigma_\delta \\
dr &= \sigma_r \\
\Rightarrow \quad \frac{dy}{\nu} &= \sigma_y \sum_i \rho_{yi} dz_i \\
\delta \sum_i \rho_{yi} dz_i &= \sigma_\delta \\
dr &= \sigma_r
\end{align*}
\]

where \( dz_v, dz_\delta \) and \( dz_r \) are increments to standard Wiener processes with correlations \( \rho_{yi}, \ [i = v, \delta, \tau] \) and \( \sigma_v, \sigma_\delta \) and \( \sigma_r \) are the standard deviation of the respective time series. In this formulation, a negative [positive] demand shock is the occurrence of a \( z_v \) that is negative [positive] and large in absolute value. This formulation excludes the possibility of negative property values and risk free rates of interest, but it does not exclude the possibility of a negative cash flow yield.\(^{29}\)

Under the risk-neutral metric, the above model of property values is replaced by:

\[
\begin{align*}
\frac{dy}{\nu} &= \sigma_y \sum_i \rho_{yi} dz_i \\
\delta \sum_i \rho_{yi} dz_i &= \sigma_\delta \\
dr &= \sigma_r
\end{align*}
\]

Notice that the process for \( \delta \) does not undergo a change under the risk-neutral metric because the cash flow yield [or dividend yield] is not an investment asset by itself and the risk-neutral preferences of the agents have no implication for its evolution over time.

\(^{29}\) The risk free rate of interest is considered because of its relationship to the mortgage rate, and because it is used in risk neutral valuation.
Appendix A provides a formal rational for this conclusion. Going from a risky to a risk-neutral world will result in the correct value of a derivative based upon a traded asset because both the expected return and discount rate used to evaluate the cash flows are adjusted [Hull, 1997].

The LSMC method starts by simulating price paths for the underlying asset [e.g., the value of an apartment unit], the cash flow yield and the risk free rate of interest. In our implementation of the LSMC we generate 10,000 price paths for a typical apartment unit for 120 months into the future. The simulated price paths are then used to estimate, by least squares, the pay-off for the portfolios conditional upon the state variables, $v$, $\delta$, and $r$ at each point in time. The results of the regression are then used to estimate the expected value of continuation. The value of the portfolio is estimated by assuming that a risk-neutral seller or buyer acts optimally given the holding and liquidity costs. The value of the option that represents the owner’s interest in the property then equals the expected present value [at the risk free rate] of the cash flows associated with optimal exercise of the option by sale of the property or default on the mortgage loan.

The data needed to simulate the price paths are the risk free rate of interest, the variance-covariance matrix of the innovations of the processes given by Equation (21), and the parameters $a_{ij}$.

B. Estimates of the Stochastic Processes

Transaction data [1988-2000], provided by CosStar Comps, for the Los Angeles County apartment building market was used to estimate the joint stochastic process (20)

---

29 During the late 1980s many Los Angeles County properties had, as levered investments, negative cash flows.
through the use of seemingly unrelated regression. Appendix B provides the details of the data sources and the empirical estimation of the property value and cash flow yield series used in this estimation. The parameter estimates of process (20) are reported in Tables 1 and 2. These results are statistically significant and consistent with our expectations.

The estimates of volatility of the price appreciation and cash flow yield reported in Table 1 contain an upward bias because the estimated rates of price appreciation and change in cash flow yield used to calculate them are subject to sampling error. Thus, even if the true appreciation rates and changes in the cash flow yield are perfectly explained by the model, the estimated volatility of the innovations will be positive simply due to the sampling error. Cauley and Pavlov [2002] derive a bias correction technique for similarly derived estimates of home price appreciation. This is the technique we used to derive our bias-corrected estimates of volatility. Table 3 reports the bias-corrected estimate of volatility of the rate of price appreciation and change in cash flow yield. Note the risk free rate is not estimated. Consequently the estimated volatility does not have to be adjusted for bias. It can be easily shown that the covariance estimate does not need to be adjusted either.

Our bias-corrected estimate of the volatility of the monthly appreciation rates and cash flow volatility are .0026 and .002 respectively [48 percent and 23 percent reductions relative to the “naïve” estimates presented in Table 1]. Our estimates of the value of the portfolio of assets that correspond to the owner’s interest in a property will be based upon these values.

*30 We are assuming a 10-year, interest only balloon payment loan. Analysis of the optimal exercise of the option results in the conclusion that extending the life of the option, say to 30 years, has little effect on the estimates of value.*
V. Estimates of the Effect of a Non-Recourse Loan on a Seller’s Reservation Price

Above we showed that the effect of a non-recourse feature on the seller’s reservation price is the difference between the market value of the seller’s mortgages with and without a non-recourse feature. In the following example, based upon the LA County data, we illustrate the magnitude of this effect. We assume:

- The mortgage loan is non-amortizing\(^{31}\);
- Refinancing to obtain a lower interest rate is not possible;
- The loan is due [i.e., the option expires] in 10 years and the potential seller can exercise the option [default] at the end of each month [i.e., when the next payment is due];
- There are no costs associated with mortgage loan default;\(^{32}\)
- The current risk-free rate of interest is 4%;
- There are no transaction costs associated with the sale of the property.

Throughout our analysis we derive the market mortgage interest rate from the risk free rate generated by equation (21). The key to computing the equilibrium fixed or adjustable mortgage rate [the spreads over the risk free rate] is the assumption that at origination the value of the mortgage equals the outstanding loan balance. In other words, the mortgage interest rate, either variable or fixed, exactly compensates the lender for the put option imbedded in the loan and for the interest rate risk associated with a FRM. If the expected cost to the creditor of providing the non-recourse feature equals the expected benefits to the borrower, then in a competitive lending industry, our

\(^{31}\) Loan amortization increases carrying costs and reduces the exercise price of the option [i.e., loan balance] over time. The shorter the time to maturity the larger the fraction of the payment is to principal. As will be seen below, amortization has no qualitative effect on our conclusions.
estimates would be the equilibrium mortgage rate. Table 4 presents our estimate of mortgage rates when the risk-free rate is 4%, properties are bought with a 80 percent LTV, refinancing to obtain a lower interest rate is not possible, and there are no liquidity costs. Estimates are provided for both variable and fixed interest rate mortgage loans.\textsuperscript{33} The assumption that all apartment purchases are financed with 80 percent LTV loans is a simplification from the rich mosaic of financing used in practice.

A. Base Case

The base case assumes that the owner incurs no liquidity costs if the property is not sold. Figure 1 depicts the relationship between the owner’s equity, as represented by the property’s market LTV, and the effect of the non-recourse feature on a seller’s reservation price. Our estimates are the difference, in terms of \textit{percent of the investment value}, between the value of the portfolio with and without the non-recourse feature. In this analysis the investment value of the property, \( V_0 \), can be thought of as given.

Variations in LTV are equivalent to variations in the exercise price of the put [i.e., loan balance] that are associated with variations in the acquisition date of the property.\textsuperscript{34} The volatility of property values is by far the most important determinant of the effect of the non-recourse feature on a seller’s reservation price. Figure 1 depicts the effect of the non-recourse feature under two assumptions regarding this volatility. The top line depicts the effect under monthly volatility of 2.6%, which is our best estimate, for Los Angeles County apartments, as reported in Table 3. As a form of sensitivity

\textsuperscript{32} Given the ownership entities used for commercial real estate default for a commercial mortgage is much less costly than it is for a residential loan.

\textsuperscript{33} The LSMC approach is used to find the mortgage rate, FRM or ARM, that equates the mortgage value with the outstanding balance at origination.

\textsuperscript{34} Implicitly we are assuming that all mortgage loans are made at a common initial LTV [e.g., 80%]. In practice the initial LTV may be greater than or less than this amount.
analysis we also report the estimated effect under assumption of volatility of one-half of our estimate, i.e., 1.25%. 35 Under our best estimate of volatility [2.6%], Figure 1 shows that if a person has zero equity [100% LTV], the effect of the non-recourse feature is the greatest and exceeds 7 percent of the properties investment value.36 For example, if the investment value of a complex was $50,000 per unit, and the owner’s loan balance was also $50,000 per unit, the seller’s reservation price would be in excess of $53,500. The effect of the non-recourse on the reservation price falls until, at a LTV of 80 percent, its value is zero.37

Returning to Figure 1, we see the value of the option to put the property back to the creditor is substantial when the LTV exceeds 100 %. For example, with negative equity of 5 percent [a LTV of 115%], the seller’s reservation price is approximately 4 percent greater than the properties investment value. This helps us understand why it may be optimal to make loan payments on a property with negative equity. During the mid 1990s many Southern California properties were in this position.

Even under the extremely conservative assumption of volatility being half of our estimate for LA County, the effect exceeds 5% of the property’s investment value when the owner has a 100% LTV. From this analysis, we conclude that even if the Los Angeles County apartment market is much more volatile than typical real estate markets [as might be expected], the estimates pictured in Figure 1 strongly suggest that our

35 Note this estimate is close to Downing, Stanton, and Wallace (2007)’s estimate of the annual implied return volatility for commercial real estate of between 20 and 24 percent.
36 These results are consistent with Genesove and Mayer’s [1997] finding that, on average, list price for Boston area condominiums with a LTV of 100 percent were 4 percent greater than those with a LTV of 80 percent LTV and that owner’s received 4 percent more when sold.
37 By construction at a LTV of 80 percent the borrower is paying for the value of the put in terms of higher interest rates and the market value of the loan equals its face value.
findings are applicable to real estate markets in general.\textsuperscript{38} From the above analysis we can see how decreases in property values can generate declines in asset liquidity [lowering the probability of a match], which, in turn, generates further declines in asset liquidity.

**B. Liquidity Cost and the Reservation Price**

In the base case, the seller incurs no costs if they choose not to transact during the first period. Liquidity costs, motivation to transact, can be thought of as a per-period cost of maintaining the option to default.\textsuperscript{39} Consequently, this cost would be expected to reduce the net value of the option, thereby reducing the impact of the non-recourse feature on the seller’s reservation price. Figure 2 extends the analysis presented in Figure 1 to include liquidity cost. The monthly costs [in terms of percent of investment value] that motivate the transaction are along one of the axes, the existing owner’s market LTV ratio is along another axis, and the effect of the non-recourse provision on the reservation price of seller is along the vertical axis. Again, variations in LTV are equivalent to variations in the exercise price of the put option [loan balance] given the property’s investment value $V$. This figure indicates that the value of the non-recourse feature decreases as the seller’s LTV $[F_S/V]$ diverges from 100% and as the liquidity costs increase in absolute value.\textsuperscript{40} The range of liquidity costs where the non-recourse feature has an economically important effect on the seller’s reservation price is quite large. Zero liquidity costs indicates that the owner is not motivated to sell and can wait

\textsuperscript{38} Clearly other parameters of the model have an effect on the estimate and it is conceivable that they may reduce the magnitude of the reported impact. However, given the robustness and economic significance of our findings, we find it unlikely that our results depend on the particular parameters.

\textsuperscript{39} It should be remembered that liquidity costs are distinct from debt service. Within the simulation, the liquidity cost represents a percent of investment value $V$ as an expense to each future “node” and affects the entire continuation value.
indefinitely. Liquidity costs of $\frac{1}{2} \%$ of investment value per month are very high and represent an owner who is highly motivated to sell. Even this owner will reject an offer of $V$ if their equity position is between approximately –4 and 4 percent of the property’s investment value. In these cases, the value of waiting exceeds the liquidity costs.

The implication of demand shocks for reservation prices can be seen by considering an individual who bought a property with 20 percent equity. At a LTV of 80 percent, the effect of the non-recourse feature on the seller’s reservation price is, by construction, zero.\footnote{\text{The relevant LTV ratio is the loan balance relative to the \textit{market value} of the property.}} If a \textit{positive demand} shock occurs after the property is bought, thereby decreasing the LTV, the effect of the non-recourse feature will be still be zero.\footnote{\text{The value of the put is included in the interest rate paid on the mortgage.}} Whereas, a \textit{negative demand shock} will increase the LTV, which in turn will increase the value of the non-recourse feature of the loan and increase the difference between the seller’s reservation price and the fair market value.

\footnote{\text{Owners of commercial real estate would be expect to borrow out so that the actual LTV would be close to the maximum LTV.}}
VI. A Non-Recourse Feature and the Decision to Transact

We now return to our analysis of the relationship between non-recourse feature and a real estate market’s response to demand shocks. In equation (15), we concluded that a mutually agreeable transaction would occur if, and only if, the sum of the liquidity costs [motivation] is greater than or equal to the difference between the principal [face value] and market value of the seller’s mortgage, i.e., \( \text{EC\_FV}_{\text{MS}} \). In the remainder of this section, we use the Los Angeles County apartment data to examine the determinants of this decision for both fixed rate and variable rate mortgages.

Figure 3 represents the result of an examination of the liquidity costs [for the buyer and/or the seller] that will induce a transaction when the seller has a 6 percent fixed interest rate mortgage [e.g., the loan was originated when the risk free rate was 4 percent]. The figure depicts the sum of the liquidity costs, for the buyer and the seller, necessary to induce a transaction as a function of the seller’s LTV ratio and the current risk-free rate. In this analysis, as was previously the case, the investment value of the property, \( V \), is fixed, and variations in LTV are the result of differences in the loan balance, \( F_S \), which induce variations in \( M_S \). Notice that the sum of the liquidity costs, not their distribution between the buyer and the seller, determines whether a mutually agreeable transaction is possible. Of course, the distribution between the buyer and the seller will have an impact on the transaction price. Furthermore, we take into account not just the current but expected future liquidity costs to both parties under the optimal exercise policy.

Figure 3 shows that reductions in the market value of the mortgage, \( M \), relative to the outstanding balance would require higher liquidity costs for the buyer and/or the
seller to induce a transaction. In our model, there are two forces that reduce the market value of the mortgage:

- Decreases in the investment value of property [pictured as increases in LTV], and
- high current risk-free rate [and mortgage rates] relative to the rates that held at the time of loan origination.

Consistent with our expectation, as the LTV ratio approaches 100%, the value of the put increases which, in turn, reduces the value [burden] of the mortgage for the seller. Consequently, combined costs of approximately 1% of value per month are needed to induce a transaction in this situation. High current interest rates have a similar, although much smaller, impact. If the current owner has a FRM with a low interest rate relative to the current rates, an “interest rate lock-in”, then the market value of the mortgage is lower than the principal amount and substantial liquidity costs are necessary to induce a transaction [i.e., there is a mortgage loan lock-in]. Notice, however, that for 80% LTV ratio, the combined liquidity costs necessary to induce a transaction at 6% risk-free rate are only .05% per month. In other words, *cetris paribus*, we can expect to see only marginal declines in liquidity following an increase in interest rates. In our analysis the effect of high current interest rates is partially mitigated because our estimates of the joint stochastic process (20) produced an increasing relationship between the level of \( r \) and \( \frac{dV}{V} \), and because the interest rate process is mean-reverting. Our analysis of the fixed interest rate non-recourse mortgage example strongly suggests that a negative demand shock that results in LTV greater than the

---

*For simplicity of exposition, LTVs greater than 100 percent were not considered in this figure. Similar relationships would hold for initial mortgage loan rates greater or less than 6 percent.*
maximum LTV for a significant portion of the property owners will preclude many otherwise desirable transactions.

Analogously to Figure 3, Figure 4 depicts the sum of the buyer’s and the seller’s liquidity costs necessary to induce a transaction, if the seller holds an adjustable rate mortgage [ARM]. The implication of ARM financing is that changes in interest rates alone do not alter the value of the mortgage. While this is true for a default-free bond, the market value of the mortgage still depends on interest rates because they affect the value of the imbedded put option. In general, increase in the risk-free interest rate decreases the value of a put option for two reasons: (1) the expected growth rate of the asset price increases, and (2) the present value of future cash flow received by the holder decreases. While both of these effects hold in our application, interest rates have one additional impact in the case of ARM: the monthly carrying costs increase. Except for very high LTV ratios, the probability of delaying the sale over extended periods of time is low and the first two effects dominate. Figure 4 suggests that for LTV ratios below 98% an increase in interest rates reduces the value of the put option, which, in turn, reduces the liquidity costs necessary to induce a transaction.

For very high LTV ratios, the probability of delaying the sale over extended time periods is high, and the increase in expected carrying costs is substantial enough to overcome the first two effects. Figure 4 suggests that for LTV ratios above 98%, an increase in interest rates increases the value of the imbedded put option, which, in turn, further increases the liquidity costs necessary to induce a transaction.

Regardless of the type of mortgage and the fluctuation of the interest rates, Figures 3 and 4 suggest a clear conclusion: even small increases of the LTV ratio above 80% [e.g., from a negative demand shock] require positive and increasing liquidity.
costs [motivation] for a mutually agreeable transaction to be possible. That is, a negative demand shock can result in a situation where no mutually agreeable transaction will be possible for a portion of the potential sellers at the property’s investment value. This can be interpreted as a decrease in asset liquidity. For LTV ratios close to 100%, the required liquidity costs for a transaction are substantial and approach 1% of the asset price per month. In contrast, a positive demand shock that decreases the seller’s LTV will facilitate transactions by reducing the cost needed to motivate the transaction.

While our model has been for a specific type of property and a specific real estate market, we contend that it has identified an important general aspect of real estate markets where properties are purchased with non-recourse loans.

VII. Conclusion

Markets for income producing real estate frequently respond asymmetrically to large positive and negative demand shocks. This paper provides an explanation for this phenomenon that is consistent with individual rationality. Our explanation is based upon the rational response of sellers and potential buyers to two characteristics of their actual, or potential, mortgage loan contract: non-assumable fixed interest rate mortgages and/or non-recourse loans. The model we developed allowed us to explore the effect of non-assumable fixed interest rate mortgages and/or non-recourse loans on the conditions under which a mutually agreeable transaction will take place. Using the model, we are able to show that a negative demand shock, in conjunction with either of these features, can result in a period during which no mutually agreeable transaction is possible between liquidity motivated sellers and buyers with identical expectations about the price formation process. In contrast, we show that a positive demand shock will never
result in a decline in asset liquidity. We conclude that the non-recourse feature is sufficient to generate the asymmetric responses to positive and negative demand shocks that are observed. Consequently, while factors such as loss aversion may contribute to a market’s response to demand shocks, they are not necessary to explain this response.

While the traditional explanations, be they institutional or behavioral, of how real estate markets respond to demand shocks are likely to be important in the short run, our results strongly suggest that the prevalence of some mortgage loan features are an important determinate of the markets’ long run response. This result is important because, to the extent that non-recourse financing is responsible for the observed declines in asset liquidity, financial innovations may improve economic efficiency. This need not be true if the loss aversion is the explanation.
References


Appendix A: Cash Flow Yield and Risk-Neutral Measure

This appendix provides a formal argument that the cash flow yield [analogous to dividend yield] retains the same drift and the same correlation with the other state variables under an equivalent probability measure. Suppose the asset price and the dividend yield evolve according to

\[
\begin{align*}
\frac{dV}{V(t)dt} &= \mu dt + \sigma dW(t) \\
\frac{dV}{V(t)dt} &= \delta dt + \tau dW(t)
\end{align*}
\]

Notice that this is a slightly different representation than the one given by (A.1). In particular Equation (A.1) represents the drift of the asset price as \( \mu - \delta \). This makes the total return to holding the asset equal to \( \mu_s \).

Let us assume that

\[(A.2) \quad dzd\rho = \frac{\sqrt{\rho}}{\sqrt{1 - \rho}} \rho \quad 1\]

We apply the following transformation. Let \( A \) is a 2x2 matrix such that

\[A^2 + \Sigma = I, \quad \text{i.e.} \Sigma = A^{-1} A\]

Let \( B = A^{-1} \). Then

\[(A.3) \quad d\delta = dB = \Sigma dB = A d\delta A^\top dt\]

We can write
We can now apply Girsanov’s theorem to the normalized \( B' \)’s. Let

\[
\alpha = \frac{\theta(t) \delta}{\mu_\theta}
\]

where \( r \) is another process [interest rate in this case]. Applying Girsanov’s theorem to (A.4) we obtain

\[
\left( \frac{dV}{\mu B} \right) = \frac{\varphi(t)}{\mu_\theta} \frac{dB}{\delta} - \frac{\varphi(t)}{\mu_\theta} \frac{dt}{\delta} + \frac{\varphi(t)}{\mu_\theta} \frac{dz}{\delta}
\]

where \( \hat{B} \) is another normalized Brownian motion \([d\hat{B}'dB' = \gamma dtdz] \) with respect to an equivalent probability measure. We can now switch back to the original process. Define \( \hat{z} = \hat{B}^{-1} \). Then, \( \frac{dz}{dt} = \Sigma \), and

\[
\left( \frac{dV}{\mu B} \right) = \frac{\varphi(t)}{\mu_\theta} \frac{dB}{\delta} - \frac{\varphi(t)}{\mu_\theta} \frac{dt}{\delta} + \frac{\varphi(t)}{\mu_\theta} \frac{dz}{\delta}
\]

In short, we can switch to an equivalent probability measure such that (A.7) holds and the new Brownian motions have the same correlation as the original ones. Based on our choice of \( \alpha \), the dividend process has the same representation as before.
Appendix B: Data and Parameter Estimates

Unlike financial markets, real estate markets do not provide all of the data needed to calculate the value of the portfolio that represents the owner’s interest in a property. Specifically, the heterogeneity of properties and infrequent trading result in the absence of reliable estimates of the historical volatility of real estate appreciation.\(^4\)

To calculate this statistic we need to first estimate the time series of rates of appreciation of the underlying asset [i.e., the per unit price of LA County apartments]. We use the following semi-log hedonic value model to estimate this series:

\[
\ln[Value_{it}] = \text{Constant} + \sum_{t=1}^{T} \beta_t S_t + \alpha' C_i + \varepsilon
\]

where \(Value_{it}\) is the price paid for property \(i\) sold at time \(t\), \(C_i\) is a vector of physical characteristics that describe the building, \(S_t\) is a matrix of indicator variables for the time of sale, and \(\beta_t\) is the marginal time effect [i.e., monthly]. \(T\) is the total number of months in the sample and \(\varepsilon\) is an error term with zero expectation.\(^5\) Thus \(\beta_t\) is an estimate of the rate of appreciation for time period \(t\). The mean of the vector \(\beta\) provides an estimate of the expected monthly rate of apartment appreciation.

As evident from Equation (18), following a negative demand shock transaction prices do not decline as much as the underlying property value, \(V\). While we acknowledge this effect, we note that it will lead to a lower volatility estimate relative to

\(^4\) Conceptually the statistic should be the prospective standard deviation of the rate of appreciation of the property in question.

\(^5\) The \(\beta\) are estimated as follows: if a transaction occurred during January 1989 [i.e., \(t=1\)], all time indicator variables are assigned a value of zero; if a transaction occurred during the second month, the first time indicator variable is assigned a value of one and all other time indicator variables are set to zero; if a
the true property value volatility. In that sense, we are underestimating any empirical
effects reported below.

To estimate the cash flow yield, $\delta$, we use a similar regression to Equation (B.1),
except the dependent variable is not the log but the level of the cash flow yield. This is
ddictated by the specification of the stochastic process for the cash flow yield (19). In
particular, since the changes of the cash flow yield are assumed to be additive, not
proportional, the correct specification is:

\[
(\text{B.2}) \quad \delta_{it} = \text{Constant} + \sum_{t=2}^{T} \gamma_{t} S_{t} + \alpha^{'} C_{i} + \epsilon
\]

where $\delta_{it}$ represents the cash flow yield of property $i$ when sold at time $t$. Analogously
to the property appreciation rates, the parameters $\gamma$ in Equation (B.2) are estimates of
the monthly change in the before tax cash flow yield.

To estimate the appreciation rates and the cash flow yields for a property we use
transaction data. The data come from CoStar COMPS. The firm produces high quality
transactions data for a wide range of income producing properties. The firm has
provided data for all transactions in Los Angeles County apartment buildings that
occurred between October 1989 and July 2001, a total of 18,167 observations. Table
B1 provides summary statistics describing the transactions that occurred during the
period. The mean and median per unit price during this period were a little more than
$50,000 per unit. The mean and median cash flow yields at the time of sale were approximately 9 percent. As can be seen from the table, the typical LA County apartment complex is relatively small and the distribution of complex size is positively skewed, with a median of 10 and a mean of 18 units.

Tables B2 and B3 report the parameter estimates and implied appreciation rates obtained by estimating Equation (B.1). The parameter estimates presented in Table B2 have the expected signs and are highly statistically significant. From Table B3 we can see that our model estimated an average [ex-post] monthly appreciation rate of nearly zero with a standard deviation of more than six percent.

The late 1980s and 1990s were a boom/bust period for Southern California property values. Between the 1990 peak and February 1997 trough, per unit prices fell by more than 54 percent. By the middle of 2001, per unit prices were more than 78 percent above their 1997 low.

Next, we estimated Equation (B.2) to obtain estimates of the annualized cash flow yield. The results are presented in Tables B4 and B5.

---

48 The cash flow yield is calculated from the NOI at the time of sale as reported by CoStar COMPS. Note that data to impute the cash flow yield was available for approximately 86 percent of the transactions.
Table 1: Joint Stochastic Process Driving the Property Markets

<table>
<thead>
<tr>
<th></th>
<th>δ</th>
<th>r</th>
<th>V</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>dV/V</td>
<td>1.02</td>
<td>-8.56</td>
<td>1.01</td>
<td>-.67</td>
</tr>
<tr>
<td></td>
<td>(5.95)</td>
<td>(-5.84)</td>
<td>(2.85)</td>
<td>(-6.66)</td>
</tr>
<tr>
<td>dδ</td>
<td>.005</td>
<td>-.05</td>
<td>-.033</td>
<td>.0026</td>
</tr>
<tr>
<td></td>
<td>(2.64)</td>
<td>(-2.38)</td>
<td>(-1.80)</td>
<td></td>
</tr>
<tr>
<td>Dr</td>
<td>-.003</td>
<td>.06</td>
<td>-.02</td>
<td>.009</td>
</tr>
<tr>
<td></td>
<td>(-2.48)</td>
<td>(3.62)</td>
<td>(-1.25)</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the estimates from Equation (20):

\[
\begin{align*}
\frac{dv}{v} &= \delta \sigma d\delta \Delta t + \delta \sigma d\sigma \Delta t + \rho d\sigma d\delta \\
\Delta t &= \sqrt{d} \\
\end{align*}
\]

All observations are monthly.
Table 2: Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>dV/V</th>
<th>δδ</th>
<th>dr</th>
</tr>
</thead>
<tbody>
<tr>
<td>dV/V</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>δδ</td>
<td>-0.72</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>dr</td>
<td>-0.13</td>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

This table reports the correlation coefficients of the innovations as estimated by Equation (20):

\[
\frac{d\delta}{\delta} = \frac{d\delta}{\delta} \quad \text{and} \quad \frac{dr}{\delta} = \frac{dr}{\delta}
\]

\[
\begin{align*}
\delta \sigma &= \sqrt{d} \\
\delta \sigma &= \sqrt{d} \\
\delta \sigma &= \sqrt{d}
\end{align*}
\]
Table 3: Bias-corrected estimate of the volatility of the monthly appreciation rates and cash flow yield

<table>
<thead>
<tr>
<th></th>
<th>Bias-corrected estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dV/V$</td>
<td>0.026</td>
</tr>
<tr>
<td>$d\delta$</td>
<td>0.002</td>
</tr>
<tr>
<td>$dr$</td>
<td>0.009</td>
</tr>
</tbody>
</table>

This table reports the bias-corrected estimates of the volatilities of the three joint stochastic process of Equation (20):

\[
\begin{align*}
\frac{dv}{v} &= \mu d\delta + \sigma d\sigma + \sqrt{\sigma^2 + \sigma^2} d\Delta v \\
\frac{d\delta}{\delta} &= \mu d\delta + \sigma d\sigma + \sqrt{\sigma^2 + \sigma^2} d\Delta \delta \\
\frac{dr}{r} &= \mu d\delta + \sigma d\sigma + \sqrt{\sigma^2 + \sigma^2} d\Delta r
\end{align*}
\]
Table 4: Equilibrium Interest Rates

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
<td>4%</td>
</tr>
<tr>
<td>Adjustable rate 10-year mortgage</td>
<td>5.5%</td>
</tr>
<tr>
<td>Fixed rate 10-year mortgage</td>
<td>6%</td>
</tr>
</tbody>
</table>

This table reports the equilibrium interest rates for adjustable and fixed-rate mortgages, conditional on the risk-free rate of 4%. These rates include a default premium based on 20% home equity [i.e., 80% LTV]. These rates make the market and the face value of the mortgage equal.
This table provides some descriptive statistics for the Los Angeles County apartment building transaction data.

<table>
<thead>
<tr>
<th></th>
<th>Price per unit</th>
<th>Cash Flow Yield</th>
<th>Age</th>
<th>Parking Spaces</th>
<th>Number of Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$56,174</td>
<td>.09</td>
<td>34</td>
<td>21</td>
<td>18</td>
</tr>
<tr>
<td>Median</td>
<td>$51,600</td>
<td>.09</td>
<td>34</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$26,744</td>
<td>.03</td>
<td>18</td>
<td>40</td>
<td>26</td>
</tr>
</tbody>
</table>
Table B2: Parameter Estimates for the Asset Value Index

<table>
<thead>
<tr>
<th>N=18,167</th>
<th>Age</th>
<th>Age Squared</th>
<th># parking spots</th>
<th># parking squared</th>
<th># units</th>
<th># units squared</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-.0105</td>
<td>8 E-5</td>
<td>.0068</td>
<td>-5 E-6</td>
<td>-.0133</td>
<td>2 E-5</td>
</tr>
<tr>
<td>St. Error</td>
<td>.0004</td>
<td>5 E-6</td>
<td>.0002</td>
<td>2 E-7</td>
<td>.0003</td>
<td>6 E-6</td>
</tr>
</tbody>
</table>

This table reports the parameter estimates using Equation (B.1):

\[
\ln[Value_{it}] = \text{Constant} + \sum_{t=1}^{T} \beta_t S_t + \alpha' C_i + \varepsilon
\]

where \(Value_{it}\) denotes the per unit value of apartment building \(i\) sold at time \(t\), \(C_i\) denotes the physical characteristics of that apartment building at the time of sale, and \(S_t\) is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Table B3: Implied monthly rates of price appreciation

<table>
<thead>
<tr>
<th>Average Appreciation Rate</th>
<th>Standard Deviation of the Appreciation Rate</th>
<th>Median Appreciation Rate</th>
<th>Skewness of the Appreciation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\alpha}$</td>
<td>$0.0528$</td>
<td>$\bar{\alpha}$</td>
<td>$-0.1463$</td>
</tr>
</tbody>
</table>

This table reports the implied appreciation rates estimates from estimating Equation (B.1):

$$\ln[Value_{it}] = \text{Constant} + \sum_{t=2}^{T} \beta_i S_t + \alpha' C_i + \varepsilon$$

where $Value_{it}$ denotes the per unit value of apartment building $i$ sold at time $t$, $C_i$ denotes the physical characteristics of that apartment building at the time of sale, and $S_t$ is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Table B4: Parameter Estimates for the Cash Flow Yield Index

<table>
<thead>
<tr>
<th></th>
<th>N=14,879</th>
<th>( R^2 = .39 )</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>14,879</td>
<td>.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td></td>
<td>Age Squared</td>
<td></td>
<td></td>
</tr>
<tr>
<td># parking spots</td>
<td></td>
<td>1.3 E-4</td>
<td>9 E-8</td>
<td>3.3 E-4</td>
</tr>
<tr>
<td># parking spots squared</td>
<td></td>
<td>1.9 E-5</td>
<td>4 E-8</td>
<td></td>
</tr>
<tr>
<td>Alpha</td>
<td></td>
<td>Age Squared</td>
<td></td>
<td></td>
</tr>
<tr>
<td># units</td>
<td></td>
<td>1.3 E-4</td>
<td>9 E-8</td>
<td>3.3 E-4</td>
</tr>
<tr>
<td># units squared</td>
<td></td>
<td>1.9 E-5</td>
<td>4 E-8</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the parameter estimates using Equation (B.2):

\[
\delta_{it} = \text{Constant} + \sum_{t=1}^{T} \gamma_t S_t + \alpha I_{C_i} + \epsilon
\]

where \( \delta_{it} \) denotes the cash flow of building \( i \) sold at time \( t \), \( C_i \) denotes the physical characteristics of that apartment building at the time of sale, and \( S_t \) is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Table B5: Implied monthly rates of change in cash flow yield

<table>
<thead>
<tr>
<th>Average change</th>
<th>Standard Deviation of the Change</th>
<th>Median Rate of Change</th>
<th>Skewness of the Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1E-4/Month</td>
<td>0.003</td>
<td>1E-4</td>
<td>-0.19</td>
</tr>
</tbody>
</table>

This table reports the implied appreciation rates estimates from estimating Equation (B.2):

\[ \delta_{it} = \text{Constant} + \sum_{t=2}^{T} S_t \gamma_t + \alpha' C_i + \varepsilon \]

where \( \delta_{it} \) denotes the cash flow of building \( i \) sold at time \( t \), \( C_i \) denotes the physical characteristics of that apartment building at the time of sale, and \( S_t \) is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Figure 1: Effect of Non-Recourse Feature on Seller's Reservation Price

Figure 1 depicts the effect of the non-recourse feature of a mortgage loan on the seller’s reservation price assuming no liquidity costs. The top and the bottom line depict this effect assuming volatility of the asset price of 2.6% and 1.25%, respectively.
Figure 2 depicts the effect of the non-recourse feature on the seller’s reservation price as a function of home equity [LTV ratio] and liquidity costs [percent of asset value per month]. Zero liquidity costs generate the largest effect. Even with very high liquidity costs, the seller’s reservation price is above the investment value of the asset near the 100% LTV region.
Figure 3 depicts the sum of the buyer’s and the seller’s liquidity costs necessary to induce a transaction as a function of the LTV ratio and the risk-free rate for a Fixed Rate Mortgage. This figure takes into account not only the current period liquidity costs, but all expected future liquidity costs for both the buyer and the seller under the optimal exercise policy.
Figure 4 depicts the sum of the buyer’s and the seller’s liquidity costs necessary to induce a transaction as a function of the LTV ratio and the risk-free rate for an Adjustable Rate Mortgage. This figure takes into account not only the current period liquidity costs, but all expected future liquidity costs for both the buyer and the seller under the optimal exercise policy.