WHY REAL ESTATE PRICES DON’T DECLINE FOLLOWING DEMAND SHOCKS

Revised

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Markets for income producing real estate frequently respond asymmetrically to demand shocks. Following negative shocks, asset liquidity declines but transaction prices change relatively little, whereas, following positive shocks, the same markets respond with increases in prices. This paper develops a model that provides an explanation for the asymmetric response of market for income producing real estate to demand shocks. Specifically, we provide an explanation for the phenomenon that is based upon the rational response of sellers and potential buyers to two characteristics of their actual, or potential, mortgage loan contract: non-assumable fixed interest rate mortgages and/or non-recourse loans. We conclude that the non-recourse feature is sufficient to explain the above phenomena. Unlike previous research, our model is based upon traditional utility maximization theory.
I. Introduction

In the short run the supply of real estate, both income producing and owner occupied housing, is fixed and the market response to demand shocks should be symmetric: positive shocks resulting in price increases and negative shocks resulting in price decreases. However, these markets typically respond to large negative demand shocks with long periods during which asset liquidity declines but transaction prices change relatively little. In contrast, the same markets respond to positive shocks with increases in prices. This paper provides an explanation for the phenomenon that is based upon the rational response of sellers and potential buyers to two characteristics of their actual, or potential, mortgage loan contract: non-assumable fixed interest rate mortgages and/or non-recourse loans.

When income-producing property is bought with a non-recourse loan, the owner’s interest in the property is equivalent to a portfolio comprised of the property, a mortgage loan, and a put option that gives the owner the right to sell the property to the lender at a price equal to the loan balance [i.e., by defaulting on the loan]. Consequently, a seller’s reservation prices and potential buyers’ bid prices depend on the economic characteristics of the assets that makes up the portfolio, not just the investment value of the property.

For organized financial markets, the characteristics of an asset that is purchased are the same as the characteristics of the asset that is sold. This need not be true for the sale of an equity interest in an income producing property. For example, following a large negative demand shock, a maximum loan to value requirement frequently results in the exercise price of the option that is being sold [the seller’s loan balance] being greater than the exercise price of the option that would be acquired [the maximum mortgage
loan for a buyer]. As we will see, consideration of these options is essential to understanding the response of real estate markets to demand shocks. Furthermore, differences between the market and contracted rate of interest on the owner’s mortgage loan can create a wedge between the value of the portfolio to the seller and the potential buyer. If, for example, the market rate is greater than the contracted rate of interest, the face value of the mortgage [the balance] will exceed the market value. Typically commercial mortgages are not assumable and this increment in value will not be transferable and the property owners can be said to be subject to an interest rate “locked-in.”

We conclude that either a non-recourse loan or an interest rate “lock-in” can result in a motivated seller’s reservation price exceeding a motivated potential buyer’s bid price following a negative demand shock. This is true even when the parties have identical expectations about the property’s investment value and the price formation process. Using data for the Los Angeles County apartment building market, we estimate through a simulation the effect of the two features on the dollar value of the motivation that is needed to induce a transaction. Our model suggests that when both features are present the non-recourse feature is likely to be more important than the interest rate “lock-in.”

The major contribution of this paper is the integration of theoretical and empirical models of a market for income producing real estate to provide both a qualitative and quantitative examination of the response of these markets to demand shocks. We do not contend that ours is the only explanation for the observed phenomenon, but we show that traditional utility maximizing behavior is sufficient to cause asymmetric responses to demand shock. Our results suggest that analysis of real
estate markets should start with the recognition that a levered purchase of income producing real estate is acquisition of a portfolio of assets, not just a piece of real estate.

This paper starts by reviewing alternative explanations of the decline in asset liquidity that follows a negative demand shock. Next we develop a model of a market for income producing properties to investigate the effect of the fixed interest rates and/or non-recourse loans on the conditions under which a mutually agreeable transaction takes place. Specifically, we investigate their effect on: (1) a seller’s reservation price, (2) a potential buyer’s bid price, (3) the dollar value of the motivation needed to induce a mutually agreeable transaction, and (4) the price at which transactions will take place. Finally, we use data for the Los Angeles County apartment building market, to evaluate the empirical relevance of the effects that our model has identified.

II. Why Liquidity Declines

Explanations for the decline in asset liquidity that follows a negative demand shock include market imperfections that result from high information cost and/or the combination of unrealistic expectations on the part of owners and the inability to sell real estate short. While these are important characteristics of real estate markets, they provide a more convincing explanation of short-term market responses than they do the long-term conditions that sometimes occur. Given the importance of markets for owner occupied housing it is not surprising that recent research has attempted to provide more convincing explanations for the long-term response of these markets to negative demand shocks. One thread of literature is based upon the observation that for a large fraction of transactions a portion of the equity used to finance the purchase of a home comes from the sale of the buyer’s existing home. Stein [1995] develops a theoretical model that
illustrates how homeowners may be “locked-in,” by an equity-constraint. This occurs because the sale of their existing home, at post shock values, would result in the owner having inadequate equity to purchase another home. Stein concludes that the “lock-in” can exacerbate the effect of a negative demand shock on housing markets [i.e., price decreases beget further price increases].

Genesove and Mayer [1997] investigate the empirical relevance of the “equity lock-in” using data for the Boston condominium market during the early 1990s. The starting point of their analysis is the observation that if the “equity lock-in” was important, homeowners with high loan to value ratios [e.g., over 80 percent after the shock] should have responded differently to the demand shock than homeowners with low LTVs. They find that there was a positive relationship between a condo’s LTV and: (1) the seller’s asking price, (2) time on market [i.e., a decline in liquidity], and (3) price received if the property was sold. Genesove and Mayer conclude that these results are consistent with sellers being equity constrained following a negative demand shock. While an “equity lock-in” may be important for owner occupied housing, it is not for income producing properties. The authors recognize that investors are not constrained to purchase another property after a sale. Genesove and Mayer’s sample included both owner occupied and non-owner occupied [income producing] units. They note that for investors loan default is equivalent to a put option on the property. They further conjecture that this option will trigger a similar relationship between LTV, asking price, time on market and transaction price for non-owner occupied units. Our model formalizes this idea and extends it to include the possibility of an interest rate lock-up.

Cauley and Pavlov [2002], take a different tack to understanding the dynamics of a housing market’s response to demand shocks. They start with the observation that first
home mortgages are almost always non-recourse loans. In this paper they consider the implications of the put option embedded in a non-recourse mortgage loan on a homeowner’s response to a negative demand shock. Specifically, they develop a statistical model to estimate the value of delaying the sale of a home and show, through an example, that it may be optimal for a homeowner to delay the sale, even when the homeowner derives no valuable services from the home. For owner occupied housing a non-recourse loan feature and an equity lock-in are not mutually exclusive explanations for a market’s response to a demand shock. The greater the decline in home value the larger the value of the put option and the more binding the equity “lock-in.” In practice, both are likely to influence the response of housing markets to demand shocks.

In a later article Genesove and Mayer [2001] apply prospect theory, or nominal loss aversion, to explain the empirical response of the Boston condominium market [the same data set they used in their earlier paper] to a negative demand shock. Nominal loss aversion is the observation that individuals are more sensitive to losses in asset value than they are to equal size gains. For this reason, the authors hypothesize that sellers who have experienced nominal losses in condominium values will set higher reservation prices, spend a longer time on market and receive a higher selling price than sellers who have not. Genesove and Mayer conclude that the empirical evidence for nominal loss aversion, for both owner occupied and investors owned units, is very strong. They estimate the effect for owner occupied units is approximately twice that of non-owner occupied units. Again the empirical implication of nominal loss aversion and our explanations for the response of market to demand shocks are not mutually exclusive, they work in the same direction and are observationally equivalent. In fact both are likely to be present in markets for income producing real estate.
III. A Model of a Real Estate Market

In this section we develop a model of a real estate market designed to abstract from all other considerations, so that we can explore the impact of the form of the loan contract on the response of a real estate market to a demand shocks. Within the model we: (1) derive the effect of an existing below market rate mortgage [an interest rate “lock-in”] and non-recourse financing on a seller’s reservation price; (2) determine the conditions under which a mutually advantageous transaction will take place; (3) show that interest rate “lock-in” and a non-recourse loan feature are each sufficient to generate asymmetric responses to demand shocks.

The market we consider is comprised of $N$ identical liquidity motivated sellers, who own identical income producing properties [e.g., 20 unit apartment complexes]. There are $K [K=N]$ identical, liquidity motivated, potential buyers for the properties. There are no transactions costs, and information is symmetric and complete. Sellers or potential buyers receive no direct utility from owning a property. There is agreement as to the price formation and cash flow generation processes. The future is comprised of a countable infinity of periods. At the beginning of a period, sellers and potential buyers are randomly allocated. They then negotiate over the sale of the property. After the negotiation is completed, the realizations of the asset price and cash flow stochastic processes for the period are observed. If the parties have not transacted during the first period, they are reallocated and negotiations start over. At the beginning of a period, sellers and potential buyers agree that $V$ is the investment value of the property [e.g., the present value of all future cash flows for a 100% equity investment]. To represent [quantify] the owner’s motivation to sell we
assume that if the property is not sold a constant per-period “liquidity” cost, $c_s > 0$, is incurred. Analogously, by transacting, potential buyers are able to avoid a constant per period “liquidity” cost $c_b \geq 0$. Analytically, these costs provide the motivation to transact.

In the absence of debt, the seller’s reservation price, $R$, is the investment value of the property minus the liquidity costs that would be born if the property were not sold [i.e., $V - c_s$]. Without debt, the maximum [bid] price a potential buyer will pay, $B$, equals the property’s investment value plus the costs that are avoided if a property is bought ($V + c_b$). When the sum of the liquidity costs [motivations] of the seller and potential buyer are non-negative, a mutually agreeable transaction will always occur at a price between $V - c_s$ and $V + c_b$. In our model, in the absence of debt, a demand shock will never preclude a mutually agreeable transaction.

A. Seller’s Reservation Price With Debt

We now introduce non-amortizing debt. At the start of the first period, the seller is assumed to have an existing, non-amortizing, mortgage loan with an outstanding balance [face value] of $F_s$ and market value of $M_s$. The market and face values may differ because: (1) the market and contracted interest rates are not equal; and/or (2) the value of the put option embedded in a non-recourse loan has changed. Note, the value of the put incorporated in the value of the mortgage, $M_s$, includes future liquidity costs, $c_s$, that would be incurred if the property is not sold this period and the seller acted optimally with respect to the mortgage loan [by waiting to defaulted or sell] in the future. It follows that increases in the future liquidity costs decrease the current
value of the option, which in turn increases the market value of the mortgage loan [i.e., \\
\frac{\partial M_s}{\partial \bar{c}_s} > 0].

The seller’s surplus from a potential transaction at a price \( S \) equals the \textit{net} value received plus the liquidity cost avoided if a sale occurs. Here the \textit{net} value equals the sale price less the mortgage balance \((S - \bar{F}_s)\), minus the value of the portfolio representing the seller’s interest in the property \((V - M_s)\). For a sale to be acceptable the seller’s surplus must be non-negative. That is,

\begin{equation}
(S - \bar{F}_s) - (V - M_s) + \bar{c}_s \geq 0
\end{equation}

Consequently, the seller’s reservation price, \( R \), equals:

\begin{equation}
R = V + (\bar{F}_s - M_s) - \bar{c}_s
\end{equation}

The above expression suggests how a below market rate mortgage can “lock-in” a property owner with a fixed interest rate loan. Specifically, if the market rate is greater than the contracted rate then, \( M_s < \bar{F}_s \). Consequently, if the property is sold the owner would have to pay off a loan balance that is greater than the market value of the mortgage loan.\(^{13}\) The greater the difference between the market and contracted rate of interest, the greater the “interest rate lock-in.” The owner’s reservation price may, in fact, exceed the property’s investment value [i.e., if \((\bar{F}_s - M_s) > \bar{c}_s\)].\(^{14}\)
Equation (2) also allows us to determine the effect of a non-recourse feature on a seller’s reservation price.\textsuperscript{15} Let subscript $n$ denote all variables related to a non-recourse loan and subscript $f$ denote the corresponding values for a full recourse loan. The effect of a non-recourse feature on a seller’s reservation price is:

\begin{equation}
R_n - R_f = \left[ V + (\bar{F}_n - M_n) - \bar{c}_s \right] - \left[ V + (\bar{F}_f - M_f) - \bar{c}_s \right]
\end{equation}

Assuming the outstanding balance is independent of loan type [i.e., $\bar{F}_n = \bar{F}_f$], equation (3) reduces to:

\begin{equation}
R_n - R_f = M_f - M_n
\end{equation}

Thus, the effect of a non-recourse feature on the seller's reservation price is the difference between the \textit{market} value of the mortgage with and without this feature. This difference is determined by the [non-negative] value of the put option incorporated in the non-recourse loan.\textsuperscript{16} When the value of the put is positive, $M_f > M_n$, a seller’s reservation price with a non-recourse loan \textit{will exceed the reservation price of an identical individuals whose ownership is financed with a full recourse mortgage}. The larger the value of the put incorporated in the non-recourse loan [e.g., because $dV < 0$] the greater the effect of the non-recourse feature on a seller’s reservation price.

We now investigate the effect of a demand shock on a seller’s reservation price. The seller’s pre shock reservation price, $R^0$, equals:
and their reservation price after the shock, \( R^a \), is

\[(6) \quad R^a = V^a + \bar{F}_s - M^a - \bar{c}_s. \]

It follows that the change in reservation price because of the demand shock is:

\[(7) \quad \Delta R = R^a - R^p = (V^a + \bar{F}_s - M^a - \bar{c}_s) - (V^p + \bar{F}_s - M^p - \bar{c}_s) \]

Simplifying, we obtain:

\[(8) \quad \Delta R = (V^a - V^p) + (M^p - M^a) \]

For a negative demand shock the issue is, which declines more the reservation price or the properties investment value. By the definition of a negative demand shock \((V^a - V^p) < 0\), consequently, the answer depends upon the relationship between \(M^p\) and \(M^a\).

We will consider demand shocks from two sources: (1) changes in interest rates; and (2) changes in factors other than interest rates [e.g., a regional recession]. If the seller’s mortgage is full recourse, changes in interest rates effects the present value of mortgage loan payments whereas changes in property values do not.\(^{17}\) Thus when a
negative demand shock is triggered by an increase in interest rates \([M^p > M^a]\) the change in reservation price for sellers who have full recourse loans is smaller, in absolute value, than the change in the properties investment value. When the demand shock is triggered by factors other than changes in interest rates the seller’s reservation price and the properties investment value change dollar for dollar in response to a demand shock.

We now consider the relationship between demand shocks and a seller’s reservation price when ownership was financed with a non-recourse loan. To analyze the effect of a demand shock stemming from factors other than changes in interest rates we assume the market rate equals the contracted mortgage rate. In this case, changes in the market value of the mortgage loan, \(M\), occur because of changes in the value of the option to put the property to the lender [e.g., \(dV < 0\) and \(dF^* = 0\) imply the put becomes more valuable and \(dM < 0\)]. It follows from option pricing theory that, \(\frac{\partial M}{\partial V} > 0\), and \(\frac{\partial^2 M}{\partial V^2} < 0\). Consequently, a negative shock, holding interest rates constant, results in \(M^p > M^a\) and the reservation price will decline less than the property’s investment value. In addition, the fact that \(\frac{\partial^2 M}{\partial V^2} < 0\) implies the larger the negative demand shock, the greater, in absolute value, the difference between the decline in seller’s reservation price and investment value.

For positive demand shocks, holding interest rates constant, the issue is, which increases more the reservation price or investment value? Following the logic used above, we can conclude that the seller’s reservation price will increase less than the property’s investment value. The larger the positive shock the smaller the difference
between the increase in investment value and reservation price. For example, when an owner has substantial equity, [i.e., the value of the put option is small] the seller’s reservation price will increase almost dollar-for-dollar with the property’s investment value.

Next we consider demand shocks that are the result of unanticipated changes in interest rates. We know that changes in interest rates are negatively related to both property values, $V$, and the present value of the payments associated with a fixed interest mortgage. If we view $V$ as a function of $r$:

\[
(9) \quad \frac{dM(V(r), r)}{dr} = \frac{\partial M}{\partial V} \frac{\partial V}{\partial r} + \frac{\partial M}{\partial r}
\]

where $\frac{\partial M}{\partial V} > 0$, $\frac{\partial V}{\partial r} < 0$, and $\frac{\partial M}{\partial r} < 0$. This implies that $\frac{dM(V(r), r)}{dr} < 0$.

This relationship implies that after a negative demand shocks, either exogenously determined or resulting from a increase in interest rates, the seller’s reservation price will decline less than the property’s investment value. As was previously the case, the reservation price would increase by less than the investment value following a positive demand shock.

Summarizing, we have shown in the context of our model that:

1. Regardless of loan type, if a demand shock is caused by a change in interest rates, a potential seller’s reservation price will change by less than the property’s investment value.
(2) For a non-recourse mortgage loan, when a demand shock is caused by factors other than an increase in interest rates, a potential seller’s reservation price will change by less than the property’s investment value.

(3) For a full-recourse mortgage loan, when a demand shock is caused by factors other than an increase in interest rates, a potential seller’s reservation price will change dollar per dollar with the property’s investment value.

**B. Buyer’s Maximum Bid Price With Debt**

If a transaction occurs, the buyer will finance the purchase with a non-amortizing, non-recourse loan with face value $F_a$ and market value $M_B$. The buyer’s surplus from a transaction is the value of the portfolio representing their interest in the property acquired minus the price $S$ paid for the property. For a transaction to be agreeable from the perspective of a potential buyer it must result in a non-negative surplus. That is,

\begin{equation}
V - M_B + \bar{c}_b - (S - F_a) \geq 0
\end{equation}

This implies that the maximum price the potential purchaser will pay, $B$, is:

\begin{equation}
B = V + F_B - M_B + \bar{c}_B
\end{equation}

We can now explore the effect of a demand shock on a potential buyer’s maximum bid price. Assuming the face and market value of loans used to purchase a
property are equal, \( F_s = M_a \). The maximum price a buyer will pay for a property becomes:\(^{21}\)

\[
(12) \quad B = V + \overline{c}_B
\]

Equation (12) shows that a negative [positive] demand shock will lower [increase], dollar for dollar, the maximum price a purchaser will pay for a property [i.e., \( \Delta B = \Delta V \)].

C. Conditions for a Transaction

If a mutually agreeable transaction it to take place, the seller’s reservation price must be less than or equal to the buyer’s maximum bid price:

\[
(13) \quad V + \overline{F}_s - M_s - \overline{c}_s \leq V + \overline{F}_B - M_B + \overline{c}_B
\]

Upon simplification, Equation (13) reduces to the following condition for a transaction:

\[
(14) \quad \overline{c}_s + \overline{c}_B \geq (\overline{F}_s - M_s) - (\overline{F}_B - M_B)
\]

That is, the sum of the liquidity costs that would be born if a transaction does not occur must be at least as large as the difference between the net values of the potential transactors’ mortgage loans. If, as would be expected, mortgage interest rates are set so that at origination the borrower pays for the put option inherent in a non-recourse loan,
then at origination the loan’s face value will equal its market value i.e., \( F_B = M_B \). Thus, for a mutually agreeable transaction to take place, the sum of the liquidity costs [motivation to transact] must exceed the difference between the face value [principal] and the market value of the seller’s mortgage loan:

\[
(15) \quad c_s + c_B \geq (F_B - M_s) = \Delta M_s,
\]

where \( \Delta M_s \) denotes the change in the market value of the seller’s non-amortizing mortgage from origination to date.

Conceptually, we can decompose the market value of the mortgage, \( M_S \), into the present value of the mortgage loan payments evaluated at a default-free [but not risk free] rate of interest and the value of a put option whose exercise price is \( F_s \). If property values and interest rates were independent, then holding the loan balance constant, declines [increases] in property values and increases [reduction] in interest rates reduce [increase] the market value of the mortgage, \( M \). This in turn, increases [reduces] the liquidity costs needed to induce a mutually agreeable transaction. Changes in property values \( V \) are, however, negatively related to interest rate changes. As was shown in equation (9) unanticipated increases [decreases] in interest rates, will increase [decrease] the liquidity costs needed to induce a mutually agreeable transaction.

**D. Demand Shocks and Mutually Agreeable Transaction**

Within our model, the effect of demand shocks on asset liquidity is revealed by our analysis of the effect of demand shocks on the maximum bid price of potential
buyers and sellers’ reservation price. We concluded that, a demand shock will lower [increase], dollar for dollar, the maximum price a purchaser will pay for a property [i.e., $\Delta B = \Delta V$]. In contrast, a seller’s reservation price will change less, in absolute value, than the change in a property’s investment value [i.e., $|\Delta R| < |\Delta V| = |\Delta B|$].

Consequently, negative demand shocks will tend to eliminate otherwise mutually agreeable transactions and positive shocks will tend to facilitate transactions. In terms of our model, where sellers are identical and buyers are identical, the outcome of each negotiation will be the same and we can conclude a sufficiently large negative demand shock will preclude any mutually agreeable transactions. In practice, at any given time, there will be distributions of loan balances and liquidity costs. Consequently, only a portion of potential transactions will be eliminated by a negative demand shock. The larger the shock the greater the fraction of potential transactions that will be eliminated and the greater the decline in asset liquidity [i.e., increase in expected time to favorable match] that will follow. In contrast, a positive demand shock will make it easier to achieve a transaction. Thus, at least theoretically, the form of the mortgage loan contract, non-recourse and/or fixed interest rate, is sufficient [but not necessary] to generate asymmetric response to large demand shocks.

E. Demand Shocks and Transaction Prices

Now we investigate the implications of non-recourse financing and fixed rate loans on sales prices. Specifically we show that our model implies the empirical observation that transaction prices fall less than the properties investment value following a negative demand shock.
We start by assuming that if a transaction is to take place \([B > R]\) the parties “split” the difference between the seller’s reservation price \([R]\) and the potential buyer’s maximum bid price \([B]\) according to a proportion \(\alpha\), \(0 < \alpha < 1\). That is, the transaction price \(p\) equals:

\[
p = \alpha(V + (F_s - M_s) - c_s) + (1 - \alpha)(V + (F_b - M_b) + c_b)
\]

Simplifying, and assuming \(F_b = M_b\), we obtain:

\[
p = V + (1 - \alpha)c_b + \alpha(F_s - M_s - c_s)
\]

Taking the derivative of the transaction price with respect to the property’s investment value, \(V\), we obtain:

\[
\frac{dp}{dV} = 1 - \alpha \frac{dM_s}{dV}
\]

Because \(\frac{\partial M_s}{\partial V} > 0\) and \(0 < \alpha < 1\), when a transaction is possible, the transaction price will fall less than the properties investment value. Note, this conclusion holds as long as the proportional split is independent of the change in \(V\).\(^{25}\)

**IV. Empirical Model of a Real Estate Market**

In this section we describe the model and techniques we use to assess the empirical relevance of the theoretical results presented in the previous section.\(^{26}\)
Specifically, we use the least-squares Monte-Carlo [LSMC] approach developed by Longstaff and Schwartz [2001] to value the portfolio of assets that is represented by an owner’s interest in an apartment building financed by a non-recourse loan. The LSMC approach combines the principals of risk-neutral valuation with least-squares regression to estimate the value of complex American options.

A. Joint Stochastic Processes

Without loss of generality, our analysis will be couched in terms of a single apartment unit. Underlying our implementation of the LSMC approach is the assumption that the per unit price of an apartment building, \( V \), the per unit before tax cash flow yield, \( \circ \), and the risk free rate of interest, \( r \), are described by the following system of differential equations:\(^{27}\)

\[
\begin{align*}
\frac{dv}{v} &= \mu_v dt + \sigma_v dz_v \\
\frac{d\delta}{\delta} &= \mu_\delta dt + \sigma_\delta dz_\delta \\
\frac{dr}{r} &= \mu_r dt + \sigma_r dz_r \\
\end{align*}
\]  

(19)

where the parameters \( \mp_{i,j}, [i=v, \circ, r] \) are at most functions of the state variables \( v, \circ, \) and \( r \) and \( dz_i \) are increments to Wiener processes. The correlation coefficients between the increments to the Wiener processes are denoted by \( \gamma_{ij} \) \( [i,j=v, \circ, r] \). In our model, as described below, the risk free rate of interest uniquely determines the equilibrium mortgage interest rate.
In order to estimate the joint stochastic process (19) we need to specify the functional form of the drift and diffusion coefficients. The basic assumption we make is that the expected rate of property appreciation, the drift of the cash flow yield, and the changes in the risk free interest rate are linear functions of the state variables. This specification implies that the joint stochastic process may be written as:

\[
\begin{align*}
\frac{dv}{v} &= (a_{v1} + a_{v2}\delta + a_{v3}v + a_{v4}r)dt + \sigma_v dz_v \\
d\delta &= (a_{\delta1} + a_{\delta2}\delta + a_{\delta3}r)dt + \sigma_\delta dz_\delta \\
dr &= (a_{r1} + a_{r2}\delta + a_{r3}r)dt + \sigma_r \sqrt{r} dz_r,
\end{align*}
\]

where \(dz_v, dz_\circ, \) and \(dz_r\) are increments to standard Wiener processes with correlations \(-\sigma_{ij}\) \(i = v, \circ, r\) and \(-\sigma_{v\circ}, -\sigma_{v\circ}\) and \(-\sigma_{r\circ}\) are the standard deviation of the respective time series. In this formulation, a negative [positive] demand shock is the occurrence of a \(z_v\) that is negative [positive] and large in absolute value. This formulation excludes the possibility of negative property values and risk free rates of interest, but it does not exclude the possibility of a negative cash flow yield.\(^{28}\)

Under the risk-neutral metric, the above model of property values is replaced by:

\[
\begin{align*}
\frac{dv}{v} &= (r - \delta)dt + \sigma_v dz_v \\
d\delta &= (a_{\delta1} + a_{\delta2}\delta + a_{\delta3}r)dt + \sigma_\delta dz_\delta \\
dr &= (a_{r1} + a_{r2}\delta + a_{r3}r)dt + \sigma_r \sqrt{r} dz_r.
\end{align*}
\]
Notice that the process for $\odot$ does not undergo a change under the risk-neutral metric because the cash flow yield [or dividend yield] is not an investment asset by itself and the risk-neutral preferences of the agents have no implication for its evolution over time. [Appendix A provides a formal rational for this conclusion]. Going from a risky to a risk-neutral world will result in the correct value of a derivative based upon a traded asset because both the expected return and discount rate used to evaluate the cash flows are adjusted [Hull, 1997].

The LSMC method starts by simulating price paths for the underlying asset [e.g., the value of an apartment unit], the cash flow yield and the risk free rate of interest. In our implementation of the LSMC we generate 10,000 price paths for a typical apartment unit for 120 months into the future. The simulated price paths are then used to estimate, by least squares, the pay-off for the portfolios conditional upon the state variables, $v$, $\odot$, and $r$ at each point in time. The results of the regression are then used to estimates the expected value of continuation. The value of the portfolio is estimated by assuming that a risk-neutral seller or buyer acts optimally given the holding and liquidity costs. The value of the option that represents the owner’s interest in the property then equals the expected present value [at the risk free rate] of the cash flows associated with optimal exercise of the option by sale of the property or default on the mortgage loan. The data needed to simulate the price paths are the risk free rate of interest, the variance-covariance matrix of the innovations of the processes given by Equation (21), and the parameters $a_{ij}$. 
B. Estimates of the Stochastic Processes

Transaction data [1988-2000], provided by CosStar Comps, for the Los Angeles County apartment building market was used to estimate the joint stochastic process \((20)\) through the use of seemingly unrelated regression. Appendix B provides the details of the data sources and the empirical estimation of the property value and cash flow yield series used in this estimation. The parameter estimates of process \((20)\) are reported in Tables 1 and 2. These results are statistically significant and consistent with our expectations.

The estimates of volatility of the price appreciation and cash flow yield reported in Table 1 contain an upward bias because the estimated rates of price appreciation and change in cash flow yield used to calculate them are subject to sampling error. Thus, even if the true appreciation rates and changes in the cash flow yield are perfectly explained by the model, the estimated volatility of the innovations will be positive simply due to the sampling error. Cauley and Pavlov [2002] derive a bias correction technique for similarly derived estimates of home price appreciation. This is the technique we used to derive our bias-corrected estimates of volatility. Table 3 reports the bias-corrected estimate of volatility of the rate of price appreciation and change in cash flow yield. Note the risk free rate is not estimated. Consequently the estimated volatility does not have to be adjusted for bias. It can be easily shown that the covariance estimate does not need to be adjusted either.

Our bias-corrected estimate of the volatility of the monthly appreciation rates and cash flow volatility are .0026 and .002 respectively [48 percent and 23 percent reductions relative to the “naïve” estimates presented in Table 1]. Our estimates of the
value of the portfolio of assets that correspond to the owner’s interest in a property will be based upon these values.

V. Estimates of the Effect of a Non-Recourse Loan on a Seller’s Reservation Price

Above we showed that the effect of a non-recourse feature on the seller’s reservation price is the difference between the market value of the seller’s mortgages with and without a non-recourse feature. In the following example, based upon the LA County data, we illustrate the magnitude of this effect. We assume:

- The mortgage loan is non-amortizing
- Refinancing to obtain a lower interest rate is not possible,
- The loan is due [i.e., the option expires] in 10 years and the potential seller can exercise the option [default] at the end of each month [i.e., when the next payment is due];
- There are no costs associated with mortgage loan default;
- The current risk-free rate of interest is 4%;
- There are no transaction costs associated with the sale of the property.

Throughout our analysis we derive the market mortgage interest rate from the risk free rate generated by equation (21). The key to computing the equilibrium fixed or adjustable mortgage rate [the spreads over the risk free rate] is the assumption that at origination the value of the mortgage equals the outstanding loan balance. In other words, the mortgage interest rate, either variable or fixed, exactly compensates the lender for the put option imbedded in the loan and for the interest rate risk associated with a FRM. If the expected cost to the creditor of providing the non-recourse feature equals the expected benefits to the borrower, then in a competitive lending industry, our
estimates would be the equilibrium mortgage rate. Table 4 presents our estimate of mortgage rates when the risk-free rate is 4%, properties are bought with a 80 percent LTV, refinancing to obtain a lower interest rate is not possible, and there are no liquidity costs. Estimates are provided for both variable and fixed interest rate mortgage loans. The assumption that all apartment purchases are financed with 80 percent LTV loans is a simplification from the rich mosaic of financing used in practice.

A. Base Case

The base case assumes that the owner incurs no liquidity costs if the property is not sold. Figure 1 depicts the relationship between the owner’s equity, as represented by the property’s market LTV, and the effect of the non-recourse feature on a seller’s reservation price. Our estimates are the difference, in terms of percent of the investment value, between the value of the portfolio with and without the non-recourse feature. In this analysis the investment value of the property, \( V_0 \), can be thought of as given. Variations in LTV are equivalent to variations in the exercise price of the put [i.e., loan balance] that are associated with variations in the acquisition date of the property.

The volatility of property values is by far the most important determinant of the effect of the non-recourse feature on a seller’s reservation price. Figure 1 depicts the effect of the non-recourse feature under two assumptions regarding this volatility. The top line depicts the effect under monthly volatility of 2.6%, which is our best estimate, for Los Angeles County apartments, as reported in Table 3. As a form of sensitivity analysis we also report the estimated effect under assumption of volatility of one-half of our estimate, i.e., 1.25%. Under our best estimate of volatility [2.6%], Figure 1 shows that if a person has zero equity [100% LTV], the effect of the non-recourse feature is the
greatest and exceeds 7 percent of the properties investment value.\textsuperscript{34} For example, if the investment value of a complex was $50,000 per unit, and the owner’s loan balance was also $50,000 per unit, the seller’s reservation price would be in excess of $53,500. The effect of the non-recourse on the reservation price falls until, at a LTV of 80 percent, its value is zero.\textsuperscript{35}

Returning to Figure 1, we see the value of the option to put the property back to the creditor is substantial when the LTV exceeds 100\%. For example, with negative equity of 5 percent [a LTV of 115\%], the seller’s reservation price is approximately 4 percent greater than the properties investment value. This helps us understand why it may be optimal to make loan payments on a property with negative equity. During the mid 1990s many Southern California properties were in this position.

Even under the extremely conservative assumption of volatility being half of our estimate for LA County, the effect exceeds 5\% of the property’s investment value when the owner has a 100\% LTV. From this analysis, we conclude that even if the Los Angeles County apartment market is much more volatile than typical real estate markets [as might be expected], the estimates pictured in Figure 1 strongly suggest that our findings are applicable to real estate markets in general.\textsuperscript{36} From the above analysis we can see how decreases in property values can generate declines in asset liquidity [lowering the probability of a match], which, in turn, generates further declines in asset liquidity.

\textbf{B. Liquidity Cost and the Reservation Price}

In the base case, the seller incurs no costs if they choose not to transact during the first period. Liquidity costs, motivation to transact, can be thought of as a per-period
cost of maintaining the option to default.\textsuperscript{37} Consequently, this cost would be expected to reduce the net value of the option, thereby reducing the impact of the non-recourse feature on the seller’s reservation price. Figure 2 extends the analysis presented in Figure 1 to include liquidity cost. The monthly costs [in terms of percent of investment value] that motivate the transaction are along one of the axes, the existing owner’s market LTV ratio is along another axis, and the effect of the non-recourse provision on the reservation price of seller is along the vertical axis. Again, variations in LTV are equivalent to variations in the exercise price of the put option [loan balance] given the property’s investment value $V$. This figure indicates that the value of the non-recourse feature decreases as the seller’s LTV $F_S/V$ diverges from 100% and as the liquidity costs increase in absolute value.\textsuperscript{38} The range of liquidity costs where the non-recourse feature has an economically important effect on the seller’s reservation price is quite large. Zero liquidity costs indicates that the owner is not motivated to sell and can wait indefinitely. Liquidity costs of $\frac{1}{2}$ % of investment value per month are very high and represent an owner who is highly motivated to sell. Even this owner will reject an offer of $V$ if their equity position is between approximately $-4$ and $4$ percent of the property’s investment value. In these cases, the value of waiting exceeds the liquidity costs.

The implication of demand shocks for reservation prices can be seen by considering an individual who bought a property with 20 percent equity. At a LTV of 80 percent, the effect of the non-recourse feature on the seller’s reservation price is, by construction, zero.\textsuperscript{39} If a \textit{positive demand} shock occurs after the property is bought, thereby decreasing the LTV, the effect of the non-recourse feature will be still be zero.\textsuperscript{40} Whereas, a \textit{negative demand shock} will increase the LTV, which in turn will increase
the value of the non-recourse feature of the loan and increase the difference between the seller’s reservation price and the fair market value.
VI. A Non-Recourse Feature and the Decision to Transact

We now return to our analysis of the relationship between non-recourse feature and a real estate market’s response to demand shocks. In equation (15), we concluded that a mutually agreeable transaction would occur if, and only if, the sum of the liquidity costs [motivation] is greater than or equal to the difference between the principal [face value] and market value of the seller’s mortgage, i.e., $c_s + c_b \geq (F_s - M_s)$. In the remainder of this section, we use the Los Angeles County apartment data to examine the determinants of this decision for both fixed rate and variable rate mortgages.

Figure 3 represents the result of an examination of the liquidity costs [for the buyer and/or the seller] that will induce a transaction when the seller has a 6 percent fixed interest rate mortgage [e.g., the loan was originated when the risk free rate was 4 percent]. The figure depicts the sum of the liquidity costs, for the buyer and the seller, necessary to induce a transaction as a function of the seller’s LTV ratio and the current risk-free rate. In this analysis, as was previously the case, the investment value of the property, $V$, is fixed, and variations in LTV are the result of differences in the loan balance, $F_s$, which induce variations in $M_s$. Notice that the sum of the liquidity costs, not their distribution between the buyer and the seller, determines whether a mutually agreeable transaction is possible. Of course, the distribution between the buyer and the seller will have an impact on the transaction price. Furthermore, we take into account not just the current but expected future liquidity costs to both parties under the optimal exercise policy.

Figure 3 shows that reductions in the market value of the mortgage, $M$, relative to the outstanding balance would require higher liquidity costs for the buyer and/or the
seller to induce a transaction. In our model, there are two forces that reduce the market value of the mortgage:

- Decreases in the investment value of property [pictured as increases in LTV], and
- high current risk-free rate [and mortgage rates] relative to the rates that held at the time of loan origination.

Consistent with our expectation, as the LTV ratio approaches 100%, the value of the put increases which, in turn, reduces the value [burden] of the mortgage for the seller. Consequently, combined costs of approximately 1% of value per month are needed to induce a transaction in this situation. High current interest rates have a similar, although much smaller, impact. If the current owner has a FRM with a low interest rate relative to the current rates, an “interest rate lock-in”, then the market value of the mortgage is lower than the principal amount and substantial liquidity costs are necessary to induce a transaction [i.e., there is a mortgage loan lock-in]. Notice, however, that for 80% LTV ratio, the combined liquidity costs necessary to induce a transaction at 6% risk-free rate are only .05% per month. In other words, \textit{ceteris paribus}, we can expect to see only marginal declines in liquidity following an increase in interest rates. In our analysis the effect of high current interest rates is partially mitigated because our estimates of the joint stochastic process (20) produced an increasing relationship between the level of $r$ and $dV/V$, and because the interest rate process is mean-reverting. Our analysis of the fixed interest rate non-recourse mortgage example strongly suggests that a negative demand shock that results in LTV greater than the maximum LTV for a significant portion of the property owners will preclude many otherwise desirable transactions.
Analogously to Figure 3, Figure 4 depicts the sum of the buyer’s and the seller’s liquidity costs necessary to induce a transaction, if the seller holds an adjustable rate mortgage [ARM]. The implication of ARM financing is that changes in interest rates alone do not alter the value of the mortgage. While this is true for a default-free bond, the market value of the mortgage still depends on interest rates because they affect the value of the embedded put option. In general, increase in the risk-free interest rate decreases the value of a put option for two reasons: (1) the expected growth rate of the asset price increases, and (2) the present value of future cash flow received by the holder decreases. While both of these effects hold in our application, interest rates have one additional impact in the case of ARM: the monthly carrying costs increase. Except for very high LTV ratios, the probability of delaying the sale over extended periods of time is low and the first two effects dominate. Figure 4 suggests that for LTV ratios below 98% an increase in interest rates reduces the value of the put option, which, in turn, reduces the liquidity costs necessary to induce a transaction.

For very high LTV ratios, the probability of delaying the sale over extended time periods is high, and the increase in expected carrying costs is substantial enough to overcome the first two effects. Figure 4 suggests that for LTV ratios above 98%, an increase in interest rates increases the value of the embedded put option, which, in turn, further increases the liquidity costs necessary to induce a transaction.

Regardless of the type of mortgage and the fluctuation of the interest rates, Figures 3 and 4 suggest a clear conclusion: even small increases of the LTV ratio above 80%, [e.g., from a negative demand shock] require positive and increasing liquidity costs [motivation] for a mutually agreeable transaction to be possible. That is, a negative demand shock can result in a situation where no mutually agreeable transaction
will be possible for a portion of the potential sellers at the property's investment value. This can be interpreted as a decrease in asset liquidity. For LTV ratios close to 100%, the required liquidity costs for a transaction are substantial and approach 1% of the asset price per month. In contrast, a positive demand shock that decreases the seller's LTV will facilitate transactions by reducing the cost needed to motivate the transaction.

While our model has been for a specific type of property and a specific real estate market, we contend that it has identified an important general aspect of real estate markets where properties are purchased with non-recourse loans.

VII. Conclusion

Markets for income producing real estate frequently respond asymmetrically to large positive and negative demand shocks. This paper provides an explanation for this phenomenon that is consistent with individual rationality. Our explanation is based upon the rational response of sellers and potential buyers to two characteristics of their actual, or potential, mortgage loan contract: non-assumable fixed interest rate mortgages and/or non-recourse loans. The model we developed allowed us to explore the effect of non-assumable fixed interest rate mortgages and/or non-recourse loans on the conditions under which a mutually agreeable transaction will take place. Using the model, we are able to show that a negative demand shock, in conjunction with either of these features, can result in a period during which no mutually agreeable transaction is possible between liquidity motivated sellers and buyers with identical expectations about the price formation process. In contrast, we show that a positive demand shock will never result in a decline in asset liquidity. We conclude that the non-recourse feature is sufficient to generate the asymmetric responses to positive and negative demand shocks.
that are observed. Consequently, while factors such as loss aversion may contribute to a market’s response to demand shocks, they are not necessary to explain this response.

While the traditional explanations, be they institutional or behavioral, of how real estate markets respond to demand shocks are likely to be important in the short run, our results strongly suggest that the prevalence of some mortgage loan features are an important determinate of the markets’ long run response. This result is important because, to the extent that non-recourse financing is responsible for the observed declines in asset liquidity, financial innovations may improve economic efficiency. This need not be true if the loss aversion is the explanation.
References


Appendix A: Cash Flow Yield and Risk-Neutral Measure

This appendix provides a formal argument that the cash flow yield [analogous to dividend yield] retains the same drift and the same correlation with the other state variables under an equivalent probability measure. Suppose the asset price and the dividend yield evolve according to

\[
\begin{pmatrix}
    dV(t) \\
    d\delta(t)
\end{pmatrix} = \begin{pmatrix}
    (\mu_v - \delta(t))V(t)dt \\
    \mu_\delta dt
\end{pmatrix} + \begin{pmatrix}
    \sigma_v V \\
    \sigma_\delta
\end{pmatrix} \begin{pmatrix}
    dz_v \\
    dz_\delta
\end{pmatrix} = \mu dt + \sigma dz
\]

Notice that this is a slightly different representation than the one given by (A.1). In particular Equation (A.1) represents the drift of the asset price as \( \mu_v - \delta(t) \). This makes the total return to holding the asset equal to \( \mp S \).

Let us assume that

\[
dz \cdot dz' = \begin{pmatrix}
    1 & \rho \\
    \rho & 1
\end{pmatrix} := \Sigma
\]

We apply the following transformation. Let \( A \) is a 2x2 matrix such that \( A \Sigma A' = I \), i.e. \( \Sigma = A^{-1}(A^{-1})' \). Let \( B = Az \). Then

\[
\begin{pmatrix}
    dB \\
    dB'
\end{pmatrix} = Adz \cdot dz' = A\Sigma dt A' = dt
\]

We can write
We can now apply Girsanov’s theorem to the normalized $B$’s. Let

\[
(A.5) \quad \alpha = \begin{pmatrix} (r(t) - \delta(t))V(t) \\ \mu_\delta \end{pmatrix}
\]

where $r$ is another process [interest rate in this case]. Applying Girsanov’s theorem to (A.4) we obtain

\[
(A.6) \quad \begin{pmatrix} dV(t) \\ d\delta(t) \end{pmatrix} = \alpha dt + \sigma A^{-1}dB
\]

where $\hat{B}$ is another normalized Brownian motion [$d\hat{B} \cdot d\hat{B} = dt$] with respect to an equivalent probability measure. We can now switch back to the original process.

Define $\hat{\delta} = A^{-1}\hat{B}$. Then, $d\hat{\delta} \cdot d\hat{\delta} = \Sigma dt$, and

\[
(A.7) \quad \begin{pmatrix} dV(t) \\ d\delta(t) \end{pmatrix} = \alpha dt + \sigma d\hat{\delta} = \begin{pmatrix} (r(t) - \delta(t))V(t)dt \\ \mu_\delta dt \end{pmatrix} + \begin{pmatrix} \sigma V \\ \sigma_\delta \end{pmatrix} \begin{pmatrix} d\hat{\delta}_V \\ d\hat{\delta}_\delta \end{pmatrix}
\]
In short, we can switch to an equivalent probability measure such that (A.7) holds and the new Brownian motions have the same correlation as the original ones. Based on our choice of $\mapsto$, the dividend process has the same representation as before.
Appendix B: Data and Parameter Estimates

Unlike financial markets, real estate markets do not provide all of the data needed to calculate the value of the portfolio that represents the owner’s interest in a property. Specifically, the heterogeneity of properties and infrequent trading result in the absence of reliable estimates of the historical volatility of real estate appreciation.\(^{42}\) To calculate this statistic we need to first estimate the time series of rates of appreciation of the underlying asset [i.e., the per unit price of LA County apartments]. We use the following semi-log hedonic value model to estimate this series:

\[
\ln[Value_{it}] = \text{Constant} + \sum_{t=2}^{T} \uparrow_t \cdot S_t + \leftrightarrow' C_i + \odot
\]

where \(Value_{it}\) is the price paid for property \(i\) sold at time \(t\), \(C_i\) is a vector of physical characteristics that describe the building, \(S_t\) is a matrix of indicator variables for the time of sale, and \(\uparrow_t\) is the marginal time effect [i.e., monthly]. \(T\) is the total number of months in the sample and \(\odot\) is an error term with zero expectation.\(^{43}\) Thus \(\uparrow_t\) is an estimate of the rate of appreciation for time period \(t\). The mean of the vector \(\uparrow\) provides an estimate of the expected monthly rate of apartment appreciation.

As evident from Equation (18), following a negative demand shock transaction prices do not decline as much as the underlying property value, \(V\). While we acknowledge this effect, we note that it will lead to a lower volatility estimate relative to the true property value volatility. In that sense, we are underestimating any empirical effects reported below.
To estimate the cash flow yield, \( \circ \), we use a similar regression to Equation (B.1), except the dependent variable is not the log but the level of the cash flow yield. This is dictated by the specification of the stochastic process for the cash flow yield (19). In particular, since the changes of the cash flow yield are assumed to be additive, not proportional, the correct specification is:

\[
\circ_{it} = \text{Constant} + \sum_{t=2}^{T} \bullet_i S_t + \leftarrow C_i + \circ
\]

where \( \circ_{it} \) represents the cash flow yield of property \( i \) when sold at time \( t \). \(^{44}\)

Analogously to the property appreciation rates, the parameters \( \bullet \) in Equation (B.2) are estimates of the monthly change in the before tax cash flow yield.

To estimate the appreciation rates and the cash flow yields for a property we use transaction data. The data come from CoStar COMPS. The firm produces high quality transactions data for a wide range of income producing properties. The firm has provided data for all transactions in Los Angeles County apartment buildings that occurred between October 1989 and July 2001, a total of 18,167 observations. \(^{45}\) Table B1 provides summary statistics describing the transactions that occurred during the period. The mean and median per unit price during this period were a little more than $50,000 per unit. The mean and median cash flow yields at the time of sale were approximately 9 percent. As can be seen from the table, the typical LA County
apartment complex is relatively small and the distribution of complex size is positively skewed, with a median of 10 and a mean of 18 units.

Tables B2 and B3 report the parameter estimates and implied appreciation rates obtained by estimating Equation (B.1). The parameter estimates presented in Table B2 have the expected signs and are highly statistically significant. From Table B3 we can see that our model estimated an average [ex-post] monthly appreciation rate of nearly zero with a standard deviation of more than six percent.

The late 1980s and 1990s were a boom/bust period for Southern California property values. Between the 1990 peak and February 1997 trough, per unit prices fell by more than 54 percent. By the middle of 2001, per unit prices were more than 78 percent above their 1997 low.

Next, we estimated Equation (B.2) to obtain estimates of the annualized cash flow yield. The results are presented in Tables B4 and B5.
Table 1: Joint Stochastic Process Driving the Property Markets

<table>
<thead>
<tr>
<th></th>
<th>○</th>
<th>r</th>
<th>V</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>dV/V</td>
<td>1.02</td>
<td>-8.56</td>
<td>1.01</td>
<td>-67</td>
</tr>
<tr>
<td></td>
<td>(5.95)</td>
<td>(-5.84)</td>
<td>(2.85)</td>
<td>(-6.66)</td>
</tr>
<tr>
<td>d○</td>
<td>.005</td>
<td>-.05</td>
<td>-.033</td>
<td>.0026</td>
</tr>
<tr>
<td></td>
<td>(2.64)</td>
<td>(-2.38)</td>
<td>(-1.80)</td>
<td></td>
</tr>
<tr>
<td>Dr</td>
<td>-.003</td>
<td>.06</td>
<td>-.02</td>
<td>.009</td>
</tr>
<tr>
<td></td>
<td>(-2.48)</td>
<td>(3.62)</td>
<td>(-1.25)</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the estimates from Equation (20):

\[
\frac{dy}{v} = (a_{v1} + a_{v2} \delta + a_{v3} v + a_{v4} r) dt + \sigma_v dz_v
\]

(20)

\[
d \delta = (a_{\delta1} + a_{\delta2} \delta + a_{\delta3} r) dt + \sigma_\delta dz_\delta
\]

\[
dr = (a_{r1} + a_{r2} \delta + a_{r3} r) dt + \sigma_r \sqrt{d}z_r
\]

All observations are monthly.
Table 2: Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>dV/V</th>
<th>dO</th>
<th>dr</th>
</tr>
</thead>
<tbody>
<tr>
<td>dV/V</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dO</td>
<td>-.72</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>dr</td>
<td>-.13</td>
<td>.1</td>
<td>1</td>
</tr>
</tbody>
</table>

This table reports the correlation coefficients of the innovations as estimated by Equation (20):

\[
\frac{dv}{v} = (a_{v1} + a_{v2}\delta + a_{v3}\nu + a_{v4}r)dt + \sigma_v dz_v
\]

\[
d\delta = (a_{\delta1} + a_{\delta2}\delta + a_{\delta3}r)dt + \sigma_\delta dz_\delta
\]

\[
dr = (a_{r1} + a_{r2}\delta + a_{r3}r)dt + \sigma_r \sqrt{r} dz_r
\]
### Table 3: Bias-corrected estimate of the volatility of the monthly appreciation rates and cash flow yield

<table>
<thead>
<tr>
<th></th>
<th>Bias-corrected estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dV/V$</td>
<td>.026</td>
</tr>
<tr>
<td>$d\circ$</td>
<td>.002</td>
</tr>
<tr>
<td>$dr$</td>
<td>.009</td>
</tr>
</tbody>
</table>

This table reports the bias-corrected estimates of the volatilities of the three joint stochastic process of Equation (20):

\[
\begin{align*}
\frac{dv}{v} &= (a_{v1} + a_{v2}\delta + a_{v3}y + a_{v4}r)dt + \sigma_y dz_y \\
\delta &= (a_{\delta1} + a_{\delta2}\delta + a_{\delta3}r)dt + \sigma_\delta dz_\delta \\
r &= (a_{r1} + a_{r2}\delta + a_{r3}r)dt + \sigma_r \sqrt{r} dz_r
\end{align*}
\]
Table 4: Equilibrium Interest Rates

<table>
<thead>
<tr>
<th>Rate Type</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
<td>4%</td>
</tr>
<tr>
<td>Adjustable rate 10-year mortgage</td>
<td>5.5%</td>
</tr>
<tr>
<td>Fixed rate 10-year mortgage</td>
<td>6%</td>
</tr>
</tbody>
</table>

This table reports the equilibrium interest rates for adjustable and fixed-rate mortgages, conditional on the risk-free rate of 4%. These rates include a default premium based on 20% home equity [i.e., 80% LTV]. These rates make the market and the face value of the mortgage equal.

<table>
<thead>
<tr>
<th></th>
<th>Price per unit</th>
<th>Cash Flow Yield</th>
<th>Age</th>
<th>Parking Spaces</th>
<th>Number of Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$56,174</td>
<td>.09</td>
<td>34</td>
<td>21</td>
<td>18</td>
</tr>
<tr>
<td>Median</td>
<td>$51,600</td>
<td>.09</td>
<td>34</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$26,744</td>
<td>.03</td>
<td>18</td>
<td>40</td>
<td>26</td>
</tr>
</tbody>
</table>

This table provides some descriptive statistics for the Los Angeles County apartment building transaction data.
Table B2: Parameter Estimates for the Asset Value Index

<table>
<thead>
<tr>
<th></th>
<th>Age</th>
<th>Age Squared</th>
<th># parking spots</th>
<th># parking squared</th>
<th># units</th>
<th># units squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=18,167</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R² = .50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>-.0105</td>
<td>8 E-5</td>
<td>.0068</td>
<td>-5 E-6</td>
<td>-.0133</td>
<td>2 E-5</td>
</tr>
<tr>
<td>St. Error</td>
<td>.0004</td>
<td>5 E-6</td>
<td>.0002</td>
<td>2 E-7</td>
<td>.0003</td>
<td>6 E-6</td>
</tr>
</tbody>
</table>

This table reports the parameter estimates using Equation (B.1):

(B.1) \[ \ln[Value_{it}] = \text{Constant} + \sum_{t=2}^{T} t S_t + \rightarrow C_t + \nabla \]

where \( Value_{it} \) denotes the per unit value of apartment building \( i \) sold at time \( t \), \( C_t \) denotes the physical characteristics of that apartment building at the time of sale, and \( S_t \) is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Table B3: Implied monthly rates of price appreciation

<table>
<thead>
<tr>
<th>Average Appreciation Rate</th>
<th>Standard Deviation of the Appreciation Rate</th>
<th>Median Appreciation Rate</th>
<th>Skewness of the Appreciation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>≈ 0</td>
<td>.0528</td>
<td>≈ 0</td>
<td>-.1463</td>
</tr>
</tbody>
</table>

This table reports the implied appreciation rates estimates from estimating Equation (B.1):

\[
(B.1) \quad \ln[Value_{it}] = Constant + \sum_{t=2}^{T} \uparrow_t S_t + \leftrightarrow C_i + \circ
\]

where \( Value_{it} \) denotes the per unit value of apartment building \( i \) sold at time \( t \), \( C_i \) denotes the physical characteristics of that apartment building at the time of sale, and \( S_t \) is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Table B4: Parameter Estimates for the Cash Flow Yield Index

<table>
<thead>
<tr>
<th>N=14,879</th>
<th>Age</th>
<th>Age</th>
<th># parking</th>
<th># parking</th>
<th># units</th>
<th># units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Squared</td>
<td>spots</td>
<td>squared</td>
<td></td>
<td>squared</td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>-7.5 E-5</td>
<td>3 E-7</td>
<td>1.3 E-4</td>
<td>9 E-8</td>
<td>3.3 E-4</td>
<td>5 E-7</td>
</tr>
<tr>
<td>St. Error</td>
<td>2.5 E-5</td>
<td>3 E-8</td>
<td>1.3 E-5</td>
<td>1 E-8</td>
<td>1.9 E-5</td>
<td>4 E-8</td>
</tr>
</tbody>
</table>

This table reports the parameter estimates using Equation (B.2):

\[
\delta_{it} = Constant + \sum_{i=2}^{T} \beta_i S_i \rightarrow C_i + \epsilon
\]

where \(\delta_{it}\) denotes the cash flow of building \(i\) sold at time \(t\), \(C_i\) denotes the physical characteristics of that apartment building at the time of sale, and \(S_i\) is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Table B5: Implied monthly rates of change in cash flow yield

<table>
<thead>
<tr>
<th>Average change</th>
<th>Standard Deviation of the Change</th>
<th>Median Rate of Change</th>
<th>Skewness of the Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1E-4/Month</td>
<td>0.003</td>
<td>1E-4</td>
<td>-.19</td>
</tr>
</tbody>
</table>

This table reports the implied appreciation rates estimates from estimating Equation (B.2):

\[ \delta_{it} = \text{Constant} + \sum_{t=2}^{T} \gamma_t S_t + \rightarrow C_i + \epsilon \]

where \( \delta_{it} \) denotes the cash flow of building \( i \) sold at time \( t \), \( C_i \) denotes the physical characteristics of that apartment building at the time of sale, and \( S_t \) is an indicator variable taking the value of 1 for all months before and including the transaction and zero thereafter.
Figure 1 depicts the effect of the non-recourse feature of a mortgage loan on the seller’s reservation price assuming no liquidity costs. The top and the bottom line depict this effect assuming volatility of the asset price of 2.6% and 1.25%, respectively.
Figure 2 depicts the effect of the non-recourse feature on the seller’s reservation price as a function of home equity [LTV ratio] and liquidity costs [percent of asset value per month]. Zero liquidity costs generate the largest effect. Even with very high liquidity costs, the seller’s reservation price is above the investment value of the asset near the 100% LTV region.
Figure 3 depicts the sum of the buyer’s and the seller’s liquidity costs necessary to induce a transaction as a function of the LTV ratio and the risk-free rate for a Fixed Rate Mortgage. This figure takes into account not only the current period liquidity costs, but all expected future liquidity costs for both the buyer and the seller under the optimal exercise policy.
Figure 4: Liquidity Costs Necessary to Induce a Transaction for an ARM

Figure 4 depicts the sum of the buyer’s and the seller’s liquidity costs necessary to induce a transaction as a function of the LTV ratio and the risk-free rate for an Adjustable Rate Mortgage. This figure takes into account not only the current period liquidity costs, but all expected future liquidity costs for both the buyer and the seller under the optimal exercise policy.
Following the negative demand shock of the late 1980s the median “time on market” [a frequently used measure of the liquidity] for homes that sold in California increased from approximately 4 weeks during 1989 to 13 weeks during 1993 [California Association of Realtors]. During the same period the median sales price declined by 4.1 percent [California Association of Realtors]. While comparable data are not available for investment properties, conversations with experienced market participants suggest that declines in liquidity are even larger for investment properties.

A non-recourse loan precludes a deficiency judgment following loan default. If a legal entity owns a property [e.g., a limited partnership where the general partner is a corporation] the possibility of discharging a recourse loan though bankruptcy would have the same economic effect. Previous research [G Kau and Kim, 1994, Cornell et. al, 1996, Genesove and Mayer, 1997 and Cauley and Pavlov, 2002] has recognizes that the option to default may delay a transaction.

It should be remembered that declines in the volume of transactions are not equivalent to declines in asset liquidity. This distinction is important because declines in liquidity imply decreases in economic efficiency where as declines in volume do not.

Their sample included approximately 1300 owner occupied and 1060 investor owned units.

A difficulty associated with estimating the effect of the put option on the insensitive to delay the sale of a home is the unknown value of the housing services received by the homeowner.

An asymmetric response to demand shocks can be thought of as an example of the disposition effect, that is the tendency to keep losers and sell winners [Shefrin and Statman, 1985].

Agents can be classified as information or liquidity motivated. Information motivated agents think they have an information advantage regarding a property’s value, whereas, liquidity motivated agents do not. Their motivation is to change the composition of their portfolio because of factors such as tax considerations, change in risk tolerance and/or to the need to rebalance the owner’s portfolio.

Implicitly continuous time, as reflected in the asset price and cash flow stochastic processes, is broken into a sequence of non-overlapping fixed length periods [e.g., month].

The assumption that $K=N$ guarantees that if an owner does not sell their property during a period, he or she will be matched with a buyer during subsequent periods [similar results can be obtained if $K>N$].

Liquidity costs are the sum of the explicit and the dollar value of the implicit costs of not transacting, for example, the cost of having a portfolio that is out of balance.

None of our conclusions would be changed qualitatively if debt were amortizing.

Increases [decreases] in the value of the put option decrease [increase] the market value of a non-recourse mortgage.

Commercial mortgages are almost never assumable at the contracted rate of interest.

This is true for both full and non-recourse loans. For a variable rate mortgage the market value is independent of changes in interest rates.

While a large fraction of loans on commercial real estate are formally non-recourse we use the term to represent any legal arrangement where a property owner’s maximum potential loss is less than the loan’s principal $F$. For example, when the general partners in a limited partnership is a corporation.

This may not be strictly true because the holding period may be a function of the mortgage type.

For a full recourse loan the seller’s mortgage contract does not include a put option and $\frac{dM}{dV} = 0$.

The present value of the payments associated with the loan are constant but $M$ changes because of changes in the value of the put.

This result provides an explanation for Genesove and Mayer’s [2001] finding that condominium owners who are subject to greater nominal losses set higher asking prices. Their explanation of this phenomenon is loss aversion.

As noted above, $F_b$ will typically equal $M_b$ at the time of purchase. Reasons why $F_b \neq M_b$ include seller financing at a below market interest rate.

The buyer would be paying for the put option through a higher interest rate at loan origination.

The value of the mortgage is not strictly additive because exercising the put results in extinguishing the loan.

For a full recourse mortgage its value is independent of changes in the properties investment value.

Excepting a full recourse loan, where holding interest rates constant, $|\Delta R| = |\Delta V|$. 
In practice the parties bargaining power, as represented by their demand shocks, may be effected by demand shocks.

The approach used here builds upon techniques developed by Cauley and Pavlov [2002] to analyze housing markets.

The risk free rate of interest is considered because of its relationship to the mortgage rate, and because it is used in risk neutral valuation.

During the late 1980s many Los Angeles County properties had, as levered investments, negative cash flows.

We are assuming a 10-year, interest only balloon payment loan. Analysis of the optimal exercise of the option results in the conclusion that extending the life of the option, say to 30 years, has little effect on the estimates of value.

Loan amortization increases carrying costs and reduces the exercise price of the option [i.e., loan balance] over time. The shorter the time to maturity the larger the fraction of the payment is to principal. As will be seen below, amortization has no qualitative effect on our conclusions.

Given the ownership entities used for commercial real estate default for a commercial mortgage is much less costly than it is for a residential loan.

The LSMC approach is used to find the mortgage rate, FRM or ARM, that equates the mortgage value with the outstanding balance at origination.

Implicitly we are assuming that all mortgage loans are made at a common initial LTV [e.g., 80%]. In practice the initial LTV may be greater than or less than this amount.

These results are consistent with Genesove and Mayer’s [1997] finding that, on average, list price for Boston area condominiums with a LTV of 100 percent were 4 percent greater than those with a LTV of 80 percent LTV and that owner’s received 4 percent more when sold.

By construction at a LTV of 80 percent the borrower is paying for the value of the put in terms of higher interest rates and the market value of the loan equals its face value.

Clearly other parameters of the model have an effect on the estimate and it is conceivable that they may reduce the magnitude of the reported impact. However, given the robustness and economic significance of our findings, we find it unlikely that our results depend on the particular parameters.

It should be remembered that liquidity costs are distinct from debt service. Within the simulation, the liquidity cost represents a percent of investment value $V$ as an expense to each future “node” and affects the entire continuation value.

The relevant LTV ratio is the loan balance relative to the market value of the property.

The value of the put is included in the interest rate paid on the mortgage.

Owners of commercial real estate would be expect to borrow out so that the actual LTV would be close to the maximum LTV.

For simplicity of exposition, LTV’s greater than 100 percent were not considered in this figure. Similar relationships would hold for initial mortgage loan rates greater or less than 6 percent.

Conceptually the statistic should be the prospective standard deviation of the rate of appreciation of the property in question.

The $t_i$ are estimated as follows: if a transaction occurred during January 1989 [i.e., $t=1$], all time indicator variables are assigned a value of zero; if a transaction occurred during the second month, the first time indicator variable is assigned a value of one and all other time indicator variables are set to zero; if a transaction occurred during the third month, the first two indicators are set to one and all others are set to zero.

It should be noted that using data collected at the time of sale may bias the reported cash flow yield upward [i.e., data may have been manipulated to make the property more attractive]. The effect on the variable of interest, the change in cash flow yield, is likely to be small.

The transaction data were screened for outliers and influential observations.

The cash flow yield is calculated from the NOI at the time of sale as reported by CoStar COMPS. Note that data to impute the cash flow yield was available for approximately 86 percent of the transactions.