Abstract

This paper studies a dynamic economy in which a risk-averse strategic investor trades with a continuum of competitive risk-averse uninformed investors. The strategic investor receives a random endowment of a nontradeable with payoffs correlated with the dividend on the traded stock. She trades with the uninformed to share the endowment risk. Due to her concern with price impact, the strategic investor responds sluggishly to shocks and her risky asset holdings exhibit inertia. This inertia effectively modifies the net supply of the asset, changing the amount of risk borne by the uninformed and feeding into the conditional risk premium. Gradual and predictable flows of capital as the strategic trader adjusts her portfolio therefore lead to predictable changes in the risk premium. In the short run, this generates momentum since the strategic investor’s trades are positively autocorrelated. In the long run reversals materialize as the nontradeable endowment of the informed reverts to its unconditional mean and there is less aggregate risk to be borne by the uninformed. The above results are first illustrated in a model with symmetric information, and they are later shown to hold when the strategic trader also has private information about the dividends on the stock. Because she has a larger concern for price impact in this setting, momentum effects are even more extreme in magnitude.

Keywords: return predictability, price drift, momentum, reversals

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1 Introduction

There is now substantial evidence that stock returns are predictable using past returns. There is remarkable consistency that returns tend to exhibit momentum at short horizons Jegadeesh and Titman (1993) but reversals at longer horizons Fama and French (1988); Poterba and Summers (1988). These effects appear across many different asset classes Moskowitz, Ooi, and Pedersen (2012) and persist despite wide knowledge of their existence.¹

In this paper, I show that short-run momentum and long-run reversals arise naturally when some investors are strategic and trade slowly to mitigate price impact. At first this may seem unsurprising. A trader who slows her trading would seem naturally to lead prices to respond sluggishly to shocks. However, the presence of rational, competitive investors means that any predictability in returns that is not due to predictable changes in risk will be immediately arbitraged away. Rather, the intuition for predictability is more subtle. The strategic investor does respond sluggishly to shocks, which means that her risky asset holdings exhibit inertia relative to a competitive setting, underweighting new information and overweighting her current portfolio. The inertia is isomorphic to a change in the net supply of the asset, which changes the amount of risk borne by the uninformed investor and therefore the conditional risk premium. Because changes in informed holdings exhibit positive autocorrelation, they impart positive autocorrelation to returns in the short run. In the long run, the properties of the nontradeable endowment dominate, and since it is assumed to be mean reverting, tends to produce negative autocorrelation.

In order to illustrate more precisely the intuition for why the strategic trader’s holdings enter the price directly, it is useful to give a brief description of the key ingredients in the model. There is a risky asset in zero net supply. This asset pays a stream of dividends $D_t$ over an infinite horizon. For now, suppose the dividend follows a random walk. There are two types of investors, both with CARA utility over their respective consumption streams. The first investor is a strategic and accounts for her price impact when trading the risky asset. This trader is endowed with $Z_t$ units of a nontradeable asset whose dividend is perfectly correlated with that on the risky asset. One can think of this investor as large institutional

¹Moskowitz, Ooi, and Pedersen (2012) study time-series predictability, in which an asset’s own past returns forecast its future returns. This is, in principle, distinct from the cross-sectional momentum of Jegadeesh and Titman (1993) in which stocks that have done relatively well in the past continue to do relatively well in the near future. However Moskowitz, Ooi, and Pedersen (2012) argue that cross-sectional momentum can be largely explained by the time-series autocorrelation of returns.
trader. The other investor is competitive and has no outside endowment. Despite the fact that the risky asset is in zero net supply, it is useful for sharing the risk of the nontradeable asset, so both investors will utilize it. As will be shown later, the optimal portfolio for the informed investor will take the form

$$\theta_{t+1} = \beta_2 D_t + \beta_4 Z_t - \gamma P_t + \frac{1}{\gamma} \theta_t.$$ 

Hence, regardless of the realizations of the divided, endowment shock, and asset price, at each time $t+1$ trader optimally chooses to hold on to fraction $\frac{1}{\gamma} \frac{1}{\gamma+1}$ of her previous portfolio $\theta_t$. This effectively reduces the free float of the asset, which reduces the amount of risk that must be borne by the uninformed trader and therefore reduces the conditional risk premium. Moreover, because changes in $\theta_t$ are autocorrelated (and are correlated with $Z_t$) it leads to predictable changes in risk premia.

As suggested by the above discussion, the size and importance of momentum effects depends on the trader’s concern about price impact. The symmetric information setting is therefore troublesome for at least two reasons. First, momentum effects may be attenuated relative to what we would expect in an arguably more realistic setting with asymmetric information in which traders are particularly concerned with price impact due to information leakage. Secondly, positing that there are two otherwise identical investors, one of whom accounts for price impact and one of whom does not may seem contrived. Hence, I consider a full version of the model in which the strategic trader also observes a private signal about next period’s dividend, which provides a additional motivation for strategic behavior on her part. This causes her to further slow her trading speed and increases quantitatively the size and duration of momentum, which has the potential to better match stylized facts quantitatively. The model with asymmetric information is also of independent theoretical interest since it is, to my knowledge, the first model of dynamic trading in which investors behave strategically and are risk-averse, prices are set by market-clearing, and asymmetric information is preserved in equilibrium. Therefore, the model can be seen as an extension of Wang (1994) to incorporate strategic behavior on the part of the informed investor, or a two-investor version of Vayanos (1999) in which the equilibrium price does not perfectly reveal all private information.

The model makes novel predictions relative to existing theories of return predictability. The results imply that short-run momentum should be larger in magnitude and longer lasting
in settings in which price impact is of greater concern to traders. For instance, settings in
which large traders control a large amount of capital, in which informational heterogeneity
is large, or in which there is less residual uncertainty. The model also does not require one
to consider the risky asset as equity. This is especially appealing in light of recent work
documenting momentum across many diverse asset classes Moskowitz, Ooi, and Pedersen
(2012), which may be hard to reconcile with some existing models that are specific to equity
in individual firms.

While this work provides one potential explanation for observed return predictability. Its
predictions are not mutually exclusive with those of other theoretical models. Such works can
be broadly classified in into risk-based (‘rational’) explanations, and explanations based on
various behavioral biases. Among the first type are Sagi and Seasholes (2007) who show that
realistic return autocorrelation patterns emerge from firms’ optimal decisions in a model of
investment under uncertainty, Vayanos and Woolley (2013) who demonstrate momentum and
reversal can result from flows between active and passive funds, and Albuquerque and Miao
(2014) who appeal to advance information as a source of momentum. On the behavioral
side, Daniel, Hirshleifer, and Subrahmanyam (1998) study a model of investor over- and
underreaction, and Hong and Stein (1999) consider a setting in which some investors neglect
the information in prices and information diffuses slowly across the population.

The paper proceeds as follows. In Section 2 I set up the full version of the model in which
the strategic trader has private information on both her endowment and the dividend on the
risky asset. Section 3 solves the special case in which there is no private information about
the dividend and therefore, in equilibrium, the investors have symmetric information. The
solution is much simpler in this case and the intuition comes through more clearly. Section
4 considers the version with asymmetric information, and Section 5 concludes. Appendix A
presents a general solution of the dynamic programming problem faced by the agents in the
model and is used to simplify the solution of the various cases, and most proofs are relegated
to Appendix B.

2 Model

2.1 Setup

Trade takes place at a set of discrete dates \( \{th\}_{t \in \mathbb{Z}} \), where \( h > 0 \) is a constant.
There are two assets in the economy, a risk-free asset with gross per-period return $R_h = e^{\rho_h}$ and a risky asset with per-period dividend $hD_t$, where the rate $D_t$ follows

$$D_{t+1} = (1 + \phi_D h) D_t + \phi_D G h G_t + \sigma_D \sqrt{h} \varepsilon_{D_{t+1}}.$$ 

The dividend has a persistent component $G_t$ that follows an AR(1) process

$$G_t = (1 + \phi_G h G_{t-1}) + \sigma_G \sqrt{h} \varepsilon_{G_t}.$$ 

The risky asset is in zero net supply. As the focus of the paper is on the dynamics of conditional expected returns but not their unconditional level, this is without loss of generality.

There are two types of agents in the economy. There is a single informed trader $I$. This investor has discounted CARA utility over her consumption

$$-\sum_{s=0}^{\infty} h \exp\{-\rho hs - \alpha_I c_{Is}\}.$$ 

The informed agent is endowed with $Z_t$ units of a nontradeable asset whose payoff is perfectly correlated with the dividend, $D_{t+1}$. The endowment follows the process

$$Z_{t+1} = 1 + \phi_Z h Z_{t-1} + \sigma_Z \sqrt{h} \varepsilon_{Z_t}.$$ 

To ensure existence of a stationary equilibrium, it is a maintained assumption that all of $a_D, a_G,$ and $a_Z$ are strictly less than the gross risk free rate $R_h$. Also, for tractability, and similar to Vayanos (1999), I assume that the informed trader also receives an endowment of cash

$$-Z_t (ha_D D_t + ha_D G_t)$$

which is equal to the negative of the expected dividend next period.

The informed agent observes $G_t$ and $Z_t$ directly, and she is allowed to condition on realized dividends and prices. Formally, an informed trader’s filtration is $\mathcal{F}_t^I = \sigma(\{G_t, Z_t, D_t, P_t\})$.

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<sup>2</sup>It is straightforward to extend the model to incorporate multiple informed traders, as long as they all observe the same private information. Without this assumption, the infinite regress problem renders the problem intractable (Makarov and Rytchkov, 2012; Singleton, 1986). The qualitative results with multiple informed traders results are similar but the analysis is more complicated, so I do not present them here.
There is a competitive uninformed trader with CARA utility over consumption

\[ -\sum_{s=0}^{\infty} h \exp\{-\rho hs - \alpha_U c_{us}\}. \]

This trader can be considered as a representative agent for an underlying unit mass of infinitesimal uninformed investors, all with the same utility function.

The uninformed trader does not observe the dividend growth rate \( G_t \) nor the informed trader’s private endowment \( Z_t \). I entertain the possibility that he can observe a noisy public signal about \( G_t \)

\[ S_{U,t} - S_{U,t-1} = hG_t + \sigma_U \sqrt{h} \varepsilon_{U,t}. \]

This nests the special case of no signal, \( \sigma_U \rightarrow \infty \), or perfect observation of \( G_t \), \( \sigma_U = 0 \). The uninformed trader can also condition on realized dividends and the asset price. Formally, the uninformed trader’s filtration is \( \mathcal{F}_{U,t} = \sigma(\{S_{U,t}, D_t, P_t\}) \).

### 2.2 Equilibrium definition

Following Kyle (1989) and Vayanos (1999) I search for Nash equilibria in linear demand schedules with constant coefficients.\(^3\) Each period, traders submit demand schedules to a Walrasian auctioneer, who aggregates the schedules and announces a market-clearing price at which trade takes place. Immediately before the risky-asset market opens at time \( t \), the informed trader observes the realization of the dividend growth rate, \( G_t \), as well as her endowment of the nontradeable asset \( Z_t \). All traders also collect (and thereby observe) the time-\( t \) dividend \( hD_t \) on their risky asset holdings from the previous round of trading.

Let \( \theta_{jt} \) denote the number of shares of the risky asset and \( M_{jt} \) the amount of cash (holdings of the risk-free asset) held by agent \( j \) entering the time-\( t \) trading round. Let \( \hat{G}_t \equiv \mathbb{E}[G_t | \mathcal{F}_{U,t}] \) and \( \hat{Z}_t \equiv \mathbb{E}[Z_t | \mathcal{F}_{U,t}] \) denote the uninformed agent’s conditional expectations.

I conjecture that both types \( I \) and \( U \) follow linear strategies that specify their time-\( t \) trades

\(^3\)There may exist other equilibria in which agents follow trigger (punishment) strategies. However, the existence and characteristics of such equilibria are, to my knowledge, still an unexplored question and are outside the scope of this paper.
The dependence on lagged dividends, public signals, and expectations is unconventional. As will become clear in the derivation below, this dependence follows from the strategic behavior of the informed trader. At each time-$t$, she understands that her trade will result in the uninformed trader updating his conditional expectations. In order to compute the new values of those expectations she must keep track of both their most recent values as well as the most recent values of the other random quantities observed by the uninformed.

**Definition 1** (Equilibrium). An equilibrium in the economy is a collections of demand functions $\{x_{jt}\}_{t=0}^{\infty}$ and consumption rates $\{c_{jt}\}_{t=0}^{\infty}$, measurable with respect to $\mathcal{F}_{jt}$, and a sequence of prices $\{P_t\}_{t=0}^{\infty}$ such that demands and consumptions solve the traders’ optimization problems

$$\max_{\{x_{jt},c_{jt}\}} E_{j0} \left[ -\sum_{s=0}^{\infty} h \exp \{-\rho hs - \alpha_j c_{us}\} \right] j \in \{I,U\},$$

s.t. $M_{jt+1} = R_h(M_{jt} - c_{jt} h - x_{jt} P_t(x_{jt})) + (\theta_{jt} + x_{jt}) h D_{t+1} + 1_{\{j=I\}} Z_t(h D_{t+1})$

$$\theta_{jt+1} = \theta_{jt} + x_{jt},$$

and the risky asset market clears at each time $t$

$$x_{It} + x_{Ut} = 0, \ \forall t.$$

## 3 Symmetric Information

As a benchmark, it is useful to characterize the equilibrium when both traders observe $G_t$ and $Z_t$. For consistency, I will continue to refer to trader $I$, who is strategic and receives the random endowment $Z_t$, as informed, despite the fact that she has no information advantage in this setting.

I conjecture and verify that traders follow linear strategies of the form

$$x_{jt} = \beta_j' X_{jt} - \gamma_j P_t$$ (3)
where
\[ X_{jt} = (1 \theta_{jt} D_t G_t Z_t)' , \]
and \( \beta_j \) is a \( 5 \times 1 \) vector. The equilibrium price will follow from imposing the market clearing condition.

I solve the traders’ problems by dynamic programming. The candidate value functions take the form
\[ V_j(M_{jt}, X_{jt}) = -\exp\left\{ -\alpha Q_{j0} M_{jt} - \frac{1}{2} \alpha X'_{jt} Q_j X_{jt} \right\} , \tag{4} \]
where \( Q_{j0} \) is a positive constant, and \( Q_j \) is a symmetric matrix with generic element \( Q_{jik}, i, k \in \{0, \ldots, n\} \). The following transversality condition guarantees that a function satisfying the Bellman equation characterizes an optimum
\[ \lim_{s \to \infty} E_0 \left[ \exp\{ -\rho hs \} V_j(M_{js}, X_{js}) \right] . \]

### 3.1 Informed trader’s problem

The informed trader behaves strategically. Following Kyle (1989) I solve a less-restrictive problem in which she is allowed to optimize against the residual demand schedule of the uninformed trader, and then I show that this strategy can be implemented by submitting a demand schedule.

**Proposition 3.1.** Suppose that both traders observe \( G_t \) and \( Z_t \) directly. The informed investor’s optimal demand can be implemented by submitting a demand schedule with coefficients
\[ \beta'_I = \left[ \alpha_I B'_{Ix} Q_I B_{Ix} + Q_{I0} R_h p_{Ix} - a'_{Ix} Q_I a_{Ix} \right]^{-1} \left[ Q_{I0} C'_{Ix} a_I + a'_I Q_I a_I - \alpha_I B'_{Ix} Q_I B_{Ix} \right] \tag{5} \]
\[ \gamma_I = \left[ \alpha_I B'_{Ix} Q_I B_{Ix} + Q_{I0} R_h p_{Ix} - a'_{Ix} Q_I a_{Ix} \right]^{-1} Q_{I0} R_h \tag{6} \]
where
\[ Q_{I0} = \frac{1}{h} \frac{R_h - 1}{R_h} , \]
and \( Q_I \) solves system
\[ Q_I = \frac{1}{R_h} \left[ A_I - \left( (Q_{I0}(a'_I C_{Ix} - R_h p_I) + a'_I Q_I a_{Ix})' - \alpha_I B'_{Ix} Q_I B_{Ix} \right)' C^{-1} \right] \times \left( (Q_{I0}(a'_I C_{Ix} - R_h p_I) + a'_I Q_I a_{Ix})' - \alpha_I B'_{Ix} Q_I B_{Ix} \right) \tag{7} \]
\[ + \frac{2}{\alpha_I} \epsilon_1 e'_1 \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_I) \right) . \]
The matrices \( A_U, B_U, \text{ and } C_U \) are defined in the Appendix.

I sketch the proof here, which is established rigorously in the Appendix. To determine the residual demand schedule at time \( t \), substitute the conjectured uninformed demand function (3) into the market clearing condition and rearrange to yield

\[
P_t(x_{It}) = \frac{1}{\gamma_U} (x_{It} + \beta_U' X_{Ut})
\]

\[
= \frac{1}{\gamma_U} [-\beta_U \theta_{It} + \beta_U D_t + \beta_U G_t + \beta_U Z_t + x_{It}]
\]

\[
= p'_t X_{It} + p_{tx} x_{It},
\]

where the second equality follows from the market clearing condition, which implies \( \theta_{It} = -\theta_{UIt} \), and where \( p_I = \frac{1}{\gamma_U} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \beta_U \) and \( p_{tx} = 1/\gamma_U \).

The informed trader’s cash holdings therefore follow

\[
M_{It+1} = R_h (M_{It} - c h - x(p'_t X_{It} + p_{tx} x)) + (\theta_{It} + x + Z_t) h D_{t+1},
\]

and the non-cash state variables follow

\[
X_{It+1} = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array} \right) X_{It} + \left( \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \sigma_V \sqrt{\gamma} & 0 \\ 0 & \sigma_V \sqrt{\gamma} \end{array} \right) \varepsilon_{t+1} + \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) x.
\]

Setting up the Bellman equation and plugging into the conjectured value function in eq. (4) allows one to solve for the optimal demand and consumption

\[
x^* = [\alpha_t B'_t \tilde{Q}_t B_{tx} + 2Q_{t0} R_h p_{tx} - d'_{tx} Q_t a_{tx}]^{-1} \left[ Q_{t0} C'_t a_t + a'_{tx} Q_t a_{tx} \right] X_{It}
\]

\[
- [\alpha_t B'_t \tilde{Q}_t B_{tx} + 2Q_{t0} R_h p_{tx} - d'_{tx} Q_t a_{tx}]^{-1} Q_{t0} R_h p'_t X_{It}
\]

\[
c^* = \frac{1}{\alpha_1 R_h} \left[ \rho h - \log \left( \frac{R_h - 1}{h} \right) + \frac{1}{2} \log d_t + \alpha_t \frac{R_h - 1}{h} M_{It} + \frac{1}{2} \alpha_t X_{It} B'_t C^{-1}_t B_t X_{It} \right].
\]

Plugging this back into the Bellman equation and matching terms yields the system in eq. (7).

While the optimal demand above was derived by allowing the trader to optimize against the residual demand schedule of the uninformed traders, it can be implemented by submitting a demand schedule to the Walrasian auctioneer. In equilibrium, one has \( P_t(x^*) = p'_t X_{It} + p_{tx} x^* \). Plugging in and rearranging to isolate \( x^* \) yields

\[
x^* = [\alpha_t B'_t \tilde{Q}_t B_{tx} + Q_{t0} R_h p_{tx} - a'_{tx} Q_t a_{tx}]^{-1}
\]

\[
\times \left( [Q_{t0} C'_t a_t + a'_{tx} Q_t a_{tx} - \alpha_t B'_t \tilde{Q}_t B_{tx}] X_{It} - Q_{t0} R_h P_t(x^*) \right)
\]

Matching with the initial conjecture yields the system equations for the demand coefficients.
3.2 Uninformed investor’s problem

The uninformed trader behaves competitively. Hence, she takes the equilibrium price as given when forming her optimal demand. Because of this, her residual demand schedule is simply equal to the equilibrium price, and one can solve her problem using standard methods with no additional effort to express her optimal demand as a schedule.

**Proposition 3.2.** Suppose that both traders observe $G_t$ and $Z_t$ directly. The uninformed trader’s optimal demand function has coefficients that satisfy

\[
\beta'_U = \left[ \alpha_U B_{Ux}' \tilde{Q}_U B_{Ux} - a'_{Ux} Q_U a_{Ux} \right]^{-1} \left[ Q_U 0 C_{Ux}' a_U + a'_{Ux} Q_U a_U - \alpha_U B_{Ux}' \tilde{Q}_U B_{Ux} \right] \tag{8}
\]

\[
\gamma_U = \left[ \alpha_U B_{Ux}' \tilde{Q}_U B_{Ux} - a'_{Ux} Q_U a_{Ux} \right]^{-1} Q_U 0 R_h. \tag{9}
\]

where

\[
Q_U 0 = \frac{1}{R_h} \frac{R_h - 1}{R_h},
\]

and $Q_U$ solves the system

\[
Q_U = \frac{1}{R_h} \left[ A_U - ((Q_U 0 (a'_U C_{Ux} - R_h P_U)) + a'_U Q_U a_{Ux})' - \alpha_U B_{Ux}' \tilde{Q}_U B_{Ux} \right] C_U^{-1} \tag{10}
\]

\[
+ \frac{2}{\alpha_U} e_1 e_1' \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{R_h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_t) \right).
\]

The matrices $A_U, B_U, C_U$ are defined in the Appendix.

To determine the equilibrium price at time $t$, substitute the conjectured demand functions (3) into the market clearing condition and rearrange to yield

\[
P_t = \frac{1}{\gamma_I + \gamma_U} (\beta'_I X_I + \beta'_U X_U)
\]

\[
= \frac{1}{\gamma_I + \gamma_U} [((\delta_I - \delta_U) \theta U_t + (\beta_I + \beta_U) G_t + (\beta_I + \beta U) Z_t + (\beta I + \beta U) D_t]
\]

\[
= p_U' X_U
\]

where $p_U = \frac{1}{\gamma_I + \gamma_U} \left( \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array} \right) \beta_I + \beta U$.

The uninformed trader’s cash holdings follow

\[
M_{Ut+1} = R_h (M_{Ut} - c h - x p_U' X_U) + (\theta U_t + x U_t) h D_{t+1}.
\]
and the non-cash state variables follow

\[
X_{U_{t+1}} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}_U \begin{pmatrix} X_{U_t} \\ \sigma_D \sqrt{k} \\ \sigma_G \sqrt{k} \\ \sigma_Z \sqrt{k} \\ \alpha_C \end{pmatrix}_U \varepsilon_{U_{t+1}} + \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1
\end{pmatrix}_U x.
\]

The optimal demand and consumption are

\[
x^* = -C_U B_U X_{U_t}
\]

\[
= \left[ \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} - a'_{U,x} Q_U a_{U,x} \right]^{-1} \left[ Q_{U0}(C'_{U,x} a_U - R_h p_U') + a'_{U,x} Q_U a_U - \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} \right] X_{U_t}
\]

\[
c^* = \frac{1}{\alpha_U R_h} \left[ \rho h - \log \left( \frac{R_h - 1}{h} \right) + \frac{1}{2} \log d_U + \alpha_U \frac{R_h - 1}{h} M_{U_t} + \frac{1}{2} \alpha_U X'_{U_t} B_U C_U^{-1} B_U X_{U_t} \right].
\]

Plugging back into the Bellman equation yields \( Q_{U0} \) in closed form

\[
Q_{U0} = \frac{1}{h} \frac{R_h - 1}{R_h},
\]

and a system of equations for the matrix \( Q_U \)

\[
Q_U = \frac{1}{R_h} (A_U - B'_U C_U^{-1} B_U)
\]

\[
+ \frac{2}{\alpha_U} e_1 e'_1 \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_U) \right)
\]

\[
= \frac{1}{R_h} \left( A_U - \left( \left( Q_{U0}(a'_U C_{U,x} - R_h p_U') + a'_U Q_U a_{U,x}' \right) - \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} \right)' C_U^{-1}
\]

\[
\left( \left( Q_{U0}(a'_U C_{U,x} - R_h p_U') + a'_U Q_U a_{U,x}' \right) - \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} \right) \right)'
\]

\[
+ \frac{2}{\alpha_U} e_1 e'_1 \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_U) \right)
\]

In equilibrium \( P_t = p'_U X_{U_t} \), so it is immediate that the optimal demand takes the form of a demand schedule

\[
x^* = \left[ \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} - a'_{U,x} Q_U a_{U,x} \right]^{-1} \left[ Q_{U0}(C'_{U,x} a_U + a'_{U,x} Q_U a_U - \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} x) \right] X_{U_t}
\]

\[
- \left[ \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} - a'_{U,x} Q_U a_{U,x} \right]^{-1} Q_{U0} R_h P_t(x^*_{U_t}).
\]

To match the initial conjectured demand, one requires

\[
\beta'_U = \left[ \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} - a'_{U,x} Q_U a_{U,x} \right]^{-1} \left[ Q_{U0}(C'_{U,x} a_U + a'_{U,x} Q_U a_U - \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} x) \right] \tag{12}
\]

\[
\gamma_U = \left[ \alpha_U B'_{U,x} \tilde{Q}_U B_{U,x} - a'_{U,x} Q_U a_{U,x} \right]^{-1} Q_{U0} R_h. \tag{13}
\]
3.3 Equilibrium

Equilibrium is characterized by a solution to the informed trader demand equations (5) and (6), informed Bellman conditions (7), the uninformed demand equations (12) and (13), and the uninformed Bellman conditions (11).

Due to the assumption of symmetric information, many of the equilibrium coefficients can be solved in closed form. The following proposition characterizes the equilibrium.

Proposition 3.3 (Symmetric information equilibrium). Supposing that both traders observe $G_t$ and $Z_t$ directly the equilibrium demand function coefficients are

\[
\beta = \gamma_I \left( -Q_{II1} + \alpha_I \frac{Q^2 R^2 h^2}{(R_h - a_D)^2} \left( \frac{a^2_{DG} b^2_G}{(R_h - a_G)^2} + b^2_D \right) + \frac{Q^2_{II4} b^2_G}{1 + \alpha_I Q_{144} b^2_Z} \right) \]

\[
\beta = \gamma_I \left( -Q_{U11} + \alpha_U \frac{Q^2 R^2 h^2}{(R_h - a_D)^2} \left( \frac{a^2_{DG} b^2_G}{(R_h - a_G)^2} + b^2_D \right) + \frac{Q^2_{II4} b^2_G}{1 + \alpha_U Q_{144} b^2_Z} \right) \]

where the Bellman coefficients $Q_{jik}$ are those that solve (7) and (11).

The following Corollary applies the market clearing condition and characterizes the equilibrium price.

Corollary 3.4. Supposing that both traders observe $G_t$ and $Z_t$ directly, the equilibrium price of the risky asset is

\[
P_t = \frac{a_D}{R_h - a_D} hD_t + \frac{a_{DG} R_h}{(R_h - a_D)(R_h - a_G)} hG_t + A_Z Z_t + A_\theta \theta_{1t},
\]

where

\[
A_Z = \beta_{II4} + \beta_{14} \frac{\gamma_I + \gamma_U}{\gamma_I + \gamma_U} < 0
\]

\[
A_\theta = \frac{1}{\gamma_U \gamma_I + \gamma_U} > 0.
\]
With symmetric information, the equilibrium price is equal to the present value of expected dividends, minus an additive discount to compensate the uninformed traders for bearing a portion of the risk associated with the endowment \( \dot{Z}_t \). These features are also true in a competitive environment (Wang, 1994). Strategic behavior also leads to a novel dependence on the current holdings of the informed trader, which is key for generating short-run momentum.\(^4\)

To understand why this term arises, it is useful to analyze the demand functions of the two types of traders. Define \( \hat{\gamma}_I \) as the informed trader’s price coefficient in the absence of price impact, \( \frac{1}{\gamma_I} = \frac{1}{\gamma_I} - \frac{1}{\gamma_U} \), and use the fact that \( \beta_{\gamma_I} = -1 + \frac{\gamma_I}{\gamma_U} = -\frac{\gamma_I}{\gamma_I} - \frac{1}{\gamma_U} \) to write the demand of the informed trader as

\[
x_{It} = \beta_{I2}D_t + \beta_{I3}G_t + \beta_{I4}Z_t - \gamma_I P_t - \frac{1}{\gamma_I} + \frac{1}{\gamma_U} \theta_{It}
\]

\[
= \frac{1}{\gamma_I} + \frac{1}{\gamma_U} \left( \frac{1}{\gamma_I} \beta_{I2}D_t + \frac{1}{\gamma_I} \beta_{I3}G_t + \frac{1}{\gamma_I} \beta_{I4}Z_t - P_t - \frac{1}{\gamma_I} \theta_{It} \right)
\]

\[
= \frac{1}{\gamma_I} + \frac{1}{\gamma_U} \left( \frac{\gamma_I}{\gamma_I} \beta_{I2}D_t + \frac{\gamma_I}{\gamma_I} \beta_{I3}G_t + \frac{\gamma_I}{\gamma_I} \beta_{I4}Z_t - \gamma_I P_t - \theta_{It} \right).
\]

Compare with the demand function for the uninformed trader

\[
x_{Ut} = \beta_{U2}D_t + \beta_{U3}G_t + \beta_{U4}Z_t - \gamma_U P_t - \theta_{Ut}.
\]

Both agents trade to close the gap between their current portfolio \( \theta_{jt} \) and a target portfolio that would be optimal in a competitive setting. Because the uninformed trader is competitive, she does so immediately, leading to a post-trade portfolio

\[
\theta_{Ut+1} = \beta_{U2}D_t + \beta_{U3}G_t + \beta_{U4}Z_t - \gamma_U P_t.
\]

Due to her concern with price impact, the informed trader closes the gap slowly. Her new portfolio is a weighted average of the target portfolio and her previous holdings

\[
\theta_{It+1} = \frac{1}{\gamma_I} + \frac{1}{\gamma_U} \left( \frac{\gamma_I}{\gamma_I} \beta_{I2}D_t + \frac{\gamma_I}{\gamma_I} \beta_{I3}G_t + \frac{\gamma_I}{\gamma_I} \beta_{I4}Z_t - \gamma_I P_t \right) + \frac{1}{\gamma_I} + \frac{1}{\gamma_U} \theta_{It}.
\]

Regardless of the realizations of the state variables governing the dividend and regardless of the asset price, the trader optimally chooses to hold \( \frac{1}{\gamma_I} + \frac{1}{\gamma_U} \theta_{It} \) units. This effectively changes

\(^4\)Equivalently, since \( \theta_{It} + \theta_{Ut} = 0 \) in equilibrium, the price could be written as a function of the uninformed trader’s holdings.
the net supply of the asset, which in turn, reduces the amount of risk that must be borne by the uninformed trader. This translates into an additional discount (premium) in the asset price when the informed holds a short (long) position. Moreover, because \( \theta_{It} \) is both autocorrelated and correlated with \( Z_t \), it leads to predictable dynamics in returns.

Let

\[
R^e_{t+1} = P_{t+1} + D_{t+1} - R_h P_t
\]

denote the excess (dollar) return on the risky asset in period \( t + 1 \). From Wang (1993) and Albuquerque and Miao (2014) it is known that in a competitive setting, returns exhibit negative autocorrelation at all horizons when \( a_Z < 1/R_h \), positive autocorrelation when \( a_Z > 1/R_h \), and no correlation when \( a_Z = 1/R_h \). In a competitive setting, \( Z_t \) represents the amount of aggregate risk that must be borne in the economy and any return predictability comes from this predictability in risk.

With slow trading by the informed investor, the dynamics of aggregate risk borne by the uninformed are no longer purely exogenous and are affected by the dynamics of the informed inventory. The following Proposition characterizes the conditional expectation of \( R^e_{t+k} \) given \( R^e_t \) when \( a_Z = 1/R_h \). It is possible to characterize the conditional expectation in the general case, which I do in the proof, but the expression is sufficiently complicated that presenting the expression here provides little insight. I will return to the general case in numerical results.

**Proposition 3.5.** Suppose that \( a_Z = 1/R_h \). The conditional expectation of the period \( t + k \) excess return given the period \( t \) excess return is

\[
\mathbb{E}[R^e_{t+k} | R^e_t] = \frac{A_{\theta} A_Z \text{Cov}(\theta_{It+k} - R\theta_{It+k-1}, Z_t - R_h Z_{t-1}) + A^2_{\theta} a_{\theta}^k \text{Cov}(\theta_{It+k} - R_h \theta_{It+k-1}, \theta_{It} - R_h \theta_{It-1})}{\text{Var}(R^e_t)} R^e_t
\]

\[
= a_{\theta}^{k-1} \frac{A_Z \beta I_{\frac{1}{U}} - \beta U I_{\frac{1}{U}} \text{Var}(Z_t - R_h Z_{t-1}) + A^2_{\theta} \text{Var}(\theta_{It} - R_h \theta_{It-1})}{\text{Var}(R^e_t)} R^e_t,
\]

where \( a_{\theta} = \frac{1}{\gamma_U + \gamma_I} \) is the persistence of the informed investor’s position in the risky asset. The coefficient on \( R^e_t \) is strictly positive for all \( k \) and decays geometrically towards zero.

The Proposition clarifies the mechanism by which strategic behavior leads to momentum. There are two effects, one direct and the other indirect. The direct effect comes from the fact that, innovations in the informed trader’s holdings are themselves autocorrelated. Suppose
that the informed starts with a net short position. If the informed trader reduces her holdings today, the uninformed bear less risk (take a smaller long position) and therefore the price increases. Because of the natural persistence in informed holdings, this is also associated with reduced holdings in the near future, which are associated with further price increases. The indirect effect comes from the fact that shocks to the endowment of the nontradable forecast additional buildup/drawdown of her holdings in the future. Supposing that $Z_t$ increases, the informed immediately sell an additional amount of the risky asset to hedge their exposure. This pushes the price down. Because of slow trading, she is also expected to continue reducing her holdings in the near future, thereby requiring the uninformed to bear more risk and further depressing the price.

As indicated by the Proposition, strategic trading alone leads only to momentum. It is natural to expect that momentum effects will be stronger when the informed faces greater price impact and therefore trades more slowly. Figure 3.3 plots return autocorrelations at various horizons $k$ for a baseline calibration as well as for various deviations from the baseline. The reported baseline parameters are based loosely on Campbell and Kyle (1993), who calibrated them using annual data. When producing the plots, I have converted all parameters to a monthly frequency, to better match the horizons over which momentum studies are conducted. In Panel (b), a doubling of the residual standard deviation $\sigma_D$ leads to momentum effects that are both larger in magnitude and take longer to converge. In Panel (c), I fix the aggregate risk tolerance (‘capital’) equal to that in the baseline, but give the informed trader only 10% of total capital. As her concern with price impact is now smaller, momentum is attenuated. Finally Panel (d) shows that a riskier endowment, for which the uninformed require a larger risk premium, increases price impact and leads to stronger momentum effects.

In general, when $a_Z < 1/R$, momentum no longer appears at all horizons. Rather, the (relatively) strong mean reversion in the endowment $Z_t$ dominates at longer horizons. Figure ??

4 Asymmetric information

This section characterizes the equilibrium in the full version of the model with asymmetric information about both dividend growth and the endowment of the informed trader. Be-
Figure 1: Autocovariance function of one-period returns. Panel (a): Baseline calibration. Parameters: $h = 1, \alpha_I = 9, \alpha_U = 3, R_h = 1.05, a_D = 1, a_{DG} = 1, a_G = 1/2, a_Z = 1/1.05, \sigma_D = 1, \sigma_G = 2,$ and $\sigma_Z = 1/2.$ Panel (b): Residual variance $\sigma_D = 2,$ Panel (c): Informed have 10% of agg. risk tol, $\alpha_I = 45/2, \alpha_U = 5/2,$ Panel (d): Riskier nontradeable endowment, $\sigma_Z = 1.$
cause the derivation is sufficiently novel, I walk through how to set up the state equations and discuss the uninformed investor’s inference problem in the main text rather than the Appendix.

At a high level, the derivation is analogous to that in the case of symmetric information. One must solve the traders’ consumption/portfolio problems using dynamic programming. Having characterized the optimal demand functions, the equilibrium is determined by matching the coefficients with those conjectured in eq. (1) and (2). The equilibrium price follows by imposing the market clearing condition. The derivation is similar to that in Vayanos (1999), complicated by the fact that the uninformed investor is not able to perfectly infer \( G_t \) from the equilibrium price, or Wang (1994), complicated by the fact that the informed trader behaves strategically.

### 4.1 Uninformed investor’s problem

Before solving the informed trader’s problem, I determine a random variable that is informationally equivalent to the ‘incremental’ information conveyed by the time-\( t \) price and solve his learning problem using the Kalman filter.

Given the conjectured demands in eq. (1) and (2), the equilibrium price is

\[
P_t = \frac{1}{\gamma_I + \gamma_U} \left[ \beta_{I1}\theta_{It} + \beta_{U1}\theta_{Ut} + (\beta_{I2} + \beta_{U2})D_t + \beta_{I3}G_t + \beta_{I4}Z_t + (\beta_{I5} + \beta_{U5})S_{Ut} \\
+ (\beta_{I6} + \beta_{U6})D_{t-1} + (\beta_{I7} + \beta_{U7})\hat{G}_{t-1} + (\beta_{I8} + \beta_{U8})\hat{Z}_{t-1} + (\beta_{I9} + \beta_{U9})S_{Ut-1} \right]
\]

\[
= \frac{1}{\gamma_I + \gamma_U} \left[ (\beta_{I1} + \beta_{U1})\theta_{Ut} + (\beta_{I2} + \beta_{U2})D_t + \beta_{I3}G_t + \beta_{I4}Z_t + (\beta_{I5} + \beta_{U5})S_{Ut} \\
+ (\beta_{I6} + \beta_{U6})D_{t-1} + (\beta_{I7} + \beta_{U7})\hat{G}_{t-1} + (\beta_{I8} + \beta_{U8})\hat{Z}_{t-1} + (\beta_{I9} + \beta_{U9})S_{Ut-1} \right]
\]

\[
\equiv \frac{1}{\gamma} \left[ \beta_1\theta_{Ut} + \beta_2D_t + \beta_{I3}G_t + \beta_{I4}Z_t + \beta_5S_{Ut} + \beta_6D_{t-1} + \beta_7\hat{G}_{t-1} + \beta_8\hat{Z}_{t-1} + \beta_9S_{Ut-1} \right] \tag{14}
\]

where the second equality uses the fact that in equilibrium \( \theta_{It} + \theta_{Ut} = 0 \) since the asset is in zero net supply.

It follows from eq. (14) that when used in combination with the current dividend and public signal, as well as past realizations of these variables, the asset price allows the uninformed traders to infer the statistic

\[
S_{Pt} \equiv \gamma P_t - \beta_1\theta_{Ut} - \beta_2D_t - \beta_5S_{Ut} - \beta_6D_{t-1} - \beta_7\hat{G}_{t-1} - \beta_8\hat{Z}_{t-1} - \beta_9S_{Ut-1} \\
= \beta_{I3}G_t + \beta_{I4}Z_t.
\]
Since the uninformed does not observe, $G_t$ or $Z_t$, she must filter them from observation of realized dividends, prices, and public signals, $\{D_t, S_{Pt}, S_{Ut}\}$. The following proposition characterizes the conditional mean and variance matrix of $(G_t, Z_t)$ using results from Liptser and Shiryaev (2001).

**Proposition 4.1.** The conditional distribution of $(G_t, Z_t)$ given $\{D_t, S_{Pt}, S_{Ut}\}$ is normal with mean $(\hat{G}_t, \hat{Z}_t)$ and covariance matrix $\nu = \begin{pmatrix} \nu_G & \nu_{GZ} \\ \nu_{GZ}^T & \nu_Z \end{pmatrix}$.

The conditional mean satisfies the difference equation

$$
\begin{pmatrix}
\hat{G}_t \\
\hat{Z}_t
\end{pmatrix} =
\begin{pmatrix}
a_G & 0 \\
0 & a_Z
\end{pmatrix}
\begin{pmatrix}
\hat{G}_{t-1} \\
\hat{Z}_{t-1}
\end{pmatrix}
+ K
\begin{pmatrix}
D_t - a_D D_{t-1} - a_D G_t \\
S_{Pt} - \beta_{13} a_G \hat{G}_{t-1} - \beta_{14} a_Z \hat{Z}_{t-1}
\end{pmatrix},
$$

where $K$ is a $2 \times 3$ matrix with elements $K_{i,j}$ given by

$$K_{1,1} = \frac{a_D b_D^2 (a_G b_D^2 + a_G b_G^2) \nu_G}{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G} + \left( \left( (a_G - a_Z)^2 b_G^2 + a_D b_G^2 \right)^2 \right) \beta_{13} \nu_G + \left( b_D^2 (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G \right)^2
$$

$$K_{1,2} = \frac{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G}{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G} + \left( \left( (a_G - a_Z)^2 b_G^2 + a_D b_G^2 \right)^2 \right) \beta_{13} \nu_G + \left( b_D^2 (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G \right)^2
$$

$$K_{1,3} = \frac{\nu_G}{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G} + \left( \left( (a_G - a_Z)^2 b_G^2 + a_D b_G^2 \right)^2 \right) \beta_{13} \nu_G + \left( b_D^2 (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G \right)^2
$$

$$K_{2,1} = \frac{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G}{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G} + \left( \left( (a_G - a_Z)^2 b_G^2 + a_D b_G^2 \right)^2 \right) \beta_{13} \nu_G + \left( b_D^2 (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G \right)^2
$$

$$K_{2,2} = \frac{\nu_G}{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G} + \left( \left( (a_G - a_Z)^2 b_G^2 + a_D b_G^2 \right)^2 \right) \beta_{13} \nu_G + \left( b_D^2 (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G \right)^2
$$

$$K_{2,3} = \frac{-b_D^2 (b_G^2 + b_D^2 + b_G^2) \nu_G}{b_D (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G} + \left( \left( (a_G - a_Z)^2 b_G^2 + a_D b_G^2 \right)^2 \right) \beta_{13} \nu_G + \left( b_D^2 (b_G^2 + b_D^2 + b_G^2) \beta_{14} \nu_G \right)^2
$$

The conditional variance of $G_t$ satisfies

$$
\left( \left( (a_G - a_Z)^2 b_D^2 + a_D^2 b_G^2 \right)^2 b_G^2 + b_D^2 \left( a_G^2 b_D^2 b_G^2 \beta_{14} \nu_G \right) + b_D \left( a_G^2 b_D^2 b_G^2 \beta_{14} \nu_G \right)^2 \right) \nu_G
$$

and

$$\nu_{GZ} = \frac{\beta_{13}}{\beta_{14}} \nu_G
$$

$$\nu_Z = \left( \frac{\beta_{13}}{\beta_{14}} \right)^2 \nu_G.$$
Having solved the uninformed trader’s filtering problem, it remains to solve for her optimal portfolio and consumption rules. Augmenting with a constant, the vector of non-cash state variables for the uninformed investor is

$$X_{Ut} = \begin{pmatrix} 1 & \theta_{Ut} & D_t & S_{Pt} & S_{Ut} & D_{t-1} & \hat{G}_{t-1} & \hat{Z}_{t-1} & S_{Ut-1} \end{pmatrix}' .$$

In Appendix ??, I solve the dynamic programming problem for a CARA investor given arbitrary dynamics for the state variables. One can apply the results there once the dynamics of $M_{Ut}$ and $X_{Ut}$ are written in form in eq. (??).

Let $\varepsilon_{Ut+1}^U$ be a $3 \times 1$ vector composed of the forecast errors of the uninformed investor

$$\varepsilon_{Dt+1}^U = D_t - a_D D_{t-1} - a_D \hat{G}_{t-1}$$
$$\varepsilon_{Pt+1}^U = S_{Pt} - \beta_{I3} a_G \hat{G}_{t-1} - \beta_{I4} a_Z \hat{Z}_{t-1}$$
$$\varepsilon_{Ut+1}^U = S_{Ut} - S_{Ut-1} - h a_G \hat{G}_{t-1} .$$

It follows from Proposition 4.1 that under the uninformed investors’ filtration, $\varepsilon_{Ut+1}^U$ is iid over time and is normally distributed with mean zero and covariance matrix

$$\Sigma_U = \begin{pmatrix} a_D^2 \nu_G + b_D^2 & a_D (a_G - a_Z) \beta_{I3} \nu_G & a_D a_G h \nu_G \\ a_D (a_G - a_Z) \beta_{I3} \nu_G & \beta_{I3}^2 (a_G - a_Z)^2 \nu_G + \beta_{I4}^2 b_Z^2 + \beta_{I4}^2 b_G^2 & h \beta_{I3} (a_G (a_G - a_Z) \nu_G + b_G^2) \\ a_D a_G h \nu_G & h \beta_{I3} (a_G (a_G - a_Z) \nu_G + b_G^2) & h^2 (a_G^2 \nu_G + b_G^2) + b_Z^2 \end{pmatrix} .$$

First, consider his cash holdings $M_{Ut}$. The equilibrium price in eq. (14), which depends on the state variables $X_{Ut}$ can be written compactly as $p_U' X_{Ut}$, where $p_U$ is a $9 \times 1$ vector with elements given by the coefficients in the price function (14).

$$M_{Ut+1} = R_h (M_{Ut} - c_{Ut} h - x_{Ut} P_t) + (\theta_{Ut} + x_{Ut}) h D_{t+1}$$
$$= R_h (M_{Ut} - c_{Ut} h - x_{Ut} p_U' X_{Ut}) + (\epsilon_{2} X_{Ut} + x_{Ut}) h \epsilon_{3} X_{Ut+1}$$
$$= R_h (M_{Ut} - c_{Ut} h - x_{Ut} p_U' X_{Ut}) + X_{Ut} \epsilon_{2} \epsilon_{3} h X_{Ut+1} + x_{Ut} h \epsilon_{3} X_{Ut+1} .$$
The non-constant elements of $X_{U_t+1}$ obey the difference equations

$$\theta_{U_t+1} = \theta_{U_t} + x_{U_t}$$

$$D_{t+1} = a_D D_t + a_{DG} \dot{G}_t + \xi_D^{U_{t+1}}$$

$$S_{Pt+1} = \beta_{13} a_G \dot{G}_t + \beta_{14} a_Z \dot{Z}_t + \xi_P^{U_{t+1}}$$

$$S_{Ut+1} = S_{Ut} + h a_G \dot{G}_t + \xi_{U_{t+1}}$$

$$D_t = D_t$$

$$\dot{G}_t = a_G \dot{G}_{t-1} + K_{11}(D_t - a_D D_{t-1} - a_{DG} \dot{G}_{t-1}) + K_{12}(S_{Pt} - \beta_{13} a_G \dot{G}_{t-1} - \beta_{14} a_Z \dot{Z}_{t-1}) + K_{13} (S_{Ut} - S_{Ut-1} - h a_G \dot{G}_{t-1})$$

$$\dot{Z}_t = a_Z \dot{Z}_{t-1} + K_{21}(D_t - a_D D_{t-1} - a_{DG} \dot{G}_{t-1}) + K_{22}(S_{Pt} - \beta_{13} a_G \dot{G}_{t-1} - \beta_{14} a_Z \dot{Z}_{t-1}) + K_{23} (S_{Ut} - S_{Ut-1} - h a_G \dot{G}_{t-1})$$

$$S_{Ut} = S_{Ut}$$

where

$$S_{Pt} = \gamma P_t - \beta_1 \theta_{Ut} - \beta_2 D_t - \beta_3 S_{Ut} - \beta_4 D_{t-1} - \beta_5 \dot{G}_{t-1} - \beta_6 \dot{Z}_{t-1} - \beta_7 S_{Ut-1}.$$
one can write the dynamics of the state vector compactly as

\[ X_{Ut+1} = \hat{a}_U X_{Ut} + b_U x^U_{t+1} + e_2 x_{Ut} + a_U P_t \]

\[ = \hat{a}_U X_{Ut} + b_U x^U_{t+1} + e_2 x_{Ut} + a_U p_U X_{Ut} \]

\[ = a_U X_{Ut} + b_U x^U_{t+1} + a_U x_{Ut} , \]

with \( a_U = \hat{a}_U + a_U p_U x_0 \) and \( a_{Ux} = e_2 \).

One can now appeal to the general formulation of the dynamic programming problem in Appendix A and the solution proceeds similarly to the symmetric information case.

4.2 Informed investor’s problem

Since the informed investor directly observes all random variables in the model, her state vector \( X_{It} \) follows immediately. As with the uninformed investor, it is convenient to also augment with a constant

\[ X_{It} = (1 \ \theta_{It} \ D_t \ G_t \ Z_t \ S_{Ut} \ D_{t-1} \ \hat{G}_{t-1} \ \hat{Z}_{t-1} \ S_{U(t-1)})' . \]

The informed trader is strategic, and following Kyle (1989) her optimal demand can be determined by allowing her to optimize against the residual demand schedule of the uninformed trader. Under the conjectured function in (14), her residual demand schedule \( P_t(x_{It}) \) follows from rearranging the market clearing condition

\[ x_{It} + \beta_{U1} \theta_{Ut} + \beta_{U2} D_t + \beta_{U3} S_{Ut} \]

\[ + \beta_{U5} D_{t-1} + \beta_{U7} \hat{G}_{t-1} + \beta_{U8} \hat{Z}_{t-1} + \beta_{U9} S_{U(t-1)} - \gamma_U P_t = 0 . \]

to yield

\[ P_t(x_{It}) = \frac{1}{\gamma_U} \left[ x_{It} + \beta_{U1} \theta_{Ut} + \beta_{U2} D_t + \beta_{U3} S_{Ut} + \beta_{U5} D_{t-1} + \beta_{U7} \hat{G}_{t-1} + \beta_{U8} \hat{Z}_{t-1} + \beta_{U9} S_{U(t-1)} \right] \]

Since market clearing requires \( \theta_{It} + \theta_{Ut} = 0 \), this can be written purely as a function of random variables directly observed by the trader

\[ P_t(x_{It}) = \frac{1}{\gamma_U} \left[ x_{It} - \beta_{U1} \theta_{It} + \beta_{U2} D_t + \beta_{U3} S_{Ut} + \beta_{U5} D_{t-1} + \beta_{U7} \hat{G}_{t-1} + \beta_{U8} \hat{Z}_{t-1} + \beta_{U9} S_{U(t-1)} \right] \]

(15)

The residual demand schedule can be written compactly as

\[ P_t(x_{It}) = p'_I X_{It} + p_{Ix} x_{It} . \]
where the vector $p_t$ and scalar $p_{tx}$ are easily computed from the expressions above.

The informed trader’s cash position evolves as

$$M_{jt+1} = R_h(M_{jt} - c_{jt}h - x_{jt}P_t) + (\theta_{jt} + x_{jt})hD_{t+1} + Z_tD_{t+1}$$

The non-constant variables in the informed state vector obey

$$\theta_{It+1} = \theta_{It} + x_{It}$$
$$D_{t+1} = a_D D_t + a_D G_t + b_D \varepsilon_{Dt+1}$$
$$G_{t+1} = a_G G_t + b_G \varepsilon_{Gt+1}$$
$$Z_{t+1} = a_Z Z_t + b_Z \varepsilon_{Zt+1}$$
$$S_{Ut+1} = S_{Ut} + h a_G G_t + h b_G \varepsilon_{Gt+1} + b_S \varepsilon_{Ut+1}$$
$$D_t = D_t$$
$$\hat{G}_t = a_G \hat{G}_{t-1} + K_{11}(D_t - a_D D_{t-1} - a_D G_{t-1}) + K_{12}(S_{Pt} - \beta_{13} a_G \hat{G}_{t-1} - \beta_{14} a_Z \hat{Z}_{t-1})$$
$$+ K_{13}\left(S_{Ut} - S_{Ut-1} - h a_G \hat{G}_{t-1}\right)$$
$$\hat{Z}_t = a_Z \hat{Z}_{t-1} + K_{21}(D_t - a_D D_{t-1} - a_D G_{t-1}) + K_{22}(S_{Pt} - \beta_{13} a_G \hat{G}_{t-1} - \beta_{14} a_Z \hat{Z}_{t-1})$$
$$+ K_{23}\left(S_{Ut} - S_{Ut-1} - h a_G \hat{G}_{t-1}\right)$$
$$S_{Ut} = S_{Ut}$$

where

$$S_{Pt} = \gamma P_t(x_{It}) - \beta_1 \theta_{Ut} - \beta_2 D_t - \beta_5 S_{Ut} - \beta_6 D_{t-1} - \beta_7 \hat{G}_{t-1} - \beta_8 \hat{Z}_{t-1} - \beta_9 S_{Ut-1}.$$  

and

$$P_t(x_{It}) = p'_{It} X_{It} + p_{tx} x_{It}$$

so that the trader correctly understands that she controls the uninformed trader’s belief update via the $P_t$ term in $S_{Pt}$ and takes as given the conditional expectations $\hat{G}_{t-1}$ and $\hat{Z}_{t-1}$ held by the uninformed trader entering the period.
Let

\[ \hat{a}_I = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_D & a_{DG} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_Z & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}, \]

\[ b_I = \begin{pmatrix}
0 & 0 & 0 & 0 & b_D & 0 & 0 & 0 & b_G & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}, \]

and

\[ a_{IP} = (0, 0, 0, 0, 0, 0, K_{12}, K_{22}, 0)'. \]

Let \( \varepsilon_t^I = (\varepsilon_{D,t}, \varepsilon_{G,t}, \varepsilon_{Z,t}, \sigma_{Ut}) \) and write the dynamics of the state vector as

\[ X_{t+1} = \hat{a}_t X_t + b_I \varepsilon_{t+1}^I + e_2 x_{t+1} + a_{IP} p_t(x_{t+1}) \]

\[ = \hat{a}_t X_t + b_I \varepsilon_{t+1}^I + e_2 x_{t+1} + a_{IP} (p_t' X_t + p_{Ix} x_{t+1}) \]

\[ = a_I X_t + b_I \varepsilon_{t+1}^I + a_{Ix} x_{t+1} \]

with \( a_I = \hat{a}_I + a_{IP} p_t' \) and \( a_{Ix} = e_2 + a_{IP} p_{Ix} \).

One can now appeal to the general formulation of the dynamic programming problem in Appendix A and the solution proceeds similarly to the symmetric information case.

5 Conclusion

This paper has studied return predictability in a model of dynamic trading. Results indicate that when investors slow their trading due to a concern with price impact it leads naturally...
to short-run momentum in returns. The mechanism is simple: the portfolio of a strategic trader exhibits inertia, which effectively changes the net supply of the asset available to other traders in the market. Gradual and predictable flows of capital as the strategic trader adjusts her portfolio lead to predictable changes in risk and therefore in risk premia. Numerical calibration suggests that the size and duration of momentum effects are more pronounced in settings in which concern for price impact is larger, such as when informed traders control a larger fraction of capital in the market or there is greater residual uncertainty in asset payoffs. The model also predicts that returns should revert in the long-run due to mean reversion in non-informational motives for trade. Here, those motives were captured by a correlated nontradeable, but the intuition should be robust to other mechanisms.

While I have relied on numerical solutions for most of the results, it would be interesting to further explore whether analytical results are available, even if only in special cases. This would help further illuminate the economic mechanisms. It would also be useful to carefully calibrate the model to various markets other than US equities and see how well it can reconcile the cross-market evidence in Moskowitz, Ooi, and Pedersen (2012). I leave these problems for future work.
A General dynamic programming problem

In this section, I solve the infinite-horizon consumption-portfolio problem for a trader with discounted CARA utility over consumption and who faces asset prices driven by an \( n + 1 \) dimensional state vector. The analysis nests the cases in which the trader does and does not have price impact.

**Proposition A.1.** Consider a trader with utility \( \sum_{s=0}^{\infty} h \exp \{ -\rho hs - \alpha c_s \} \) whose cash holdings evolve as in eq. (17), driven by a \( (n + 1) \) vector of observable state variables \( X_t \), the dynamics of which are specified in eq. (16).

Supposing that the matrix equation
\[
Q = \frac{1}{a_{11}} (A - B'C^{-1}B) + \\
+ \frac{2}{\alpha} e_1 \left( \log(a_{11}) + \frac{1 - a_{11}}{a_{11}} \log \left( \frac{a_{11} - 1}{h} \right) - \frac{1}{a_{11}} (\rho h + \frac{1}{2} \log d) \right) e_1
\]

(where \( A, B, \) and \( C \) are defined below) has a symmetric solution in \( Q \) and that the matrix \( C \) is negative semidefinite, then the trader’s optimal portfolio and consumption choices exist and are given by
\[
\hat{x}^* = -C^{-1}B\hat{X}_t \\
c^* = \frac{1}{\alpha a_{11}} \left[ \rho h - \log \left( \frac{a_{11} - 1}{h} \right) + \frac{1}{2} \log d + \alpha \frac{1 - a_{11}}{a_{c11}} a_{11} M_t - \frac{1}{2} \alpha \hat{X}'_t (A - B'C^{-1}B)\hat{X}_t \right],
\]
and her value function is
\[
V(M_t, X_t) = -\exp \left\{ -\alpha Q_0 M_t - \frac{1}{2} \alpha X'_t Q X_t \right\},
\]
where
\[
Q_0 = \frac{1 - a_{11}}{a_{c11}}
\]

Let \( M_t \) denote the trader’s cash position. Let \( X_t \) denote the \( (n + 1) \)-dimensional vector of non-cash state variables. Let \( x \in \mathbb{R}^m \) be the vector of non-consumption choice variables.

Suppose that \( X_t \) has dynamics
\[
X_{t+1} = aX_t + b\varepsilon_{t+1} + a_xx
\]
where \( a \) is an \( (n + 1) \times (n + 1) \) matrix, \( b \) is an \( (n + 1) \times k \) matrix, \( a_x \) is an \( (n + 1) \times m \) matrix, all of which can depend on \( h \). Suppose further that the first element of \( \hat{X} \) is the constant 1.
Suppose that the dynamics of $M_t$ are

$$M_{t+1} = a_{11}M_t + a_1X_t + b_1\varepsilon_{t+1} + a_{c11}c + a_{x1}x + (c_XX_t)'x - \frac{1}{2}x'c_xx + \frac{1}{2}X_t'c_{XX}X_t'X_t + 1 \quad (17)$$

$$+ (C_XX_t + C_xx)'X_{t+1} + \frac{1}{2}X_{t+1}'C_{XX}X_{t+1}, \quad (18)$$

where $a_{11}$ and $a_{c11}$ are constants, $a_1$ is $1 \times (n+1)$, $b_1$ is $1 \times k$, $a_{x1}$ is $1 \times m$, $c_X$ is $m \times (n+1)$, $c_x$ is $m \times m$, $c_{XX}$ is a symmetric $(n+1) \times (n+1)$ matrix, $C_X$ is $(n+1) \times (n+1)$, and $C_x$ is $(n+1) \times m$, and $C_{XX}$ is a symmetric $(n+1) \times (n+1)$ matrix.

Consider the problem of the investor. Conjecture that her value function $V(\cdot)$ is exponential-linear in $M_t$ and exponential-quadratic in the other state variables, $\hat{X}_t$

$$V(M_t, X_t) = -\exp\left\{-\alpha Q_0 M_{t+1} - \frac{1}{2}\alpha X_{t+1}'QX_{t+1}\right\},$$

where $Q_0$ is a constant and $Q$ is an $(n + 1) \times (n + 1)$ symmetric positive semidefinite matrix. The Bellman equation is

$$V(M_t, X_t) = \max_x \left\{-h \exp\{-\alpha c\} - E_t \left[\exp\left\{-\rho h - \alpha Q_0 M_{t+1} - \frac{1}{2}\alpha X_{t+1}'QX_{t+1}\right\}\right]\right\}. \quad (19)$$
Using the equations for $X_{t+1}$, the expression in the exponent in eq. (19) becomes

$$-\alpha Q_0 \left( a_{11} M_t + a_1 X_t + b_1 \varepsilon_{t+1} + a_{e1} e + a_x x + (c_X X_t)' x - \frac{1}{2} a' c_x x + \frac{1}{2} X_t' c_{XX} X_t \\
+ (C_X X_t + C_x)' (a X_t + b \varepsilon_{t+1} + a_x x) \right) - \alpha Q_0 \left( \frac{1}{2} (a X_t + a_x x)' c_{XX} (a X_t + a_x x) + [C_{XX} (a X_t + a_x x)]' b \varepsilon_{t+1} + \frac{1}{2} [b \varepsilon_{t+1}]' C_{XX} b \varepsilon_{t+1} \right) - \frac{1}{2} \alpha (a X_t + a_x x)' Q (a X_t + a_x x) - \alpha Q_0 (a X_t + a_x x)' b \varepsilon_{t+1} - \frac{1}{2} \alpha [b \varepsilon_{t+1}]' Q b \varepsilon_{t+1}$$

$$= -\alpha Q_0 a_{11} M_t - \alpha Q_0 a_{e1} e - \frac{1}{2} \alpha Q_0 X_t' ((e_1 a_1 + C'_X a) + (e_1 a_1 + C'_X a)' + c_{XX}) X_t \\
- \alpha Q_0 X_t' (e_1 a_{x1} + c'_X + C'_X a_x + a' C_x) x - \frac{1}{2} \alpha Q_0 x' (C'_x a_x + a' C_x - c_x) x \\
- \alpha Q_0 ((b'_1 e'_1 + C_X) X_t + C_x)' b \varepsilon_{t+1} - \frac{1}{2} \alpha (a X_t + a_x x)' (Q + Q_0 C_{XX}) (a X_t + a_x x) \\
- \alpha [(Q + Q_0 C_{XX}) (a X_t + a_x x)]' b \varepsilon_{t+1} - \frac{1}{2} \alpha [b \varepsilon_{t+1}]' (Q + Q_0 C_{XX}) b \varepsilon_{t+1}$$

$$= -\alpha Q_0 a_{11} M_t - \alpha Q_0 a_{e1} e - \frac{1}{2} \alpha X_t' (Q_0 ((e_1 a_1 + C'_X a) + (e_1 a_1 + C'_X a)' + c_{XX}) + a'(Q + Q_0 C_{XX}) a) X_t \\
- \alpha X_t' (Q_0 (e_1 a_{x1} + c'_X + C'_X a_x + a' C_x) + a'(Q + Q_0 C_{XX}) a_x) x \\
- \frac{1}{2} \alpha x' (Q_0 (C'_x a_x + a' C_x - c_x) + a' (Q + Q_0 C_{XX}) a_x) x \\
- \alpha [(Q_0 (b'_1 e'_1 + C_X) + (Q + Q_0 C_{XX}) a) X_t + (Q_0 C_x + (Q + Q_0 C_{XX}) a_x) x]' b \varepsilon_{t+1} \\
- \frac{1}{2} \alpha [b \varepsilon_{t+1}]' (Q + Q_0 C_{XX}) b \varepsilon_{t+1}$$

$$= -\alpha Q_0 a_{11} M_t - \alpha Q_0 a_{e1} e - \frac{1}{2} \alpha X_t' A_{XX} X_t - \alpha X_t' A'_{Xx} x - \frac{1}{2} \alpha x' A_{xx} x \\
- \alpha (B_X X_t + B_x x)' b \varepsilon_{t+1} - \frac{1}{2} \alpha [b \varepsilon_{t+1}]' (Q + Q_0 C_{XX}) b \varepsilon_{t+1},$$

with

$$A_{XX} = Q_0 ((e_1 a_1 + C'_X a) + (e_1 a_1 + C'_X a)' + c_{XX}) + a'(Q + Q_0 C_{XX}) a \\
A_{Xx} = (Q_0 (e_1 a_{x1} + c'_X + C'_X a_x + a' C_x) + a'(Q + Q_0 C_{XX}) a_x)' \\
A_{xx} = Q_0 (C'_x a_x + a' C_x - c_x) + a' (Q + Q_0 C_{XX}) a_x \\
B_X = Q_0 (b'_1 e'_1 + C_X) + (Q + Q_0 C_{XX}) a \\
B_x = Q_0 C_x + (Q + Q_0 C_{XX}) a_x.$$
Taking the expectation using Lemma ?? gives

\[ \mathbb{E}_t \left[ \exp \left\{ -\rho h - \alpha Q_0 M_{t+1} - \frac{1}{2} \alpha X_{t+1}' Q X_{t+1} \right\} \right] \]

\[ = \mathbb{E}_t \left[ \exp \left\{ -\rho h - \alpha Q_0 a_{11} M_t - \alpha Q_0 a_{c11} c - \frac{1}{2} \alpha X_t' A_{XX} X_t - \frac{1}{2} \alpha X_t' A_{Xx} x - \frac{1}{2} \alpha x' A_{xx} x - \frac{1}{2} \log d 
+ \frac{1}{2} \alpha^2 (B_{X} X_t + B_{x} x)' \tilde{Q} (B_{X} X_t + B_{x} x) \right\} \right] \]

where \( \Sigma = bb', d = |I + \alpha (Q + Q_0 C_{XX}) \Sigma| \), and \( \tilde{Q} = \Sigma (I + \alpha (Q + Q_0 C_{XX}) \Sigma)^{-1} \). Letting

\[ A = A_{XX} - \alpha B_{X}' \tilde{Q} B_{X} \]
\[ B = A_{Xx} - \alpha B_{x}' \tilde{Q} B_{x} \]
\[ C = A_{xx} - \alpha B_{x}' \tilde{Q} B_{x} \]

the expectation can be written compactly as

\[ \exp \left\{ -\rho h - \frac{1}{2} \log d - \alpha Q_0 a_{11} M_t - \alpha Q_0 a_{c11} c - \frac{1}{2} \alpha X_t' A_{XX} X_t - \alpha X_t' A_{Xx} x - \frac{1}{2} \alpha x' A_{xx} x - \frac{1}{2} \alpha x' C x \right\} . \]

Hence, the Bellman equation can be written

\[ V(M_t, X_t) = \max_c \left\{ -\exp\{-\alpha c + \log h\} \right. \]
\[ - \exp\{-\rho h - \frac{1}{2} \log d - \alpha Q_0 a_{11} M_t - \alpha Q_0 a_{c11} c - \frac{1}{2} \alpha X_t' A_{XX} X_t - \alpha X_t' A_{Xx} x - \frac{1}{2} \alpha x' A_{xx} x - \frac{1}{2} \alpha x' C x \} \right\} \]

It is clear that \( x \) can be solved for separately from \( c \). The first-order condition with respect to \( \hat{x} \) is

\[ B_{X} X_t + C x = 0, \]

with associated second-order condition, evaluated at the \( x \) satisfying the FOC,

\[ C \leq 0. \]

Rearranging the FOC yields the optimal \( \hat{x} \)

\[ \hat{x}^* = -C^{-1} B_{X} X_t. \]

Substituting back into the Bellman equation leaves

\[ V(M_t, X_t) = \max_c \left\{ -\exp\{-\alpha c + \log h\} \right. \]
\[ - \exp\{-\rho h - \frac{1}{2} \log d - \alpha Q_0 (a_{11} M_t + a_{c11} c) - \frac{1}{2} \alpha X_t' A_{XX} X_t + \frac{1}{2} \alpha X_t' B' C^{-1} B_{X} X_t\} \right\} \]

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The first order condition for $c$ is
\[
\alpha \exp\{-\alpha c + \log h\} + \alpha a_{c11}Q_0 \exp\{-\rho h - \frac{1}{2} \log d - \alpha Q_0(a_{11}M_t + a_{c11}c) - \frac{1}{2} \alpha X'_t(A - B'C^{-1}B)X_t\} = 0,
\]
and the second-order condition is
\[
- \alpha^2 \exp\{-\alpha c + \log h\} - (\alpha a_{c11}Q_0)^2 \exp\{-\rho h - \frac{1}{2} \log d - \alpha Q_0(a_{11}M_t + a_{c11}c) - \frac{1}{2} \alpha X'_t(A - B'C^{-1}B)X_t\} \leq 0,
\]
which is always satisfied.

Rearranging the FOC for $c$ yields
\[
c^* = \frac{1}{\alpha(1 - a_{c11}Q_0)} \left[ \rho h - \log(-a_{c11}Q_0) + \frac{1}{2} \log d + \log h + \alpha Q_0a_{11}M_t + \frac{1}{2} \alpha X'_t(A - B'C^{-1}B)X_t \right]
\]
Finally, plug back into the Bellman equation
\[
V(X_t) = -\exp \left\{ \log(1 - a_{c11}Q_0) + \frac{a_{c11}Q_0}{1 - a_{c11}Q_0} \log \left( \frac{-a_{c11}Q_0}{h} \right) - \frac{1}{1 - a_{c11}Q_0}(\rho h + \frac{1}{2} \log d) - \alpha Q_0a_{11} \frac{1}{1 - a_{c11}Q_0}X_t(A - B'C^{-1}B)X_t \right\}
\]
Matching coefficients with the original conjecture yields a system of equations for $Q_0$ and the submatrix $\hat{Q}$
\[
Q_0 = \frac{a_{11}Q_0}{1 - a_{c11}Q_0} \quad \text{(20)}
\]
\[
Q = \frac{1}{1 - a_{c11}Q_0}(A - B'C^{-1}B) \quad \text{(21)}
\]
\[
+ \frac{2}{\alpha} e_1 \left( \log(1 - a_{c11}Q_0) + \frac{a_{c11}Q_0}{1 - a_{c11}Q_0} \log \left( \frac{-a_{c11}Q_0}{h} \right) - \frac{1}{1 - a_{c11}Q_0}(\rho h + \frac{1}{2} \log d) \right) e_1' \quad \text{(22)}
\]
Note that the constant $Q_0$ can be solved explicitly as.
\[
Q_0 = \frac{1 - a_{11}}{a_{c11}}
\]
Plugging this back into the equation for $\hat{Q}$ yields
\[
Q = \frac{1}{a_{11}}(A - B'C^{-1}B)
\]
\[
+ \frac{2}{\alpha} e_1 \left( \log(a_{11}) + \frac{1 - a_{11}}{a_{11}} \log \left( \frac{a_{11} - 1}{h} \right) - \frac{1}{a_{11}}(\rho h + \frac{1}{2} \log d) \right) e_1'
\]
The expression for $Q_0$, also delivers the optimal consumption in terms of model primitives and the remaining elements of $Q$

$$c^* = \frac{1}{\alpha a_{11}} \left[ \rho h - \log \left( \frac{a_{11} - 1}{h} \right) \right] + \frac{1}{2} \log d + \alpha \frac{1 - a_{11}}{a_{c11}} M_t - \frac{1}{2} \alpha \hat{X}_t'(A - B'C^{-1}B)\hat{X}_t. $$

### B Proofs

**Proof of Proposition 3.1.** I will apply Proposition A.1 from Appendix A to solve the trader’s problem. To apply that result one must write the cash dynamics in the form of eq. (17)

$$M_{t+1} = a_{11} M_t + a_1 X_t + b_1 b \varepsilon_{t+1} + a_{c11} c + a_{x1} x + (c_X X_t)' x - \frac{1}{2} x' c_x x + \frac{1}{2} X_t' c_{XX} X_t + (C_X X_t + C_x x)' X_{t+1} + \frac{1}{2} X_{t+1}' C_{XX} X_{t+1}. $$

Adding $I$ subscripts, this is achieved by defining

$$a_{I11} = R_h, \quad a_{Ix11} = -R_h h,$$

$$c_{IX} = -R_h p_I', \quad c_{Ix} = 2R_h p_{Ix}, \quad c_{IXX} = 0$$

$$C_{IX} = \begin{pmatrix} 0 \\ h \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad C_{Ix} = \begin{pmatrix} 0 \\ 0 \\ h \\ 0 \\ 0 \end{pmatrix}, \quad C_{IXX} = 0$$

$$a_{I11} = 0, b_{I1} = 0, a_{Ix1} = 0.$$

Further define the matrices

$$A_{IXX} = Q_{I0} (C_{IX} \hat{a}_I + (C'_{IX} \hat{a}_I)'), \quad A_{IX} = \left( Q_{I0} (\hat{a}_I' C_{Ix} - R_h p_I) + \hat{a}_I' \hat{Q}_I \hat{a}_{Ix} \right)'$$

$$A_{Ixx} = -2Q_{I0} R_h p_{Ix} + \hat{a}_I' \hat{Q}_I \hat{a}_{Ix}$$

$$B_{IX} = Q_{I0} C_{IX} + \hat{Q}_I \hat{a}_I$$

$$B_{Ix} = Q_{I0} C_{Ix} + \hat{Q}_I \hat{a}_{Ix}$$

$$\Sigma_I = b_I b_I'$$

$$\hat{Q}_I = \Sigma_I (I + \alpha_I Q_I \Sigma_I)^{-1},$$
and
\[ A_I = A_{IX} - \alpha_I B_{IX} Q_I B_{1X}, \]
\[ B_I = A_{IX} - \alpha_I B_{IX} Q_I B_{1X}, \]
\[ C_I = A_{IX} - \alpha_I B_{IX} Q_I B_{1X}. \]

Applying Proposition A.1 the optimal demand and consumption are
\[ x^* = -C_I^{-1} B_I X_t \]
\[ = \left[ \alpha_I B'_{IX} Q_I B_{1X} + 2Q_{10} R_h p_{IX} - \dot{a}'_{IX} Q^t \dot{a}_{IX} \right]^{-1} \left[ Q_{10} (C'_{IX} \dot{a}_I - R_h \hat{p}_I') + \dot{a}'_{IX} \dot{Q}_I \dot{a}_I - \alpha_I B'_{IX} Q_I B_{1X} \right] X_t \]
\[ c^* = \frac{1}{\alpha_I R_h} \left[ \rho h - \log \left( \frac{R_h - 1}{h} \right) + \frac{1}{2} \log d_I + \alpha_I R_h - 1 M_t + \frac{1}{2} \alpha_I X_t^t B'C_I^{-1} B_I X_t \right]. \]

Substituting the optimal demand and consumption into the Bellman equation and equating terms yields \( Q_{10} \) in closed form
\[ Q_{10} = \frac{1}{h} \frac{R_h - 1}{R_h}, \]
and produces a matrix equation that \( Q_I \) must solve
\[ Q_I = \frac{1}{R_h} (A_I - B_IC_I^{-1} B_I) \]
\[ + \frac{2}{\alpha_I} e_1 e_1' \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_I) \right) \]
\[ = \frac{1}{R_h} \left( A_I - ((Q_{10}(a'I C_{IX} - R_h p_I) + a'I Q_I a_{IX})' - \alpha_I B'_{IX} Q_I B_{1X})' C_I^{-1} \right. \]
\[ \left. + \frac{2}{\alpha_I} e_1 e_1' \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_I) \right) \right). \]

\[ \square \]

Proof of Proposition. To place the cash dynamics in the form of Proposition A.1, let
\[ a_{U11} = R_h, \quad a_{Ux11} = -R_h h \]
\[ c_{UX} = -R_h p_{U}, \quad c_{Ux} = 0 \]
\[ C_{UX} = \begin{pmatrix} 0 \\ h \\ 0 \\ 0 \end{pmatrix}, \quad C_{Ux} = \begin{pmatrix} 0 \\ 0 \\ h \\ 0 \end{pmatrix} \]
\[ a_{U1} = 0, b_{U1} = 0, a_{Ux1} = 0. \]
Using the above expressions, define the matrices

\[ A_{UX} = Q_U \left( C'_{UX} a_U + \left(C'_{UX} a_U\right)' \right) + a_U' Q_U a_U \]
\[ A_{UX} = \left(Q_U \left(c'_{UX} + a_U' C_{UX}\right) + a_U' Q_U a_U\right)' \]
\[ A_{XX} = -Q_U c_{IX} + a_{U,x} Q_U a_{U,x} \]
\[ B_{UX} = Q_U C_{UX} + Q_U a_U \]
\[ B_{Ux} = Q_U C_{Ux} + Q_U a_{Ux} \]
\[ \Sigma_U = b_U b_U' \]
\[ \tilde{Q}_U = \Sigma_U (I + \alpha U Q_U \Sigma_U)^{-1} \]

and

\[ A_U = A_{UX} - \alpha U B_{UX} \tilde{Q}_U B_{UX} \]
\[ B_U = A_{UX} - \alpha U B_{Ux} \tilde{Q}_U B_{Ux} \]
\[ C_U = A_{XX} - \alpha U B_{Ux} \tilde{Q}_U B_{Ux} \]

The optimal demand and consumption are

\[ x^* = -C_U B_U X_U t \]
\[ c^* = \frac{1}{\alpha U R_h} \left[ R_h - \log \left( \frac{R_h - 1}{h} \right) \right] + \frac{1}{2} \log d_U + \alpha U R_h - \frac{1}{R_h} M_{Ut} + \frac{1}{2} \alpha U X_{Ut} B_{Ux} C^{-1} B_{UX} X_{Ut} \]

Plugging back into the Bellman equation yields \( Q_U \) in closed form

\[ Q_U = \frac{1}{R_h - 1} \frac{R_h - 1}{R_h} \]

and a system of equations for the matrix \( Q_U \)

\[ Q_U = \frac{1}{R_h} (A_U - B_U C_U^{-1} B_U) \]
\[ + \frac{2}{\alpha U} e_1 e_1' \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_U) \right) \]
\[ = \frac{1}{R_h} \left( A_U - ((Q_U (a_U' C_{UX} - R_h p_U) + a_U' Q_U a_{UX})' - \alpha U B_{UX} \tilde{Q}_U B_{UX})' \right) C_U^{-1} \]
\[ + \frac{2}{\alpha U} e_1 e_1' \left( \log(R_h) + \frac{1 - R_h}{R_h} \log \left( \frac{R_h - 1}{h} \right) - \frac{1}{R_h} (\rho h + \frac{1}{2} \log d_U) \right) \]
In equilibrium $P_t = p'_U X_{Ut}$, so it is immediate that the optimal demand can be implemented by submitting a demand schedule satisfying

$$x^* = \left[ \alpha U B'_{Ux} \hat{Q} U B_{Ux} - a'_{Ux} Q_U a_{Ux} \right]^{-1} \left[ Q_U C'_{Ux} a_U + a'_{Ux} Q_U a_U - \alpha U B'_{Ux} \hat{Q} U B_{Ux} \right] X_{Ut}$$

$$- \left[ \alpha U B'_{Ux} \hat{Q} U B_{Ux} - a'_{Ux} Q_U a_{Ux} \right]^{-1} Q_U R_h P_t (x^*_{Ut}).$$

To match the initial conjectured demand, one requires

$$\beta'_U = \left[ \alpha U B'_{Ux} \hat{Q} U B_{Ux} - a'_{Ux} Q_U a_{Ux} \right]^{-1} \left[ Q_U C'_{Ux} a_U + a'_{Ux} Q_U a_U - \alpha U B'_{Ux} \hat{Q} U B_{Ux} \right]$$

$$\gamma_U = \left[ \alpha U B'_{Ux} \hat{Q} U B_{Ux} - a'_{Ux} Q_U a_{Ux} \right]^{-1} Q_U R_h.$$

\[\square\]

**Proof of Proposition 3.3.** The result follows directly from substituting into eqs. (7) – (13) and doing some tedious algebra, so I suppress the details for brevity. Other than $Q_{j11}$, $Q_{j14}$, and $Q_{j44}$ Bellman terms are available in closed form

$$Q_{j12} = Q_0 \frac{h a_D}{(a_D - R_h)}$$

$$Q_{j13} = Q_0 \frac{h a_D g R_h}{(R_h - a_D)(R_h - a_G)}$$

$$Q_{jik} = 0, \quad (i, k) \in \{(2, 2), (2, 3), (3, 3), (2, 4), (3, 4)\}.$$

The six equations for $Q_{j11}$, $Q_{j14}$, and $Q_{j44}, j \in \{I, U\}$ are obtained by substituting the above $Q$ coefficients, as well as the demand function coefficients from the statement of the Proposition, into eqs. (7) and (11).

\[\square\]

**Proof of Corollary 3.4.** Summing up the demand functions from Prop. 3.3 and imposing the market clearing condition gives

$$P_t = \frac{1}{\gamma_I + \gamma_U} \left[ \beta'_I X_{It} + \beta'_U X_{Ut} \right]$$

$$= \frac{1}{\gamma_I + \gamma_U} \left[ \beta'_I + \beta'_U \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right] X_{It}$$

$$= \frac{1}{\gamma_I + \gamma_U} \left[ (\beta_{I1} - \beta_{U1}) \theta_{It} + (\beta_{I2} + \beta_{U2}) D_t + (\beta_{I3} + \beta_{U3}) G_t + (\beta_{I4} + \beta_{U4}) Z_t \right],$$

where the second equality follows because $\theta_{Ut} = -\theta_{It}$ in equilibrium. Substituting in from Prop. 3.3 yields the coefficients on $D_t$, $G_t$, and $Z_t$ immediately. To derive the coefficient on
It, note that the expression for $\beta_{I1}$ can be written
\[ \beta_{I1} = -\gamma_I \left( \frac{1}{\gamma_I} - \frac{1}{\gamma_U} \right) = -1 + \frac{\gamma_I}{\gamma_U}, \]
and one has $\beta_{U1} = -1$. Therefore,
\[ \beta_{I1} - \beta_{U1} = \frac{\gamma_I}{\gamma_U}. \]

Proof of Proposition 3.5. To determine the return covariances, it will be helpful to first derive the dynamics for the informed trader’s holdings $\theta_I$. Substituting in for the equilibrium demand and price functions, the equilibrium dynamics of $\theta_I$ are
\[
\theta_{t+1} = x_{t} + \theta_{t} \\
= \beta_I x_{t} - \gamma_I P_t + \theta_{t} \\
= \beta_{I1} Z_t - \left(1 - \frac{\gamma_I}{\gamma_U}\right) \theta_t - \gamma_I \left( \frac{\beta_{I1} + \beta_{U1}}{\gamma_I + \gamma_U} Z_t + \frac{1}{\gamma_U} \frac{\gamma_I}{\gamma_I + \gamma_U} \theta_t \right) + \theta_t \\
= \frac{\beta_{I1} - \beta_{U1}}{\gamma_I + \gamma_U} Z_t - \frac{\gamma_U}{\gamma_I + \gamma_U} \theta_t + \theta_t \\
= \frac{\gamma_I}{\gamma_I + \gamma_U} Z_t + \frac{1}{\gamma_I + \gamma_U} \theta_t.
\]

Therefore, $\theta_{t+k}$ can be written
\[
\theta_{t+k} = \frac{\beta_{I1} - \beta_{U1}}{\gamma_I + \gamma_U} \sum_{j=0}^{\infty} a_\theta^j Z_{t+k-1-j} \\
= \text{(terms involving $\varepsilon_{Zt+1}, \ldots, \varepsilon_{Zt+k}$)} + \frac{\beta_{I1} - \beta_{U1}}{\gamma_I + \gamma_U} a_\theta^k Z_t + a_\theta^{k-1} \theta_t
\]

The excess return at $t+k$ is
\[
R^e_{t+k} = A_Z(Z_{t+k} - R_h Z_{t+k-1}) + A_\theta(\theta_{t+k} - R \theta_{t+k-1}) \\
+ \frac{R_h}{R_h - a_D} h b_D \varepsilon_{Dt+k} + \frac{R_h a_{DG}}{(R_h - a_D)(R_h - a_G)} h b_G \varepsilon_{Gt+k},
\]
and that at $t$ is
\[
R^e_{t} = A_Z(Z_t - R_h Z_{t-1}) + A_\theta(\theta_t - R \theta_{t-1}) \\
+ \frac{R_h}{R_h - a_D} h b_D \varepsilon_{Dt} + \frac{R_h a_{DG}}{(R_h - a_D)(R_h - a_G)} h b_G \varepsilon_{Gt}.
\]
Hence, for $k \geq 1$,
\[
\mathbb{E}[R_{t+k}^{e}|R_t^{e}] = \frac{\text{Cov}(R_{t+k}^{e}, R_{t}^{e})}{\text{Var}(R_t^{e})} R_t^{e}
\]
\[
= \frac{\text{Cov}(A(Z_{t+k} - R_h Z_{t+k-1}) + A_\theta(\theta_{It+k} - R\theta_{It+k-1}), A(Z_t - R_h Z_{t-1}) + A_\theta(\theta_{It} - R\theta_{It-1}))}{\text{Var}(R_t^{e})} R_t^{e}.
\]

Compute
\[
\text{Cov}(Z_{t+k} - R_h Z_{t+k-1}, Z_t - R_h Z_{t-1}) = \text{Cov}((a_Z - R_h)Z_{t+k-1}, Z_t - R_h Z_{t-1})
\]
\[
= (a_Z - R_h) \frac{a_Z^{k-1}}{1 - a_Z^2} b_Z^2 - R_h(a_Z - R_h) \frac{a_Z^k}{1 - a_Z^2} b_Z^2
\]
\[
= (R_h a_Z - 1)(R_h - a_Z) \frac{a_Z^{k-1}}{1 - a_Z^2} b_Z^2,
\]
followed by
\[
\text{Cov}(Z_{t+k} - R_h Z_{t+k-1}, \theta_{It} - R_h \theta_{It-1})
\]
\[
= \text{Cov} \left( (a_Z - R_h)Z_{t+k-1}, \frac{\beta_{14}}{\gamma_I} - \frac{\beta_{14}}{\gamma_U} Z_{t-1} + (a_\theta - R_h) \theta_{It-1} \right)
\]
\[
= (a_Z - R_h) \frac{\beta_{14}}{\gamma_I} - \frac{\beta_{14}}{\gamma_U} \sum_{j=0}^{\infty} a_\theta^j Z_{t-2-j}
\]
\[
= (a_Z - R_h) \frac{\beta_{14}}{\gamma_I} - \frac{\beta_{14}}{\gamma_U} \sum_{j=0}^{\infty} a_\theta^j a_Z^{k+1+j}
\]
\[
= (a_Z - R_h) \frac{\beta_{14}}{\gamma_I} - \frac{\beta_{14}}{\gamma_U} \frac{R_h a_Z - 1}{1 - a_Z^2} a_\theta a_Z - 1.
\]

Next,
\[
\text{Cov}(\theta_{It+k} - R\theta_{It+k-1}, Z_t - R_h Z_{t-1})
\]
\[
= \frac{\beta_{14}}{\gamma_I} - \frac{\beta_{14}}{\gamma_U} \sum_{j=0}^{\infty} a_\theta^j \text{Cov} \left( Z_{t+k-1-j} - R_h Z_{t+k-2-j}, Z_t - R_h Z_{t-1} \right)
\]
\[
(24)
\]

Note that eq. (23) implies that for the sum over terms $j \leq k - 2$,
\[
\sum_{j=0}^{k-2} a_\theta^j \text{Cov} \left( Z_{t+k-1-j} - R_h Z_{t+k-2-j}, Z_t - R_h Z_{t-1} \right)
\]
\[
= (R_h a_Z - 1)(R_h - a_Z) \frac{b_Z^2}{1 - a_Z^2} \sum_{j=0}^{k-2} a_\theta^j a_Z^{2-j}
\]
\[
= (R_h a_Z - 1)(R_h - a_Z) \frac{b_Z^2}{1 - a_Z^2} \frac{a_Z^{k-1} - a_\theta^{k-1}}{a_Z - a_\theta}.
\]
For $j = k - 1$, direct computation yields
\[ \text{Cov}(Z_t - R_hZ_{t-1}, Z_t - R_hZ_{t-1}) = \left[R_h^2 + (R_ha_Z - 1)\right] \frac{b_Z^2}{1 - a_Z^2}, \]

while for $j \geq k$ eq. (23) once again implies
\[
\sum_{j=k}^{\infty} a_j^k \text{Cov}(Z_t - R_hZ_{t-1}, Z_{t+k-1-j} - R_hZ_{t+k-2-j}) = (R_ha_Z - 1)(R_h - a_Z) \frac{b_Z^2}{1 - a_Z^2} \sum_{j=k}^{\infty} a_j^k a_j^{-k} \\
= (R_ha_Z - 1)(R_h - a_Z) \frac{b_Z^2}{1 - a_Z^2} a_k^k \frac{a_k^k}{1 - a_\theta a_Z}.
\]

Combining the above results, eq. (24) becomes
\[
\text{Cov}(\theta_{It+k} - R\theta_{It+k-1}, Z_t - R_hZ_{t-1}) = \frac{\beta_1}{\gamma_I} - \frac{\beta_2}{\gamma_I} \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} \left[ (R_ha_Z - 1)(R_h - a_Z) \frac{b_Z^2}{1 - a_Z^2} \left( \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} + \frac{a_\theta^k}{1 - a_\theta a_Z} \right) \right. \\
\left. \quad + a_\theta^k \left[ R_h^2 + (R_ha_Z - 1) \right] \frac{b_Z^2}{1 - a_Z^2} \right]
\]

Finally,
\[
\text{Cov}(\theta_{It+k} - R\theta_{It+k-1}, \theta_{It} - R\theta_{It-1}) = \text{Cov} \left( \frac{\beta_1}{\gamma_I} - \frac{\beta_2}{\gamma_I} \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} (Z_t - R_hZ_{t-1}) + a_\theta^k (\theta_{It} - R_h\theta_{It-1}), \theta_{It} - R\theta_{It-1} \right) \\
= \text{Cov} \left( \frac{\beta_1}{\gamma_I} - \frac{\beta_2}{\gamma_I} \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} (Z_t - R_hZ_{t-1}), \theta_{It} - R\theta_{It-1} \right) + a_\theta^k \text{Cov} (\theta_{It} - R_h\theta_{It-1}, \theta_{It} - R\theta_{It-1}).
\]

One has
\[
\frac{\beta_1}{\gamma_I} - \frac{\beta_2}{\gamma_I} \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} \text{Cov} (Z_t - R_hZ_{t-1}, \theta_{It} - R_h\theta_{It-1}) \\
= \frac{\beta_1}{\gamma_I} - \frac{\beta_2}{\gamma_I} \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} \sum_{j=0}^{\infty} a_j^k \text{Cov} (Z_t - R_hZ_{t-1}, Z_{t-1-j} - R_hZ_{t-2-j}) \\
= \frac{\beta_1}{\gamma_I} - \frac{\beta_2}{\gamma_I} \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} (R_ha_Z - 1)(R_h - a_Z) \sum_{j=0}^{\infty} a_j^k \frac{a_j^k}{1 - a_Z^2} b_Z^2 \\
= \frac{\beta_1}{\gamma_I} - \frac{\beta_2}{\gamma_I} \frac{a_k^k - a_\theta^k}{a_Z - a_\theta} \frac{R_ha_Z - 1}{1 - a_\theta a_Z} (R_h - a_Z) \frac{b_Z^2}{1 - a_Z^2}.
\]
and
\[
\text{Cov}(\theta_{It} - R_h\theta_{It-1}, \theta_{It} - R_h\theta_{It-1}) = (1 + R_h^2) \text{Var}(\theta_{It}) - 2R_h \text{Cov}(\theta_{It}, \theta_{It-1})
\]
\[
= (1 + R_h^2) \frac{1}{1 - a_\theta^2} \left( \frac{\beta_{14}}{\gamma_l} - \frac{\beta_{14}}{\gamma_w} \right)^2 \frac{b_Z^2}{1 - a_Z^2} \left( 1 + a_\theta a_Z \right)
\]
\[
- 2R_h \left( \frac{\beta_{14}}{\gamma_l} - \frac{\beta_{14}}{\gamma_w} \right)^2 \frac{b_Z^2}{1 - a_Z^2} \left[ \frac{a_\theta}{1 - a_\theta^2} \left( 1 + a_\theta a_Z \right) + \frac{a_Z}{1 - a_\theta a_Z} \right].
\]
Hence,
\[
\text{Cov}(\theta_{It+k} - R\theta_{It+k-1}, \theta_{It} - R\theta_{It-1})
\]
\[
= \frac{\beta_{14}}{\gamma_l} - \frac{\beta_{14}}{\gamma_w} \frac{a_Z^k - a_\theta^k}{a_Z - a_\theta} R_h a_Z - 1 \left( R_h - a_Z \right) \frac{b_Z^2}{1 - a_Z^2} + a_\theta^k - 1 \text{Var}(\theta_{It} - R_h\theta_{It-1}).
\]

Combining all of the above results, it follows that for $a_Z = 1/R_h$,
\[
\text{Cov}(R_{t+k}^e, R_t^e) = \text{Cov}(\theta_{It+k} - R\theta_{It+k-1}, Z_t - R_h Z_{t-1}) + \text{Cov}(\theta_{It+k} - R\theta_{It+k-1}, \theta_{It} - R\theta_{It-1})
\]
\[
= \frac{\beta_{14}}{\gamma_l} - \frac{\beta_{14}}{\gamma_w} \frac{a_Z^k - a_\theta^k}{a_Z - a_\theta} \text{Var}(Z_t - R_h Z_{t-1}) + A_\theta^k a_\theta^{-1} \text{Var}(\theta_{It} - R_h\theta_{It-1}) > 0.
\]

\[\square\]

**References**


