Profit-Sharing, Information Aggregation, and Theory of the Firm

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Abstract

Existing studies on rational expectation views the equilibrium market price as an information aggregator empowering individuals of wisdom of the crowd. However, other endogenous variables such as total profit could also have this information aggregation function. In particular, this paper shows that an appropriately designed profit-sharing contract (i.e. equity-based compensation and decentralized control) aggregates dispersed information. This result parallels theories of financial markets built on rational expectation, and sheds light on the nature of the firm. On a macro-level, joint-stock companies endogenously emerge to complete the market; while on a micro-level, profit-sharing design speaks to optimal corporate governance structures.

Keywords: corporate governance, incomplete financial market, information aggregation, financial intermediation, rational expectation, theory of the firm, wisdom of the crowd


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Hayek (1944, 1945) highlight the importance of recognizing wisdom of the crowd in economic analysis. Standing on the giant’s shoulders, this paper is an attempt to study profit-sharing through the lens of information aggregation. Profit-sharing is an important contracting relationship. First, it is a defining element of joint-stock companies that build up modern economies (Hansmann (2009)) and a cornerstone in classic financial intermediation theories (e.g. Ramakrishnan and Thakor (1984)). A fresh perspective on profit-sharing contracts may shed new light on several old questions: what is the nature of the firm, and why do firms arise in a market economy (Coase (1937))? Revisiting the theory of the firm foundations may also help guide future corporate finance studies (Zingales (2000)). Second, ideological arguments over profit-sharing abound in various cultures throughout history.\footnote{For example, the Bible champions equal pay regardless of the amount of contribution (Matthew 20: The Parable of the Workers in the Vineyard); the Jewish classic Babylonian Talmud suggests a quasi-equal “loss” sharing principle in bankruptcy settlement (Kethuboth 93a); the socialism principle “to each according his contribution” (Marx (1875)), applied to an investment setting, suggests splitting profits according to individual investment amount (pro-rata sharing).} A rigorous theory, micro-founded on the most widely-accepted economic principle of private benefit maximization,\footnote{The assumption of individual selfishness is by no means the only one describing human behaviors, nor does wisdom of the crowd apply in all economic settings. Hence the theory developed here is not meant to exactly match the reality, but rather to provide a rational benchmark.} could provide a point of departure for further economic analysis.\footnote{Cooperative game theory provides useful tools for many profit-sharing problems (See e.g. Nash Jr (1950) and Shapley (1952) for original references, Aoki (1984) for a review of applications in the theory of the firm, Brandenburger and Nalebuff (2011) for a popular introduction, and Aumann and Maschler (1985) for a cooperative game theoretical analysis for the Talmud bankruptcy problem. My solution, however, is entirely based on non-cooperative game theory. Compared to cooperative game theory in which the value for a particular subset of players is exogenously specified, in my analysis the value created by any subset of players is endogenous, dependent on their particular sharing contract.} This paper is a small step toward these goals.

Consider a simple thought experiment. Alice and Bob are two identical deep-pocketed and risk-averse investors. They individually decide on how much money to invest in one risky business opportunity. Alice and Bob have different opinions on the return from the business opportunity, as they rely on independent yet unbiased private information sources to update their posterior beliefs. Although neither investor has access to the other’s private
information, it is common knowledge that Alice’s private signal is more accurate than Bob’s.\footnote{To be mathematically precise, assume that the return and private signals all follow normal distributions, i.e. return $\tilde{r} \sim \mathcal{N}(\tilde{r}, \tau^{-1}_r)$ while investor $i$’s signal $s_i = r + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \tau^{-1}_i)$, $\epsilon_i \perp \tilde{r}$, $i \in \{A, B\}$, and $r$ denotes the realization of $\tilde{r}$. Information sources being independent indicates that $\epsilon_A \perp \epsilon_B$, while Alice’s information being more accurate indicates that $\tau_A > \tau_B$. Since both investors are deep-pocketed, their preferences feature no wealth effect, and could be summarized by a constant absolute risk-aversion (CARA) utility function $u(W) = -e^{-\rho W}$ for some $\rho > 0$. Such CARA-normal setup is standard among investment models in finance started by Lintner (1965)).}

Our question is, given that Alice enjoys superior private information, should she ever agree to share investment profits with Bob? Would she ever benefit from profit-sharing? If indeed profit-sharing is preferred, what is the optimal profit-sharing rule for Alice that reflects her information advantage?

Section 1 will analyze this example in detail. The crux of our question lies in recognizing that an appropriately designed profit-sharing contract acts as an information aggregator. Specific to the Alice-Bob example, regardless of her information quality it is always optimal for Alice to agree to go fifty-fifty with Bob. Indeed as a Nash equilibrium outcome, an equal division of profits is also preferred by Bob. If Alice and Bob agree to go fifty-fifty, each of them will earn a profit exactly the same as if he or she had access to the other’s private information (even though he/she actually does not). Because a fifty-fifty contract obtains the first best outcome (i.e. a symmetric-information benchmark), other contracts will only lead to inferior outcomes.

To understand why profit-sharing serves as an information aggregator, notice that when investors write ex ante contracts to share total profits (to be realized ex post), they explicitly link their compensation to others’ investment decisions, which in turn are functions of others’ private information. This linkage created by profit-sharing indirectly transmits private information via action choices.\footnote{This intuition is indeed not so unfamiliar. While conventional wisdom already tells us that “actions speak louder than words”, signaling models further formalize this insight (e.g. Spence (1973) and Leland and Pyle (1977)). The concept of rational expectation is also built on the fact that some endogenous public signals in an economic system could serve as summary statistics for actions of all agents in the economy. The revelation principle in mechanism design presents another incarnation of this insight.} Given a properly designed sharing rule, such indirect
transmission is perfect. Section 2 discusses further necessary conditions for profit-sharing to be a perfect information aggregator.

The insight from the Alice-Bob thought experiment inspires a general framework to model joint-stock companies, where joint-owners provide productive inputs to and share profits from a particular technology. Section 3 proves that sharing profits properly could still serve as a perfect information aggregator for an arbitrary number of heterogeneous players. I characterize the optimal sharing rule under this general setting, and illustrate the rise of (inefficient) “moonlighting” incentives when a profit-sharing contract is not well-designed.

The framework developed in Section 3 provides a laboratory to address several classic questions in the theory of the firm, such as the relationship between a firm and the market (Section 4) and the boundary of the firm (Section 5). Firms arise as an institutional innovation in response to market incompleteness, similar to the role of new securities/markets discussed in the literature on financial innovation (e.g. Grossman (1977), Allen and Gale (1994)); the boundary of a firm is delineated by a trade-off between the benefits from wisdom of the crowd and costs due to free-riding or oligopsony. Variations to the workhorse model also illustrates how information aggregation becomes imperfect in the presence of free-riding, market power, or economy of scale, etc., yet profit-sharing still remains a preferred governance structure.

The rest of the paper is organized as follows. Section 1 analyzes the solution to the Alice-Bob question. Section 2 contains more discussions on sufficient conditions for perfect information aggregation. Section 3 sets up a workhorse model for joint-stock companies, and derives the optimal profit-sharing contract. Section 4 explores the relationship between a firm and the market within my framework and illustrates the endogenous arise of a firm in an incomplete market. Section 5 investigates forces that shape firm boundaries. Section 6 discusses general implications on various corporate governance topics. Section 7 discusses empirical implications of the result. Section 8 relates existing literature. Section 9 concludes.
1 Analysis of the Alice-Bob Example

I first prove that, despite more accurate private signal, to maximize her own expected utility, Alice should accept Bob’s proposal and agree to equally share their total investment profit.

As a recap of the example, both Alice and Bob have preferences summarized by a utility function \( u(W) = -e^{-\rho W} \) for some \( \rho > 0 \), and they individually decide on how much money to invest in a business opportunity with return \( \tilde{r} \sim N(\bar{r}, \tau_r^{-1}) \). The two investors have different opinions on \( \tilde{r} \), as they rely on independent yet unbiased private information sources to update posterior beliefs. Mathematically, investor \( i \) has an unbiased private signal of the project return \( s_i = r + \epsilon_i \), where \( \epsilon_i \sim N(0, \tau_i^{-1}) \), \( \epsilon_i \perp \tilde{r} \), \( i \in \{A, B\} \), and \( r \) denotes the realization of \( \tilde{r} \). Although neither investor has access to the other’s private information, it is common knowledge that Alice’s private signal is more accurate than Bob’s, i.e. \( \tau_A > \tau_B \).

Given an equal division of investment profits, investor \( i \)'s problem is given by choosing investment amount \( x_i \) given \( s_i \) such that

\[
x_i(s_i) = \arg\max_{x_i} \mathbb{E}[e^{-\frac{\rho}{2} \tilde{r}[x + \tilde{r}_{-i}(s_{-i})]} | s_i],
\]

where \( i \in \{A, B\} \) and \( -i = \{A, B\} \backslash \{i\} \). Because the optimum to the right hand side depends on \( i \)'s belief of \( x_{-i}(s_{-i}) \), the solution is given in a Nash equilibrium.

**Definition** A Nash Equilibrium under equal profit-sharing in the Alice-Bob example consists of two investment functions \( x_A(\cdot) \) and \( x_B(\cdot) \) such that

\[
x_i(s_i) = \arg\max_{x_i} \mathbb{E}[e^{-\frac{\rho}{2} \tilde{r}[x + \tilde{r}_{-i}(s_{-i})]} | s_i], \tag{1}
\]

where \( i \in \{A, B\} \) and \( -i = \{A, B\} \backslash \{i\} \).

I will prove the existence and uniqueness of the Nash equilibrium under a more general setting in Section 2. However, in a special case in which all random variables are normally
distributed, a linear Nash equilibrium could be easily obtained via guess and verify. Focusing on symmetric equilibrium and assume

\[ x_i(s_i) = \alpha + \beta_i s_i \]

\[(1) \Rightarrow \alpha + \beta_i s_i = \arg\max_x -E[e^{-\frac{1}{2}\rho \tilde{r}} | x + \alpha + \beta_{-i} \tilde{s}_{-i} | s_i]. \tag{2} \]

The conditional expectation on the right hand side is similar to the moment-generating function of a non-central \(\chi^2\)-distributed random variable (because both \(-\frac{1}{2}\rho \tilde{r}\) and \(x + \alpha + \beta_{-i} \tilde{s}_{-i}\) an affine transformation of the normal variable \(\tilde{s}_{-i}\), follow normal distributions), which has a closed-form expression given by the following lemma.

**Lemma 1.1.** If \[
\begin{bmatrix}
\tilde{y}_1 \\
\tilde{y}_2
\end{bmatrix}
\sim \mathcal{N}\left(
\begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix},
\begin{bmatrix}
\sigma_1^2 & \rho \sigma_1 \sigma_2 \\
\rho \sigma_1 \sigma_2 & \sigma_2^2
\end{bmatrix}
\right),
\]
then

\[
E[e^{\tilde{y}_1 \tilde{y}_2}] = \exp\left(\frac{\left(\theta_1^2 \sigma_1^2 - 2 \rho \theta_1 \theta_2 \sigma_1 \sigma_2 + \theta_2^2 \sigma_2^2 + 2 \theta_1 \theta_2\right)/\left(2[(\rho \sigma_1 \sigma_2 - 1)^2 - \sigma_1^2 \sigma_2^2]\right)}{\sqrt{(\rho \sigma_1 \sigma_2 - 1)^2 - \sigma_1^2 \sigma_2^2}}\right).
\]

**Proof.** Standard integration. \(\square\)

Plug in \(-\frac{1}{2}\rho \tilde{r}\) and \(x + \alpha + \beta_{-i} \tilde{s}_{-i}\) into Lemma 1.1, and notice that conditional on \(s_i\),

\[
\begin{bmatrix}
-\frac{1}{2}\rho \tilde{r} \\
x + \alpha + \beta_{-i} \tilde{s}_{-i}
\end{bmatrix}\bigg|_{s_i} \sim \mathcal{N}\left(
\begin{bmatrix}
-\frac{\rho \tau_{ri} + \tau_{ii} s_i}{2(\tau_{ri} + \tau_{ii})} \\
x + \alpha + \beta_{-i} \frac{\tau_{ri} + \tau_{ii} s_i}{(\tau_{ri} + \tau_{ii})}
\end{bmatrix},
\begin{bmatrix}
\frac{\rho^2}{4(\tau_{ri} + \tau_{ii})} & \frac{-\rho \beta_{-i}}{2(\tau_{ri} + \tau_{ii})} \\
\frac{-\rho \beta_{-i}}{2(\tau_{ri} + \tau_{ii})} & \frac{\beta_{-i}^2}{(\tau_{ri} + 1)(\tau_{ri} + \tau_{ii})}
\end{bmatrix}
\right),
\]

thus the expectation on the right hand side of (2) is equal to
\[
\exp \left\{ \frac{(x + \alpha + \beta - \frac{\tau r + \tau_i s_i}{\tau + \tau_i})^2}{4(\tau + \tau_i)} - \frac{\rho}{2} \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \left( x + \alpha + \beta - \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \right) \beta^{-i} \left( \frac{1}{\tau + \tau_i} + \frac{1}{\tau_i} \right) \right\}
\]

\[
\sqrt{\left( \frac{1}{2} \frac{\rho_\beta^{-i}}{\tau + \tau_i} - 1 \right)^2 - \frac{\rho^2}{4(\tau + \tau_i)} \beta^{-i} \left( \frac{1}{\tau + \tau_i} + \frac{1}{\tau_i} \right)}
\]

Notice that \( x \), the variable we maximize over, only enters the numerator of the exponent in the above expression in a linear-quadratic function, thus (2) leads to

\[
\alpha + \beta_i s_i = \arg\min_x \left\{ \left( x + \alpha + \beta - \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \right)^2 \frac{\rho^2}{4(\tau + \tau_i)} - \frac{\rho}{2} \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \left( x + \alpha + \beta - \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \right) \beta^{-i} \right\}
\]

\[
+ \left( \frac{\rho}{2} \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \right)^2 \beta^{-i} \left( \frac{1}{\tau + \tau_i} + \frac{1}{\tau_i} \right) - \frac{\rho}{2} \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \left( x + \alpha + \beta - \frac{\tau r + \tau_i s_i}{\tau + \tau_i} \right)
\]

\[
= \frac{2}{\rho} (\tau r + \tau_i s_i) - \alpha.
\]

Matching coefficients gives \( \alpha = \frac{1}{\rho} \tau r \) and \( \beta_i = \frac{2}{\rho} \tau_i \), leading to

\[
\begin{align*}
    x_A &= \frac{1}{\rho} (\tau r + 2\tau_A s_A) \\
    x_B &= \frac{1}{\rho} (\tau r + 2\tau_B s_B)
\end{align*}
\]

Thus for any particular joint realization of project return and private signals, investor \( i \)'s payoff under equal profit-sharing is

\[
x_r \frac{x_A(s_A) + x_B(s_B)}{2} = \frac{r}{\rho} (\tau r + \tau A s_A + \tau B s_B).
\]

Let’s compare this outcome with a symmetric-information benchmark. When Alice could (hypothetically) make independent investment decisions based on both her private signal and
Bob’s, her investment amount would be given by

\[ x'_A(s_A, s_B) = \arg \max_x \mathbb{E}[-e^{-\rho x} | s_A, s_B] \]
\[ = \arg \max_x -e^{-\rho \mathbb{E}(r | s_A, s_B) x + \frac{1}{2} \text{Var}(r | s_A, s_B) \rho^2 x^2} \]
\[ \Rightarrow x'_A(s_A, s_B) = \frac{\mathbb{E}(r | s_A, s_B)}{\rho \text{Var}(r | s_A, s_B)} \]
\[ = \frac{1}{\rho} (\tau_r \bar{r} + \tau_A s_A + \tau_B s_B). \]

We thus get the following observation.

**Theorem 1.2.** For all realizations of the state of nature \( \{r, s_A, s_B\} \), each investor’s payoff under equal division of profits is always equal to that under a symmetric-information benchmark. So does the expected utility. A well-designed profit-sharing contract serves as a perfect information aggregator, leading to best outcomes for both investors.

## 2 Profit-sharing as an Information Aggregator

This section relaxes the normality assumption in the previous example, and explores sufficient conditions for profit-sharing to be a perfect information aggregator.

Denote \( u(W) = -e^{-\rho W} \), and consider general distributions of \( r \) and \( s_i, i \in \{A, B\} \). Under a symmetric-information benchmark \( x_i(s_i, s_{-i}) \) maximizes

\[
\mathbb{E}[u(xr)|s_i, s_{-i}]
\]
\[= \int u(xr) f(r|s_i, s_{-i}) dr \]
\[= \int u(xr) f(s_{-i}|r, s_i) f(s_i|r) \frac{1}{f(s_i, s_{-i})} dr \]
\[= \int u(xr) f(s_{-i}|r) f(s_i|r) \frac{1}{f(s_i, s_{-i})} dr, \quad \because s_i \perp s_{-i}|r, \]

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or equivalently \( x(s_i, s_{-i}) \) maximizes

\[
\int u(x) f(s_{-i}|r)f(s_i|r)f(r)dr
\]  

(3)

Assume the profit-sharing agreement stipulates that investor \( i \) gets \( \alpha_i \) of the total profit \( \sum \alpha_i = 1 \), then under profit-sharing \( x_i(s_i) \) maximizes (in a Nash equilibrium)

\[
\mathbb{E} \left[ u \left( \alpha_i x + \alpha_i x_{-i}(s_{-i}) \mathbb{r} \right) \right] | s_i \\
= \iint u(\alpha_i x + \alpha_i x_{-i}(s_{-i}) f(r, s_{-i}|s_i) ds_{-i} dr \\
= \iint u(\alpha_i x + \alpha_i x_{-i}(s_{-i}) f(s_{-i}|r)f(s_i|r)f(r) \frac{1}{f(s_i)} ds_{-i} dr
\]

or equivalently \( x(s_i) \) maximizes

\[
\iint u(\alpha_i x + \alpha_i x_{-i}(s_{-i}) f(s_{-i}|r)f(s_i|r)f(r)ds_{-i}dr
\]  

(4)

Taking first-order conditions we have that

(3) \( \Rightarrow \) \[
\int u'(x_i(s_i, s_{-i}) r) f(s_{-i}|r)f(s_i|r)f(r) dr = 0
\]  

(5)

(4) \( \Rightarrow \) \[
\iint u'(\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r f(s_{-i}|r)f(s_i|r)f(r) ds_{-i} dr = 0,
\]  

(6)

where (with some abuse of notation) \( x(s_i, s_{-i}) \) denotes the optimal investment amount given signal \( s_i \) and \( s_{-i} \) under the full information benchmark, while \( x_i(s_i) \) denotes investor \( i \)'s investment amount given signal \( s_i \) in the profit-sharing Nash equilibrium.

In order to keep tractability, in the spirit of Breon-Drish (2015), I further assume that the likelihood function of \( r \) conditioning on private signals \( s_i, i \in \{A, B\} \) lies in an exponential family, an assumption extensively used in Bayesian statistics and decision theories to preserve
closed-form expression. Precisely, assume that

\[ f(s_i|r) = h_i(s_i)e^{r_{k_is_i}}g(r) \]

for some constant \( k_i \) and positive function \( h_i(\cdot) \). then

\[
\begin{align*}
(5) \quad &\Rightarrow \quad \int e^{-\rho x_i(s_i,s_{-i})}r h_{-i}(s_{-i})e^{r_{k_{i}s_{-i}}}g(r)h_i(s_i)e^{r_{k_is_i}}g(r)f(r)dr = 0 \\
(6) \quad &\Rightarrow \quad \int \int e^{-(\rho x_i(s_i)+\alpha_i z_{-i}(s_{-i}))r} h_{-i}(s_{-i})e^{r_{k_{i}z_{-i}}}g(r)h_i(s_i)e^{r_{k_is_i}}g(r)f(r)ds_{-i}dr = 0
\end{align*}
\]

thus (factoring out \( h_i(s_i) \))

\[
\begin{align*}
(7) \quad &\Rightarrow \quad \int e^{-\rho x_i(s_i,s_{-i})+r_{k_{i}s_{-i}}+r_{k_is_i}}g^2(r)f(r)dr = 0 \\
(8) \quad &\Rightarrow \quad \int \int e^{-(\rho x_i(s_i)+\alpha_i z_{-i}(s_{-i}))r} h_{-i}(s_{-i})e^{r_{k_{i}z_{-i}}+r_{k_is_i}}g^2(r)f(r)ds_{-i}dr = 0 \\
&\Rightarrow \quad \int e^{-\rho x_i(s_i)+r_{k_is_i}}g^2(r)f(r) \left( \int e^{-\rho x_i(s_{-i})}h_{-i}(s_{-i})e^{r_{k_{i}s_{-i}}}ds_{-i} \right)dr = 0
\end{align*}
\]

We thus have the following result

**Theorem 2.1.** Under the symmetric-information benchmark, equation (9) has a unique solution, which is linear in \( s_i \) and \( s_{-i} \). Similarly, under any profit-sharing agreement (i.e. for any given \( \alpha_i \)), equation (10) has a unique Nash equilibrium, in which investor \( i \)'s strategy is linear in \( s_i, \forall i \). When the profit-sharing agreement is optimally designed, profit-sharing obtains the same payoff as in the symmetric-information benchmark. In other words, profit-sharing serves as a perfect information aggregator.

**Proof.** Consider the equation of \( x \)

\[ \int e^{\tau r}g^2(r)f(r)dr = 0. \]  

\[ ^6 \text{E.g. exponential family is particularly useful for deriving conjugate priors.} \]
Taking derivative with respect to \(x\) immediately tells that equation (11) has at most one solution, denoted as \(X\). Compared to equation (9) we get \(x_i(s_i, s_{-i}) = \frac{1}{\rho}(k_{-i}s_{-i} + k_is_i - X)\).

Similarly, consider the equation of \(x\)

\[
\int e^{rx}rg^2(r)f(r)H_{-i}(r)dr = 0,
\]

where \(H_{-i}(r) = \int e^{-\rho\alpha_i x_{-i}(s_{-i}) x}h_{-i}(s_{-i})e^{rk_{-i}s_{-i}}ds_{-i} > 0\). Taking derivative with respect to \(x\) immediately tells that the equation features at most one solution (for a given \(x_{-i}(s_{-i})\)). Compared to equation (10) we get that \(x_i(s_i) = \frac{k_is_i - C'}{\rho\alpha_i}\), where \(C'\) is a constant such that

\[
\int e^{rC'}rg^2(r)f(r)H_{-i}(r)dr = 0. \tag{12}
\]

By the same logic, \(x_{-i}(s_{-i}) = \frac{k_{-i}s_{-i} - C''}{\rho\alpha_{-i}}\), where \(C''\) is also a constant such that

\[
\int e^{rC'}rg^2(r)f(r)H_{i}(r)dr = 0, \tag{13}
\]

where \(H_{i}(r) = \int e^{-\rho\alpha_{-i}x_{i}(s_i)} h_{i}(s_i)e^{rk_{i}s_i}ds_i > 0\). Plug in \(x_i\) and \(x_{-i}\) into (12), we have

\[
\int e^{rC'}rg^2(r)f(r) \int e^{\frac{\alpha_i}{\rho\alpha_{-i}}(C''-k_{-i}s_{-i})}h_{-i}(s_{-i})e^{rk_{-i}s_{-i}}ds_{-i}dr = 0. \tag{14}
\]

If \(\alpha_i = \alpha_{-i} = \frac{1}{2}\), equation (14) simplifies into (after factoring out \(\int h_{-i}(s_{-i})ds_{-i}\))

\[
\int e^{r(C'+C')}rg^2(r)f(r)dr = 0. \tag{15}
\]

Since equation (11) has at most one solution, we have \(C + C' = X\). Thus under profit-sharing
the payoff to investor $i$ for a given realization of $r$ and private signals is

$$\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r$$

$$= \alpha_i \frac{k_i s_i - C}{\rho \alpha_i} r + \alpha_i \frac{k_{-i} s_{-i} - C'}{\rho \alpha_{-i}} r$$

$$= \frac{r}{\rho} (k_i s_i - C + k_{-i} s_{-i} - C'), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2}$$

$$= \frac{r}{\rho} (k_i s_i + k_{-i} s_{-i} - X), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2},$$

which exactly equals to $rx(s_i, s_{-i})$, or the payoff under a full information benchmark. □

Theorem 2.1 can be easily extended to scenarios with more than two investors of heterogeneous risk preferences. The exact math is tedious but straightforward, and is omitted for the sake of brevity.\footnote{I conjecture yet has been able to prove that under more general settings nonlinear sharing rules exist to obtain perfect information aggregation.} Section 3.1, however, provides an illustrative example with normal distributions, which also embeds a workhorse model to describe a joint-stock company.

3 Workhorse Model for a Joint-Stock Company

The structure of modern corporations have evolved into prohibitive complicacy, yet they do share some common features. A given firm features a particular risky production technology, to which a set of “owners” provide (usually relatively homogeneous) production inputs (capital, labor, or raw ingredients, etc.), and from which the same set of owners share residual earnings according to a pre-specified rule. The governance structure of a particular firm is determined its owners’ assessments of the firm’s business prospect. This primitive description of firms with emphases on profit-sharing and risk-taking is consistent with the corporate legal literature traditions (Hansmann (2009)), and is partially reflected in two synonyms of
firms as “companies” and “ventures”. Firms in this form has been accompanying human history ever since Queen Elizabeth granted the East India Company its first Royal Charter on December 31st, 1600 AD, and in today still underlies partnerships (e.g. private equity firms), producer cooperatives, joint-ventures, and (at more hidden level) all firms except for sole-proprietorship.

Insights learned from the Alice-Bob example suggest a possibility that joint-stock companies thrive because they empowers their owners of wisdom of the crowd. This section formalizes this thought.

3.1 The Firm as a Profit-Sharing Coalition

A firm features a charter, which stipulates a compensation scheme among its \( n \) owners (players). Upon firm creation in period \( t = 0 \), the charter entitles owner \( i \), who has a constant absolute risk aversion parameter \( \rho_i \), of \( a_i \) of the firm’s residual earning to be realized by the end of period \( t = 1 \), where \( \sum_{i=1}^{n} a_i = 1 \). The firm has a constant-return-to-scale production technology \( Y = vX \), where \( Y \) is the total earning, \( v \sim \mathcal{N}(\bar{v}, \tau_v^{-1}) \) is a stochastic factor productivity, and \( X \) is total amount of productive input contributed by all the owners, i.e. \( X = \sum_{j=1}^{n} x_i \), where \( x_i \) is player \( i \)’s productive input contribution.\(^8\) For the moment I do not consider any private cost to each player’s input supply, a point I will address in Section 5. The assumption of wisdom of the crowd indicates that each player has some private knowledge in assessing the stochastic factor productivity. Assume player \( i \)’s private knowledge \( s_i = v + e_i \), where \( v_{\parallel} e_i \) and \( e_i \sim \mathcal{N}(0, \tau_j^{-1}) \). Each player independently decides how much production input to put into the firm Depending on context, I will use “player”, “partner”, “stakeholder”, “investor”, “owner”, or “entrepreneur” interchangeably.

\(^8\)By assuming a constant-return-to-scale production technology, I shut down any complementarity in players’ inputs which would mechanically favor firm creation. For different modeling purposes, existing literature usually assumes non-separable production technologies, e.g. Alchian and Demsetz (1972). In these models agents’ productive input choices impose (usually positive) externalities on each other. Such externalities can either come from output (e.g. Kandel and Lazear (1992)) or cost (e.g. Edmans, Goldstein, and Zhu (2011)).
The input provided by the \( n \) owners of the firm is given in a Nash equilibrium. In particular, entrepreneur \( i \) chooses \( x_i \) to maximize

\[
\mathbb{E} \left[ -\exp \left( -\rho_i \left( a_i v(x_i + \sum_{k \neq i} x_k) \right) \right) | s_i \right],
\]

(16)
given her perception of other players’ equilibrium productive input \( x_k, k \neq i \). The following theorem provides a linear Nash equilibrium solution for a given player.

**Theorem 3.1.** A linear Nash equilibrium exists only when \( a_i = \frac{1}{\sum_{i=1}^{n} \pi_i} \), and player \( i \)’s equilibrium productive input is given by

\[
\frac{\tau_i v}{\rho_i} + \left( \sum_{i=1}^{n} \frac{1}{\rho_i} \right) \tau_i s_i,
\]

(17)

**Proof.** A linear symmetric equilibrium is given by \( x_k = \pi_k + \gamma_k s_k \) for some \( \pi_k \) and \( \gamma_k \). Because

\[
\begin{bmatrix}
-a_i \rho_i v \\
x_i + \sum_{k \neq i} x_k
\end{bmatrix}
\]

\[
\sim
\begin{bmatrix}
-\rho_i a_i \mathbb{E}(v|s_i) \\
x_i + \sum_{k \neq i} \pi_k + \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i)
\end{bmatrix}
\]

\[
\mathcal{N}\begin{bmatrix}
\rho_i^2 a_i^2 \mathbb{E}(v|s_i) + \rho_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) + \gamma_k^2 \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) \\
-\rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) - \rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) - \rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) - \rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) - \rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) - \rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i)
\end{bmatrix}
\]

by Lemma 1.1, entrepreneur \( i \) equivalently minimizes

\[
\theta_2 \rho_i^2 a_i^2 \mathbb{E}(v|s_i) + 2\theta_1 \rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) + \theta_1^2 \left[ \left( \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) \right) + \gamma_k^2 \sum_{k \neq i} \gamma_k \right] + 2\theta_1 \theta_2
\]

\[
\Rightarrow 2\theta_2 \rho_i^2 a_i^2 \mathbb{E}(v|s_i) + 2\theta_1 \rho_i a_i \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) + 2\theta_1 = 0,
\]

where \( \theta_1 = -\rho_i a_i \mathbb{E}(v|s_i) \) and \( \theta_2 = x_i + \sum_{k \neq i} \pi_k + \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) \).

14
Plugging in $x_i = \pi_i + \gamma_i s_i$ leads to

$$\left[ \sum_{k \neq i} \pi_k + \pi_i + \gamma_i s_i + \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) \rho_k \eta_i^2 \text{Var}(v|s_i) - \rho_i a_i \mathbb{E}(v|s_i) \right] = 0$$

and matching coefficients renders $\gamma_i = \frac{\pi_i}{\tau_v \rho_i} \Pi, \quad \Pi \equiv \sum_{i=1}^n \pi_i$, and thus $a_i = \frac{\gamma_i}{\sum_{i=1}^n \frac{1}{\rho_i}} \Pi = \left( \sum_{i=1}^n \frac{1}{\rho_i} \right) \tau_v \bar{v}$. Notice that $\pi_k$ is indeterminate. It is natural to look at one particular equilibrium in which $\pi_k = \frac{\tau_v \bar{v}}{\rho_k}$, leading to $x_i = \frac{\tau_v \bar{v}}{\rho_i} + \left( \sum_{i=1}^n \frac{1}{\rho_i} \right) \tau_i s_i$. \hfill \Box

Given no complementarities in productive inputs, one may be tempted to think that entrepreneurs should be indifferent between running a sole proprietorship or taking part in a firm, nor would firm creation affect real allocation. However, Theorem 3.1 shows that a player becomes more dedicated to the project when in a firm whenever she has positive assessment of the project prospect (i.e., $x_i$ increases with $n$ when $s_i > 0$). Because a player’s assessment are more likely to be positive for a high-value ($v$) project, creating a firm (rather than keeping multiple sole proprietorships) helps a good project to receive (probablistically) higher total productive input. Firm creation improves input allocation.

This result stems from the fact that a firm serves as a de facto information aggregation mechanism. Notice that if entrepreneur $i$ has full information, her input supply would be

$$x_i = \frac{\tau_v \bar{v}}{\rho_i} + \frac{\sum_{i=1}^n \tau_i s_i}{\rho_i}, \text{ where } s_i = \frac{\sum_{i=1}^n \tau_k s_k}{\sum_{k=1}^n \tau_k}.$$

Thus entrepreneur $i$’s payoff is

$$v \left[ \frac{\tau_v \bar{v}}{\rho_i} + \frac{\sum_{i=1}^n \tau_i \hat{s}_i}{\rho_i} \right]$$

under both full information and in a firm. This is summarized in the following theorem.

**Theorem 3.2.** Given her private project assessment, an entrepreneur’s expected utilities of participating in a properly structured $n$-owner firm is identical to as if she could obtain other
n − 1 entrepreneurs’ private knowledge without cost while running a sole proprietorship.

One direct implication of this result is that firm creation raises entrepreneurs’ expected utilities, and thus is a voluntary outcome of economic evolution. An alternative way to understand the de facto information aggregation effect comes in the corporate setting recognizes that a firm’s charter changes its stakeholders’ risk taking incentives (compared to when running as sole-proprietors). In a CARA-normal setting, each stakeholder’s productive input decision is given by a mean-variance trading-off. When in a firm, stakeholder i’s compensation partially comes from $a_i$ of the expected value of her contribution bearing only $a_i^2$ of its variance. Her productive input is thus more responsive to her knowledge because the sharing rule in a firm puts her in front of a (as if) less risky project. This effect is reminiscent of what has become conventional wisdom since the seminal work of Markowitz (1952), Sharpe (1964), and Lintner (1965) that proper diversification achieves optimal return-risk trade-off. Firm creation provides means for human-capital diversification.

However, the de facto information aggregation channel is different from traditional diversification arguments. First, diversification as in portfolio theory relies on pooling multiple assets and forming portfolios, yet in a profit-sharing firm only one single investment project is needed, and the the de facto information aggregation is achieved through teaming agents. Second, portfolio theory diversification does not require asymmetric information, yet the de facto information aggregation channel is built on stakeholders having dispersed private information. In fact, if there is no private knowledge (i.e., $\tau_e = 0$), profit-sharing would make no difference. The relation between information aggregation and risk-sharing, however, suggests an alternative interpretation of profit-sharing as institutional innovation, compared to “financial innovation” based on security design à la Allen and Gale (1994).
3.2 “Moonlighting” under a Suboptimal Sharing Contract

If a firm disregard each partner’s risk preference and stipulates arbitrary sharing rules, its stakeholders will have incentives to “moonlight”, or to contribute to private enterprises in addition to the firm in which they jointly own. To see this, consider an entrepreneur $i$ who has $a_i$ shares in a firm choosing $x_i$ and $X_i$ to maximize

$$
\mathbb{E} \left[ - \exp \left( -\rho_i \left( a_j v(x_i + \sum_{k \neq i} x_k) + vX_i \right) \right) \bigg| s_i \right],
$$

(18)
given her (correct) anticipation of other stakeholders’ equilibrium input provision to the firm $x_k, k \neq i$. The following theorem provides a linear Nash equilibrium solution.

**Theorem 3.3.** Each stakeholder’s expected utility is maximized when ownership shares in the firm is divided according to stakeholders’ risk preferences, i.e. $a_i = \frac{1}{\rho_i}$. In the resulting linear Nash equilibrium, stakeholders do not moonlight.

**Proof of Theorem 3.3.** A linear symmetric equilibrium is given by $x_k + \frac{X_k}{a_k} = \pi_k + \gamma_k s_k + \frac{\pi_k + \Gamma_k s_k}{\alpha_k}$ for some $\pi_k$ and $\gamma_k$. Because

$$
\begin{bmatrix}
-a_i \rho_i v \\
x_i + \frac{X_i}{a_i} + \sum_{k \neq i} x_k \\
\end{bmatrix}
\sim
\begin{bmatrix}
-a_i \rho_i v \\
x_i + \frac{X_i}{a_i} + \sum_{k \neq i} x_k \\
\end{bmatrix}
| s_i
$$

$$
\mathcal{N}
\begin{pmatrix}
-\rho_i a_i \mathbb{E}(v|s_i) \\
\rho_i^2 a_i^2 \text{Var}(v|s_i) \\
-\rho_i a_i \sum_{k \neq i} \gamma_k \text{Var}(v|s_i) \\
\rho_i^2 a_i^2 \text{Var}(v|s_i) \\
-\rho_i a_i \sum_{k \neq i} \gamma_k \text{Var}(v|s_i) \\
\end{pmatrix}
\begin{pmatrix}
-\rho_i a_i \sum_{k \neq i} \gamma_k \text{Var}(v|s_i) \\
-\rho_i a_i \sum_{k \neq i} \gamma_k \text{Var}(v|s_i) \\
(\sum_{k \neq i} \gamma_k)^2 \text{Var}(v|s_i) + \sum_{k \neq i} \gamma_k^2 \tau_k^{-1} \\
\end{pmatrix}
\)
by Lemma 1.1, entrepreneur \(i\) equivalently minimizes

\[
\theta_2^2 \rho_i a_i^2 \text{Var}(v|s_i) + 2 \theta_2 \rho_i a_i \sum_{k \neq i} \gamma_k \text{Var}(v|s_i) + \theta_1^2 \left[ \left( \sum_{k \neq i} \gamma_k \right)^2 \text{Var}(v|s_i) + \sum_{k \neq i} \gamma_k^2 \tau_k^{-1} \right] + 2 \theta_2 \theta_2 \text{FOC} \Rightarrow 2 \theta_2 \rho_i a_i^2 \text{Var}(v|s_i) + 2 \theta_1 \rho_i a_i \sum_{k \neq i} \gamma_k \text{Var}(v|s_i) + 2 \theta_1 = 0,
\]

where \(\theta_1 = -\rho_i a_i E(v|s_i)\) and \(\theta_2 = x_i + \frac{X_i}{a_i} + \sum_{k \neq i} \pi_k + \sum_{k \neq i} \gamma_k E(v|s_i)\).

Plugging in \(x_i + \frac{X_i}{a_i} = \pi_i + \gamma_i s_i + \frac{\pi_k + \Gamma_k s_k}{a_k}\) leads to

\[
\left( \sum_{k \neq i} \pi_k + \pi_i + \gamma_i s_i + \frac{\pi_k + \Gamma_k s_k}{a_k} + \sum_{k \neq i} \gamma_k E(v|s_i) \right) \rho_i a_i^2 \text{Var}(v|s_i) - \rho_i a_i E(v|s_i) \rho a_i \sum_{k \neq i} \gamma_k \text{Var}(v|s_i) - \rho_i a_i E(v|s_i) = 0
\]

and matching coefficients renders \(\gamma_i + \frac{\Gamma_i}{a_i} = \frac{\pi_i}{\tau_i \theta} (\Pi + \frac{\pi_i}{a_i}), \frac{1}{\tau_i \theta} (\Pi + \frac{\pi_i}{a_i}) \rho a_i = 1 (\Pi \equiv \sum_{i=1}^{n} \pi_i)\).

Thus expected utility is given by

\[
-\frac{\sqrt{\tau_v + \tau_i} \exp \left\{ -\frac{(\tau_v + \tau_i) E^2(v|s_i)}{2} \right\}}{\sqrt{\tau_v + \tau_i + 2 \rho_i a_i \sum_{k \neq i} \gamma_k - \rho_i a_i^2 \sum_{k \neq i} \gamma_k^2 \tau_k^{-1}}},
\]

which is maximized at \(\gamma_i = \frac{\pi_i}{\rho_i a_i}\). Plugging in \(\gamma_i + \frac{\Gamma_i}{a_i} = \frac{\pi_i}{\tau_i \theta} (\Pi + \frac{\pi_i}{a_i})\) and \(\frac{1}{\tau_i \theta} (\Pi + \frac{\pi_i}{a_i}) \rho a_i = 1\) lead to the at the optimal \(\gamma_i, \Gamma_i, \rho\) = 0. Thus for any given sharing rule \(a_k (k = 1, \cdots, n)\), there exists a linear equilibrium in which each entrepreneur optimally chooses her amount of input supply both within and outside of a firm. In particular, when \(a_i\) is chosen to be \(\frac{1}{\sum_{i=1}^{n} \pi_i}\), input supply in the firm can be stipulated so that no entrepreneur has incentive to work outside of the firm.\(^9\)

\(^9\)In particular, when the entrepreneurs have homogeneous risk preference (but possibly heterogeneous knowledge precisions), a \(\frac{1}{n}\) equal sharing rule is optimal.
4 Arise of a Joint-Stock Firm in a Market Economy

An important question in the theory of the firm concerns the relationship between a firm and the market a large. To this end, this section generalizes the workhorse model by adding a productive input market, and allow the cost of the productive input to be determined as an equilibrium market outcome. The setup for the productive input market resembles classic noisy rational expectation models à la Hellwig (1980) and Diamond and Verrecchia (1981). I show that while the market also communicates information through equilibrium price, it is nevertheless dominated by profit-sharing due to the presence of market “noise”.\footnote{See Black (1986) for a comprehensive assessment of “noise”. Existing literature often attributes the non-informative “noise” to quantity shocks (or noise traders), but this is not necessary. The noise could be interpreted as a reduced-form description of investors’ incomplete knowledge about market architecture. Alternatively, it could be viewed as a partial equilibrium outcome, in which some un-modeled outside market also influences price (this is indeed a justification for treating the risk-free rate as exogenous in most noisy rational expectation equilibrium models).} In this sense, a joint-stock company arises in response to market incompleteness caused by asymmetric information.

The market for the productive input consists of a continuum of players with player $i$ having a constant absolute risk aversion (CARA) utility of risk aversion $\rho_i$, $i \in [0, 1]$. There are two-periods. On $t = 0$, a risky business opportunity with factor of productivity $v \sim \mathcal{N}(\bar{v}, \tau_v^{-1})$ emerges. Player $i$ decides on $x_i$, the optimal amount of productive input to provide to the investment opportunity. When making decisions, player $i$ has a private signal of the project valuation $s_i = v + e_i$, where $v$ and $e_i$ are independent and $e_i \sim \mathcal{N}(0, \tau_i^{-1})$. A quantity noise $z \sim \mathcal{N}(\bar{z}, \sigma_z^2)$ measures the aggregate demand for the productive input in alternative uses other than the new business opportunity. Assume that $z$ carries no information about the new business opportunity, i.e., $z \perp v$.

Assume that players $1, 2, \cdots, n$ agree to created a firm and share profits. I first present a general result of their optimal input provisions and corresponding expected utilities given that the equilibrium input price is determined in an arbitrary linear price system.
**Theorem 4.1.** In an economy in which the equilibrium input cost follows a linear function \( p = \mu + \pi v - \gamma z \), the optimal input provision amount of an entrepreneur \( i \) in a firm of size \( n \) is given by

\[
x_i = \frac{1}{\rho_i} \left[ \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma^2} (\mu - \gamma \bar{z}) \right] + \left( \sum_{k=1}^{n} \frac{1}{\rho_k} \right) \tau_i s_i - \frac{1}{\rho_i} \left[ \sum_{k=1}^{n} \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma^2} - \frac{\pi}{\gamma^2 \sigma^2} \right] p \tag{19}
\]

and her expected utility

\[
\exp \left( -\frac{1}{2} \frac{1}{\tau_i + \frac{x^2}{\gamma^2 \sigma^2} + \tau_v} \left[ \tau_i s_i - \frac{\pi}{\gamma^2 \sigma^2} (\mu - \gamma \bar{z}) + \tau_v \bar{v} - (\tau_i + \tau_v + \frac{\pi^2}{\gamma^2 \sigma^2} - \frac{\pi}{\gamma^2 \sigma^2})p \right]^2 \right) \nonumber
\]

\[
= \sqrt{\frac{\sum_{k=1}^{n} \tau_k + \frac{x^2}{\gamma^2 \sigma^2} + \tau_v}{\tau_i + \frac{x^2}{\gamma^2 \sigma^2} + \tau_v}}
\]

**Proof.** See Appendix. \( \square \)

In comparison, under a (hypothetically) symmetric information benchmark, each entrepreneur will base her decision on the weighted average of the private signals of all \( n \) entrepreneurs. Denote \( \sum_{k=1}^{n} \tau_k s_k / \sum_{k=1}^{n} \tau_k \) as \( s^* = \bar{v} + e^* \), then \( v \| e^* \) and \( e^* \sim \mathcal{N}(0, \frac{1}{\sum_{k=1}^{n} \tau_k}) \).

With a linear cost system in which \( p = \mu + \pi v - \gamma z \), the input provision by member \( i \) in an signal-sharing alliance is given by maximizing \( \mathbb{E} \left[ -\exp(-\rho_i (v - p) x_i | s^*, p) \right] \), and thus

\[
x_i' = \frac{\mathbb{E}(v|s^*, p) - p}{\rho_i \text{Var}(v|s^*, p)}
\]

\[
= \frac{1}{\rho_i} \left[ \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma^2} (\mu - \gamma \bar{z}) + \sum_{k=1}^{n} \tau_k s^* - (\sum_{k=1}^{n} \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma^2} - \frac{\pi}{\gamma^2 \sigma^2})p \right] \tag{20}
\]

(Notice that \( \{ \begin{align*}
\mathbb{E}(v|s^*, p) &= \frac{\gamma^2 \sigma^2 \sum_{k=1}^{n} \tau_k s^* + \pi (\mu - \gamma \bar{z}) + \gamma^2 \sigma^2 \tau_v \bar{v}}{\gamma^2 \sigma^2 \sum_{k=1}^{n} \tau_k + \pi^2 + \gamma^2 \sigma^2 \tau_v} \\
\text{Var}(v|s^*, p) &= \frac{\gamma^2 \sigma^2}{\gamma^2 \sigma^2 \sum_{k=1}^{n} \tau_k + \pi^2 + \gamma^2 \sigma^2 \tau_v}
\end{align*} \})

Compared (19) with (20), it is easy to verify that the total (ex post) input provision from a firm and that from players under symmetric information benchmark are identical.
Hence equilibrium market price is the same across both structures. In another word, there is an isomorphism in terms of price between under profit-sharing and under symmetric information. This result leads to a similar isomorphism in terms of \((interim)\) expected utility, as summarized below.

**Theorem 4.2.** An entrepreneur’s interim expected utilities conditioning on her own private signal are the same in a joint-stock company and under a symmetric information benchmark.

*Proof.* See Appendix. \(\square\)

**Comment:** The expected utility equivalence between forming a partnership and direct communication illustrates that from an information aggregation perspective, a competitive market with dispersed private information is intrinsically unstable. Rational agents will always have incentive to partner with others.

The equivalence result, however, carries over to much broader settings. As long as the economy features the wisdom of crowds, a linear equilibrium within a partnership achieves *de facto* information aggregation.\(^{11}\) Furthermore, as long as market participants also perceive the economy of featuring wisdom of crowds, creating a partnership will be a voluntary outcome of market participants’ utility maximization behavior.\(^{12}\)

As compared to rational expectation inference from the market price, profit-sharing provides a non-price based mechanism for information aggregation that helps complete the market with dispersed information. Indeed, the idea of rational expectation in its most general form assumes that when agents make decisions, they rely on not only their own

---

\(^{11}\)Although widely true, the wisdom of crowds is occasionally violated due to either behavioral biases (e.g. the above-average effect is observed in some individualist cultures in which people rate themselves as being above average, say, a better driver than the average) or principle-agent problems (client short-termism leading to herding among fund manager.

\(^{12}\)The perception of of the economy featuring wisdom of crowd is one the underpinnings of the entire rational expectation thoughts. As in one the earliest paper on rational expectation, Muth (1961) points out that rational expectation implicitly assume certain “humbleness” among economic agents. However widely accepted assumption might not be universal. For example, many studies assume that economic agents possess over-confidence or agree to disagree (thus violating the Harsanyi principle).
information, but also further inference from any endogenous variables in the economic system. In a financial market, such endogenous variables are usually thought of as equilibrium market prices. When the market price is absent, creating a new market (thus creating new prices) facilitates information aggregation, and thus completes the market under asymmetric information. This is the essence of financial innovation. A notable example is Grossman (1977), who in his dissertation provides an explanation for the existence of futures markets based on this insight. Institutional innovations such as creating a firm resemble security design in a financial market.

When we go beyond the financial market, however, a lot of other endogenous variables arise. Indeed, rational expectation in its most general form is never confined to market prices. In particular, any publicly observable outcomes of a set of economic interactions among multiple parties could serve as the endogenous variable concerned (because it is affected by the behaviors of all involved parties). Outputs of a joint project provide excellent examples.

As we have seen that the output of a joint project could be a good endogenous variable to contingent on, then why not explicitly link each stakeholder’s compensation to the output? Such practice is essentially equity-based compensation, or the sharing of residual earnings, which defines ownership à la Hansmann (2009). A general theory of the firm arises in this sense. The CARA-normal case, describing agricultural cooperatives, gives a concrete example for the general insight above.

The complementarity between a co-operative firm and the market in terms of information aggregation is related to the complementarity between banks and markets discussed in Boot and Thakor (1997). In a dynamic version of that model, Song and Thakor (2010) highlight that banks and markets exhibit three forms of interaction: competition, complementarity and co-evolution. My paper thus provides an alternative perspective on the complementarity between market and more general institutional structures in a different context.

There are extensive discussions in the finance literature on how a well-functioning cap-
ital market aggregates (dispersed) information among market participants. Despite its very power, rational expectation is not without limitations. The effectiveness of rational expectation crucially depends on the capability to take actions contingent on market prices. In an organized stock market, price-contingent investment could be implemented via posting a demand schedule consisting of a spectrum of limit orders at every incremental price. However, once we move beyond secondary market trading, price-contingent actions are not always available. Private equity/venture capital investment provides a good example.

Furthermore, for many real sector decisions, the supply of certain production inputs has to be determined one period ahead, and in absolute rather than price contingent amounts. For example, in the agricultural sector, at the beginning of each growing season, a farmer has to determine the exact amount of crops to plant (rather than an amount contingent on the spot price to be realized at the next harvest). Because of the impossibility of price-contingency, the farmer has to rely solely on his imprecise information about the next period’s crop demand, of which the inherited risk would deter him from planting all but a trivial amount. A well-organized agricultural cooperative, as a profit-sharing mechanism among (potentially a large number of) farmers, could effectively aggregate information dispersed among its members, reduce each member’s perceived risk, and thus enable more efficient planting decisions. Grossman (1977) uses a similar information aggregation argument to develop a theory for the existence of futures markets. My analysis reveals a similarity between futures markets and jointly-owned firms on information aggregation, consistent with the coexistence of futures markets and producer cooperatives across various commodities.

Because a partnership serves an information aggregation device, the following result follows immediately.

**Corollary 4.3.** *When the partnership size is adequate small, each partner’s expected utility*
strictly increases in the partnership size \( n \).

The optimal firm size is infinity in our setup so far because we have only considered the benefit of wisdom of the crowd. Once other frictions related to a large firm are considered, the boundary of the firm could be delineated.

5 Forces that Shape Firm Boundaries

Corollary 4.3 suggests that firm creation features (locally) “the more the merrier”. However, as a firm looms large, several natural forces would kick in to restrict firm size. An optimal firm size could thus be determined.

5.1 The Boundary of the Wise Crowd

Because in our theory the benefit of firm creation comes from wisdom of the crowd, a straightforward force to restrict firm size is a boundary of the crowd. Such a boundary is plausible, if the particular business in question features agree to disagree or overconfidence, etc. Since overconfident players believe that they are the smartest and having nothing to learn from others, they have no incentive to form a firm with others. Indeed as Muth (1961) in his seminal article on rational expectation points out, rational expectation is built on the assumption of people’s humbleness. If this assumption fails, then there is no room for rational expectation, nor for a theory of the firm based on rational expectation.\(^\text{14}\) Although whether a particular business line features wisdom of the crowd or not is an empirical question, we can conclude that it would be schizophrenic if we study the market with rational expectation, while refusing to look at firms via the perspective of information aggregation.

\(^\text{14}\)Even in an economy where the wisdom of the crowd applies, search frictions (i.e. it takes time to find another entrepreneurs interested in the same business) naturally limits firm size. Note that search dynamic is a key element in information percolation models built on direct communication.
5.2 Costly Information Acquisition

Another force shaping the firm boundary arises when players have to incur some acquisition cost to get their private information. When such acquisition cost is private and not contractible, profit-sharing would lead to a free-riding problem. Free-riding costs trade-off benefits from information aggregation, and determines an optimal firm size. We shall note that such free-riding channel is also present when we study the financial market. Grossman and Stiglitz (1980) rely on this free-riding problem to prove the impossibility of strong form market efficiency. Thus this form of free-riding problem shall not concern us if our focus is on comparing the relative role of a firm versus the market in knowledge aggregation.

5.3 Private Cost in Providing the Productive Input

Another form of free-riding cost arises when the cost of the productive input is private. Such cost resembles the moral hazard in teams problem studied by Holmström (1982). In this case, the decision to form a firm involves a trade-off between the benefit from de facto knowledge aggregation and a free-riding problem. I illustrate below.\footnote{I assume $\rho_i = \rho$ for simplicity.}

If the cost of productive input has to be born by player $i$ in its entirety, she would choose $x_i$ to maximize

$$\mathbb{E} \left[ -\exp \left( -\rho \left[ \frac{1}{n} v(x_i + \sum_{k \neq i} x_k) - p x_i \right] \right) \middle| s_i \right],$$

given her anticipation of other stakeholders’ equilibrium productive input level $x_k$, $k \neq i$. Using similar solution technique, the stakeholder $i$’s equilibrium dedication to the firm is given by

$$x_i = \frac{\tau_v \bar{v}}{\rho} - \frac{1}{\rho} n p \left( \tau_v + n \tau_e \right) + \frac{n \tau_e}{\rho} s_i$$

(21)
The equilibrium dedication level consists of three parts. The first part \( \frac{\tau_{e}}{\rho} \) is a constant, the second part \( -\frac{1}{\rho} np \left( \tau_{o} + n \tau_{e} \right) \) represents a free-riding effect, and the third part \( \frac{n p^2}{\rho} s_{i} \) represents the \textit{de facto} knowledge aggregation effect. Under this framework, the relative magnitudes of the two effects determines optimal firm size.\(^\text{16}\)

In the context of information gathering agencies, \textit{Millon and Thakor} (1985) also analyze how moral hazard related intrafirm costs within a partnership pin down a finite optimal size of the firm. In addition to differences in context, in \textit{Millon and Thakor} (1985) the benefit of forming a partnership comes from \textit{direct} information communication, while in my paper the benefit comes from \textit{de facto} aggregation of wisdom of the crowd.\(^\text{17}\)

The case where private cost \( c \) equals 0 corresponds to costless intrafirm monitoring. This assumption appears in \textit{Ramakrishnan and Thakor} (1984), who develop a theory of financial intermediation, where information producers also write \textit{ex ante} contracts on \textit{ex post} payoffs. The specific sharing rule in the current paper is similar their independent (not IMJC) contract. The difference is that, in \textit{Ramakrishnan and Thakor} (1984), it is the information producers who are producing information about entrepreneurs. In contrast in my paper, entrepreneurs directly form a coalition. This rules out the kinds of joint contracts that \textit{Ramakrishnan and Thakor} (1984) consider.

5.4 Market Powers

The firm size could also be restricted by both the decline in value of private information and increased price impact. When a non-zero measure of entrepreneurs become \textit{effectively}

\(^{16}\)That said, if we combine profit-sharing with a “massacre” or "scapegoat" penalizing mechanism as introduced in \textit{Rasmussen} (1987), first best result may still be maintained. Some further discussion on moral hazard in teams: \textit{Williams and Radner} (1988) develop examples showing how partnerships preserve efficiency when the joint output is uncertain. \textit{Legros and Matsushima} (1991) present a necessary and sufficient condition for achieving efficiency in partnerships. \textit{Strausz} (1999) studies how sequential partnerships sustain efficiency. These considerations, however, are beyond the scope of the current paper.

\(^{17}\)For example on how partnering (partially) incentivizes truthful \textit{direct} communication, see \textit{Garicano and Santos} (2004) on how partnership encourages efficient case referral among lawyers.
more informed via creating a firm, equilibrium price in the market changes. Because when in a firm, entrepreneurs become more responsive to her own private information relative to the (imperfectly rationally inferred) public information from equilibrium market price, the equilibrium price aggregates more private information (i.e., becomes more efficient), lowering the value of private information. Furthermore, when the partnership size is sufficiently large, the absolute value of each entrepreneur’s demand enlarges. The price impact of each partner’s input provision amount is no longer negligible, forcing them to “shred their orders”. In this case, the value of a partnership is further lowered, interior optimal sizes of partnerships are obtained, and “boundaries of partnership firms” are drawn.

To illustrate the first channel, I calculate the price efficiency, input provision behavior, and expected utility when all entrepreneurs in the continuum join their respective partnerships of finite size n. Because in this case a non-zero measure (indeed a measure one) of entrepreneurs respond more aggressively to their private information when making input provision amount decisions, equilibrium price changes. Yet the finiteness of n still justifies assuming each entrepreneur within their respective firms as price-takers.18 For ease of exposition, I assume that $\forall i, \rho_i = \rho$. The results are summarized below.

**Corollary 5.1.** When all entrepreneurs in the continuum join their respective partnerships of finite size n, the equilibrium price becomes

$$\begin{align*}
\begin{cases}
p &= \mu + \pi \nu - \gamma z \\
\mu &= \frac{\sigma^2 \rho^2}{(n\tau_e + \tau_v)^2 \rho^2 + n^2 \tau_v^2} (\nu^2 v - \frac{n\tau_e}{\sigma^2 \rho^2}) \\
\pi &= \frac{-n\tau_e}{\rho} \\
\gamma &= \frac{-n\tau_e (n\tau_e + \sigma^2 \rho^2)}{(n\tau_e + \tau_v)^2 \rho^2 + n^2 \tau_v^2} \\
\end{cases}
\end{align*}
$$

(23)

---

18 Appendix D further relaxes the price-taking assumption.
and entrepreneur $i$’s equilibrium demand is given by

$$
\frac{1}{p} \left[ \tau_v \bar{U} - n \tau_c - \frac{\tau_v \bar{U} + \rho \bar{Z}}{\tau_c + \sigma_Z^2 \rho^2} + n \tau_c s_i - (n \tau_c + \frac{\rho^2 \sigma_Z^2 \tau_v}{\rho^2 \sigma_Z^2 + n \tau_c}) p \right],
$$

which gives entrepreneur $i$’s expected utility as

$$
\exp \left( \frac{-\rho^2 \sigma_Z^2 \tau_v \bar{U} + n \rho \sigma_Z \tau_v \tau_c + (n \tau_c + \rho^2 \sigma_Z^2 \tau_v + \rho^2 \sigma_Z^2 \tau_v) p}{2(n \tau_c + \rho^2 \sigma_Z^2) + \rho^2 \sigma_Z^2 (\tau_v + \tau_c)} \right)
$$

Equation (25) gives an interior optimal $n^*$.

6 General Implications for Corporate Finance

On a micro level, an information-based view of the firm speaks to optimal corporate governance. Within-firm information aggregation is important in a knowledge economy. In reality, however, many obstacles deter effective direct communication. First, truthful-telling may not be incentive compatible. The fear of one's valuable information being abused, as well as the jeopardy of unintentional divulgence or blatant re-sale by others, often deter truthful communication.\(^{19}\) Second, communication often takes time. If a market opportunity is short-lived and requires immediate reaction, delays might negate benefits from communic-

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\(^{19}\)Preventing knowledge theft is empirically a serious concern, as Bhide (1994) reports that 71% of the firms included in the Inc 500 (a list of young, fast-growing firms) were founded by people who replicated or modified an idea encountered in their previous employment. Theoretically the concern over critical knowledge theft is also the departure point in Rajan and Zingales (1998). In the context of financial markets, resale has brought up in discussions on information production (e.g. Hirschleifer (1971)). Controlling information usage motivates studies on optimal selling of information (e.g. Admati and Pfleiderer (1986) and Admati and Pfleiderer (1990)). Allen (1990) develops a theory of financial intermediation based on indirect selling of information. See Dewatripont and Tirole (2005) for further discussions on potential costs and biases in knowledge transfers.
tion. Delays in information transmission are even more severe problems when the number of people involved increases. Since profit-sharing obtains de facto information aggregation without actually incurring communication, it overcomes the obstacles discussed above.

We shall particularly note that under both the private cost and the oligopsony case discussed in Section 5.3 and 5.4, profit-sharing no longer aggregates information perfectly. Were the symmetric information benchmark indeed feasible, it would well dominate profit-sharing. However in these two cases symmetric information benchmark is not obtainable. Even if players are allowed to communicate with each other freely, they won’t have incentives to tell the truth. This is because in an oligopsonistic market, players have an downplay incentive. Profit-sharing still turns out to be dominant.

Viewing firms as de facto knowledge aggregators sheds new light on several corporate finance topics. Corporate governance, for example, is defined by Shleifer and Vishny (1997) as a study that “deals with the ways in which suppliers of finance to corporations assure themselves of getting a return on their investment”, with “a straightforward agency perspective, sometimes referred to as separation of ownership and control”. The de facto knowledge aggregation theory of the firm, however, suggests another possible side of the coin.

In an age of information explosion, business success increasingly asks for knowledge and talent possessed by not only a single individual but possibly a large team. Compared to physical assets, knowledge and talent are inalienable, rendering a governance theory based on property rights and asset ownership reallocation (e.g. Grossman and Hart (1986)) less applicable. It is then natural to ask what governance structures best glue human capital

\footnote{Bolton and Dewatripont (1994) consider the time involved in information transmission. The “information percolation” literature explicitly models the slow diffusion process of information in the financial market over repeated direct communication, see Duffie, Giroux, and Manno (2010) and Andrei and Cujean (2013), etc.}

\footnote{Furthermore, conversations, meetings, and discussions take time, at the cost of leisure, actual work, and missing opportunities; misinterpretation and oblivion create additional attrition to communication; “soft” information like haphazard know-hows, amorphous business acumen, and tacit knowledge à la Grant (1996) are simply too hard to codify and impossible to convey; cognitive capacity limits cap the amount of knowledge an individual can possess (see the rational inattention literature as in e.g. Veldkamp (2011)).}
together, and whether such governance structures are ever superior to pure market allocations. The *de facto* knowledge aggregation view is thus a response to his call for “search of new foundations” (of corporate finance / theory of the firm) in Zingales (2000), emphasizing that existing models are in need of reexamination as traditional asset intensive firms are now being gradually peripheralized by human capital intensive ones. The *de facto* knowledge aggregation view also adds to the rising topic on internal governance of the firm (see e.g. Acharya, Myers, and Rajan (2011) and Rajan (2012)).

The profit-sharing perspective in my analysis features a meritocracy-based governance structure. This assumption follows the spirit of Aghion and Tirole (1997), in which formal authority is distinguished from real authority. An agent with formal authority will exercise her power if and only if she acquires the necessary knowledge to do so, or otherwise she delegates decision-making to her more knowledgeable subordinates. Aghion and Tirole focus on how the allocation of formal authority alters agents’ *ex ante* knowledge acquisition incentives. In both revenue-sharing models of my paper, entrepreneurs make decisions without others’ interference, as their private knowledge (either about the mapping from the information set to the optimal action or the information set itself) grant them real authority. This decentralized governance structure lines up with the flat organization, teamwork focus, and advocated “workforce democracy” found in most human capital intensive firms. It also differentiates my model from the social choice problem of Wilson (1968).

Viewing firms as sharing mechanisms also helps interpret some recent trends in capital structure changes. In a recent discussion on “secular stagnation” among industrialized economies, Summers (2014) pinpoints the “reductions in demand for debt-financed investment”, and contends that “probably to a greater extent, it is a reflection of the changing

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22 Gluing human capital and preventing talent attrition is an important consideration in modern corporate governance. The consequence of neglecting it is vividly illustrated in the case of the British advertising agency Saatchi and Saatchi documented in Rajan and Zingales (2000).

character of productive economic activity.”

Traditional asset-intensive industries are debt-friendly, as assets serve as stable collateral, and allow outside investors to take a passive role in the firm’s operation (except in default). Firms with intensive knowledge inputs, however, require a more active role of all input providers, e.g. the more active roles played by venture capitalists than commercial banks, the adoption of equity-based employee compensation, and little involvement of passive creditors in purely knowledge-based firms (law firms, strategic management (but not IT) consulting firms, etc.) – even though such industry might be most subject to insider moral hazard or unverifiable cash flow, which traditional theories (e.g. Innes (1990) and Townsend (1979)) would predict in favor of debt-financing.

In a broader sense, my theory relates to the partnership model of outside equity investors in Myers (2000).

The knowledge aggregation effect of a firm also provide new perspectives on valuation. The nexus of explicit contracts view of the firm à la Alchian and Demsetz (1972) and Jensen and Meckling (1979) assumes that the compensation to all stakeholders but shareholders could be explicitly contracted, and thus maximizing shareholder value equates to maximizing social welfare of all firm stakeholders, since equity holders are the only residual rights owner. This view is no long valid once we take into consideration of employee human capital, as residual rights owners can no longer be summarized as one representative person, and their internal relations do matter. This is particularly true for a knowledge-based firm.\footnote{Willamson (1988) sympathizes this perspective.}

\footnote{Summers elaborates further: "Ponder that the leading technological companies of this age – I think, for example, of Apple and Google – find themselves swimming in cash and facing the challenge of what to do with a very large cash hoard. Ponder the fact that WhatsApp has a greater market value than Sony, with next to no capital investment required to achieve it. Ponder the fact that it used to require tens of millions of dollars to start a significant new venture, and significant new ventures today are seeded with hundreds of thousands of dollars. …"}

\footnote{Several recent papers have investigated the valuation implication of firm’s non-tangible assets. For example, Eisfeldt and Papanikolaou (2013) document that firms with more organization capital have average returns that are 4.6% higher than firms with less organization capital. Zhang (2014) studies the implications of employee’s limited commitment to the firm on cash flow volatility.}

\footnote{See also Berk, Stanton, and Zeckner (2010) and Berk and Walden (2013) on the implications of human capital on capital structure and asset pricing.}
7 Empirical Implications

The profit-sharing perspective on the theory of the firm generates several empirical implications that are helpful for guiding future research.

First, profit-sharing as an organization structure for a firm should be more common in sectors where information is more dispersedly held. For example, producer cooperatives are particularly prevalent in the production of commodity goods (e.g. crops), where producers across a large region possess dispersed information. On the other hand, if production-related information is concentrated in a small set of stakeholders, or if agents are overconfident or agree to disagree, profit-sharing will be less likely.

Second, profit-sharing as an organization structure for a firm should be more common in sectors featuring intense private information. For example, partnership structures dominate human capital intensive industries including professional services or investment firms (e.g. private equity/venture capital). On the other hand, because manual labor usually involves little private information, profit-sharing is seldom seen among manual labors. This perspective sheds light on the observation that while both physical or human capital providers are seen as owners of manufacturing or service firms, respectively, granting ownership to manual labors is rarely successful.

Third, from the perspective of information aggregation, profit-sharing is most effective when the production technology is better captured by a linear technology, like in the provision of physical capital, or (homogeneous) commodity inputs. Investment funds or producer cooperatives provide examples. When production technology features features salient non-linearity, direct communication in addition to profit-sharing is called for, and other organizational structures (e.g. hierarchies as in Bolton and Dewatripont (1994)) will arise.

Fourth, when private information and private cost both exist, optimal firm size will be determined by a trade-off between between information aggregation and worse free-riding.
The particular sharing profit-sharing rule within a firm will also be affected by the cost of
direct communication. When direct communication is costly, profit-sharing will be more akin
to what is described by Theorem 3.3, while otherwise direct communication is more likely
to happen while profit-sharing rule will be more like as if a collection of sole-proprietors.

Of course, we shall note that because the current paper focuses on information aggrega-
tion, it necessarily abstracts from many other confounding issues that determines optimal
organization structures (e.g. the tangibility of assets). All implications listed above are thus
built on a *ceteris paribus* basis and should be taken with a grain of salt. Careful empirical
design for testing the above implications are left for future research.

8 Other Related Literature

The theoretical results of my paper is related to studies on the equilibrium and efficient use
of information. In a linear-quadratic setup featuring asymmetric information and strate-
gic complementarity/substitutability, *Angeletos and Pavan (2007)* show that redistribution
among individuals can achieve efficient information aggregation as an equilibrium outcome.
In my profit-sharing setup, I obtain a similar result, although strategic complementar-
ity/substitutability are not present.

The indirect information transmission with profit-sharing can also be linked to a series
of papers by on indirect information sale in the financial market.\(^{28}\) What Admati and
Pfeiferer mean by indirect sale of information is that when informed investors manage
delegated portfolios on behalf of those uninformed for a fee, they are essentially selling their
information. Following this logic, when information in the economic system is dispersed,
investors would have incentives to delegate their wealth to each other. The result of this
mutual delegation is exactly the profit-sharing mechanism seen in a firm. Admati and

\(^{28}\)See *Admati and Pfeiferer (1990)*, etc.
Pfeifferer uses their indirect information sale insight to explain the rise of institutional investors. In this sense, we can also view their results as a specific yet important example for a more general theory of the firm.  

Third, from a purely technical perspective, a cooperative provides a game-theoretical implementation of a rational expectation equilibrium. So my result is connected to the implementation theory literature in mechanism design, whose focus is on designing mechanisms to achieve one equilibrium outcome via another equilibrium concept. Palfrey (2002) provides a nice introduction to implementation theory, while Blume and Easley (1990) studies implementation of Walrasian expectations equilibrium in a general setting.

The second part of my paper also contributes the literature on the theory of the firm. Over 70 years’ academic endeavors on this fundamental topic makes an exhaustive reference a daunting job, so I only attempt to classify some well-known contributions around several major lines of thoughts and connect them with the revenue-sharing theory.

The neoclassic theory views firms simply as production technology sets, and firms per se are void of meaningful definitions. The first and foremost question, raised by the seminal work of Coase (1937), asks what essentially defines a firm, and how within-firm organization is distinguished from market contracting. Coase identifies authority, which is useful when contracting is costly, as the defining feature that differs within-firm transactions apart from market contracting. Two questions remain to be answered though in Coase’s original argument, the first being what constitutes contracting costs, and the second being a formal definition of authority.

On contracting costs, Williamson (1975), Klein, Crawford, and Alchian (1978), Williamson (1979), and Williamson (1985) identifies ex post haggling as a source of cost to contracting. In my humble opinion, another important friction to contracting lies in the nonexistence of Pareto optimal, incentive compatible, and budget-balancing bilateral bargaining outcomes

\footnote{Also see García and Vanden (2009) on wealth delegation with endogenous information acquisition.}
under two-sided asymmetric information (Myerson and Satterthwaite (1983)), although to my best knowledge no resolutions have yet been proposed in this direction.

The formalization of authority spearheads the development of the incomplete contracting approach. Under this theme, Grossman and Hart (1986) and Hart and Moore (1990) argue that asset ownership determines the allocation of residual rights (or authority). In this property rights theory of the firm, physical assets play vital roles for the very existence of firms as they entangle other production inputs around it and give birth to firms. Hart (1995) even goes further and argues that “a firm’s non-human assets, then, simply represent the glue that keeps the firm together . . . If non-human assets do not exist, then it is not clear what keeps the firm together” (p. 57). However, as human capital intensive firms arise in the knowledge economy, Rajan and Zingales (1998) point out the narrowness of property rights theory and develop a new theory based on “access” to critical resources.

The Coase authority-cost paradigm is not the only framework for understanding firms. For example, the “nexus of contracts” theory views a firm as a legal illusion no more than a central contracting party to subsume a complex of multilateral contracts (Alchian and Demsetz (1972)). Jensen and Meckling (1979) specifically focus on the principal-agent contracting problem between a firm owner and the management subject to moral hazard. In the market intermediary theory, Spulber (1999) interprets firms as centralized exchanges to reduce market search costs. A few papers take a knowledge perspective on the essence of the firm. Demsetz (1988) emphasizes information cost reduction as a foundation of firms, and Grant (1996) proposes a knowledge-based theory of the firm. However, both papers take information cost reduction and knowledge aggregation with a firm as given, abstracting from concrete micro-founded mechanisms supporting the effects. Bolton and Dewatripont (1994) analyze how organizations minimize information processing and communicating costs, their

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30See Hart (1989) and Holmström and Tirole (1989) for reviews.
31This view is partially supported by Kaplan, Sensoy, and Strömberg (2009).
32See also Winter (2006), Winter (2010).
study, however, abstracts from firm stakeholders’ communication incentives, which is emphasized here. While my paper is built on dispersed information, Dicks and Fulghieri (2014) build on disagreement between owners and managers endogenously generated by ambiguity aversion and study optimal governance structure.

My paper attempts to add to the general discussions on the theory of the firm, with specific relevance in our current knowledge economy. This specific focus is also found in Rajan and Zingales (2001), who espouse the “critical resource theory” for human-capital intensive firms; in Zingales (2000), who connects this trend to the change of landscape in corporate finance studies; and in the AFA presidential address of Rajan (2012), who further highlights the linkage between firm organization and corporate finance in the context of the life-cycle of entrepreneurial firms.

9 Conclusion

The important concept of rational expectation tells that the market could be viewed as an information aggregation mechanism. The current paper points out that profit-sharing as in a joint-stock company could also aggregate information. When technological constraints limit price-contingent actions, when noises intrinsic to a market prevent full revelation, or when incentive problems or costs shut down direct communication, profit-sharing shows superiority and complements the market to take advantage of the wisdom of the crowd. A competitive market does not exist in isolation, and firms arise endogenously out of a market economy. The results provide a new perspective on the theory of the firm.

Further implications of the information aggregation role of profit-sharing are left for future research. For example, my result is related to other non-price-based information aggregation mechanisms. How auctions serve as information aggregation devices has long been studied. In political science, voting as an information aggregation is also frequently discussed (see
e.g. Feddersen and Pesendorfer (1997)). My non-cooperative game-theoretical result also contrasts existing cooperative-game studies on surplus-sharing.

The de facto information aggregation result could potentially describe the formation of networks (e.g. Stanton, Walden, and Wallace (2015)) and the rise of “too-big-to-fail” banks (e.g. Erel (2011)). As stated earlier, my result could provide an alternative explanation for mutual fund families, and an incentive-compatibility alternative for information percolation. I leave these application to future research.

Last but not least, the equivalence between payoff-sharing and sharing information might suggest an often neglected de facto information channel. For example, how effective are Chinese walls between firm subdivisions that are exposed to conflict of interests? Does profit-sharing camouflage insider trading? What implications this paper’s findings have on the governance of PE/VC, the organization of R&D activities, the formation of banking network, or furthermore optimal taxation? Is there a theory for financial disintermediation (i.e. crowd-funding) down the road (as opposed to theories of financial intermediation, e.g. Diamond (1984), Ramakrishnan and Thakor (1984))? Does the investigation into the relationship between the firm and market inspire a connection between Welfare Theorem and Coase Theorem? Is there an equivalence between capitalism and socialism in terms of information aggregation? More thorough development is scheduled for future research.

Appendix

A Proof of Theorem 4.1

Proof of Theorem 4.1: When \( n \) entrepreneurs agree to partner, partner \( i \)'s investment in the risky project maximizes

\[
\mathbb{E} \left[ - \exp(-\rho_i a_i(v - p)(x_i + \sum_{k \neq j} x_k)) | s_i, p \right]
\]

(26)
Focusing on symmetric linear equilibria, assume \( x_k = \alpha_0 + \alpha_{1k}s_k + \alpha_{2k}p \). Notice that
\[
    x_i + \sum_{k \neq i} x_k = x_i + \sum_{k \neq i} \alpha_0 + \sum_{k \neq i} \alpha_{1k} v + \sum_{k \neq i} \alpha_{2k} p + \sum_{k \neq i} \alpha_{1k} c_k,
\]
thus
\[
    \left( -\rho_i a_i(v-p) \right)_{|s_i,p} \sim \mathcal{N} \left( \begin{array}{c}
    \frac{-\rho_i a_i(\mathbb{E}(v|s_i,p) - p)}{ho_i^2 a_i^2 \text{Var}(v|s_i,p)} \\
    \frac{-\rho_i a_i \alpha_0 + \sum_{k \neq i} \alpha_{1k} \mathbb{E}(v|s_i,p) + \sum_{k \neq i} \alpha_{2k} p}{\rho_i^2 a_i^2 \text{Var}(v|s_i,p)} \\
    \frac{-\rho_i a_i \sum_{k \neq i} \alpha_{1k} \text{Var}(v|s_i,p)}{\rho_i^2 a_i^2 \text{Var}(v|s_i,p)} \\
    \frac{\rho_i a_i \sum_{k \neq i} \alpha_{1k} \text{Var}(v|s_i,p)}{\rho_i^2 a_i^2 \text{Var}(v|s_i,p)}
\end{array} \right) \right)
\]
(28)

By Lemma 1.1, the certainty equivalent of (26) is
\[
    -\frac{\exp(A)}{\sqrt{B}},
\]
(29)

where
\[
    A = \rho_i^2 a_i^2 (\mathbb{E}(v|s_i,p) - p)^2 [\left( \sum_{k \neq i} \alpha_{1k} \right)^2 \text{Var}(v|s_i,p) + \sum_{k \neq i} \alpha_{1k}^2 \tau^{-1}_k] \\
    + \left[ x_i + \sum_{k \neq i} \alpha_0 + \sum_{k \neq i} \alpha_{1k} \mathbb{E}(v|s_i,p) + \sum_{k \neq i} \alpha_{2k} p \right] \rho_i a_i^2 \text{Var}(v|s_i,p) \\
    - 2 \rho_i a_i (\mathbb{E}(v|s_i,p) - p) x_i + \sum_{k \neq i} \alpha_0 + \sum_{k \neq i} \alpha_{1k} \mathbb{E}(v|s_i,p) + \sum_{k \neq i} \alpha_{2k} p [1 + \rho_i a_i \sum_{k \neq i} \alpha_{1k} \text{Var}(v|s_i,p)]
\]
\[
    B = [1 + \rho_i a_i \sum_{k \neq i} \alpha_{1k} \text{Var}(v|s_i,p)]^2 \\
    - \rho_i^2 a_i^2 \text{Var}(v|s_i,p) [\left( \sum_{k \neq i} \alpha_{1k} \right)^2 \text{Var}(v|s_i,p) + \sum_{k \neq i} \alpha_{1k}^2 \tau^{-1}_k]
\]

Taking FOC w.r.t. \( x_i \) we get
\[
\left[ x_i + \sum_{k \neq i} \alpha_0 + \sum_{k \neq i} \alpha_{1k} \mathbb{E}(v|s_i,p) + \sum_{k \neq i} \alpha_{2k} p \right] \rho_i a_i \text{Var}(v|s_i,p)
\]
(30)
\[
= (\mathbb{E}(v|s_i,p) - p) [1 + \rho_i a_i \sum_{k \neq i} \alpha_{1k} \text{Var}(v|s_i,p)] \\
\]
(31)

Given \( p = \mu + \pi v - \gamma z \), we have
\[
\begin{align*}
    \mathbb{E}(v|s_i,p) &= \bar{v} + \frac{\gamma^2 \sigma^2 \tau^{-1}_v (s_i - \bar{v}) + \pi \tau^{-1}_v r^{-1}_v (p - \mu + \pi \bar{v} - \gamma z)}{\gamma^2 \sigma^2 \tau^{-1}_v + \pi \tau^{-1}_v r^{-1}_v + \gamma^2 \sigma^2 \tau^{-1}_v} \\
    \text{Var}(v|s_i,p) &= \frac{\tau^{-1}_v - \frac{\gamma^2 \sigma^2 \tau^{-1}_v + \pi \tau^{-1}_v r^{-1}_v + \gamma^2 \sigma^2 \tau^{-1}_v}{\gamma^2 \sigma^2 \tau^{-1}_v + \pi \tau^{-1}_v r^{-1}_v + \gamma^2 \sigma^2 \tau^{-1}_v}}{\gamma^2 \sigma^2 \tau^{-1}_v + \pi \tau^{-1}_v r^{-1}_v + \gamma^2 \sigma^2 \tau^{-1}_v}
\end{align*}
\]
(32)
In equilibrium \( x_t = \alpha_{0j} + \alpha_{1j} s_t + \alpha_{2j} p_t \), and Equation (31) leads to

\[
\alpha_{1j} s_t + \frac{\sum_{k=1}^{n} \alpha_0 + \sum_{k \neq i}^{n} \alpha_{2k} p_t}{\rho_i a_i \text{Var}(v|s_i, p)} = \frac{(\mathbb{E}(v|s_i, p) - p)[1 + \rho_i a_i \sum_{k \neq i} \alpha_{1k} \text{Var}(v|s_i, p)]}{\rho_i a_i \text{Var}(v|s_i, p)},
\]

thus

\[
\rho_i a_i \text{Var}(v|s_i, p) \left[ \alpha_{1j} s_t + \sum_{k=1}^{n} \alpha_0 + \sum_{k \neq i}^{n} \alpha_{2k} p_t \right] = \mathbb{E}(v|s_i, p) - p - \rho_i a_i \sum_{k \neq i} \alpha_{1k} \text{Var}(v|s_i, p) p
\]

Plug in (32),

\[
\rho_i a_i \gamma^2 \sigma_v^2 \tau_i^{-1} \tau_v^{-1} \left[ \alpha_{1j} s_t + \sum_{k=1}^{n} \alpha_0 + \sum_{k \neq i}^{n} \alpha_{2k} p_t \right] = \gamma^2 \sigma_v^2 \tau_i^{-1} s_t + \pi \tau_i^{-1} \tau_v^{-1} (p - \mu + \gamma \bar{v}) + \gamma^2 \sigma_v^2 \tau_i^{-1} \bar{v}
\]

\[
- (\gamma^2 \sigma_v^2 \tau_i^{-1} + \pi \tau_i^{-1} \tau_v^{-1} \gamma^2 \sigma_v^2) p - \rho_i a_i \sum_{k \neq i} \alpha_{1k} \gamma^2 \sigma_v^2 \tau_i^{-1} \tau_v^{-1} p
\]

Equalizing coefficients:

\[
\begin{align*}
\rho_i a_i \sum_{k=1}^{n} \alpha_0 &= \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma_v^2} (\mu - \gamma \bar{v}) \\
\rho_i a_i \alpha_{1j} &= \tau_i \\
\rho_i a_i \sum_{k \neq i}^{n} \alpha_{2k} &= -(\tau_i + \tau_v + \frac{\pi^2}{\gamma^2 \sigma_v^2}) + \frac{\tau_v}{\gamma^2 \sigma_v^2} - \rho_i a_i \sum_{k \neq i} \frac{\tau_v}{\rho_k a_k}
\end{align*}
\]

\[
\Rightarrow
\begin{align*}
\alpha_i &= \frac{1}{\sum_{k=1}^{n} \frac{1}{\rho_k a_k}} \\
\sum_{k=1}^{n} \alpha_0 &= \left( \sum_{k=1}^{n} \frac{1}{\rho_k} \right) \left[ \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma_v^2} (\mu - \gamma \bar{v}) \right] \\
\alpha_{1j} &= \left( \sum_{k=1}^{n} \frac{1}{\rho_k} \right) \tau_i \\
\sum_{k=1}^{n} \alpha_{2k} &= \left( \sum_{k=1}^{n} \frac{1}{\rho_k} \right) \left[ -(\sum_{k=1}^{n} \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma_v^2}) + \frac{\pi}{\gamma^2 \sigma_v^2} \right]
\end{align*}
\]

Thus

\[
x_t = \frac{1}{\rho_i} \left[ \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma_v^2} (\mu - \gamma \bar{v}) \right] + \left( \sum_{k=1}^{n} \frac{1}{\rho_k} \right) \tau_i s_i - \frac{1}{\rho_i} \left[ \sum_{k=1}^{n} \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma_v^2} \right] p
\]

Plug (36) in (29) shows that

\[
\mathbb{E}(v|s_i, p) = \frac{\tau_i s_i + \frac{\pi}{\gamma^2 \sigma_v^2} (p - \mu + \gamma \bar{v}) + \tau_v \bar{v}}{\tau_i + \frac{\pi^2}{\gamma^2 \sigma_v^2} + \tau_v}
\]

\[
\text{Var}(v|s_i, p) = \frac{1}{\tau_i + \frac{\pi^2}{\gamma^2 \sigma_v^2} + \tau_v}
\]

39
A = -(E(v|s_i, p) - p)^2 \left( \sum_{k=1}^{n} \tau_k + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v \right)

B = \frac{\sum_{k=1}^{n} \tau_k + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v}{\tau_i + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v},

thus the expression for the expected utility of entrepreneur i is given by

\exp \left( - \frac{(E(v|s_i, p) - p)^2 (\tau_i + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v)}{2} \right)
\frac{\sqrt{\sum_{k=1}^{n} \tau_k + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v}}{\tau_i + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v}
\exp \left( -\frac{1}{2} \frac{1}{\tau_i + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v} \left[ \tau_i s_i - \frac{\pi}{\gamma^2 \sigma^2} (\mu - \gamma \bar{z}) + \tau_v \bar{v} - (\tau_i + \tau_v + \frac{\pi^2}{\gamma^2 \sigma^2} - \frac{\pi}{\gamma^2 \sigma^2}) p \right]^2 \right)
\frac{\sqrt{\sum_{k=1}^{n} \tau_k + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v}}{\tau_i + \frac{\pi^2}{\gamma^2 \sigma^2} + \tau_v}

Notice that this is identical to the expected utility achieved under complete information (but no partnership creation). □

B Proof of Theorem 4.2

Proof. In a symmetric information benchmark, entrepreneur i’s expected utility is given by

- \exp \left\{ - \left[ \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma^2} (\mu - \gamma \bar{z}) + \sum_{k=1}^{n} \tau_k s^* - (\sum_{k=1}^{n} \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma^2} - \frac{\pi}{\gamma^2 \sigma^2}) p \right] [E(v|s^*, p) - p] + \frac{1}{2} \left[ \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma^2} (\mu - \gamma \bar{z}) + \sum_{k=1}^{n} \tau_k s^* - (\sum_{k=1}^{n} \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma^2} - \frac{\pi}{\gamma^2 \sigma^2}) p \right]^2 \text{Var}(v|s^*, p) \right\}

and expected utility before entering symmetric information benchmark conditional on s_i is

(38)
\[-\mathbb{E} \left\{ \exp \left( - \frac{\mu - \gamma \bar{Z}}{\gamma^2 \sigma_Z^2} + \frac{n}{\gamma^2 \sigma_Z^2} (\mu + \gamma \bar{Z}) + \sum_{k=1}^{n} \tau_k s^* - \left( \sum_{k=1}^{n} \tau_k + \tau_v + \frac{n^2}{\gamma^2 \sigma_Z^2} - \frac{n}{\gamma^2 \sigma_Z^2} \right) p \right) \right\} \right\}
\frac{1}{2} \left[ \tau_v \tilde{v} - \frac{\pi^2}{\gamma^2 \sigma^2} (\mu - \gamma \bar{Z}) + \sum_{k=1}^{n} \tau_k s^* - \left( \sum_{k=1}^{n} \tau_k + \tau_v + \frac{n}{\gamma^2 \sigma_Z^2} - \frac{1}{\gamma^2 \sigma_Z^2} \right) p \right] \right\}
\frac{\text{Var}(v|s^*,p)}{s_i,p} \}
\right\}
\exp \left( - \frac{1}{2} \frac{\pi^2}{\gamma^2 \sigma^2} (\mu - \gamma \bar{Z}) + \sum_{k=1}^{n} \tau_k s^* - \left( \sum_{k=1}^{n} \tau_k + \tau_v + \frac{n}{\gamma^2 \sigma_Z^2} - \frac{1}{\gamma^2 \sigma_Z^2} \right) p \right)^2 \right\}
\exp \left( - \frac{1}{2} \frac{\pi^2}{\gamma^2 \sigma^2} (\mu - \gamma \bar{Z}) + \tau_v \tilde{v} - (\tau_i + \tau_v + \frac{n}{\gamma^2 \sigma_Z^2} - \frac{1}{\gamma^2 \sigma_Z^2} \right) p \right)^2 \sqrt{\sum_{k=1}^{n} \tau_k + \frac{n^2}{\gamma^2 \sigma_Z^2} + \tau_v}
\frac{1}{\tau_i + \frac{n^2}{\gamma^2 \sigma_Z^2} + \tau_v}
\right\}
\right\}
\mathcal{N} \left( \frac{\sum_{k=1}^{n} \tau_k + \frac{n^2}{\gamma^2 \sigma_Z^2} + \tau_v}{\tau_i + \frac{n^2}{\gamma^2 \sigma_Z^2} + \tau_v} \right)
\frac{\tau_i s_i - \frac{\pi^2}{\gamma^2 \sigma^2} (\mu - \gamma \bar{Z}) + \tau_v \tilde{v} - (\tau_i + \tau_v + \frac{n}{\gamma^2 \sigma_Z^2} - \frac{1}{\gamma^2 \sigma_Z^2} \right) p \left( \sum_{k=1}^{n} \frac{\tau_k + \frac{n^2}{\gamma^2 \sigma_Z^2} + \tau_v}{\tau_i + \frac{n^2}{\gamma^2 \sigma_Z^2} + \tau_v} \right) \right)
\right\}
\right\}
\left\{ \[ \begin{array}{l}
\alpha_0 = -\frac{1}{\gamma}
\end{array} \right\}
\left\{ \[ \begin{array}{l}
\alpha_1 = -\frac{1}{\gamma}
\alpha_2 = \frac{1}{\gamma}
\end{array} \right\}
\right\}
\text{Proof of Corollary 5.1:}

When all entrepreneurs create firms, in a linear equilibrium, project valuation and entrepreneur investment still follows
\begin{align*}
\hat{p} &= \mu + \pi \tilde{v} - \gamma \bar{Z} \\
x_k &= n \alpha_0 + n \alpha_1 s_k + n \alpha_2 \hat{p} 
\end{align*}

where \( \mu, \pi, \gamma \) and \( \alpha_i \) \((i = 0, 1, 2)\) are all functions of \( n \) to be determined, and by the same arguments in the proof of Corollary ??, these coefficients satisfy (??).

Integrate each individual entrepreneurs’ investment over the continuum and by market clearing, \( n \alpha_0 + n \alpha_1 + n \alpha_2 (\mu + \pi v - \gamma z) + z = 0 \), thus
\begin{align*}
n \alpha_0 + n \alpha_1 + n \alpha_2 (\mu + \pi v - \gamma z) + z &= 0, \quad \text{thus} \quad \left\{ \begin{array}{l}
\alpha_0 = -\alpha_2 \mu \\
\alpha_1 = -\alpha_2 \pi \\
\alpha_2 = \frac{1}{n \gamma}
\end{array} \right\}
\end{align*}
Plug in (??) and re-arrange terms,

\[
\begin{align*}
\mu &= \frac{\sigma^2 \rho^2}{(n\tau_e + \tau_0) \sigma^2 \rho^4 + n^2 \tau_e^2} \left( \tau_e \bar{u} - \frac{n\tau_e \bar{z}}{\rho \sigma^2} \right) \\
\pi &= -\frac{n\tau_e \gamma}{\rho} = \frac{n\tau_e (n\tau_e + \sigma^2 \rho^2)}{(n\tau_e + \tau_0) \sigma^2 \rho^4 + n^2 \tau_e^2} \\
\gamma &= -\frac{1}{(n\tau_e + \tau_0) \sigma^2 \rho^4 + n^2 \tau_e^2}
\end{align*}
\]

(43)

Plug in (37) and (29) and we get the optimal demand and expected payoff given in (24).

\[\square\]

## D Market Powers in the Productive Input Market

First consider a heterogeneous agents extension to Kyle (1989). In this case there are \(N\) investors who do not form partnerships. Perceiving a residual supply curve \(p = p_i + \lambda_i x_i\), investor \(i\) chooses \(x_i\) to maximize

\[
E[-\exp(-\rho_i (v - p(x_i))x_i) | s_i, p], \quad \text{or}
\]

\[
E[-\exp(-\rho_i (v - p_i - \lambda_i x_i)x_i) | s_i, p_i]
\]

\[\Leftrightarrow -\exp \left[ -\rho_i (E(v|s_i, p_i) - p_i)x_i + \rho_i \lambda_i x_i^2 + \frac{1}{2} \rho^2 \text{Var}(v|s_i, p_i)x_i^2 \right] \]

\[\implies x_i = \frac{(E(v|s_i, p_i) - p_i)}{2\lambda_i + \rho_i \text{Var}(v|s_i, p_i)}
\]

thus

\[
p = p_i + \lambda_i x_i
\]

\[= p_i + \frac{\lambda_i (E(v|s_i, p_i) - p_i)}{2\lambda_i + \rho_i \text{Var}(v|s_i, p_i)}
\]

\[\implies x_i = \frac{(2\lambda_i + \rho_i \text{Var}(v|s_i, p_i))p - \lambda_i E(v|s_i, p_i)}{(\lambda_i + \rho_i \text{Var}(v|s_i, p_i))}
\]

(46)

(47)

thus (given that \(p_i\) and \(p\) are informationally equivalent)

\[x_i = \frac{E(v|s_i, p) - p}{(\lambda_i + \rho_i \text{Var}(v|s_i, p))}
\]

Conjecture a linear strategy profile \(x_i = \mu_i + \beta_i s_i - \gamma_i p\), then by market clearing

\[
\sum_{k=1}^{N} \mu_k + \sum_{k=1}^{N} \beta_k s_k - \sum_{k=1}^{N} \gamma_k p + z = 0
\]

(48)
or

\[ x_i = \sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k - \sum_{k \neq i} \gamma_k p + z = 0 \]  

\[ \Rightarrow p = \frac{x_i + \sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k + z}{\sum_{k \neq i} \gamma_k} \]  

\[ \Rightarrow \lambda_i = \frac{1}{\sum_{k \neq i} \gamma_k}, \quad p_i = \frac{\sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k + z}{\sum_{k \neq i} \gamma_k} \]  

thus

\[ \mathbb{E}(v|s_i, p_i) = \bar{v} + \frac{\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2)(s_i - \bar{v}) + \sum_{k \neq i} \beta_k \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2}{\tau_i^{-1}([\sum_{k \neq i} \beta_k]^2 + \tau_i \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2} \]  

\[ \text{Var}(v|s_i, p_i) = \frac{\tau_i^{-1}([\sum_{k \neq i} \beta_k]^2 + \tau_i \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2}{\tau_i^{-1}([\sum_{k \neq i} \beta_k]^2 + \tau_i \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2} \]  

and

\[ x_i = \frac{\bar{v} + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2)(s_i - \bar{v}) + \sum_{k \neq i} \beta_k \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2}{\tau_i^{-1}([\sum_{k \neq i} \beta_k]^2 + \tau_i \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2} + \sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k + z}{\sum_{k \neq i} \gamma_k} \]  

\[ = \mu_i + \beta_i s_i - \gamma_i p + \sum_{k=1}^{N} \frac{\mu_k + \sum_{k=1}^{N} \beta_k s_k + z}{\gamma_k} \]  

Equating coefficients we get

\[ \bar{v} + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2)(\bar{v}) + \sum_{k \neq i} \beta_k \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2}{\tau_i^{-1}([\sum_{k \neq i} \beta_k]^2 + \tau_i \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2} - \sum_{k \neq i} \mu_k \]  

\[ = \mu_i - \sum_{k=1}^{N} \frac{\mu_k}{\gamma_k} \]  

\[ = \mu_i - \sum_{k=1}^{N} \frac{\mu_k}{\gamma_k} = \mu_i - \sum_{k=1}^{N} \frac{\mu_k}{\gamma_k} \]  

\[ = \beta_i - \sum_{k=1}^{N} \frac{\beta_k}{\gamma_k} \]  

\[ = \beta_i - \sum_{k=1}^{N} \frac{\beta_k}{\gamma_k} \]  

\[ = \sum_{k=1}^{N} \frac{\beta_k}{\gamma_k} \]  

\[ = \sum_{k=1}^{N} \frac{\beta_k}{\gamma_k} \]  

43
Equation (58) and (59) lead to

$$\frac{[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2](\sum_{k=1}^N \gamma_k - \sum_{k \neq i} \gamma_k \beta_i \rho_i \tau_i^{-1})}{\tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2} = 2 \beta_i$$

(60)

$$\frac{\sum_{k=1}^N \gamma_k \sum_{k \neq i} \beta_k \tau_k^{-1} + \gamma_i \rho_i \tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2]}{\tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2} = - \gamma_i + 1$$

(61)

(57) leads to

$$\tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2]v - \sum_{k \neq i} \beta_k \tau_k^{-1} z = (\mu_i - \gamma_i \sum_{k=1}^N \frac{\mu_k}{\gamma_k}) \rho_i \tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2]$$

$$+ [(\mu_i - \gamma_i \sum_{k=1}^N \frac{\mu_k}{\gamma_k}) \rho_i \tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2] + \tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_i \sigma_k^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_k^2]$$

(62)

The three equations above defines the equilibrium.

**Profit-sharing** Now consider $M \leq N$ investors agree to partner. Perceiving a residual supply curve $p = p_i + \lambda_i x_i$, partner $i$ chooses $x_i$ to maximize

$$\mathbb{E} \left[ - \exp(-\rho_i a_i (v - p_i - \lambda_i x_i) (x_i + \sum_{k \neq i, k=1}^M x_k)) | s_i, p \right],$$

Conjecture a linear strategy profile $x_i = \mu_i + \beta_i s_i - \gamma_i p$, thus

$$x_i + \sum_{k \neq i, k=1}^M x_k = x_i + \sum_{k \neq i, k=1}^M \mu_k + \sum_{k \neq i, k=1}^M \beta_k s_k - \sum_{k \neq i, k=1}^M \gamma_k p$$

(63)

and

$$\left( -\rho_i a_i (v - p_i - \lambda_i x_i) \right)_{|s_i, p} \sim N \left( \begin{array}{c} x_i + \sum_{k \neq i, k=1}^M \mu_k + \sum_{k \neq i, k=1}^M \beta_k \mathbb{E}(v|s_i, p) - \sum_{k \neq i, k=1}^M \gamma_k p \left( \frac{-\rho_i a_i (\mathbb{E}(v|s_i, p) - p_i - \lambda_i x_i)}{\rho_i^2 a_i^2 \text{Var}(v|s_i, p) + \sum_{k \neq i, k=1}^M \beta_k \text{Var}(v|s_i, p) + \sum_{k \neq i, k=1}^M \beta_k^2 \tau_k^{-1}} \right) \right)$$

whose certainty equivalent is (by Lemma 1.1)

$$-\frac{\exp(\frac{A_B}{B})}{\sqrt{B}}$$

(64)
where

\[ A = \rho_i^2 a_i^2 (\mathbb{E}(v|s_i, p) - p_i - \lambda_i x_i)^2 [\left( \sum_{k \neq i, k = 1}^M \beta_k \right)^2 \text{Var}(v|s_i, p) + \sum_{k \neq i, k = 1}^M \beta_k^2 \tau_k^{-1}] \]

\[ + \left[ x_i + \sum_{k \neq i, k = 1}^M \mu_k + \sum_{k \neq i, k = 1}^M \beta_k \mathbb{E}(v|s_i, p) - \sum_{k \neq i, k = 1}^M \gamma_k p \right]^2 \rho_i^2 a_i^2 \text{Var}(v|s_i, p) \]

\[ - 2 \rho_i a_i (\mathbb{E}(v|s_i, p) - p_i - \lambda_i x_i) [x_i + \sum_{k \neq i, k = 1}^M \mu_k + \sum_{k \neq i, k = 1}^M \beta_k \mathbb{E}(v|s_i, p) - \sum_{k \neq i, k = 1}^M \gamma_k p] \]

\[ [1 + \rho_i a_i \sum_{k \neq i, k = 1}^M \beta_k \text{Var}(v|s_i, p)] \]

\[ B = [1 + \rho_i a_i \sum_{k \neq i, k = 1}^M \beta_k \text{Var}(v|s_i, p)]^2 \]

\[ - \rho_i^2 a_i^2 \text{Var}(v|s_i, p) \left[ \left( \sum_{k \neq i, k = 1}^M \beta_k \right)^2 \text{Var}(v|s_i, p) + \sum_{k \neq i, k = 1}^M \beta_k^2 \tau_k^{-1} \right] \]

Taking FOC w.r.t. \( x_i \) we get

\[ -\lambda_i \rho_i^2 a_i^2 (\mathbb{E}(v|s_i, p) - p_i - \lambda_i x_i) \left[ \left( \sum_{k \neq i, k = 1}^M \beta_k \right)^2 \text{Var}(v|s_i, p) + \sum_{k \neq i, k = 1}^M \beta_k^2 \tau_k^{-1} \right] \]

\[ + \left[ x_i + \sum_{k \neq i, k = 1}^M \mu_k + \sum_{k \neq i, k = 1}^M \beta_k \mathbb{E}(v|s_i, p) - \sum_{k \neq i, k = 1}^M \gamma_k p \right] \rho_i^2 a_i^2 \text{Var}(v|s_i, p) \]

\[ + \rho_i a_i \lambda_i [x_i + \sum_{k \neq i, k = 1}^M \mu_k + \sum_{k \neq i, k = 1}^M \beta_k \mathbb{E}(v|s_i, p) - \sum_{k \neq i, k = 1}^M \gamma_k p] \left[ 1 + \rho_i a_i \sum_{k \neq i, k = 1}^M \beta_k \text{Var}(v|s_i, p) \right] \]

\[ - \rho_i a_i (\mathbb{E}(v|s_i, p) - p_i - \lambda_i x_i) \left[ 1 + \rho_i a_i \sum_{k \neq i, k = 1}^M \beta_k \text{Var}(v|s_i, p) \right] = 0. \]
thus plug in $x_i = \mu_i + \beta_i s_i - \gamma_i p$, and since

$$p = \frac{x_i + \sum_{k \neq i, k=1}^{N} \mu_k + \sum_{k \neq i, k=1}^{N} \beta_k s_k + z}{\sum_{k \neq i, k=1}^{N} \gamma_k} = \frac{\sum_{k=1}^{N} \mu_k + \sum_{k \neq i, k=1}^{N} \beta_k s_k + z}{\sum_{k=1}^{N} \gamma_k}$$

$$\lambda_i = \frac{1}{\sum_{k \neq i, k=1}^{N} \gamma_k}$$

$$p_i = \frac{\sum_{k \neq i, k=1}^{N} \mu_k + \sum_{k \neq i, k=1}^{N} \beta_k s_k + z}{\sum_{k \neq i, k=1}^{N} \gamma_k}$$

$$\mathbb{E}(v|s_i, p_i) = \tilde{v} + \frac{\sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2}{\sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2} \left( s_i - \tilde{v} \right) + \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} \left( \sum_{k \neq i, k=1}^{N} \beta_k (s_k - \tilde{v}) + z - \overline{z} \right)$$

$$\mathbb{E}(v|s_i, p_i) = \frac{\sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2}{\sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2} \left( s_i - \tilde{v} \right) + \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} \left( \sum_{k \neq i, k=1}^{N} \beta_k (s_k - \tilde{v}) + z - \overline{z} \right)$$

$$\text{Var}(v|s_i, p_i) = \frac{\tau_i^{-1} \left( \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2 \right)}{\tau_i^{-1} \left( \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2 \right) + \tau_i \left( \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2 \right) + \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2}$$

Equating coefficients we get

$$\begin{align*}
\left( \sum_{k=1}^{M} \mu_k + \sum_{k \neq i, k=1}^{M} \beta_k \right) \tilde{v} - \left( \sum_{k \neq i, k=1}^{M} \beta_k \right) \tilde{v} & = \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} \left( \sum_{k \neq i, k=1}^{N} \beta_k \tilde{v} + \overline{z} \right) + \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} \left( \sum_{k \neq i, k=1}^{N} \beta_k \overline{v} + \tilde{z} \right) \\
& - \sum_{k=1}^{M} \gamma_k \left( \sum_{k=1}^{N} \mu_k \sum_{k=1}^{N} \beta_k \right) \left[ \rho_i \tilde{a}_i \text{Var}(v|s_i, p) + \lambda_i + \lambda_i \rho_i \tilde{a}_i \sum_{k \neq i, k=1}^{M} \beta_k \text{Var}(v|s_i, p) \right] \\
& = \left( \tilde{v} + \frac{\sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2}{\sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} + \sigma_2^2} \left( s_i - \tilde{v} \right) + \sum_{k \neq i, k=1}^{N} \beta_k \tau_k^{-1} \left( \sum_{k \neq i, k=1}^{N} \beta_k (s_k - \tilde{v}) + z - \overline{z} \right) \right) \\
& - \sum_{k=1}^{N} \mu_k \sum_{k=1}^{N} \beta_k \left( \sum_{k \neq i, k=1}^{N} \beta_k \text{Var}(v|s_i, p) + \lambda_i \rho_i \tilde{a}_i \left( \sum_{k \neq i, k=1}^{M} \beta_k \right) \text{Var}(v|s_i, p) + \sum_{k \neq i, k=1}^{M} \beta_k \tau_k^{-1} \right) \\
& = \left[ 1 + \rho_i \tilde{a}_i \sum_{k \neq i, k=1}^{M} \beta_k \text{Var}(v|s_i, p) + \lambda_i \rho_i \tilde{a}_i \left( \sum_{k \neq i, k=1}^{M} \beta_k \right) \text{Var}(v|s_i, p) + \sum_{k \neq i, k=1}^{M} \beta_k \tau_k^{-1} \right] \end{align*}$$

(65)
\[
\begin{align*}
\beta_i + \left( \sum_{k \neq i, k=1}^{M} \beta_k \right) \frac{\gamma_k}{\tau_i - \left( \sum_{k \neq i, k=1}^{N} \beta_k \right)^2 + \tau_0 \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_0 \sigma_z^2 + \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \sum_{k=1}^{M} \frac{\beta_i}{\sum_{k=1}^{N} \gamma_k} \\
\rho_i a_i \text{Var}(v|s_i, p) + \lambda_i + \lambda_i \rho_i a_i \left( \sum_{k \neq i, k=1}^{M} \beta_k \text{Var}(v|s_i, p) \right)
\end{align*}
\]

\[
\left( \tau_i - \left( \sum_{k \neq i, k=1}^{N} \beta_k \right)^2 + \tau_0 \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_0 \sigma_z^2 + \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2 \right) - \frac{\beta_i}{\sum_{k=1}^{N} \gamma_k} \\
1 + \rho_i a_i \left( \sum_{k \neq i, k=1}^{M} \beta_k \text{Var}(v|s_i, p) \right) + \lambda_i \rho_i a_i \left( \sum_{k \neq i, k=1}^{M} \beta_k^2 \text{Var}(v|s_i, p) \right) + \sum_{k \neq i, k=1}^{M} \beta_k^2 \tau_k^{-1}
\]

\[\text{(66)}\]

\[
\begin{align*}
\beta_i + \left( \sum_{k \neq i, k=1}^{M} \beta_k \right) \frac{\gamma_k}{\tau_i - \left( \sum_{k \neq i, k=1}^{N} \beta_k \right)^2 + \tau_0 \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_0 \sigma_z^2 + \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \sum_{k=1}^{M} \frac{\beta_i}{\sum_{k=1}^{N} \gamma_k} \\
\rho_i a_i \text{Var}(v|s_i, p) + \lambda_i + \lambda_i \rho_i a_i \left( \sum_{k \neq i, k=1}^{M} \beta_k \text{Var}(v|s_i, p) \right)
\end{align*}
\]

\[
\left( \tau_i - \left( \sum_{k \neq i, k=1}^{N} \beta_k \right)^2 + \tau_0 \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_0 \sigma_z^2 + \sum_{k \neq i, k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2 \right) - \frac{1}{\sum_{k=1}^{N} \gamma_k} \\
1 + \rho_i a_i \left( \sum_{k \neq i, k=1}^{M} \beta_k \text{Var}(v|s_i, p) \right) + \lambda_i \rho_i a_i \left( \sum_{k \neq i, k=1}^{M} \beta_k^2 \text{Var}(v|s_i, p) \right) + \sum_{k \neq i, k=1}^{M} \beta_k^2 \tau_k^{-1}
\]

\[\text{(67)}\]

Special case: **ex ante** identical investors   Simplify (66) and (67) for 1 \le j \le M as well as (60) and (61) for M + 1 \le j \le N we have that under profit-sharing:

\[
\begin{align*}
\frac{1}{\gamma_1 (M - 1) + \gamma_2 (N - M)} + \frac{\rho \left( \beta_1^2 (M - 1) + \beta_2^2 (N - M) + \sigma_y^2 \right)}{\left( \beta_1 (M - 1) \rho \beta_1^2 (M - 1) + \beta_2^2 (N - M) + \sigma_y^2 \right)} \\
\frac{M \left( - 2 \beta_1 \beta_2 (M - 1) (M - N) \tau_e + \beta_1^2 (M - N) \left( -1 + M - N \right) \tau_e - \tau_o + \sigma_y^2 \tau_e (\tau_e + \tau_o) + \beta_1^2 (M - 1) (M \tau_e + \tau_o) \right)}{\left( \beta_1 (M - 1) \rho \beta_1^2 (M - 1) + \beta_2^2 (N - M) + \sigma_y^2 \right)} \\
\frac{1}{\gamma_1 (M - 1) + \gamma_2 (N - M)} + \frac{\lambda_1}{\gamma_1 M + \gamma_2 (N - M)} \\
\frac{1}{\gamma_1 M + \gamma_2 (N - M)} + \frac{\lambda_1}{\gamma_1 M + \gamma_2 (N - M)} \\
1 + \frac{M \left( - 2 \beta_1 \beta_2 (M - 1) (M - N) \tau_e + \beta_1^2 (M - N) \left( -1 + M - N \right) \tau_e - \tau_o + \sigma_y^2 \tau_e (\tau_e + \tau_o) + \beta_1^2 (M - 1) (M \tau_e + \tau_o) \right)}{\left( \beta_1 (M - 1) \rho \beta_1^2 (M - 1) + \beta_2^2 (N - M) + \sigma_y^2 \right)} \\
\frac{\beta_1}{\gamma_1 M + \gamma_2 (N - M)} + \frac{1}{\gamma_1 M + \gamma_2 (N - M)} + \frac{\lambda_1}{\gamma_1 M + \gamma_2 (N - M)} \\
\frac{\left( M - 1 \right) \rho \beta_1^2 \left( \gamma_1 (M - 1) + \gamma_2 (N - M) \right) + \sigma_y^2 \tau_e}{M \left( \gamma_1 (M - 1) + \gamma_2 (N - M) \right) \tau_e}
\end{align*}
\]
\[
\begin{align*}
&\left[ \frac{\gamma_1 M}{\gamma_1 M + \tau_2 (N - M)} + \frac{\beta_1 (M - 1) (\beta_1 (M - 1) + \beta_2 (N - M)) \tau_e}{\gamma_1 (M - 1) + \gamma_2 (N - M) + \beta_2 (M - 1) (\beta_1 (M - 1) + \beta_2 (N - M)) \tau_e + \beta_1^2 (M - 1) (\beta_1 (M - 1) + \beta_2 (N - M)) \tau_e + \beta_1^2 (M - 1) (\beta_1 (M - 1) + \beta_2 (N - M)) \tau_e}
\right]
\end{align*}
\]

Under information sharing, however equations (58) and (59) lead to

\[
\begin{align*}
\frac{[(M - 1)/M \beta_2^2 + (N - M) \beta_2^2 + \tau_e \sigma_e^2]((M \gamma_1 + (N - M) \gamma_2) M - ((M - 1) \gamma_1 + (N - M) \gamma_2) \beta_2 \rho \tau_e^{-1})}{(M \beta_1 + (N - M) \beta_2)^2 + \tau_e (M \beta_2^2 + (N - M) \beta_2^2) \tau_e^{-1} + \tau_e \sigma_e^2 + M \beta_2^2 + (N - M) \beta_2^2 + \tau_e \sigma_e^2} = 2 \beta_2 \\
\frac{[(M \gamma_1 + (N - M) \gamma_2) \beta_2 + \tau_e \sigma_e^2]((M \gamma_1 + (N - M) \gamma_2) M - ((M - 1) \gamma_1 + (N - M) \gamma_2) \beta_2 \rho \tau_e^{-1})}{(M \beta_1 + (N - M) \beta_2)^2 + \tau_e (M \beta_2^2 + (N - M) \beta_2^2) \tau_e^{-1} + \tau_e \sigma_e^2 + M \beta_2^2 + (N - M) \beta_2^2 + \tau_e \sigma_e^2} = \frac{-\gamma_2}{M \gamma_1 + (N - M) \gamma_2} + 1
\end{align*}
\]

If we further assume \(\sigma_2^2 \rightarrow \infty\) then under profit-sharing we have:

\[
\begin{align*}
&\left[ \frac{1}{\gamma_1 (M - 1) + \gamma_2 (N - M) + \tau_e \tau_\alpha} + \frac{\beta_1 (M - 1) \rho}{M \gamma_1 (M - 1) + \gamma_2 (N - M) \tau_e + \tau_\alpha} \right] + \frac{\beta_1 (M - 1) \rho}{M \gamma_1 (M - 1) + \gamma_2 (N - M) \tau_e + \tau_\alpha} \\
&= \left[ \frac{1}{\gamma_1 (M - 1) + \gamma_2 (N - M) + \tau_e \tau_\alpha} + \frac{\beta_1 (M - 1) \rho}{M \gamma_1 (M - 1) + \gamma_2 (N - M) \tau_e + \tau_\alpha} \right] + \frac{\beta_1 (M - 1) \rho}{M \gamma_1 (M - 1) + \gamma_2 (N - M) \tau_e + \tau_\alpha} \\
&= \left[ \frac{1}{\gamma_1 (M - 1) + \gamma_2 (N - M) + \tau_e \tau_\alpha} + \frac{\beta_1 (M - 1) \rho}{M \gamma_1 (M - 1) + \gamma_2 (N - M) \tau_e + \tau_\alpha} \right] + \frac{\beta_1 (M - 1) \rho}{M \gamma_1 (M - 1) + \gamma_2 (N - M) \tau_e + \tau_\alpha}
\end{align*}
\]
\[
\frac{M\gamma_1 + (N - M)\gamma_2}{\gamma_1\tau_e + (N - M - 1)\gamma_2} \beta_2 = 2\beta_2
\]
\[
\frac{\gamma_2}{\tau_e + \tau_v} = \frac{-\gamma_2}{M\gamma_1 + (N - M - 1)\gamma_2} + 1
\]

Plug (69) into (68) we get
\[
\left[ \beta_1 - \frac{\beta_1\gamma_1 M}{\gamma_1 M + \gamma_2 (N - M)} + \frac{\beta_1 (M - 1) \tau_e}{\tau_e + \tau_v} \right] = \left[ -\frac{\beta_1}{\gamma_1 M + \gamma_2 (N - M)} + \frac{\tau_e}{(\tau_e + \tau_v)} \right] \gamma_1 M
\]
\Rightarrow \quad \beta_1 = \frac{\tau_e}{M\tau_e + \tau_v} \gamma_1 M
\]

Plug (72) into (69) we get
\[
\frac{1}{\gamma_1 (M - 1) + \gamma_2 (N - M)} + \frac{\rho}{M(\tau_e + \tau_v)} + \frac{\gamma_1 (M - 1) \rho \tau_e}{[\gamma_1 (M - 1) + \gamma_2 (N - M)](\tau_e + \tau_v)(M\tau_e + \tau_v)} \gamma_1 M
\]
\Rightarrow \quad \frac{\gamma_1 M}{\gamma_1 (M - 1) + \gamma_2 (N - M)} + \frac{\rho \gamma_1}{M\tau_e + \tau_v} = 1
\]

The results are summarized in the following theorem

**Theorem D.1.** Under profit sharing, we have
\[
\beta_1 = \frac{M\tau_e \gamma_1}{M\tau_e + \tau_v}
\]
\[
\beta_2 = \frac{\tau_e \gamma_2}{\tau_v + \tau_e}
\]

while \( \gamma_1 \) and \( \gamma_2 \) are determined by
\[
\frac{\gamma_1 \rho}{M\tau_e + \tau_v} = \frac{-\gamma_1 M}{(M - 1)\gamma_1 + (N - M)\gamma_2} + 1
\]
\[
\frac{\gamma_2 \rho}{\tau_v + \tau_e} = \frac{-\gamma_2}{M\gamma_1 + (N - M - 1)\gamma_2} + 1.
\]

Under information sharing, equation (60) and (61) lead to
\[
\begin{align*}
(M\gamma_1 + (N - M)\gamma_2)M\tau_e - ((M - 1)\gamma_1 + (N - M)\gamma_2)\beta_1 \rho &= 2\beta_1 \\
\frac{\gamma_1 \rho}{\tau_v + \tau_e} &= \frac{-\gamma_1}{(M - 1)\gamma_1 + (N - M)\gamma_2} + 1 \\
[M\gamma_1 + (N - M)\gamma_2]\tau_e - (M\gamma_1 + (N - M - 1)\gamma_2)\beta_2 \rho &= 2\beta_2 \\
\frac{\gamma_2 \rho}{\tau_v + \tau_e} &= \frac{-\gamma_2}{M\gamma_1 + (N - M - 1)\gamma_2} + 1
\end{align*}
\]

(75) \Rightarrow
\[
\beta_1 = \frac{(M\gamma_1 + (N - M)\gamma_2)M\tau_e}{\rho[(M - 1)\gamma_1 + (N - M)\gamma_2] + 2(\tau_v + \tau_e)}
\]

(76) \Rightarrow \gamma_1 \rho[(M - 1)\gamma_1 + (N - M)\gamma_2] = -(\tau_v + M\tau_e)\gamma_1 + (\tau_v + M\tau_e)[(M - 1)\gamma_1 + (N - M)\gamma_2]

\therefore \quad \{ \rho[(M - 1)\gamma_1 + (N - M)\gamma_2] + (\tau_v + M\tau_e) \} \gamma_1 = (\tau_v + M\tau_e)[(M - 1)\gamma_1 + (N - M)\gamma_2]

\Rightarrow \quad \frac{\rho[(M - 1)\gamma_1 + (N - M)\gamma_2] + 2(\tau_v + M\tau_e)}{\rho[(M - 1)\gamma_1 + (N - M)\gamma_2] + 2(\tau_v + M\tau_e)} = \frac{\gamma_1}{\tau_v + \tau_e}

Plug in (79) we have
\[
\beta_1 = \frac{M\tau_e \gamma_1}{\tau_v + M\tau_e},
\]
and similarly we have
\[
\beta_2 = \frac{\tau_e \gamma_2}{\tau_v + \tau_e},
\]
while \(\gamma_1\) and \(\gamma_2\) are jointly determined by solving (76) and (78). The results are summarized below.

**Theorem D.2.** Under information sharing, we have
\[
\begin{align*}
\beta_1 &= \frac{M\tau_e \gamma_1}{M\tau_e + \tau_v} \\
\beta_2 &= \frac{\tau_e \gamma_2}{\tau_v + \tau_e}
\end{align*}
\]
while \(\gamma_1\) and \(\gamma_2\) are determined by
\[
\begin{align*}
\frac{\gamma_1 \rho}{M\tau_e + \tau_v} &= \frac{-\gamma_1}{(M - 1)\gamma_1 + (N - M)\gamma_2} + 1 \\
\frac{\gamma_2 \rho}{\tau_v + \tau_e} &= \frac{-\gamma_2}{M\gamma_1 + (N - M - 1)\gamma_2} + 1
\end{align*}
\]

Compared to the the profit-sharing case, both \(\beta_i\) and \(\gamma_i\) \((i \in \{1, 2\})\) are smaller under profit-sharing.
Under full information, investors' payoffs are $\mathbb{E} \left[ - \exp(-\rho(v - p(x_i))x_i) | s_i, p_i \right]$ when $\sigma_x^2 \to \infty$.

$$
\mathbb{E} \left[ - \exp(-\rho(v - p(x_i))x_i) | s_i, p_i \right] 
= \mathbb{E} \left[ - \exp\left(-\rho \frac{(\frac{\tau_i s_i}{\tau_0 + \tau_i} - p_i)}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right) | s_i, p_i \right] 
\exp\left[ \rho \left( p_i + \frac{1}{(M - 1)\gamma_1 + (N - M)\gamma_2} \frac{(\frac{\tau_i s_i}{\tau_0 + \tau_i} - p_i)}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right) \frac{(\frac{\tau_i s_i}{\tau_0 + \tau_i} - p_i)}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right] 
\exp\left[ -\rho^2 \frac{(\frac{\tau_i s_i}{\tau_0 + \tau_i} - p_i)^2}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right] 
+ \rho \left( p_i + \frac{1}{(M - 1)\gamma_1 + (N - M)\gamma_2} \frac{(\frac{\tau_i s_i}{\tau_0 + \tau_i} - p_i)}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right) \frac{(\frac{\tau_i s_i}{\tau_0 + \tau_i} - p_i)}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right] 
$$
thus the expected utility for those who participate in information sharing is

$$
- \exp\left[ -\rho^2 \frac{(\frac{M\tau_0 s_i}{\tau_0 + \tau_i} - p_i)^2}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right] 
+ \rho \left( p_i + \frac{1}{(M - 1)\gamma_1 + (N - M)\gamma_2} \frac{(\frac{M\tau_0 s_i}{\tau_0 + \tau_i} - p_i)}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right) \frac{(\frac{M\tau_0 s_i}{\tau_0 + \tau_i} - p_i)}{2((M - 1)\gamma_1 + (N - M)\gamma_2) + \rho \frac{1}{\tau_0 + \tau_i}} \right] 
$$
thus expected utility is

$$
- \exp\left[ \frac{(M\tau_0 s_i - p_i(M\tau_0 + \tau_0))^2(\gamma_2(N - M)\rho - 2(M\tau_0 + \tau_0) + \gamma_2^2(M - 1)\gamma_1(M - 1)\gamma_2(M\tau_0 + \tau_0) + \gamma_1(M - 1)(\rho - \gamma_2(M - N)(M\tau_0 + \tau_0))}{2(\gamma_1(M - 1) + \gamma_2(N - M))(M\tau_0 + \tau_0)(\rho + 2(\gamma_1(M - 1) + \gamma_2(N - M))(M\tau_0 + \tau_0))^2} \right] 
$$

While under profit sharing, partner $i$’s expected utility is given by

$$
- \exp\left( \frac{A_i}{\sqrt{B}} \right), \quad (80)
$$

$^{33}$Notice that the conditioning on $p_i$ is important even when $\sigma_x^2 \to \infty$, i.e. price itself does not aggregate information.
where

\[
A = \frac{\rho^2}{M^2} \left( \frac{\tau_s s_i}{\tau_v + \tau_e} - p_i - \lambda_i x_i \right)^2 \left[ ((M - 1)\beta_1) \frac{1}{\tau_v + \tau_e} + (M - 1)\beta_1^2 \tau_e^{-1} \right] \\
+ \left[ x_i + (M - 1)\beta_1 \frac{\tau_s s_i}{\tau_v + \tau_e} - (M - 1)\gamma_1 (p_i + \lambda_i x_i) \right] \frac{\rho^2}{M^2} \frac{1}{\tau_v + \tau_e} \\
- 2 \frac{\rho}{M} \left( \frac{\tau_s s_i}{\tau_e} - p_i - \lambda_i x_i \right) \left[ x_i + (M - 1)\beta_1 \frac{\tau_s s_i}{\tau_v + \tau_e} - (M - 1)\gamma_1 (p_i + \lambda_i x_i) \right] \\
\left[ 1 + \frac{\rho}{M} (M - 1)\beta_1 \frac{1}{\tau_v + \tau_e} \right]
\]

\[
B = \left[ 1 + \frac{\rho}{M} (M - 1)\beta_1 \frac{1}{\tau_v + \tau_e} \right]^2 - \frac{\rho^2}{M^2} \frac{1}{\tau_v + \tau_e} \left[ ((M - 1)\beta_1) \frac{1}{\tau_v + \tau_e} + (M - 1)\beta_1^2 \tau_e^{-1} \right]
\]

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