Average and Marginal Tobin's q as Indicators of Future Growth Opportunities, Expected Return, and Risk

by

Richard Roll and J. Fred Weston

December 3, 2008

Abstract

Contrary to popular opinion, average Tobin's q is a better indicator of future growth opportunities than marginal Tobin's q. We derive a curious relation between average and marginal q: the more profitable a new investment opportunity, the smaller will be the increase in average q when the opportunity is undertaken. Average q is inversely related to the cost of equity capital, so it represents an inverse measure of risk. The closely-related book/market ratio is also a measure of risk in the cross-section.

Acknowledgment

We are grateful for stimulating conversations with Bruce Carlin and Robert Geske.

Average and Marginal Tobin's q as Indicators of
Future Growth Opportunities, Expected Return, and Risk

by

Richard Roll and J. Fred Weston

December 3, 2008

We assert that average Tobin's [1969] q is a better reflection of future growth opportunities than marginal Tobin's q. In addition, the more profitable a new investment opportunity and the larger marginal q, the smaller will be the resulting change in average q.

For four decades, Tobin's q has played a central role in the theory of investment. More recently, it has found application in financial economics as a cross-sectional indicator of anticipated return, and, implicitly, as a proxy measure of risk. Our purpose here is first to reconsider the conceptual foundations of Tobin's q, both the average and marginal versions, as empirical indicators of future growth opportunities. Then, we derive the curious negative relation mentioned above between marginal q and the change in average q. Finally, we discuss the implications of q as an empirical indicator of return and risk.

In Tobin's original paper,[1] the q ratio was defined as the market value of a firm's assets divided by their replacement cost. This has come to be known as "average" q and has often been measured for an individual firm by the book value of assets plus the difference between the market and book values of equity, divided by some estimate of the replacement cost of assets in place. The numerator of the average q ratio should, of course, be the market value of all assets but much of the existing literature ignores any divergence between the market and book values

[1] See also Brainard and Tobin [1968], which appeared in print prior to Tobin [1969]. It is not clear to us why q shouldn't be "Brainard and Tobin's."

of debt. As for the denominator of the q ratio, some have made an earnest effort to measure replacement costs[2] while others have simply assumed them equal to the book value of assets.[3]

Almost all authors, beginning with Tobin, mention that firms should make investment decisions based on the marginal q, not the average q, where marginal q is defined as the ratio of the incremental market value of the firm from new investments divided by their costs; see, for example, Abel and Eberly [1994]. As stated by Hayashi [1982],

> If a firm can freely change its capital stock, then it will continue to increase or decrease its capital stock until [average] q is equal to unity, (p. 214).

This accords well with traditional corporate finance, which admonishes managers to accept new investment projects whose net present values are positive. The net present value is the difference between the market value of the project's cash flows and the initial project cost; (It's a market value because the cash flows are discounted at the market's opportunity cost of capital.)

Many authors have accepted the basic principle of marginal q while at the same time lamenting its unobservability. The following statements are typical:

> The "q" theory…is not operational as long as q is not observable…What we can observe is *average* q, namely the ratio of the market value of *existing* capital to its replacement cost, (Hayashi, 1982, p. 214, emphasis in original.)

> Most standard intertemporal models of investment imply that investment opportunities should be measured by an unobservable quantity: marginal q, (Whited, 2001, p. 1668.)

Interestingly, one of Hayashi's main contributions was to derive conditions under which marginal and average q are equal; viz., the firm is a price taker with constant returns to scale. Whited regards their difference as a form of measurement error. In her view, marginal q should be an indicator of investment opportunities while average q, being perhaps a poor proxy for marginal q, should be corrected for error.

---

[2] The seminal article on measuring q is by Lindeberg and Ross [1981] who made what must have been an exhausting effort to get everything right in both numerator and denominator. Lewellen and Badrinath [1997] provide an updated empirical analysis and critique of measurement methods.

[3] For example, see Whited [2001], who argues a larger sample size is worth the imperfect measurement.

In light of the voluminous literature about marginal q, one of our main points may seem rather heretical. If average q is a better indicator of future investment opportunities than marginal q, empiricists need not lose sleep over the difficulty in finding direct measures of marginal q.[4]

Our allegation is based on a simple idea; namely, the firm's market value already reflects the market's assessment of future investment opportunities, including all conceivable but as yet not-undertaken projects. Future projects belong to the firm only in the sense that they might actually be physically initiated at some point, but this is enough to affect value. At the present time, future projects can be characterized as real options - contingent assets for whom option exercise is the act of making a cash investment. Note that such contingent assets need not be limited to traditional capital investments; they might also include opportunistic takeovers. Jovanovic and Rousseau [2002] call this the "Q theory of mergers."[5]

The denominator of average q in all past literature is supposed to be the replacement cost of all physical assets already in place. Consequently, average q is a quotient that mixes up two categories of assets. While the numerator subsumes market value increments from both current physical assets and assets to be purchased in the future, the denominator includes replacement costs only for the former.

To be specific, ignoring measurement problems with replacement costs and the market value of debt, we assert that Tobin's (average) q is empirically equivalent to

$$q_0 = \frac{V_0 + \sum_{t=1}^{\infty} C_t(V_t)}{X_0}, \tag{1}$$

where the numerator, the market value of the firm's securities, consists of V_0, the present value of cash flows from assets already in place, plus a series of real options, one covering each future decision period t, with current (t=0) market value $C_t(V_t)$.

---

[4] We abstract, as many do, from the large literature on the adjustment costs of investment; see, for example, Lucas and Prescott [1971] and Abel and Eberly [1994].

[5] Interestingly, Andrade, Mitchell and Stafford [2001] report that acquiring firms generally have higher q's than target firms. The total gains from merging appear to be higher when the difference in q, bidder less target, is larger; see Servaes [1991] and Lang, Stulz and Walkling [1989], who attribute the phenomenon to superior (inferior) management in high (low) q firms.

Note that the t subscripts on $C_t(V_t)$ in (1) refer to the expiration date of the real option, beyond which the investment opportunity will no longer be available. The current (t=0) market value of the option is C_t and the current (t=0) value of the underlying state variable is V_t. At expiration, V_t will be the present value of the cash flows from the project, which will be undertaken only if it exceeds the project's cost at that time. Also, $C_t(V_t)$ should be understood as the aggregate option value of all the investment projects that can be undertaken at t. There need not exist, of course, options corresponding to every future calendar period. The denominator of Tobin's average q in (1) is X_0, the replacement cost of the firm's current assets in place.

Now imagine that the firm progresses to the next period, t=1, and that the real option expiring at that time is in-the-money. This is tantamount to that particular period's investment opportunity having a positive net present value; i.e., $V_1>X_1$ where X_1 denotes its original investment cost. After undertaking the project, Tobin's (average) q becomes

$$q_1 = \frac{V_0 + V_1 + \sum_{t=2}^{\infty} C_t(V_t)}{X_0 + X_1}. \tag{2}$$

In the traditional literature, the marginal value of Tobin's q would simply be V_1/X_1 or, in Hayashi's terms, "…the ratio of the market value of new additional investment goods to their replacement cost," (p. 214.) One might have thought this would be related to the observed incremental change in average q between the original period (t=0) and period t=1 when the new investment goods are put into place, but this turns out to be far from the case.

The incremental $\Delta q = q_1 - q_0$, between t=0 and t=1 can be more lucidly considered by holding constant the values and replacement costs of the current assets in place and the values of all future investment opportunities. Given this condition, and noting that the value at t=0 for the real option expiring at t=1 satisfies $C_1(V_1) \geq V_1 - e^{-r}X_1$, where r is the riskless rate of discount,

$$\Delta q \equiv q_1 - q_0 \leq \frac{V_1 + K}{X_0 + X_1} - \frac{V_1 - e^{-r}X_1 + K}{X_0}, \tag{3}$$

where $K = V_0 + \sum_{t=2}^{\infty} C_t(V_t)$. At issue is the relation between $q_1 - q_0$ and $V_1 - X_1$. Simplifying notation by denoting marginal q as $\alpha \equiv V_1/X_1 > 1$, the incremental change in observed average q satisfies

$$\Delta q \leq \frac{\alpha X_1 + K}{X_0 + X_1} - \frac{(\alpha - e^{-r})X_1 + K}{X_0} \tag{4}$$

and, curiously, $\partial\Delta q/\partial\alpha \leq 0$ with strict inequality for $X_1>0$. In other words, the more profitable the new investment project and the larger marginal q, the smaller will be the incremental empirical change in Tobin's average q between t=0 and t=1! Not only is the observed average q an imperfect measure of marginal q, but the change in average q, which one might have thought would more or less reflect the marginal impact of new investments, is negatively related to their incremental value.[6]

It is not difficult to understand the source of this seemingly perverse result. Prior to undertaking the new project, its full market value net of cost is reflected in the numerator of average q but nothing about the new project is accounted for in the denominator. At the expiration date of the first option (period t=1), both the denominator and the numerator are incremented by the same amount, the project cost, if it is undertaken.[7] The impact on average q is a decreasing function of the relative value of the new project. Essentially, the very formulation of (average) q is somewhat defective because the denominator measures replacement costs only for assets already in place while the numerator, being driven to some extent by market anticipations, includes the real option value of all future investment opportunities.

The empirical implications are somewhat unsettling. In a cross-section of firms investing during a given period, those undertaking more valuable projects will see their average q's decreasing relative to firms with less valuable projects!

This simple analysis abstracts from other intertemporal influences on q. Perhaps most important, the menu of available real options and their values can change from one period to another. The market's assessment of these options can, of course, induce large fluctuations in the firm's overall value. For many firms, particularly those in high growth and high tech industries, this probably constitutes the single largest cause of price volatility.

---

[6] If the t=1 investment opportunity expires out-of-the-money, average q will decline because $C_1(V_1)\geq 0$.

[7] At expiration in t=1, $q_1=(V_1+K)/(X_0+X_1)$ and q_0 becomes (approximately) $(V_1-X_1+K)/X_0$.

At any given time, however, average q incorporates the market's best guess about the current total value of future investment opportunities, including the probabilities of their eventual adoption. Even projects never adopted have an influence on average q through their option characteristics. There is nothing wrong conceptually with marginal q defined as $\Delta V/\Delta X$, the incremental change in value divided by the incremental cost; but as we have seen, this construct cannot be measured successfully by the observed intertemporal change in average q, even abstracting from those inevitable surprises that impinge on market values. The reason for this infeasibility is essentially that the numerator of marginal q, ΔV, has influenced average q long before the project is actually undertaken.

One might hesitate to use average q as an indicator of future investment opportunities on the grounds that the market has imperfect information about their value or even their existence; but a similar caveat could be directed at the market's valuation of future cash flows from assets already in place. It seems to us that average q does, in fact, contain the market's best forecast of future investment opportunities. Unfortunately, average q also contains another component: the market value of assets currently in place relative to their replacement cost, so it is not a pristine measure of future investment opportunities.

Lindenberg and Ross [1981] argued that

> …for firms engaged in positive investment, in equilibrium we expect q to exceed one by the capitalized value of the Ricardian and monopoly rents which the firm enjoys, (p. 3).[8]

Note that they make no direct mention of real investment options whose values are embedded in current asset prices, but there is an indirect recognition within the conditioning clause, "for firms engaged in positive investment." We now see that the Ricardian and monopoly rents reflected in q must arise not only from current assets in place but also from investments yet to be made.

For a given firm at a particular point in time, it is probably impossible to unambiguously disentangle the rents attributable to current assets versus future opportunities. For example,

---

[8] Ricardian rents are due to specific unique factors of production that reduce costs relative to that of the marginal firm. They could be earned by the infra-marginal firm even in a competitive industry. The replacement cost of the unique factor is presumably carried on the firm's books at cost; i.e., without including the capitalized value of rents.

Lindenberg and Ross present average q ratios for well-known individual firms calculated over the 1960-77 sample period. In their table 2, pp. 18-20, there are nine firms listed with very high q's (above four.) They are Avon Products, Coca-Cola, IBM, Eli Lilly, 3-M, Polaroid, Schering-Plough, Smithkline, and Xerox. It would be rather hard to believe that, say, Coca-Cola or Avon Products possessed a large ensemble of valuable real options on future investment opportunities. For IBM, Lilly, or 3-M, in contrast, this would be easy to accept. Polaroid or Xerox might reflect monopoly rents expected on patents or perhaps they too had many opportunities.

II. Average q, Market/Book, and Leverage; a Clarification.

Recent research and financial practice has focused on the Market/Book ratio of equity as an indicator of future growth opportunities.[9] Since q also measures these opportunities, a natural question is how these two indicia are related.

This question is easily settled if we follow Whited [2001] and assume equivalence between replacement costs and book values. In this case, q is simply

$$q = \frac{M + D}{B + D} \tag{5}$$

where M (B) is the market (book) value of equity and D is the value of debt.[10] Defining a book leverage ratio as $L \equiv D/(B+D)$, a few algebraic substitutions provides the following relation between q and M/B:

$$q = \frac{M}{B}(1 - L) + L. \tag{6}$$

[9] For example, see Fama and French [1995]. Market/Book ratios are widely used by financial practitioners. High values are thought to indicate "growth" stocks while low values indicate "value" stocks.

[10] Generally, D has been measured by the book value of debt but our analysis here goes through exactly if D happens to be the market value of debt. In this case, however, replacement costs would, curiously, be the book value of equity plus the market value of debt. Note also that if $L' = D/(M+D)$ is the market leverage ratio, then $q = L/L'$.

Thus, holding (book) leverage constant, q is a positive linear function of M/B with a coefficient equal to the complement of the leverage ratio. For an unlevered firm, q and M/B are identical. Recent empirical literature has found a negative relation between M/B and equity returns, which implies (again holding leverage constant) that high q firms will have lower returns on average. This is counter-intuitive in that high q firms are those with the more growth opportunities and presumably are subject to more risk.

Holding M/B constant, the expression above indicates that q would increase (decrease) with leverage if M<(>)B. But higher leverage increases equity risk, *ceteris paribus*, and thus M would probably be affected by a change of leverage, rendering the overall impact of leverage on q ambiguous without further analysis (which we provide in the next section.)

In the more plausible circumstance that replacement costs are not exactly equal to the book value of firm assets, q and M/B need not so closely related. The generalized version of (6) is

$$q = \frac{M}{B}(1 - L)\varphi + L\varphi\,, \tag{7}$$

where φ is the ratio of the book value of assets to their replacement cost.

As depicted by (7), there are two wedges in the cross-sectional relation between q and M/B, leverage (L) and the book value/replacement cost dichotomy (φ). For some groups of assets, such as companies in the same industry, these wedges might not matter very much and hence q and M/B could be very closely related. Similarly, if the two wedges were cross-sectionally unrelated to both q and M/B, the latter might still be highly correlated. Interestingly, for unlevered firms q and M/B are identical regardless of any divergence between the book value of assets and replacement cost.

As an empirical matter, it seems plausible to anticipate a relatively strong relation between M/B and q, however q is measured.

III. Average q, expected return, and risk.

The close correspondence between the equity market/book ratio and average q intimates a similar connection for q and the expected (required) return on equity. Perhaps the most direct path to an algebraic relation is through the well-known constant growth valuation model, commonly called the "Gordon" model,

$$P = \frac{D}{\rho - g},$$

where P is the share price, D is expected dividends per share in the next period, g is the perpetual growth rate of dividends and ρ is the risky-adjusted cost of equity, or the expected equity return. The ratio of dividends to earnings is, by definition, the payout ratio, denoted λ. The ratio of total earnings to book value of equity is, by definition, the expected operating return on book equity, denoted k. The expected operating return can be thought of as a measure of the expected cash flow relative to the initial cost of assets in place. Substituting total values for per share values and rearranging, the Gordon model implies

$$\rho = g + k\lambda \frac{B}{M}. \tag{8}$$

Consequently, holding constant the growth rate of expected dividends, the payout ratio, and the expected operating return on book equity, there is a direct linear connection between the equity book/market ratio and the required (risky) expected return on equity. From this perspective, it seems that B/M is, in fact, a risk indicator. Note that this cross-sectional relation between expected equity return and B/M should hold irrespective of the underlying asset pricing theory. It is even independent of investor preferences, provided that they are not risk neutral.[11] Hence, the relation's tautological nature suggests that B/M should be dubbed an "indicator" of risk rather than anything more profound.

The Gordon model is used most often to explain a negative relation between dividend yields and growth rates. As (8) shows, there is a corresponding negative connection between growth rates and the equity book/market ratio <u>provided</u> that the cost of equity capital is held constant. But there is no more compelling reason to hold the cost of equity constant than to hold the growth

[11] If all investors were risk neutral, there would be no cross-sectional variation in the required return on equity.

rate constant when considering the cross-section of stocks. For similarly growing firms, expected equity returns should vary positively with book/market.

Turning back to Tobin's q, solving for B/M from (7) and substituting in (8), we obtain

$$\rho = g + k\lambda \frac{1 - L}{q / \varphi - L} . \tag{9}$$

(ρ cannot be negative because $q>\varphi L$.)

Hence, holding constant operating earnings on book equity (k), leverage, the payout ratio, and the growth in earnings, there is a negative cross-sectional relation between average q and ρ, the expected return on equity. This is really nothing more than saying that high q firms must have lower costs of capital, *ceteris paribus*. Of course, in a random sample of real firms, one would expect a strong cross-sectional correlation between earnings growth and q, while the impact of leverage on growth would represent yet another confounding influence. Nonetheless, if adequate estimates of earnings growth, leverage, and q are available, (9) could conceivably be an operational model of the cost of equity capital across firms within an industry where φ is likely to be roughly constant. (The other required input, k, is known at least to the extent that accounting reports are reliable.)

IV. Conclusions.

Tobin's q, as typically measured, has some curious properties. Although marginal q is supposedly a sound measure of the value of investment decisions, average q could actually be a better reflection of future growth opportunities. Moreover, changes in average q are inversely related to the marginal value of investment projects as they are undertaken.

There is a virtually tautological relation between the required return on equity and the equity book/market ratio, which implies that the latter is an indicator of risk in the cross-section, regardless of the asset pricing model. Tobin's q and equity book/market are negatively related and the intensity of the connection is affected by leverage; consequently q also can be considered an (inverse) risk measure in the cross-section.

References

Abel, Andrew B., and Janice C. Eberly, 1994, A Unified Model of Investment Under Uncertainty, *American Economic Review* 84 (5, December), 1369-1384.

Andrade, Gregor, Mark Mitchell, and Erik Stafford, 2001, New Evidence and Perspective on Mergers, *Journal of Economic Perspectives* 15, (2, Spring), 103-120.

Brainard, William C., and James Tobin, 1968, Pitfalls in Financial Model Building, *American Economic Review* 58 (2, May), 99-122.

Fama, Eugene F. and Kenneth R. French, 1995, Size and Book-to-Market Factors in Earnings and Returns, *Journal of Finance* 50 (1, March), 131-155.

Hayashi, Fumio, 1982, Tobin's Marginal q and Average q: A Neoclassical Interpretation, *Econometrica* 50 (1, January), 213-224.

Jovanovic, Boyan, and Peter L. Rousseau, 2002, The Q-Theory of Mergers, *American Economic Review* 92 (2, May), 198-204.

Lang, Larry H.P., Rene M. Stulz and Ralph A. Walkling, 1989, Managerial performance, Tobin's Q, and the gains from successful tender offers, *Journal of Financial Economics* 24, (1, September), 137-154.

Lewellen, Wilbur G. and S. G. Badrinath, 1997, On the Measurement of Tobin's q, *Journal of Financial Economics* 44 (1, April), 77-122.

Lindenberg, Eric B. and Stephen A. Ross, Tobin's q Ratio and Industrial Organization, *Journal of Business* 54 (1, January), 1981, 1-32.

Lucas, Robert E., Jr., and Edward C. Prescott, 1971, Investment Under Uncertainty, *Econometrica* 39, (5, September), 659-681.

Servaes, Henri, 1991, Tobin's Q and the Gains from Takeovers, *Journal of Finance* 46 (1, March), 409-419.

Tobin, James, 1969, A General Equilibrium Approach to Monetary Theory, *Journal of Money, Credit and Banking* 1 (1, February), 15-29.

Whited, Toni M., 2001, Is It Inefficient Investment that Causes the Diversification Discount, *Journal of Finance* 56 (5, October), 1667-1721.