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The Market Price of Risk in Interest Rate Swaps: The Roles of Default and Liquidity Risks*

We study how the market prices the default and liquidity risks incorporated into interest rate swap spreads. We jointly model the Treasury, repo, and swap term structures using a five-factor affine framework and estimate the model by maximum likelihood. The credit spread is driven by a persistent liquidity process and a rapidly mean-reverting default intensity process. The credit premium for all but the shortest maturities is primarily compensation for liquidity risk. The term structure of liquidity premia increases steeply, while that of default premia is almost flat. Both liquidity and default premia vary significantly over time.

I. Introduction

One of the most fundamental issues in finance is how the market compensates investors for bearing credit risk. This issue is complicated by the fact that credit spreads may actually consist of both default and liquidity components, as a number of recent papers have

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shown.[1] How large are the market premia for the default and liquidity risks faced by the holder of a credit-sensitive security?

To address this issue, this article studies what is rapidly becoming one of the most important credit spreads in the financial markets—interest rate swap spreads. Since swap spreads represent the difference between swap rates and Treasury bond yields, they reflect the difference in the default risk of the financial sector quoting Libor rates and the U.S. Treasury. In addition, swap spreads may include a significant liquidity component if the relevant Treasury bond trades special in the repo market. Thus, swap spreads represent a nearly ideal data set for examining how both default and liquidity risks influence security returns. The importance of swap spreads derives from the dramatic recent growth in the notional amount of interest rate swaps outstanding relative to the size of the Treasury bond market. For example, the total amount of Treasury debt outstanding at the end of June 2003 was $6.6 trillion. In contrast, the Bank for International Settlements (BIS) estimates that the total notional amount of interest rate swaps outstanding at the end of June 2003 was $95.0 trillion, representing nearly 15 times the amount of Treasury debt (see http://www.bis.org/publ/otc_hy0311.pdf).

Our empirical approach consists of jointly modeling the Treasury, repo, and swap term structures using the reduced-form credit framework of Duffie and Singleton (1997, 1999). The liquidity component in swap spreads is identified from the difference between general collateral government repo rates (which can be viewed essentially as riskless rates) and yields on highly liquid on-the-run Treasury bonds. The default component in swap spreads can then be identified from the difference between swap and repo rates. Estimating all three curves jointly allows us to capture the interactions among the term structures. To capture the rich dynamics of the Treasury, repo, and swap curves, we use a five-factor affine term structure model that allows the swap spread to be correlated with the riskless rate. In addition, our specification allows market prices of risk to vary over time to reflect the possibility that the willingness of investors to bear default and liquidity risk may change. We estimate the parameters of the model by maximum likelihood. The data for the study span nearly the full history of the swap market. We show that both the swap and Treasury term structures are well described by the five-factor affine model.

As a preliminary, we first verify that there are both significant default and liquidity components in the swap spread. On average, the default risk component is about 31 basis points, while the liquidity risk component is about 7 basis points. The default risk component is uniformly positive, has frequent spikes, and is rapidly mean reverting. In contrast, the liquidity risk component is very persistent and was near zero for much of the 1990s but has increased

1. See, e.g., Duffie and Singleton (1997), Huang and Huang (2000), Collin-Dufresne, Goldstein, and Martin (2001), Elton et al. (2001), and Longstaff, Mithal, and Neis (2005).

dramatically in recent years. The results also suggest that little of the swap spread is attributable to tax effects.

We then turn to the central issue of how the market prices the liquidity and default risks in the swap curve. We find that there is a sizable credit premium built into the swap curve. Surprisingly, however, for horizons beyond 5 years, this credit premium is largely compensation for the variation in the liquidity component of the spread. On average, the term structure of liquidity premia is positive and steeply increasing with maturity. In contrast, the average term structure of default risk premia is flat. Both the default and liquidity premia vary significantly through time and occasionally take on negative values during the sample period.

A number of other papers have also focused on the determinants of swap spreads. In an important recent paper, Duffie and Singleton (1997) apply a reduced-form credit modeling approach to the swap curve and examine the properties of swap spreads. Our results support their finding that both default risk and liquidity components are present in swap spreads. He (2000) uses a multifactor affine term structure framework to model the Treasury and swap curves simultaneously but does not estimate the model. Other research on swap spreads includes Sun, Sundaresan, and Wang (1993), Lang, Litzenberger, and Liu (1998), Collin-Dufresne and Solnik (2001), Grinblatt (2001), Eom, Subrahmanyam, and Uno (2002), Huang, Neftci, and Jersey (2003), Afonso and Strauch (2004), and Kambhu (2004). Our article differs in a fundamental way from this literature, since, by jointly modeling and estimating the Treasury, repo, and swap curves, our approach allows us to identify both the default and liquidity components of the credit spread embedded in swap rates. Furthermore, to our knowledge, this article is the first to provide direct estimates of both the liquidity and default risk premia in the swap market.

The remainder of this article is organized as follows. Section II explains the framework used to model the Treasury, repo, and swap term structures. Section III describes the data. Section IV discusses the estimation of the model. Section V presents the empirical results. Section VI summarizes the results and makes concluding remarks.

II. Modeling Swap Spreads

To understand how the market prices credit risk over time, we need a framework for estimating expected returns implied by the swap and Treasury term structures. In this section, we use the Duffie and Singleton (1997, 1999) credit modeling approach as the underlying framework for analyzing the behavior of swap spreads. In particular, we jointly model the Treasury, repo, and swap term structures using a five-factor affine framework and estimate the parameters of the model by maximum likelihood.[2]

2. There are many recent examples of affine credit models. A few of these are Duffee (1999), He (2000), Collin-Dufresne and Solnik (2001), Duffie and Liu (2001), Duffie, Pedersen, and Singleton (2003), Huang and Huang (2003), Berndt et al. (2004), and Longstaff et al. (2005).

Recall that in the Duffie and Singleton (1997, 1999) framework, the value $D(t, T)$ of a liquid riskless zero-coupon bond with maturity date T can be expressed as

$$D(t, T) = E_Q\left[\exp\left(-\int_t^T r_s \, ds\right)\right], \tag{1}$$

where r_t denotes the instantaneous riskless rate and the expectation is taken with respect to the risk-neutral measure Q rather than the objective measure P. Assume that there are also illiquid riskless zero-coupon bonds in the market. This framework can be extended to show that the price $A(t, T)$ of an illiquid riskless zero-coupon bond can be expressed as

$$A(t, T) = E_Q\left[\exp\left(-\int_t^T r_s + \gamma_s \, ds\right)\right], \tag{2}$$

where γ_t is an instantaneous liquidity spread (or perhaps more precisely, an illiquidity spread).[3] Finally, default is modeled as the realization of a Poisson process with an intensity that may be time varying. Under some assumptions about the nature of recovery in the event of default, the value of a risky zero-coupon bond $C(t, T)$ can be expressed in the following form:

$$C(t, T) = E_Q\left[\exp\left(-\int_t^T r_s + \gamma_s + \lambda_s \, ds\right)\right], \tag{3}$$

where λ_t is the risk-neutral default intensity process. This default intensity process can also be thought of as the product of the time-varying Poisson intensity and the fraction of the loss of market value in the event of default.

In applying this credit model to swaps, we are implicitly making two assumptions. First, we assume that there is no counterparty credit risk. This is consistent with recent papers by Duffie and Singleton (1997), He (2000), and Grinblatt (2001) that argue that the effects of counterparty credit risk on market swap rates should be negligible because of the standard marking-to-market or posting-of-collateral and haircut requirements almost universally applied in swap markets.[4] Second, we make the relatively weak assumption that the credit risk inherent in the Libor rate (which determines the swap rate) can be modeled as the credit risk of a single defaultable entity. In actuality, the Libor rate is a composite of rates quoted by 16 banks and, as such, need not represent

3. This approach follows Duffie and Singleton (1997), who allow for Treasury cash flows to be discounted at a lower rate than non-Treasury cash flows, where the difference is due to a convenience yield process. Our approach accomplishes the same by adding a spread to the discount rate applied to non-Treasury cash flows.

4. Even in the absence of these requirements, the effects of counterparty credit risk for swaps between similar counterparties are very small relative to the size of the swap spread. For example, see Cooper and Mello (1991), Sun et al. (1993), Bollier and Sorensen (1994), Longstaff and Schwartz (1995), Duffie and Huang (1996), and Minton (1997).

the credit risk of any particular bank.[5] In this sense, the credit risk implicit in the swap curve can be viewed essentially as the average credit risk of the most representative banks providing quotations for Eurodollar deposits.[6]

To model the bond prices $D(t,T)$, $A(t,T)$, and $C(t,T)$, we next need to specify the dynamics of r, γ, and λ. In doing this, we work within a general affine framework. In particular, we assume that the dynamics of r, γ, and λ are driven by a vector X of five state variables, $X' = [X_1, X_2, X_3, X_4, X_5]$.

In modeling the liquid riskless rate r, we assume that

$$r = \delta_0 + X_1 + X_2 + X_3, \tag{4}$$

where δ_0 is a constant. Thus, the dynamics of the liquid riskless term structure are driven by the first three state variables. This three-factor specification of the riskless term structure is consistent with recent evidence by Dai and Singleton (2002) and Duffee (2002) about the number of significant factors affecting Treasury yields.[7]

In modeling the dynamics of the liquidity spread γ, we assume that

$$\gamma = \delta_1 + X_4, \tag{5}$$

where δ_1 is a constant. Thus, the state variable X_4 drives the variation in the yield spreads between illiquid and liquid riskless bonds. The liquidity spread γ may be correlated with both r and λ, since our framework will allow X_4 to be correlated with the other state variables.

Finally, to model the dynamics of the default intensity λ, we assume that

$$\lambda = \delta_2 + \tau r + X_5, \tag{6}$$

where δ_2 and τ are constants. This specification allows the default process λ to depend on the state variables driving the riskless term structure in both direct and indirect ways. Specifically, λ depends directly on the first three state variables through the term τr in equation (6). Indirectly, however, the default process λ may be correlated with the riskless term structure through correlations between X_5 and the other state variables. The advantage of allowing both direct and indirect dependence is that it enables us to examine in more depth the determinants of swap spreads. For example, our approach

5. The official Libor rate is determined by eliminating the highest and lowest four bank quotes and then averaging the remaining eight. Furthermore, the set of 16 banks whose quotes are included in determining Libor may change over time. Thus, the credit risk inherent in Libor may be "refreshed" periodically as low-credit banks are dropped from the sample and higher-credit banks are added. The effects of this "refreshing" phenomenon on the differences between Libor rates and swap rates are discussed in Collin-Dufresne and Solnik (2001).

6. For discussions about the economic role that interest-rate swaps play in financial markets, see Bicksler and Chen (1986), Turnbull (1987), Smith, Smithson, and Wakeman (1988), Wall and Pringle (1989), Macfarlane, Ross, and Showers (1991), Sundaresan (1991), Litzenberger (1992), Sun et al. (1993), Brown, Harlow, and Smith (1994), Minton (1997), Gupta and Subrahmanyam (2000), and Longstaff, Santa-Clara, and Schwartz (2001).

7. Also see the empirical evidence in Litterman and Scheinkman (1991), Knez, Litterman, and Scheinkman (1994), Longstaff et al. (2001), and Piazzesi (2005) indicating the presence of at least three significant factors in term structure dynamics.

allows us to examine whether the swap spread is an artifact of the difference in the tax treatment given to Treasury securities and Eurodollar deposits. Specifically, interest from Treasury securities is exempt from state income taxation while interest from Eurodollar deposits is not. Thus, if the spread λ were determined entirely by the differential tax treatment, the parameter τ would represent the marginal state tax rate of the marginal investor and might be on the order of 0.05 to 0.10. In contrast, structural models of default risk, such as Merton (1974), Black and Cox (1976), and Longstaff and Schwartz (1995), suggest that credit spreads should be inversely related to the level of r, implying a negative sign for τ. Finally, we assume that the value of γ is the same under both the objective and risk-neutral measures. The value of λ, however, can be different under the objective and risk-neutral measures. This issue is addressed later in the section on risk premia.

To close the model, we need to specify the dynamics of the five state variables driving r, γ, and λ. We assume that under the risk-neutral measure, the state variable vector X follows the general Gaussian process,

$$dX = -\beta X dt + \Sigma dB^Q, \tag{7}$$

where β is a diagonal matrix, B^Q is a vector of independent standard Brownian motions, Σ is lower diagonal (with elements denoted by σ_{ij}), and the covariance matrix of the state variables $\Sigma\Sigma'$ is of full rank and allows for general correlations among the state variables. As shown by Dai and Singleton (2000), this is the most general Gaussian or $A_5(0)$ structure that can be defined under the risk-neutral measure. Finally, Dai and Singleton (2002) argue that Gaussian models are more successful in capturing the dynamic behavior of risk premia in the class of affine models.

To study how the market compensates investors over time for bearing credit risk, it is important to allow a fairly general specification of the market prices of risk in this affine $A_0(5)$ framework. Accordingly, we assume that the dynamics of X under the objective measure are given by

$$dX = -\kappa(X - \theta)dt + \Sigma dB^P, \tag{8}$$

where κ is a diagonal matrix, θ is a vector, and B^P is a vector of independent standard Brownian motions. This specification has the advantages of being both tractable and allowing for general time-varying market prices of risk for each of the state variables.[8]

8. It is important to acknowledge, however, that even more general specifications for the market prices of risk are possible. For example, the diagonal matrix κ could be generalized to allow nonzero off-diagonal terms. Our specification, however, already requires the estimation of 10 market price of risk parameters and approaches the practical limits of our computational techniques. Adding more market price of risk parameters also raises the risk of introducing identification problems.

Given the risk-neutral dynamics of the state variables, closed-form solutions for the prices of zero-coupon bonds are given by

$$D(t,T) = \exp\left(-\delta_0(T-t) + a(t) + b'(t)X\right), \tag{9}$$

$$A(t,T) = \exp\left(-(\delta_0+\delta_1)(T-t) + c(t) + d'(t)X\right), \tag{10}$$

$$C(t,T) = \exp\left(-((1+\tau)\delta_0+\delta_1+\delta_2))(T-t) + e(t) + f'(t)X\right), \tag{11}$$

where

$$\begin{aligned} a(t) &= \frac{1}{2} L'\beta^{-1}\Sigma\Sigma'\beta^{-1}L(T-t)a \\ &\quad - L'\beta^{-1}\Sigma\Sigma'\beta^{-2}(I - e^{-\beta(T-t)})L \\ &\quad + \sum_{i,j} \frac{1-e^{-(\beta_{ii}+\beta_{jj})(T-t)}}{2\beta_{ii}\beta_{jj}(\beta_{ii}+\beta_{jj})}(\Sigma\Sigma')_{ij}L_iL_j, \\ b(t) &= \beta^{-1}(e^{-\beta(T-t)} - I)L, \end{aligned}$$

I is the identity matrix, and $L' = [1,1,1,0,0]$. The functions $c(t)$ and $d(t)$ are the same as $a(t)$ and $b(t)$, except that L' is defined as $[1,1,1,1,0]$. Similarly, the functions $e(t)$ and $f(t)$ are the same as $a(t)$ and $b(t)$, except that L' is defined as $[1+\tau, 1+\tau, 1+\tau, 1, 1]$.

III. The Data

The objective of our article is to estimate the values of the liquidity and default processes underlying swap spreads and then to identify the risk premia associated with these processes. To this end, our approach is to use data that allow these processes to be identified separately. Specifically, we use data for actively traded on-the-run Treasury bonds to define the liquid riskless term structure. To identify the liquidity component of spreads over Treasuries, we need a proxy for the yields on illiquid Treasury bonds. There are several possible candidates for this proxy. First, we could use data from off-the-run Treasury bonds. The difficulty with this approach is that even off-the-run Treasury bonds may still contain some "flight-to-liquidity" premium over other types of fixed income securities. Second, we could use data for bonds that are guaranteed by the U.S. Treasury, such as Refcorp Strips. As shown by Longstaff (2004), these bonds have the same credit risk as Treasury bonds, but they do not enjoy the same liquidity as Treasury bonds. One difficulty with this approach, however, is that data for Refcorp Strips are not available for the first part of the sample period. The third possibility, and the approach we adopt, is to use the general collateral government repo rate as a proxy for the "liquidity-adjusted" riskless rate. As argued by Longstaff (2000), this rate is virtually a riskless rate, since repo loans are almost always overcollateralized using Treasury securities as collateral. Furthermore, since repo loans are contracts rather than securities, they are less likely to be affected by the types of supply- and demand-related specialness effects that influence the prices of securities. We note that the 3-month repo rate and the yield on 3-month Refcorp Strips are within several basis points of each other throughout most of the sample period (when both rates are available). With this interpretation of the repo rate as the "liquidity-adjusted" riskless rate, the estimated value of *r* can be viewed as a proxy for the implied "special" repo rate for the highly liquid on-the-run bonds used to estimate the Treasury curve.[9] Finally, the default component of the credit spread built into swap rates can be identified using market swap rates in addition to the Treasury and repo rates.

Given this approach, the next step is to estimate the parameters of the model from market data. In doing this, we use one of the most extensive sets of U.S. swap data available, covering the period from January 1988 to February 2002. This period includes most of the active history of the U.S. swap market.

The Treasury data consist of weekly (Friday) observations of the constant maturity Treasury (CMT) rates published by the Federal Reserve in the H-15 release for maturities of 2, 3, 5, and 10 years. These rates are based on the yields of on-the-run Treasury bonds of various maturities and reflect the Federal Reserve's estimate of what the par or coupon rate would be for these maturities. The CMT rates are widely used in financial markets as indicators of Treasury rates for the most actively traded bond maturities. Since CMT rates are based heavily on the most recently auctioned bonds for each maturity, they provide accurate estimates of yields for liquid on-the-run Treasury bonds. The possibility that these bonds may trade special in the repo market is taken into account explicitly in the estimation, since the liquidity process γ_t can be viewed as a direct measure of the specialness of Treasury bonds relative to the repo rate. Finally, data on 3-month general collateral repo rates are provided by Salomon Smith Barney.

The swap data for the study consist of weekly (Friday) observations of the 3-month Libor rate and midmarket constant maturity swap (CMS) rates for maturities of 2, 3, 5, and 10 years. These maturities represent the most liquid and actively traded maturities for swap contracts. All of these rates are based on end-of-trading-day quotes available in New York to insure comparability of the data. In estimating the parameters, we are careful to take into account daycount differences among the rates, since Libor rates are quoted on an actual/360 basis while swap rates are semiannual bond equivalent yields. There are two sources for the swap data. The primary source is the Bloomberg system, which uses quotations from a number of swap brokers. The data for Libor rates and for swap rates from the pre-1990 period are provided by Salomon Smith Barney. As an independent check on the data, we also compare the rates with quotes obtained from Datastream; the two sources of data are generally very consistent.

9. For discussions of the implications of "special" repo rates for Treasury bonds, see Duffie (1996), Buraschi and Menini (2002), and Krishnamurthy (2002).

Table 1 presents summary statistics for the Treasury, repo, and swap data, as well as the corresponding swap spreads. In this article, we define the swap spread to be the difference between the CMS rate and the corresponding maturity CMT rate. Figure 1 plots the 2-year, 3-year, 5-year, and 10-year swap spreads over the sample period. As shown, swap spreads average between 40 and 60 basis points during the sample period, with standard deviations on the order of 20–25 basis points. The standard deviations of weekly changes in swap spreads are only on the order of 6–8 basis points. Note, however, that there are weeks when swap spreads narrow or widen by as much as 45 basis points. In general, swap spreads are less serially correlated than the interest rates. The first difference of swap spreads, however, displays significantly more negative serial correlation. This implies that there is a strong mean-reverting component to swap spreads.

IV. Estimating the Term Structure Model

In this section, we describe the empirical approach used in estimating the term structure model and report the maximum likelihood parameter estimates. The empirical approach closely parallels that of the recent papers by Duffie and Singleton (1997), Dai and Singleton (2000), and Duffee (2002). This approach also draws on other papers in the empirical term structure literature, such as Longstaff and Schwartz (1992), Chen and Scott (1993), Pearson and Sun (1994), Duffee (1999), and many others.

In this five-factor model, the parameters of both the objective and risk-neutral dynamics of the state variables need to be estimated. In addition, we need to solve for the value of the state variable vector X for each of the 734 weeks in the sample period. At each date, the information set consists of four points along the Treasury curve, one point on the repo curve, and five points along the swap curve. Specifically, we use the CMT2, CMT3, CMT5, and CMT10 rates for the Treasury curve, the 3-month repo rate, and the 3-month Libor, CMS2, CMS3, CMS5, and CMS10 rates for the swap curve. Since the model involves only five state variables, using 10 observations at each date provides us with significant additional cross-sectional pricing information from which the parameters of the risk-neutral dynamics can be more precisely identified.

We focus first on how the five values of the state variables are determined. As in Chen and Scott (1993), Duffie and Singleton (1997), Dai and Singleton (2000), Duffee (2002), and others, we solve for the value of X by assuming that specific rates are observed without error each week. In particular, we assume that the CMT2 and CMT10 rates, the 3-month repo rate, and the 3-month Libor and CMS10 rates are observed without error. These rates represent the shortest and longest maturity rates along each curve and are among the most liquid maturities quoted, and hence, the most likely to be observed with a minimum of error.

TABLE 1 Summary Statistics for the Data

	Levels						First Differences					
	Mean	Standard Deviation	Minimum	Median	Maximum	Serial Correlation	Mean	Standard Deviation	Minimum	Median	Maximim	Serial Correlation
Libor	5.817	1.801	1.750	5.688	10.560	.998	−.007	.113	−.800	.000	.563	.097
GC Repo	5.509	1.748	1.560	5.450	10.150	.998	−.007	.104	−.735	.000	.450	.034
CMS2	6.417	1.613	2.838	6.205	10.750	.995	−.007	.156	−.555	.000	.638	−.031
CMS3	6.668	1.511	3.466	6.395	10.560	.995	−.007	.154	−.510	−.010	.681	−.054
CMS5	6.991	1.395	4.208	6.692	10.330	.994	−.006	.148	−.464	−.007	.655	−.070
CMS10	7.369	1.305	4.908	7.053	10.130	.994	−.006	.137	−.420	−.010	.608	−.081
CMT2	6.019	1.520	2.400	5.895	9.840	.996	−.007	.142	−.500	−.010	.470	.018
CMT3	6.199	1.443	2.810	6.020	9.760	.995	−.006	.143	−.480	−.010	.480	.009
CMT5	6.471	1.330	3.580	6.240	9.650	.994	−.006	.140	−.460	−.010	.450	−.021
CMT10	6.767	1.276	4.300	6.530	9.490	.995	−.006	.129	−.380	.000	.460	−.035
SS2	.399	.198	.040	.398	.951	.916	.000	.081	−.340	.000	.358	−.450
SS3	.470	.213	.100	.499	1.009	.937	.000	.075	−.300	.000	.380	−.457
SS5	.520	.240	.120	.523	1.121	.954	.000	.073	−.290	.000	.457	−.424
SS10	.602	.253	.155	.570	1.343	.968	.000	.064	−.280	.000	.437	−.435

NOTE.—This table reports the indicated summary statistics for both the level and first difference of the indicated data series. The data consist of 734 weekly observations from January 1988 to February 2002. Libor denotes the 3-month Libor rate; GC Repo denotes the 3-month general collateral government repo rate; CMS denotes the swap rate for the indicated maturity; CMT denotes the constant maturity Treasury rate for the indicated maturity; and SS denotes the swap spread for the indicated maturity, where the swap spread is defined as the difference between the corresponding CMS and CMT rates. All rates are in percentages.

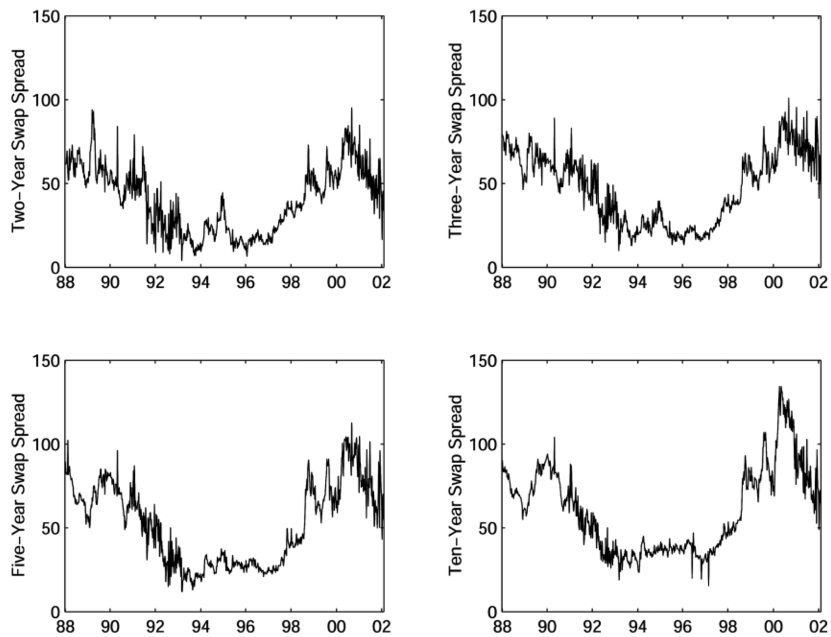

FIG. 1.—Plots of weekly time series of swap spreads measured in basis points. The sample period is January 1988 to February 2002.

Libor is given simply from the expression for a risky zero-coupon bond,

$$\text{Libor} = \frac{a}{360}\left[\frac{1}{C(t, t + 1/4)} - 1\right], \tag{12}$$

where a is the actual number of days during the next 3 months. Similarly, the repo rate is given by

$$\text{Repo} = \frac{a}{360}\left[\frac{1}{A(t, t + 1/4)} - 1\right]. \tag{13}$$

Since CMT rates represent par rates, they are also easily expressed as explicit functions of riskless zero coupon bonds:

$$\text{CMT}_T = 2\left[\frac{1 - D(t, t + T)}{\sum_{i=1}^{2T} D(t, t + i/2)}\right]. \tag{14}$$

Similarly, as in Duffie and Singleton (1997), CMS rates can be expressed as the par rates implied by the term structure of risky zero-coupon bonds:

$$CMS_T = 2\left[\frac{1 - C(t, t + T)}{\sum_{i=1}^{2T} C(t, t + i/2)}\right]. \tag{15}$$

Given a parameter vector, we can then invert the closed-form expressions for these five rates to solve for the corresponding values of the state variables using a standard nonlinear optimization technique. While this process is straightforward, it is computationally very intensive, since the inversion must be repeated for every trial value of the parameter vector utilized by the numerical search algorithm in maximizing the likelihood function.[10]

To define the log likelihood function, let $R_{1,t}$ be the vector of the five rates assumed to be observed without error at time t and let $R_{2,t}$ be the vector of the remaining five observed rates. Using the closed-form solution, we can solve for X_t from R_{1t},

$$X_t = h(R_{1,t}, \Theta), \tag{16}$$

where Θ is the parameter vector. The conditional log likelihood function for $X_{t+\Delta t}$ is

$$-\frac{1}{2}\Big((X_{t+\Delta t} - \theta - K(X_t - \theta))'\Omega^{-1}(X_{t+\Delta t} - \theta - K(X_t - \theta)) + \ln|\,\Omega\,|\Big), \tag{17}$$

where K is a diagonal matrix with ith diagonal term $e^{-\kappa_{ii}\Delta t}$ and Ω is a matrix with ijth term given by

$$\Omega_{ij} = \frac{1 - e^{-(\kappa_{ii}+\kappa_{jj})\Delta t}}{\kappa_{ii} + \kappa_{jj}}(\Sigma\Sigma')_{ij}.$$

Let $\epsilon_{t+\Delta t}$ denote the vector of differences between the observed value of $R_{2,t+\Delta t}$ and the value implied by the model.[11] Assuming that the ϵ terms are independently distributed normal variables with zero means and variances η_i^2, the log likelihood function for $\epsilon_{t+\Delta t}$ is given by

$$-\frac{1}{2}\epsilon'_{t+\Delta t}\Sigma_\epsilon^{-1}\epsilon_{t+\Delta t} - \frac{1}{2}\ln|\,\Sigma_\epsilon\,|, \tag{18}$$

where Σ_ϵ is a diagonal matrix with diagonal elements η_i^2, $i = 1, \ldots, 5$. Since $X_{t+\Delta t}$ and $\epsilon_{t+\Delta t}$ are assumed to be independent, the log likelihood function for $[X_{t+\Delta t}, \epsilon_{t+\Delta t}]'$ is simply the sum of equations (17) and (18). The final step in specifying the likelihood function consists of changing variables from the vector $[X_t, \epsilon_t]'$ of state variables and error terms to the vector $[R_{1,t}, R_{2,t}]'$ of rates actually observed. It is easily shown that the determinant of the Jacobian

10. By representing swap rates as par rates, this approach implicitly assumes that both the floating and fixed legs of a swap are valued at par initially. Since we assume that there is no counterparty default risk, however, an alternative approach might be to discount swap cash flows along a riskless curve. In this case, the value of each leg could be slightly higher than par, although both would still share the same value. This alternative approach, however, results in empirical estimates of the liquidity and default processes that are virtually identical to those we report.

11. We assume that the ϵ terms are independent. In actuality, the ϵ terms could be correlated. As is shown later, however, the variances of the ϵ terms are very small, and the assumption of independence is unlikely to have much effect on the estimated model parameters.

matrix is given by $|J_t| = |\partial h(R_{1,t})/\partial R'_{1,t}|$. Summing over all observations gives the log likelihood function for the data:

$$-\frac{1}{2}\sum_{t=1}^{T-1}\Big[(X_{t+\Delta t}-\theta-K(X_t-\theta))'\Omega^{-1}(X_{t+\Delta t}-\theta-K(X_t-\theta)) + \ln|\Omega| + \epsilon'_{t+\Delta t}\Sigma_\epsilon^{-1}\epsilon_{t+\Delta t} + \ln|\Sigma_\epsilon| + 2\ln|J_t|\Big]. \qquad (19)$$

Given this specification, the likelihood function depends explicitly on 39 parameters.

From this log likelihood function, we now solve directly for the maximum likelihood parameter estimates using a standard nonlinear optimization algorithm. In doing this, we initiate the algorithm at a wide variety of starting values to insure that the global maximum is achieved. Furthermore, we check the results using an alternative genetic algorithm that has the property of being less susceptible to finding local minima. These diagnostic checks confirm that the algorithm converges to the global maximum and that the parameter estimates are robust to perturbations of the starting values.

Table 2 reports the maximum likelihood parameter estimates and their asymptotic standard errors. As shown, there are clear differences between the objective and risk-neutral parameters. These differences have major implications for the dynamics of the default and liquidity processes that we will consider in the next section. The differences themselves reflect the market prices of risk for the state variables and also have important implications for the expected returns from bearing default and liquidity risk. One key result that emerges from the maximum likelihood estimation is that the five-factor model fits the data well, at least in its cross-sectional dimension. For example, the standard deviations of the pricing errors for the CMS2, CMS3, and CMS5 rates and the CMT3 and CMT5 rates (given by η_1, η_2, η_3, η_4, and η_5, respectively) are 9.1, 8.1, 7.5, 4.5, and 6.3 basis points, respectively. These errors are relatively small and are on the same order of magnitude as those reported in Duffie and Singleton (1997) and Duffee (2002). Note, however, that we are estimating the Treasury, repo, and swap curves simultaneously.

V. Empirical Results

In this section, we first discuss the empirical estimates of the liquidity and default components. We then present the results for the liquidity and default risk premia.

A. The Liquidity and Default Components

Since the instantaneous credit spread applied to swaps in this framework is equal to the sum $\gamma_t + \lambda_t$, it is natural to think of γ_t and λ_t as the liquidity and default components of the credit spread. Summary statistics for the es-

TABLE 2 Maximum Likelihood Estimates of the Model Parameters

Parameter	Value	Standard Error
β_1	6.97493	1.60510
β_2	.43063	.00790
β_3	−.00778	.00161
β_4	−.08669	.00296
β_5	1.47830	.10616
κ_1	2.59781	.71490
κ_2	.25373	.21606
κ_3	.37843	.19430
κ_4	1.79193	.52619
κ_5	14.39822	1.53849
θ_1	.00124	.00981
θ_2	−.01729	.03886
θ_3	.06726	.02271
θ_4	.00529	.00088
θ_5	.00032	.00089
σ_{11}	.03209	.00413
σ_{21}	.00019	.00114
σ_{22}	.01495	.00076
σ_{31}	.00000	.00077
σ_{32}	.00003	.00074
σ_{33}	.00924	.00051
σ_{41}	.00028	.00023
σ_{42}	−.00006	.00019
σ_{43}	.00039	.00017
σ_{44}	.00300	.00007
σ_{51}	.00029	.00079
σ_{52}	.00010	.00060
σ_{53}	.00037	.00048
σ_{54}	.00030	.00042
σ_{55}	.00895	.00022
δ_0	.00324	.02006
δ_1	−.00458	.00043
δ_2	.00260	.00076
τ	.00403	.00575
η_1	.00091	.00000
η_2	.00081	.00000
η_3	.00075	.00000
η_4	.00045	.00000
η_5	.00063	.00000

NOTE.—This table reports the maximum likelihood estimates of the parameters of the five-factor term structure model, along with their asymptotic standard errors. The asymptotic standard errors are based on the inverse of the information matrix computed from the Hessian matrix for the log likelihood function.

timated values of the liquidity and default components are presented in table 3. Figure 2 graphs the time series of the two components along with the time series of their sum, or equivalently, the total credit spread.

Table 3 shows that the average value of the liquidity component is 7.1 basis points. In contrast, the average value of the default component is 31.3 basis points. Thus, on average, the liquidity component represents only 18.5% of the total instantaneous credit spread.

The two components, however, vary significantly over time. The standard deviation of the liquidity component is 15.9 basis points over the sample period, while the same measure for the default component is 15.5 basis points. Figure 2 shows that the liquidity component ranges from about 10 to 20 basis points during the first part of the sample period. During the middle part of

TABLE 3 **Summary Statistics for the Implied Repo Rate, the Liquidity Component, and the Default Component**

	Mean	Standard Deviation	Minimum	Medium	Maximum	Serial Correlation
Implied repo rate	5.439	1.807	.701	5.420	10.478	.991
Liquidity component	.071	.159	−.215	.046	.539	.966
Default component	.313	.155	.012	.271	1.210	.733

Correlation Matrix

	Implied Repo Rate	Liquidity Component	Default Component
Implied repo rate	1.000		
Liquidity component	.232	1.000	
Default component	.255	.182	1.000

NOTE.—This table reports the indicated summary statistics for the implied repo rate, liquidity component, and default component. The data consist of 734 weekly observations from January 1988 to February 2002.

the sample period, however, the liquidity component is slightly negative, ranging from −5 to −10 basis points. The liquidity component is very stable during this middle period. Beginning with approximately May 1998, the period just prior to the Russian debt default, the liquidity spread becomes positive again and rises rapidly to more than 20 basis points by the beginning of the LTCM (long-term capital management) crisis in August 1998. The liquidity spread stays high for the remainder of the sample period, reaching a maximum of nearly 54 basis points in early 2000. As indicated by the serial correlation coefficient of 0.966, the liquidity component displays a high degree of persistence.

In contrast, the default component of the spread displays far less persistence; the serial correlation coefficient for the default component is 0.733. This is evident from the time series plot of the default component shown in the middle panel of figure 2. As illustrated, the default component ranges from a low of about 1 basis point (the estimated default component is never negative) to a high of 121 basis points. The default component is clearly skewed toward large values and exhibits many spikes. These spikes, however, appear to dissipate quickly consistent with the rapidly mean-reverting nature of the time series of the default component. The two largest spikes in the default component occur in late December of 1990, which was immediately before the first Gulf War, and in October 1999, which was a period when a number of large hedge funds experienced major losses in European fixed income positions. The two components of the credit spread are positively correlated with each other and with the level of interest rates.

The bottom panel of figure 2 shows the time series of the total instantaneous credit spread $\gamma_t + \lambda_t$. As illustrated, the credit spread averages about 38.4 basis points, but it varies widely throughout the sample period. Near the beginning of the sample period, the credit spread ranges from about 60 to 80 basis points. During the middle period, however, the credit spread declines

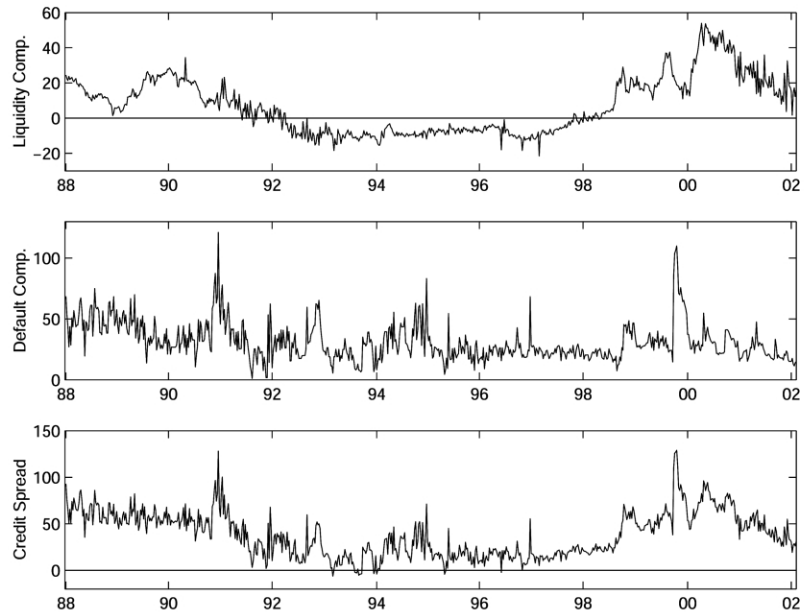

FIG. 2.—The top panel plots the liquidity component of the spread. The middle panel plots the default component of the spread. The bottom panel plots the sum of the liquidity and default components, which equals the credit spread. All time series are measured in basis points. The sample period is January 1988 to February 2002.

to near zero, ranging between zero and about 10 basis points for most of the 1991–98 period. With the hedge fund crisis of 1998, the total credit spread increases rapidly, reaching a maximum of about 129 basis points in 1999, but it then begins to decline significantly near the end of the sample period. The standard deviation of the credit spread over the sample period is 24.2 basis points. Its minimum value is -6.2 basis points, and its first-order serial correlation coefficient is 0.882.

Finally, we note that the estimate of the parameter τ for the default risk process λ_t is 0.00403. This value, however, is not statistically significant. This small value indicates that, if there is a marginal state tax effect, it is on the order of .5% or less. Thus, the effect on swap rates of the differential state tax treatment given to Treasury bonds appears negligible.

B. Liquidity and Default Risk Premia

The primary objective of this article is to examine how the market prices the default and liquidity risks in interest rate swaps. To this end, we focus on the premia incorporated into the expected returns of bonds implied by the estimated term structure model. These premia are given directly from the differences between the objective and risk-neutral parameters of the model.

To provide some perspective for these results, however, it is useful to also examine the implications of the model for the term premia in Treasury bond prices. Applying Ito's Lemma to the closed-form expression for the value of a liquid riskless zero-coupon bond $D(t, T)$ results in the following expression for its instantaneous expected return:

$$r_t + b'(t)((\beta - \kappa)X_t + \kappa\theta)). \tag{20}$$

The first term in this expression is the riskless rate, and the second term is the instantaneous term premium for the bond. This term premium is time varying, since it depends explicitly on the state variables. The term premium represents compensation to investors for bearing the risk of variation in the riskless rate, or equivalently, the risk of interest-rate-related changes in the value of the riskless bond.

Now applying Ito's Lemma to the expression for the price of a price of a illiquid riskless zero-coupon bond $A(t, T)$ gives the instantaneous expected return:

$$r_t + \gamma_t + d'(t)((\beta - \kappa)X_t + \kappa\theta)). \tag{21}$$

The first term is again the riskless rate. The second term is the liquidity spread that compensates the investor for holding an illiquid riskless security. The third term is the total risk premium, consisting of the term premium and the liquidity premium, where the liquidity premium compensates the investor for the risk of liquidity-related changes in the values of bonds that are not as liquid as Treasury bonds.

It is straightforward to show that the instantaneous expected return of a risky zero-coupon bond with price $C(t, T)$ is given by

$$r_t + \gamma_t + f'(t)((\beta - \kappa)X_t + \kappa\theta)) + \lambda - \hat{\lambda}, \tag{22}$$

where $\hat{\lambda}$ is the value of the intensity process under the objective measure. The first two terms in this expression are the same as in equation (21). The third term can be interpreted as a combined term premium, liquidity premium, and premium for variation in the default intensity λ. The last term $\lambda - \hat{\lambda}$ is the difference between the risk-neutral and objective values of the default intensity process, and it directly measures the premium for default risk.[12] Following standard practice, we refer to the sum of the premia for variation in the default intensity and for the default event itself simply as the default premium.

The objective value of the default intensity $\hat{\lambda}$ can only be identified by using additional information. Unfortunately, our econometric approach allows us to estimate λ but not $\hat{\lambda}$. Rather than to make a specific identifying assumption about $\hat{\lambda}$, however, our approach will be to provide upper and lower bounds for its value. This has the advantage of allowing us to fully explore

12. For a discussion of this last term, see Yu (2002), Amato and Remolona (2003), and Jarrow, Lando, and Yu (2005).

TABLE 4 Summary Statistics for the Premia

	Maturity in Years									
Premium	1	2	3	4	5	6	7	8	9	10
Mean:										
Term premium	.53	.98	1.29	1.51	1.67	1.79	1.90	1.98	2.05	2.12
Liquidity premium	.05	.10	.16	.22	.29	.36	.44	.53	.63	.73
Default premium lower bound	−.02	−.03	−.03	−.03	−.03	−.03	−.03	−.03	−.03	−.03
Default premium upper bound	.29	.29	.29	.29	.29	.29	.29	.29	.29	.29
Standard deviation:										
Term premium	.94	1.35	1.82	2.31	2.80	3.30	3.80	4.31	4.82	5.33
Liquidity premium	.31	.65	1.02	1.43	1.87	2.35	2.88	3.45	4.08	4.76
Default premium lower bound	1.04	1.27	1.33	1.34	1.34	1.34	1.34	1.34	1.35	1.35
Default premium upper bound	1.19	1.43	1.48	1.49	1.50	1.50	1.50	1.50	1.50	1.50

NOTE.—This table reports the means and standard deviations of the annualized term premia, liquidity premia, and upper and lower bounds for the default premia in zero-coupon bonds of the indicated horizons implied by the fitted model. The data consist of 734 weekly observations from January 1988 to February 2002.

the range of possible default premia. Clearly, the minimum value that $\hat{\lambda}$ could take is zero. In this case, the premium for default event is simply λ. At the polar extreme, the most realistic upper bound on the value of $\hat{\lambda}$ is λ itself. In this case, there is no premium at all for the default event. Given the low probability of the entire Libor basket ever being in default (see Duarte, Longstaff, and Fan 2006), our prior would be that the actual value of $\hat{\lambda}$ would be much closer to zero than to λ.

To identify the liquidity premium separately, we simply take the difference between the expected returns in equations (21) and (20) (and subtract out the liquidity component γ_t). Similarly, to identify the upper bound for the default premium, we take the difference between the expected returns in equations (22) and (21), where we assume that $\hat{\lambda} = 0$. To identify the lower bound for the default premium, we again take the difference between the expected returns in equations (22) and (21), but here we assume that $\hat{\lambda} = \lambda$. As shown, all of these premia are time varying through their dependence on the state variable vector.

Table 4 reports summary statistics for the term, liquidity, and default premia for zero-coupon bonds with maturities ranging from 1 to 10 years. Figure 3 plots the average values of these premia. As shown, the average term premia range from about 53 basis points for a 1-year horizon to 212 basis points for a 10-year horizon. Figure 3 shows that the average term premia are concave in the horizon. These estimates of average term premia are similar to those reported by Fama (1984), Fama and Bliss (1987), and others.

The average liquidity premia are all positive and range from about 5 basis points for a 1-year horizon to 73 basis points for a 10-year horizon. Thus, the average liquidity premium can be as much as one-third the size of the

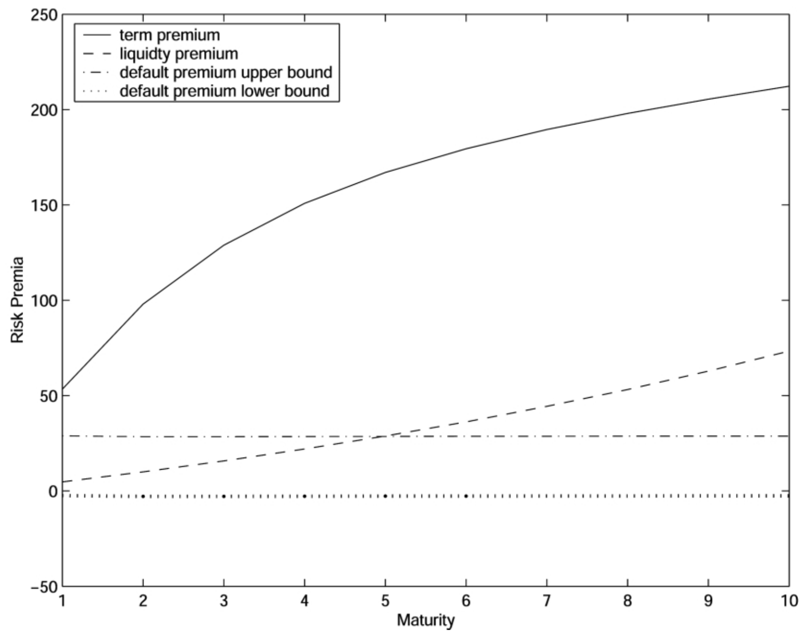

FIG. 3.—Plots of average term premium, liquidity premium, and upper and lower bounds for the default premium for zero-coupon bonds with the indicated maturities. All premia are measured in basis points. The sample period is January 1988 to February 2002.

term premium for longer maturities. Figure 3 shows that the liquidity premium is actually slightly convex in the horizon of the zero-coupon bond. Table 4 also shows that the liquidity premium is highly variable.

The results for the average upper and lower bounds for the default premia are strikingly different. As shown, all of the average lower bounds for the default risk premia are slightly negative. Numerically, their values are all close to −3 basis points. The average upper bounds for the default risk premia are about 31 basis points higher. For both the upper and lower bounds, the term structure of default premia is flat. Note that for maturities greater than 5 years, the majority of the total credit premium is due to the liquidity premium, even when we use the upper bound as the measure of the default premium.

To give some sense of the time variation in term, liquidity, and default premia, figure 4 graphs these premia for 1-year maturity zero-coupon bonds. As illustrated, the term premium displays a significant amount of variation. The term premium is usually positive, but it has generally tended downward and it has occasionally been negative during the latter part of the sample period.

The time series of the liquidity premium displays a number of interesting features. Recall that the average liquidity premium for a 1-year horizon is

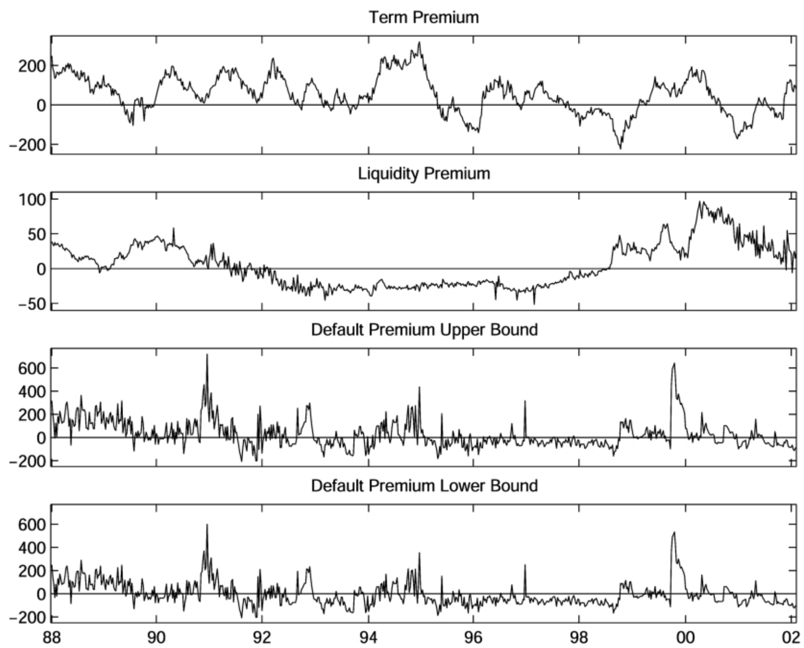

FIG. 4.—The top panel plots the conditional term premium for a 1-year zero-coupon bond. The second panel plots the conditional liquidity premium for a 1-year zero-coupon bond. The third panel plots the upper bound on the conditional default premium for a 1-year zero-coupon bond. The bottom panel plots the lower bound on the conditional default premium for a 1-year zero-coupon bond. All premia are measured in basis points. The sample period is January 1988 to February 2002.

about 5 basis points. Figure 4 shows that the conditional liquidity premium varies significantly over time and is often large in absolute terms. Most surprising, the liquidity premium is negative for nearly one-half of the sample period. The liquidity premium first becomes negative around 1992 and remains generally negative until August of 1998. Despite the variation, however, the liquidity premium is less volatile than the term premium. Although not shown, a similar pattern holds for liquidity premia in bonds with longer maturities.

Finally, the upper and lower bounds for the default premia show a pattern similar to that for the default component. In particular, the default premium displays rapid mean reversion and exhibits a number of large spikes. The default premium for a 1-year horizon is much more volatile than the term or liquidity premia. For longer maturities, however, the default premium is less volatile than the other premia. These results indicate that the conditional default premium can be substantially different from zero. Thus, there are times when the market price of default risk may be a significant determinant of swap spreads.

VI. Conclusion

This article examines how the market prices the default and liquidity risk inherent in interest rate swaps. A number of key results emerge from this analysis. We find that the credit spread in swaps consists of both a liquidity component and a default component. On average, the default component of the credit spread is larger, but the liquidity component is slightly more volatile. Both components vary significantly through time. The liquidity component displays a high level of persistence. In contrast, the default component is rapidly mean reverting. In addition, the default component exhibits a number of large but temporary spikes in its level over time.

We find that the liquidity risk inherent in swaps is compensated for by the market with a significant risk premium. In contrast, the average premium for default risk is in the range of zero to 30 basis points and the term structure of default risk premia is flat. Thus, the majority of the credit premium built into the swap curve for horizons beyond a few years is due to liquidity premia. It is curious, however, that these liquidity premia were slightly negative during the mid-1990s.

These results raise a number of intriguing questions for future research. Do the negative values for the liquidity component of the spread and the associated risk premia during the 1990s imply that swaps were viewed as even more liquid than Treasury bonds? Has the flight-to-liquidity phenomenon become more important in the post-LTCM era, thus explaining the return of the liquidity spread and its associated premium to positive values? Finally, is the default risk premium in the swap curve unique to this market, or is this feature found in corporate, sovereign, agency, or municipal bond markets as well?

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