

# VALUING CREDIT DERIVATIVES

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**C**redit derivatives are potentially one of the most important new types of financial products introduced during the past decade. Many investors have portfolios with values that are highly sensitive to shifts in the spread between risky and riskless yields, and credit derivatives offer these investors an important new tool for managing and hedging their exposure to this type of risk.

In this article, we develop a simple framework for valuing derivatives on credit spreads. It captures the major empirical properties of observed credit spreads in that it allows for spreads to be stationary and mean-reverting. We then use this framework to derive simple closed-form solutions for the prices of call and put options on the credit spread.

These closed-form solutions have a number of interesting implications for the pricing and hedging properties of credit derivatives. We show that calls and puts on the credit spread can have prices below their intrinsic value. We also show that credit spread calls can have negative convexity, and that the delta of the call can be higher for out-of-the-money and at-the-money calls than for in-the-money calls. In addition, we show that the deltas of both calls and puts converge to zero as the time until expiration increases, which implies that long-term credit derivatives may have little ability to hedge against current shifts in credit spreads.

## I. CREDIT SPREAD PROPERTIES

Before developing a valuation model for credit derivatives, it is important to examine first the properties of actual credit spreads. By incorporating these properties into the valuation model, the resulting impli-

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**EXHIBIT 1 ■ Summary Statistics for Credit Spreads over Treasuries ■ Moody's Utility and Industrial Bond Yield Averages ■ April 1977-December 1992**

	Mean of Credit Spread	Std. Dev. of Credit Spread	Mean of Relative Spread	Std. Dev. of Relative Spread	Number of Observations
Aaa Utilities	0.930	0.349	1.0975	0.034	180
Aa Utilities	1.276	0.431	1.1314	0.038	190
A Utilities	1.660	0.667	1.1696	0.054	190
Baa Utilities	2.077	0.758	1.2116	0.057	190
Aaa Industrials	0.481	0.373	1.0560	0.051	190
Aa Industrials	0.809	0.452	1.0888	0.059	190
A Industrials	1.231	0.580	1.1321	0.071	190
Baa Industrials	1.835	0.654	1.1972	0.084	190

fications for pricing and hedging credit derivatives will be more realistic.

We focus on the spread between a number of Moody's bond indexes and the long-term U.S. Treasury bond yield. Data on Moody's Aaa, Aa, A, and Baa industrial and utility bond indexes provide monthly observations for the spread over 1977-1992. Summary statistics for the spreads are presented in Exhibit 1.

Let  $X$  denote the logarithm of the credit spread. To illustrate how spreads evolve over time, Exhibits 2 and 3 plot the time series of  $X$  for the industrial and utility Baa indexes. The credit spread shown displays a significant amount of stability, the value of  $X$  in both graphs tends to stay in a fairly narrow range, and is gen-

erally between zero to one. Note that the variability of the spread is typically fairly constant.

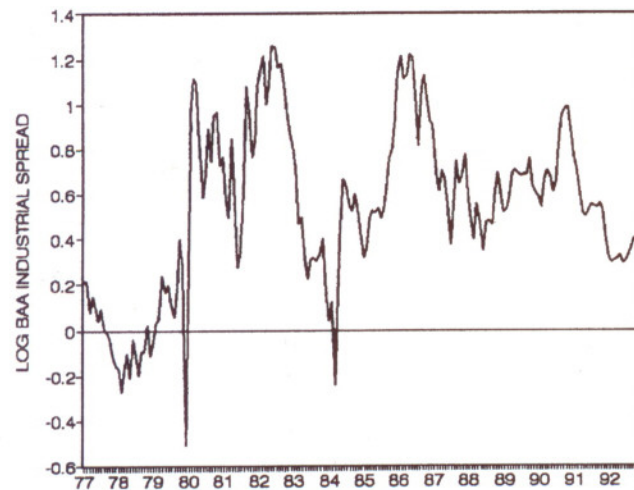
To formalize these observations, we conduct a simple regression analysis using the time series of spreads. Motivated by continuous-time models in which the process is mean-reverting, we regress the monthly change in the value of  $X$  on the value of  $X$  at the beginning of the month:

$$\Delta X_{t+1} = \gamma_0 + \gamma_1 X_t + \varepsilon_t \quad (1)$$

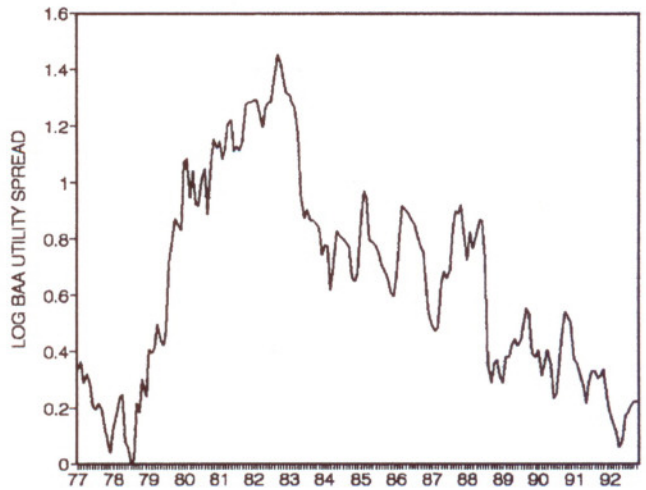
where  $\gamma_0$  and  $\gamma_1$  are the intercept and slope coefficients for the regression. The regression results are reported in Exhibit 4.

The results provide a number of important

**EXHIBIT 2 ■ Logarithm of the Credit Spread for Baa-Rated Industrial Bonds**



**EXHIBIT 3 ■ Logarithm of the Credit Spread for Baa-Rated Utility Bonds**





insights about the behavior of credit spreads. The slope coefficient is negative in all the regressions — and significantly negative in all but one or two. This indicates that the logarithm of the credit spread is mean-reverting. Annualizing the slope coefficient for the regression implies that the half-life of deviations from the long-run mean value ranges from about 0.7 to 1.0 years for the industrial bonds to about 1.5 to 4.0 years for the utility bonds. Note that the credit spreads of the lower-rated bonds display less mean reversion.

Another interesting implication of the regression results is that the logarithm of the credit spread is more volatile for the higher-rated bonds than for the lower-rated bonds. For example, the standard error of the regression for the Aaa-rated industrial bonds is 0.4229, while the same measure for the Baa-rated industrial bonds is only 0.1560. A similar pattern holds for the utility bonds. Histograms of the monthly changes in  $X$  indicate that the distribution of changes is generally well-approximated by the normal distribution.

## II. THE VALUATION MODEL

Given these empirical properties, we first assume that the dynamics of the logarithm of the credit spread  $X$  are given by the stochastic differential equation:

$$dX = (a - bX) dt + s dZ_1 \quad (2)$$

where  $a$ ,  $b$ , and  $c$  are parameters, and  $Z_1$  is a standard

Wiener process. These dynamics imply that changes in  $X$  are mean-reverting and homoscedastic, consistent with the empirical data. Furthermore, these dynamics imply that credit spreads are positive and conditionally lognormally distributed.

To allow for random interest rates in the model, we make the simple assumption that the riskless term structure is determined by the single-factor Vasicek [1977] model in which the dynamics of the short-term interest rate  $r$  are given by

$$dr = (\alpha - \beta r) dt + \sigma dZ_2 \quad (3)$$

where  $\alpha$ ,  $\beta$ , and  $\sigma$  are parameters, and  $Z_2$  is also a standard Wiener process. The correlation coefficient between  $dZ_1$  and  $dZ_2$  is  $\rho$ .

In pricing derivative claims on the credit spread, we allow for the possibility that both interest rate risk and the risk of shifts in the credit spread are priced by the market. Rather than developing a full general equilibrium model, we simply assume that the market prices of risk premiums are incorporated into the  $a$  and  $\alpha$  terms. Thus,  $a$  and  $\alpha$  are equilibrium- or risk-adjusted parameters rather than empirical parameters. This assumption is consistent with Vasicek [1977], Longstaff and Schwartz [1995], and others.

With this framework, standard valuation theory can be used to show that the price  $F(X, r, T)$  of a European-type claim on the credit spread with payoff function  $H(X)$  and time to expiration  $T$  must solve the partial differential equation:

### EXHIBIT 4 ■ Results from Regressing Monthly Changes in the Logarithm of the Credit Spread on the Logarithm of the Credit Spread at the Beginning of the Month

	$\gamma_0$	$\gamma_1$	$t_{\gamma_0}$	$t_{\gamma_1}$	$R^2$	S.E.
Aaa Utilities	-0.0085	-0.0526	-0.77	-2.23	0.022	0.1352
Aa Utilities	0.0103	-0.0546	1.14	-2.39	0.030	0.1134
A Utilities	0.0151	-0.0354	1.41	-1.88	0.019	0.1018
Baa Utilities	0.0139	-0.0222	1.17	-1.40	0.010	0.0799
Aaa Industrials	-0.1223	-0.1204	-2.48	-3.52	0.064	0.4229
Aa Industrials	-0.0558	-0.1565	-2.07	-4.08	0.077	0.3080
A Industrials	0.0108	-0.1182	0.66	-3.48	0.056	0.2212
Baa Industrials	0.0509	-0.0936	2.56	-3.06	0.043	0.1560

The term  $\Delta X$  is the change in the logarithm of the credit spread,  $X$  is the logarithm of the credit spread. S.E. is the standard error of the regression:

$$\Delta X = \gamma_0 + \gamma_1 X + \epsilon$$

$$\frac{s^2}{2} F_{XX} + \rho\sigma s F_{Xr} + \frac{\sigma^2}{2} F_{rr} + (\alpha - \beta r) F_r + (a - bX) F_X - rF - F_T = 0 \quad (4)$$

subject to the initial condition  $F(X, r, 0) = H(X)$ . Following the results in Longstaff [1990], however, the solution to this partial differential equation can be obtained in closed-form using the certainty-equivalent representation:

$$F(X, r, T) = D(r, T) E[H(X)] \quad (5)$$

where  $D(r, T)$  is the price of a riskless discount bond with maturity  $T$ , and the expectation is taken with respect to the "adjusted" risk-adjusted process for  $X$ :

$$dX = \left[ a - bX - \frac{\rho\sigma s}{\beta} \times (1 - \exp[-\beta(T - t)]) \right] dt + s dZ_1 \quad (6)$$

Thus, the value of a claim on the credit spread can be found by first taking the expectation of its payoff, and then discounting the expectation back to the present by multiplying by the riskless present value factor  $D(T)$ .

Following Longstaff and Schwartz [1995], the stochastic differential equation in (6) can be solved by making a change of variables and then integrating. The resulting solution implies that  $X_T$  is conditionally normally distributed with respect to (6) with mean  $\mu$  and variance  $\eta^2$ , where

$$\mu = \exp(-bT) X + \frac{1}{b} \left( a - \frac{\rho\sigma s}{\beta} \right) \times [1 - \exp(-bT)] + \frac{\rho\sigma s}{\beta(b + \beta)} \times [1 - \exp(-(b + \beta)T)] \quad (7)$$

$$\eta^2 = \frac{s^2 [1 - \exp(-2bT)]}{2b} \quad (8)$$

Note that as  $T \rightarrow \infty$ , the values of  $\mu$  and  $\eta^2$  con-

verge to fixed values, and the distribution of  $X_T$  converges to a steady-state stationary density.

### III. OPTIONS ON CREDIT SPREADS

Let  $C(X, r, T)$  denote the value of a European call option on the level of the credit spread, where the strike price of the option is  $K$ . Since  $X$  denotes the logarithm of the spread, the payoff function for this option is simply  $H(X) = \max(0, e^X - K)$ . Applying the certainty-equivalent valuation operator in (5) results in the closed-form solution for the value of the call option:

$$C(X, r, T) = D(r, T) \exp(\mu + \eta^2/2) N(d_1) - K D(r, T) N(d_2) \quad (9)$$

where  $N(\cdot)$  is the cumulative standard normal distribution function and

$$d_1 = \frac{-\ln K + \mu + \eta^2}{\eta}$$

$$d_2 = d_1 - \eta$$

This option pricing formula has some features that are similar to the Black-Scholes [1973] option pricing formula. In particular, both models involve the cumulative normal distribution. In general, however, the properties of the two models are quite different.

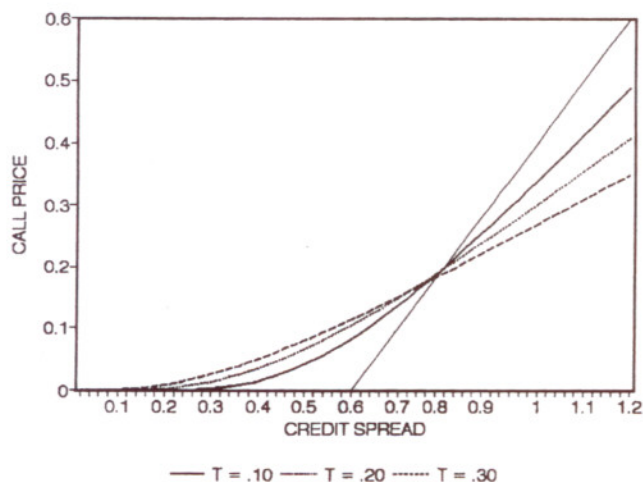
To illustrate this, Exhibit 5 graphs the value of a call option as a function of the value of the current credit spread. The call price is an increasing function of the credit spread. What is clearly different, however, is that the value of a call option on the credit spread can be less than its intrinsic value even when the call option is only slightly in the money.

The intuition for this surprising result relates to the mean reversion of the credit spread. When the credit spread is above its long-run mean, the credit spread is expected to decline over time. In turn, this means that in-the-money call options are less likely to remain in the money over time. Hence, the value of an in-the-money call option can be less than its intrinsic value because of the expected decrease in the credit spread.

Note that this cannot happen in the standard Black-Scholes model because the underlying asset must appreciate like the riskless rate in the risk-neutral valu-



**EXHIBIT 5 ■ Credit Spread Call Prices Graphed as a Function of the Underlying Credit Spread**



ation framework. Only when the underlying is not the price of traded asset can the expected change in the underlying be different from the riskless rate.

To illustrate another important feature of calls on credit spread, Exhibit 6 graphs call prices for values of T ranging from 0.50 to 1.50 years. The call price can be a concave function of the credit spread (negative convexity). The reason for this is again related to the fact that the credit spread can be mean-reverting. This means that expected percentage changes in the value of the credit spread depend on the current level of the credit spread. Hence, the dynamics of the credit spread do not satisfy the first-degree homogeneity property that Merton [1973] shows is necessary in order for the option price to be a globally convex function of the underlying.

Differentiating the valuation expression in (9) with respect to the credit spread gives the expression for the delta of a call on the credit spread:

$$D(r, T) \exp(\mu + \eta^2/2) N(d_1) \times \exp(-bT) \exp(-X) \quad (10)$$

As in the Black-Scholes formula, the delta for a call option on the credit spread is always positive. Because of the mean reversion of the credit spread, however, the delta of a call option decreases to zero as the time until expiration T increases to infinity. This is because changes in the current value of the credit spread have little effect on the expected payoff of the option if the time

until expiration is many half-lives in the future; a change in the current credit spread is expected to be nearly completely cancelled out by the effects of mean reversion if the expiration date of the call is far in the future.

Because of this property, credit spread options with long maturities may not provide a useful vehicle for hedging the risk of changes in the current credit spread. This has many important implications for contract design.

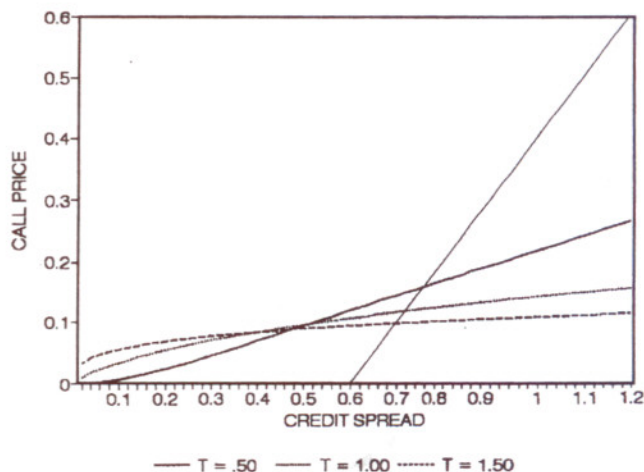
While long-term options on credit spreads may not be useful hedging vehicles, short-term option prices will move in response to changes in the current credit spread. To show this, Exhibit 7 graphs the delta of credit spread calls with differing times until expiration.

This graph indicates that, while deltas are positive, they can also be decreasing functions of the credit spread. The reason for this is again related to the concavity of the option price in the underlying credit spread. This implies that, for some credit spread calls, the option price is most sensitive to changes in the underlying when the option is out of the money, rather than in the money.

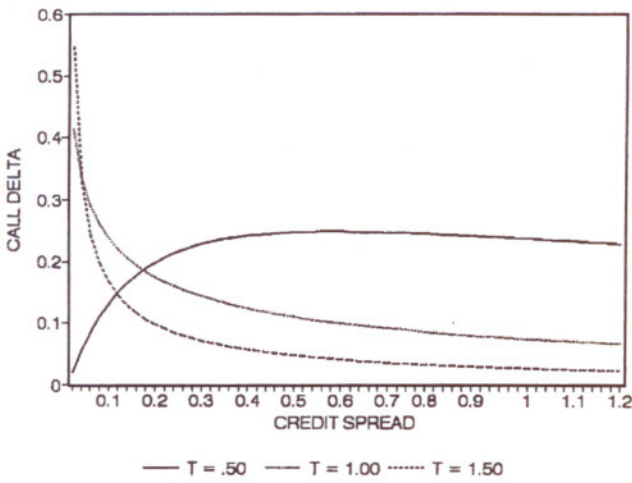
Similarly, the graph also shows an example of a credit spread call for which the delta is greatest when the option is at the money. These results illustrate that the price of a call option on the credit spread behaves very differently from those for options on traded assets.

The value of a European put option on a credit spread  $P(X, r, T)$  is found from the put-call parity relation:

**EXHIBIT 6 ■ Credit Spread Call Prices Graphed as a Function of the Underlying Credit Spread**



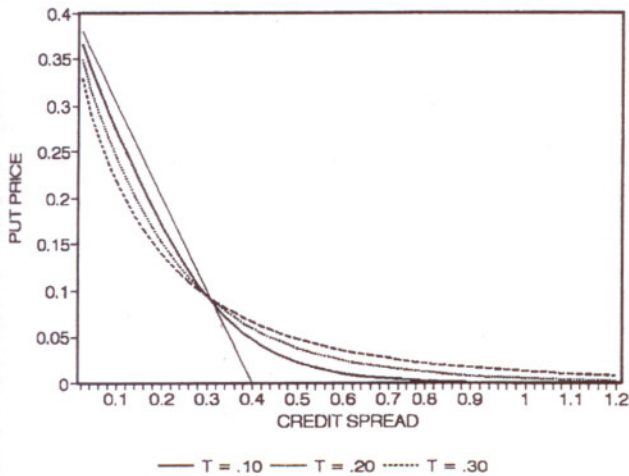
**EXHIBIT 7 ■ Credit Spread Call Deltas Graphed as a Function of the Underlying Credit Spread**



$$P(X, r, T) = C(X, r, T) + D(r, T) K - D(r, T) \exp(\mu + \sigma^2/2) \quad (11)$$

This put-call parity relation differs from the traditional put-call parity relation in that the discounted forward credit spread appears rather than the underlying value. The reason for this is that when the underlying is the price of a traded asset, the discounted forward price is simply the current price of the asset. In contrast, the discounted forward credit spread can be either greater

**EXHIBIT 8 ■ Credit Spread Put Prices Graphed as a Function of the Underlying Credit Spread**



or less than the current value of the credit spread.

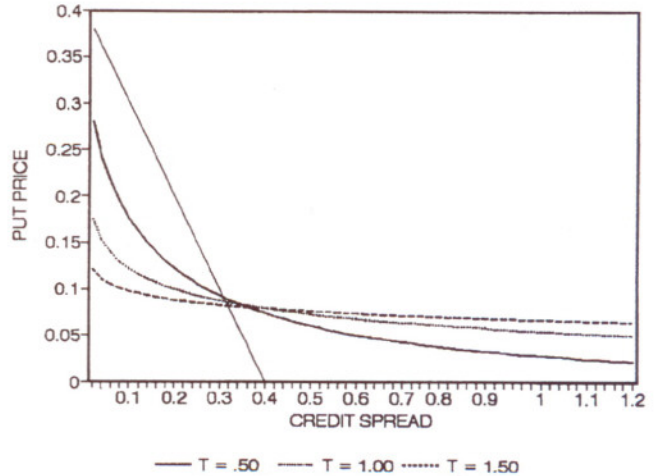
Exhibit 8 graphs the value of puts on the credit spread as a function of the underlying credit spread. Once again, the value of the put option can be less than its intrinsic value. The graph also shows that the value of the put is not always an increasing function of the time until expiration  $T$ . When the put is out of the money, a longer time until expiration leads to a higher option price, because it allows the credit spread more time to move back toward its mean, tending to make the option move closer to being in the money. The reverse is true when the put option is in the money.

Exhibit 9 graphs the put option value for values of  $T$  ranging from 0.50 to 1.50 years. The put option is always an convex function of the credit spread. In this respect, calls and puts on credit spreads are not symmetric. This graph also shows that, as the time to expiration for the option increases, the value of the put becomes less sensitive to changes in the current value of the credit spread. The intuition for this result is similar to that for the call option; mean reversion reduces the delta of long-term puts to zero as  $T \rightarrow \infty$ .

#### IV. CONCLUSION

We have presented simple closed-form valuation expressions for European calls and puts on credit spreads. While these are only two of the many possible types of credit derivatives, these results provide insights into the reasons why credit derivatives may behave very

**EXHIBIT 9 ■ Credit Spread Put Prices Graphed as a Function of the Underlying Credit Spread**





differently from derivatives on traded assets.

In particular, the mean-reverting property of credit spreads has many important implications for the pricing and hedging behavior of credit derivatives. Our results suggest that the effects of mean reversion should be carefully considered in designing credit derivatives.

#### ENDNOTE

We are grateful for discussions with Andreas Grunbichler and Stephen Figlewski. All errors are the responsibility of the authors.

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