

Systemic Credit Risk: What Is the Market Telling Us?

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The ongoing subprime crisis raises many concerns about the possibility of even more widespread credit shocks. We describe a simple linear version of a sophisticated model that can be used to extract information about macroeconomic credit risk from the prices of tranches of liquid credit indices. The market appears to price three types of credit risk: idiosyncratic risk at the level of individual companies, sectorwide risk at the level of companies within an industry, and economywide or systemic risk. We applied the model to the recent behavior of tranches in the U.S. and European credit derivatives markets and show that the current crisis has more than twice the systemic risk of the automotive-downgrade credit crisis of May 2005.

The dramatic meltdown in the subprime market during 2007–2008 raised many red flags among market participants about their potential exposure to broad, systemic credit shocks. These heightened concerns have produced dramatic declines in market liquidity and access to credit, flights to quality, sharp increases in market volatility, and rising risk premiums in many financial markets. As a result, prices of the most credit sensitive securities in the market may actually play the role of “the canary in the coal mine” in providing information about how market participants collectively assess the risk of systemic or macroeconomic credit shocks.

In this article, we describe using the prices of indexed credit derivatives to extract market expectations about the nature and magnitude of the credit risks facing financial markets. Since their inception in 2002, the indexed credit derivatives markets have exploded in size and participation. Broad indices are now traded for the U.S. (Markit CDX) and European (Markit iTraxx) credit markets, which usually have high liquidity, and indices are traded to a lesser degree for the Japanese and U.K. credit markets. As of the end of 2007, the investment-grade CDX was in its ninth generation and its European counterpart, in its eighth generation. Even more striking than the success of the indices, however, are the launch and

success of *tranches* on the indices. Tranches can be best thought of as call spreads on the credit losses of a portfolio. Investors can use tranches to control their exposure to particular loss thresholds.

To extract the information from these credit derivatives, we first developed a simple linearized version of the collateralized debt obligation (CDO) pricing model of Longstaff and Rajan (2008). They proposed a three-jump model that is directly calibrated to the traded spreads of tranches and indices. The model allows for the possibility that credit spreads might be a composite of several types of credit risk. Specifically, they found that the credit-loss distribution embedded in index tranche prices includes a component for the risk of idiosyncratic or company-specific defaults, a component for the risk of broader, sectorwide or industrywide defaults, and a component for the risk of a massive economywide default scenario.¹ In the study reported in this article, we fit the linearized version of the model to the market prices of the credit indices and tranches.

The Linearized Three-Jump Model

Following Longstaff and Rajan (2008), we let L denote the proportion of portfolio losses realized on a credit portfolio (so, $L_0 = 0$ losses). We write the proportion of portfolio losses as

$$L = \gamma_1 N_1 + \gamma_2 N_2 + \gamma_3 N_3, \quad (1)$$

where the γ_i parameters (where $i = 1, 2, 3$) denote jump sizes and N_i are independent Poisson counters that correspond to the number of jumps.² Note that Equation 1 allows for L to be greater than 1, but in practice, such large values of L are never

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realized.³ In terms of constant intensities λ_i over a period T , we can write the probability of j jumps for the i th Poisson process, P_{ij} , as

$$P_{ij} = \frac{e^{-\lambda_i T} (\lambda_i T)^j}{j!} \quad (2)$$

The risk-neutral pricing equation for an index (e.g., the CDX) of maturity T implies that we can solve for coupon C for the index by setting the value of the *premium leg*,⁴

$$C \int_0^T D(t) \{1 - E[L(t)]\} dt, \quad (3)$$

equal to the value of the *protection leg*,

$$\int_0^T D(t) E(dL). \quad (4)$$

In Expressions 3 and 4, $D(t)$ denotes the riskless discount factor for time t (for pricing the CDX, we use the swap curve) and the expected loss appears in the integral.⁵ Expanding these expressions in terms of the loss function and Poisson intensities gives

$$\begin{aligned} C \int_0^T D(t) (1 - \gamma_1 \lambda_1 t - \gamma_2 \lambda_2 t - \gamma_3 \lambda_3 t) dt \\ = \int_0^T D(t) (\gamma_1 \lambda_1 + \gamma_2 \lambda_2 + \gamma_3 \lambda_3) dt. \end{aligned} \quad (5)$$

After some straightforward algebra, we can show that the index coupon value is given by

$$C = \frac{\gamma_1 \lambda_1 + \gamma_2 \lambda_2 + \gamma_3 \lambda_3}{1 - (\gamma_1 \lambda_1 + \gamma_2 \lambda_2 + \gamma_3 \lambda_3) A}, \quad (6)$$

where A represents the duration of an annuity and is given by

$$A = \frac{\int_0^T D(t) t dt}{\int_0^T D(t) dt}. \quad (7)$$

In implementing the model, we will also find it convenient to rearrange Equation 6 to be

$$\lambda_1 = \frac{[C / (1 + AC)] - \gamma_2 \lambda_2 - \gamma_3 \lambda_3}{\gamma_1}. \quad (8)$$

Thus, given values of λ_2 and λ_3 , Equation 8 may be used to determine λ_1 as an explicit function of the market index spread (coupon), C . This approach allows the model to fit the market index spread exactly. We use the index spreads and spreads on standard tranches to numerically obtain the values of λ_2 and λ_3 and jump parameters γ_1 , γ_2 , and γ_3 .

Data and Methodology

To calibrate the model, we used the spreads for the CDX investment-grade and high-yield indices of various maturities and vintages. The data came from PIMCO and JPMorgan Chase & Co. Because the CDX indices roll every six months and come in various maturities (the most liquid point being the 5-year index, followed by the 10-year and 7-year indices), the investment-grade index derivatives markets are representative of the broad investment-grade U.S. credit market, and the same is true for the high-yield index. The index derivatives suite also allows for market participants to implement "beta" and hedge views on the credit market directly without taking the duration or liquidity risk that comes from using cash bond instruments. We used data similarly for the iTraxx index that tracks the credit risk of European investment-grade credit markets.⁶

In addition to the index data, we used data on the market spreads for standardized tranches on these indices. The U.S. CDX investment-grade tranches are broken down in terms of losses that attach and detach at the 0 percent, 3 percent, 7 percent, 10 percent, 15 percent, 30 percent, and 100 percent points. The index, by construction, is a 0–100 percent tranche. The standard 0–3 percent investment-grade equity tranche trades on a points-up-front basis with a fixed coupon of 500 bps, but the other tranches trade on the basis of a spread that changes with market conditions. The U.S. CDX high-yield tranches attach and detach at the 0 percent, 10 percent, 15 percent, 25 percent, 35 percent, and 100 percent points. The 0–10 percent and 10–15 percent tranches trade on a points-up-front basis with a zero fixed coupon; the other tranches trade on the basis of a market spread. Similarly, the European investment-grade iTraxx index has tranches with points at 0 percent, 3 percent, 6 percent, 9 percent, 12 percent, 22 percent, and 100 percent. The 0–3 percent iTraxx tranche also trades on a points-up-front basis with a 500 bp fixed coupon.

To see how we obtained the value of the parameters, note that we first assumed that jump sizes γ_i are given. For a tranche with attachment point a and detachment point b , we can write the loss on the tranche at time t , $L_{a,b}(t)$, in terms of the loss function, $L(t)$, on the underlying portfolio; that is,

$$L_{a,b} = \frac{\max(0, L - a) - \max(0, L - b)}{b - a}. \quad (9)$$

This function illustrates that the losses on the tranche can be viewed as the payoffs for a call spread. Specifically, the losses on the tranche equal the payoff of a call on L with strike a minus the payoff of a call with strike b , where the spread is

scaled by $1/(b - a)$. The tranche spread, $C_{a,b}$, is determined, as previously, by setting the value of the premium leg,

$$C_{a,b} \int_0^T D(t) \{1 - E[L_{a,b}(t)]\} dt, \tag{10}$$

equal to the value of the protection leg,

$$\int_0^T D(t) E(dL_{a,b}). \tag{11}$$

We implemented this pricing equation numerically. Specifically, we evaluated the expectations in Expressions 10 and 11 by computing the tranche loss function, $L_{a,b}(t)$, for values of N_i ranging from zero to some suitably large value and then weighting by the corresponding Poisson probabilities, P_{ij} , from Equation 2. These procedures allowed us then to solve for the model tranche spreads.⁷

To identify λ_1 , we used Equation 8 to fit the model exactly to the market index spread, C . To identify λ_2 and λ_3 , we minimized the root-mean-squared percentage pricing error between the model and the observed market tranche price for each day in the estimation period. Finally, we iterated over different values of jump parameters γ_1 , γ_2 , and γ_3 until we achieved the global minimum root-mean-squared percentage pricing error.

Once we had the λ and γ values, we could identify three types of spreads that make up the full index spread in the risk-neutral setting:

$$\text{Idiosyncratic spread} \equiv S_1 = \frac{\gamma_1 \lambda_1}{1 - (\gamma_1 \lambda_1 + \gamma_2 \lambda_2 + \gamma_3 \lambda_3) A}, \tag{12}$$

$$\text{Sectorwide spread} \equiv S_2 = \frac{\gamma_2 \lambda_2}{1 - (\gamma_1 \lambda_1 + \gamma_2 \lambda_2 + \gamma_3 \lambda_3) A}, \tag{13}$$

and

$$\text{Economywide spread} \equiv S_3 = \frac{\gamma_3 \lambda_3}{1 - (\gamma_1 \lambda_1 + \gamma_2 \lambda_2 + \gamma_3 \lambda_3) A}. \tag{14}$$

From Equation 6, the reader can see that C is equal to $S_1 + S_2 + S_3$. Thus, the sum of the three types of spreads equals the total spread of the index.

Intuitively, this simple linear model is a way to decompose the index spread into its idiosyncratic, sectoral, and economywide components. Note that all the computations were done in a risk-neutral setting, so we had no way of distinguishing how much of the spread might be the result of risk premium terms.

Finally, to compute the sensitivity of a tranche with attachment a and detachment b to spread S_i (where $i = 1, 2, 3$) as defined here, we numerically computed the change of a tranche price to a shift in the underlying spread.

Results and Strategy Implications

To implement the methodology that we have described, we first fit the model to the index values and tranche spreads by using current market data for the CDX investment-grade (IG), iTraxx IG, and CDX high-yield (HY) indices. **Table 1** shows the results and parameter values from fitting the model.

Table 1 reveals many similarities in the results for the indices. The size of the first jump, γ_1 , is in the range of 0.9 percent to 1.5 percent for all of the investment-grade indices and is roughly 2.3 percent for the high-yield indices. Thus, a realization of the first Poisson process can clearly be given the interpretation of the idiosyncratic default of one or two of the companies in the index.

Table 1. Parameter Estimates and Root-Mean-Squared Error Values from Model Fitting

Index	Maturity (years)	Horizon	γ_1	γ_2	γ_3	RMSE
CDX NA IG 8	5	20-3-07 to 31-12-07	1.132	10.008	77.058	7.519
CDX NA IG 8	7	20-3-07 to 31-12-07	1.048	9.265	62.802	5.083
CDX NA IG 8	10	20-3-07 to 31-12-07	1.167	9.269	56.840	4.016
CDX NA IG 9	5	21-9-07 to 31-12-07	0.914	9.303	62.496	4.368
iTraxx Main 7	5	20-3-07 to 31-12-07	1.511	9.143	61.265	6.674
iTraxx Main 8	5	20-9-07 to 31-12-07	1.254	9.206	55.865	4.267
CDX NA HY 8	5	27-3-07 to 31-12-07	2.272	6.822	52.591	1.444
CDX NA HY 9	5	27-9-07 to 31-12-07	2.314	6.343	50.599	1.048

Notes: Estimated jump size parameters reported are per US\$100 notional value. NA = North America. RMSE = root-mean-squared error. "Main" refers to the Europe IG index.

In contrast, the size of the second jump, γ_2 , ranges from about 6 percent to 10 percent across the different indices. Because the companies in the CDX and iTraxx indices are roughly evenly distributed over 10–12 broad industries or sectors, this second jump size is consistent with the realization of a credit event in which an entire industry or sector of the market goes into default.

Finally, the size of the third jump, γ_3 , ranges from about 50 percent to 75 percent. Thus, for all indices, a jump in the third Poisson process translates into a credit event in which the majority of companies in the index default together, implying a catastrophic systemic credit event affecting all sectors of the economy. Clearly, the realization of such an event would be so severe that even the most senior index tranches would experience significant losses. These results parallel those reported in Longstaff and Rajan (2008) and extend the analysis with more-recent data for a broader set of indices and maturities.⁸

In addition to the information about the nature and size of potential types of credit events, the model also allowed us to infer the three components of the spread: S_1 , S_2 , and S_3 . **Figure 1** and

Figure 2 provide plots of the values of these components for the CDX IG series 8 five-year and ten-year indices, respectively, for the March 2007 through December 2007 period. **Figure 3** gives plots of the iTraxx IG series 7 five-year index for the same period, and **Figure 4** does the same for the CDX HY series 8 five-year index for the March 2007 through December 2007 period.

Figures 1–4 show clearly the onset of the subprime CDO crisis in mid-2007. At approximately the start of August 2007, all three components of the indices began to increase substantially. Interestingly, however, the individual components did not all increase by the same percentage. For the investment-grade indices (Figures 1–3), the idiosyncratic component (S_1) of the spread was roughly 50–75 percent higher during the second part of 2007 than during the first part of 2007. The systemic component (S_3) of the spread, however, more than tripled after August 2007. In fact, the systemic spread's value at the end of 2007 was nearly 10 times its value in March 2007. The sectorwide component (S_2) of the spread also increased during 2007 but by less than 10 bps.

Figure 1. Components of the CDX IG Series 8 Five-Year Index, March 2007–December 2007

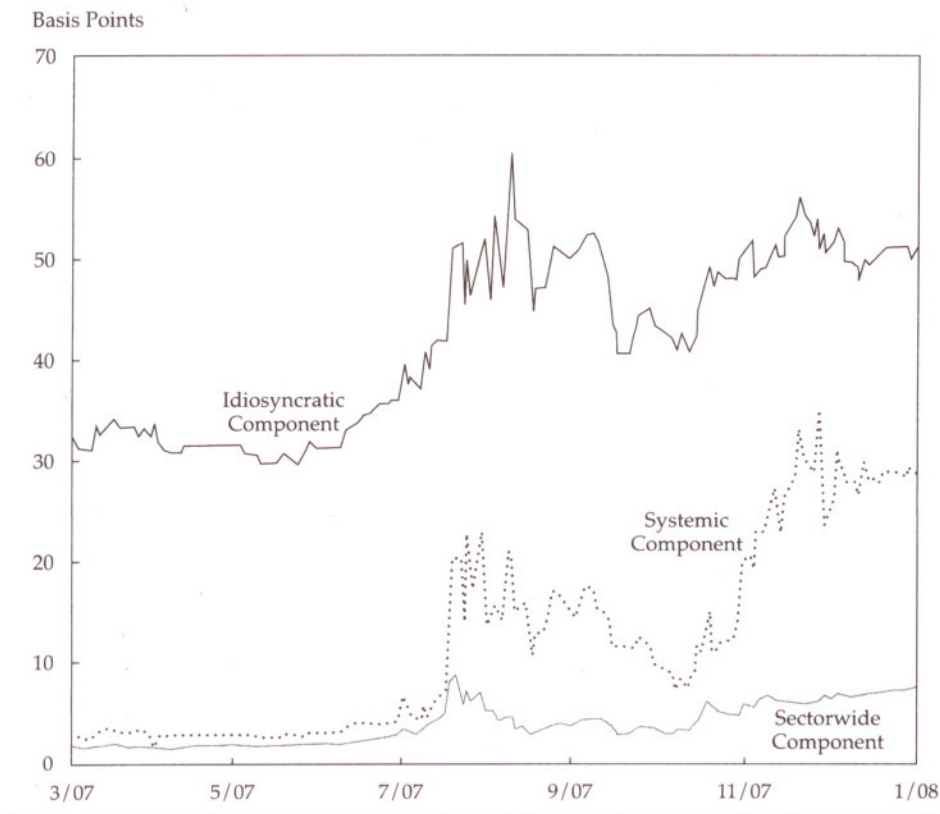


Figure 2. Components of the CDX IG Series 8 Ten-Year Index, March 2007–December 2007

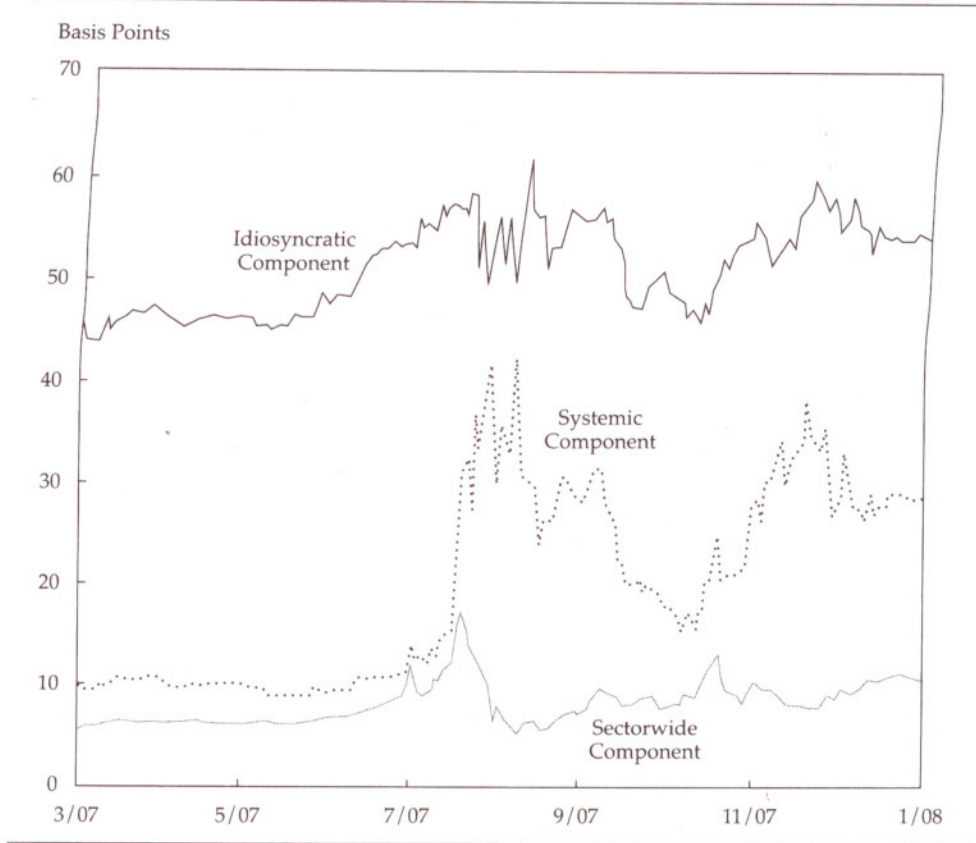


Figure 3. Components of the iTraxx IG Series 7 Five-Year Index, March 2007–December 2007

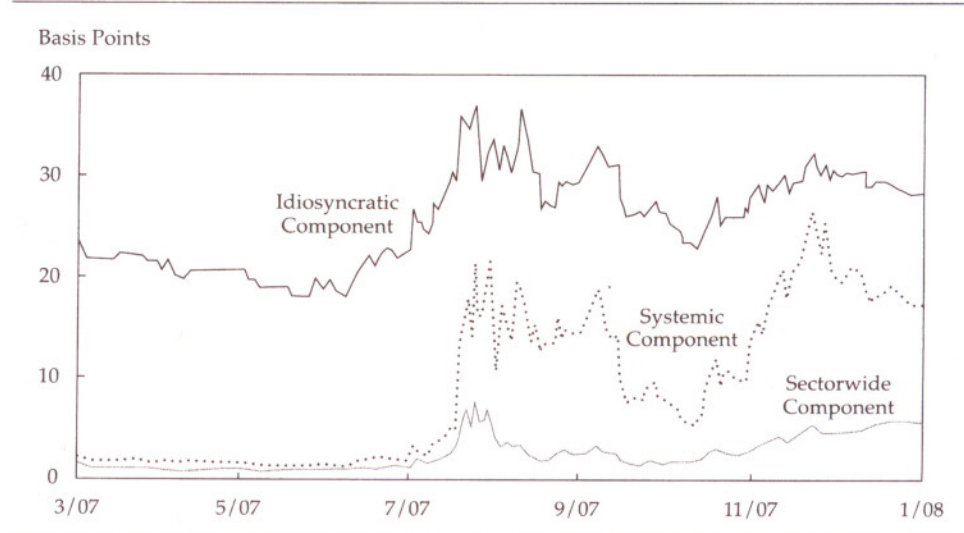


Figure 4. Components of the CDX HY Series 8 Five-Year Index, March 2007–December 2007

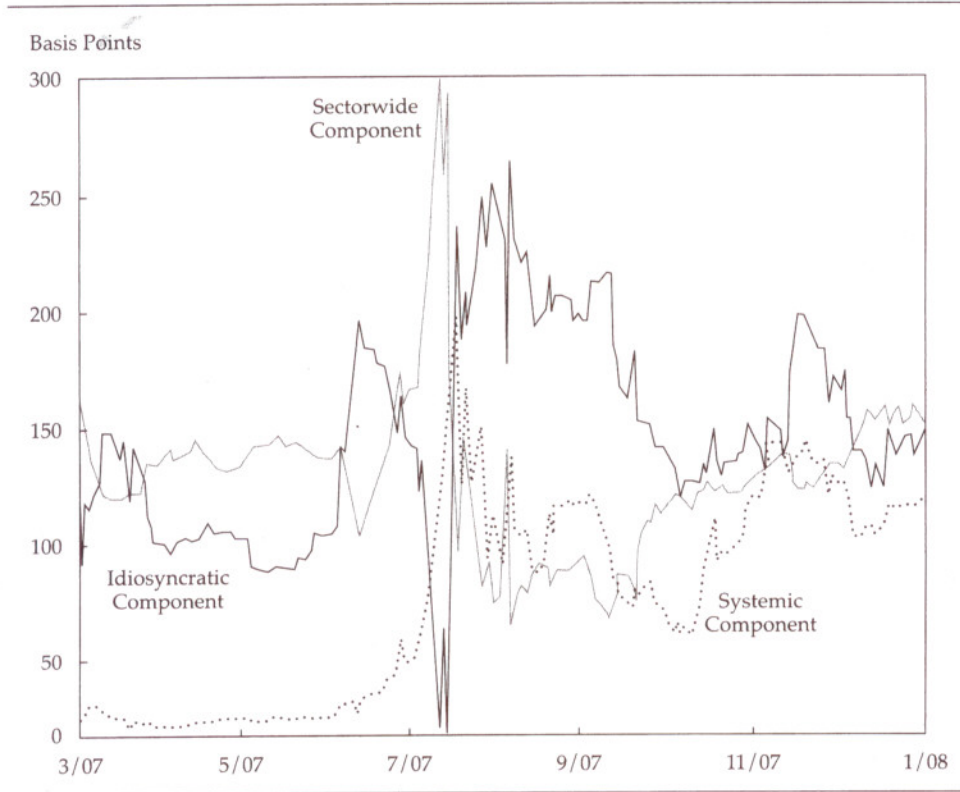


Figure 4 shows that a parallel situation held for the CDX high-yield index. Overall, the values of the idiosyncratic and sectorwide components of the high-yield spread, after peaking, were only modestly higher during the latter part of 2007. The major difference occurred for the systemic component, which rose, dipped, then rose dramatically from a level of about 25 bps during the first part of 2007 to a range of 100–150 bps during the latter part of 2007.

These results provide valuable information about the market's credit concerns. By decomposing the credit indices into three components, we can see that much of the increase in credit spreads during 2007 was driven by concerns about the risk of systemic or macroeconomic credit problems. This kind of risk is clearly much different from the credit-worthiness of individual companies, or even of an entire industry, and raises much different concerns.

To provide a longer-term perspective, we have plotted in **Figure 5** the value of the three components of the spread on the basis of fitting the model to the on-the-run CDX IG five-year index over the period of March 2005 through December 2007. (Thus, we are essentially treating the index as a continuous series.) Although the idiosyncratic risk component currently approximates its 2005 value, the *systemic* risk component is currently much larger in magnitude than in 2005.

Systemic risk is now also a larger proportion of the total risk of the index than it was in 2005, which is logical because the widening of the index in early 2005 was caused by downgrades in the automotive sector. The 2007 spread widening, however, can be traced to distress in the financial sector and a marketwide lack of liquidity, which, arguably, affects the entire economy. Of course, the widening of the senior and "super-senior" tranches (such as the CDX IG 15–30 percent tranche) means that more risk premium has come back into this part of the capital structure.

Numerous headlines have suggested that the widening of the senior and super-senior tranches is a result of the mark-to-market losses taken by specialized vehicles that had leverage exposure to these tranches. This mark-to-market loss did, indeed, weaken the balance sheets of brokers and money center banks that, in effect, had exposures through their financing to many of these vehicles. Systemic risk is usually measured in terms of the widening of the LIBOR swap spread and the credit-default swap spreads of banks and financial institutions. Therefore, it is no surprise that the period in which the super-senior tranches widened drastically in spread was accompanied by a sharp widening of swap spreads and financial sector spreads, an increase in volatility, and a general reduction in liquidity.

Figure 5. Components of the CDX IG Five-Year On-the-Run Index, March 2005–December 2007



From a strategic angle, one would like to know what combination of tranches will most directly implement a view on one of the three macro spreads. For answers, we provide in **Table 2** the exposures to the movements of the underlying spreads for the CDX IG series 8 five-year, seven-year, and ten-year spreads. To provide additional perspective, Table 2 also reports, using the standard Gaussian copula model widely used on Wall Street, the sensitivities of the tranches to a 1 bp move in the CDX index (for a discussion of the copula model, see Li 2000).

As expected, the equity tranche (0–3 percent) has the largest exposure to the idiosyncratic risk factor (S_1) and the senior and super-senior tranches have higher exposures to the systemic factor than to the other factors. Thus, the tranches have very different sensitivities and their risk has a multidimensional nature. This aspect has important implications for the risk management of these structured credit products. For example, imagine that the CDX IG five-year index increases by 1 bp. The standard copula model would imply that the price of the 0–3 percent equity tranche should change by 42.7 cents per US\$100 notional. In our model, however, the change in the value of the equity tranche depends

on the underlying source of the change in the CDX index. For example, if the change was entirely the result of an increase in the idiosyncratic risk of some companies in the index, the valuation effect would be 90.5 cents instead of 42.7 cents per US\$100 notional. If the change was entirely the result of an increase in the systemic component, the valuation effect would be only 2.2 cents. In the case of idiosyncratic risk increasing, the copula model would underestimate the valuation effect by more than 50 percent; in the case of systemic risk increasing, the copula model would overestimate the valuation effect by a factor of more than 10. The bottom line is that if index credit risk is really driven by three distinct factors, the use of a single factor for risk management is simply not adequate.

For an example of the use of these risk factors, assume that we think systemic risk will decline from its current elevated levels. If we want to go long (sell protection on) the economywide part of the index but remain unexposed to the idiosyncratic and sectorwide components while using five-year CDX IG tranches, we can go short credit (buy protection) on the 3–7 percent and 7–10 percent tranches with notionals of, respectively, US\$11.7 million and US\$1.3 million and go long credit (sell

Table 2. Sensitivity of CDX IG Index Tranches to Changes in the Component Spreads of the CDX Index

Index	Maturity (years)	DV01	DV01 for CDX IG Index Tranche					
			0–3%	3–7%	7–10%	10–15%	15–30%	30–100%
CDX IG 8	5	Copula	0.427	0.277	0.099	0.055	0.027	0.010
		S ₁	0.905	0.394	0.017	0.024	0.001	0.001
		S ₂	0.178	0.387	0.405	0.177	0.008	0.001
		S ₃	0.022	0.049	0.052	0.052	0.053	0.037
CDX IG 8	7	Copula	0.401	0.406	0.185	0.111	0.051	0.015
		S ₁	0.944	0.744	0.078	0.065	0.004	0.002
		S ₂	0.180	0.542	0.583	0.276	0.022	0.002
		S ₃	0.023	0.077	0.085	0.087	0.088	0.046
CDX IG 8	10	Copula	0.315	0.537	0.302	0.176	0.082	0.023
		S ₁	0.724	1.260	0.429	0.136	0.021	0.005
		S ₂	0.143	0.621	0.770	0.536	0.068	0.005
		S ₃	0.016	0.094	0.120	0.127	0.132	0.060

Notes: DV01 is the price change per US\$100 notional in the indicated CDX index tranche CDO with respect to a 1 bp change in the index components—S₁, S₂, and S₃. Also reported is the DV01 based on the standard Gaussian copula model. The results are based on market prices for 31 December 2007.

protection) on the 0–3 percent, 15–30 percent, and 30–100 percent tranches with notionals of, respectively, US\$2.1 million, US\$305 million, and US\$2,282 million. With this portfolio, a 1 bp decline in the economywide spread would result in a profit of \$1 million whereas moves in the other spreads would result in negligible profit and loss.

This approach may also be used to explain and exploit credit-curve risk premiums. For example, Figures 1 and 2 show the risk allocation to the three spreads before and after the August 2007 subprime crisis for the five-year and ten-year CDX IG series 8 indices. Clearly, both CDX IG indices contained more systemic risk in the second part of the period than in the first part; systemic risk increased from 10 percent to 33 percent of the total five-year spread and increased from 16 percent to 30 percent of the total ten-year spread. But even though the systemic risk embedded in the indices increased for both indices, the increase was larger in the five-year index. Thus, to sell the systemic liquidity premium, the five-year index and its tranches might be the preferred instruments.

Conclusion

We have shown how the information in credit derivatives about the market's expectations of systemic credit risk can be extracted. We did so by implementing a simple linearized version of a three-jump model and calibrating it to market index and tranche spread levels. Thus, we were

able to quantify the relative magnitudes of macro risks embedded in the liquid indexed credit derivatives. This method also provides a unique way, with an increased menu of instruments, of expressing credit views. Furthermore, our approach provides a simple yet powerful framework for valuing tranches on bespoke (custom-made) portfolios relative to the prices of standard index tranches.

The results indicate that the current subprime credit crisis is fundamentally different from previous credit crises. Specifically, systemic credit risk has become a much larger fraction of total credit risk, and idiosyncratic and sectorwide credit risk levels have remained relatively constant. These results have many important implications for the financial markets.

One important implication is that credit risk premiums in financial markets may remain at high levels for the foreseeable future, which will lead to a significantly higher cost of debt for many companies and sectors. Another implication of this shifting trend in the nature of credit risk is that traditional risk management strategies, such as portfolio diversification, may be less effective in the future in controlling credit risk exposure.

Another key implication is that some credit-modeling tools that are widely used in practice may severely underestimate the actual risk exposure of credit portfolios. Specifically, the behavior of index tranches in relation to idiosyncratic and systemic risks varies significantly across their attachment

and detachment points. Thus, the Gaussian copula model, although applicable to individual tranches, is not sufficient to capture the risks of all tranches. At best, the implied base correlation of the copula model reflects the average correlation of the underlying equities and, hence, the correlation for the most junior or equity tranche (see Bhansali 2008). Because of the lack of data on correlation during systemic defaults, extrapolating the tails to the senior part of the capital structure is difficult.

Therefore, using the copula model as a risk management tool is fraught with dangers.

We are grateful for helpful discussions with Arvind Rajan. This article contains the current opinions of the authors and not necessarily those of PIMCO (Pacific Investment Management Company LLC). These opinions are subject to change without notice.

This article qualifies for 1 CE credit.

Notes

1. Longstaff and Rajan's (2008) results are consistent with the CDO-modeling framework of Duffie and Gârleanu (2001).
2. "Poisson counters" means that the waiting time between jumps is exponentially distributed.
3. The benefit of the computational speed from linearizing the model far outweighs this technical difficulty with loss of accuracy. Furthermore, the probability of L being greater than 1 is vanishingly small for realistic calibrations.
4. Although the formulas in the article treat the cash flows from the index or the tranches as continuous for expositional simplicity, the numerical implementation of the model is based on discrete quarterly cash flows, which is consistent with the actual contractual provisions of these credit derivatives.
5. As in Longstaff and Rajan (2008), we are assuming that interest rates are uncorrelated with loss realizations.
6. Of course, the indices are not completely sufficient to hedge or control for basis risk between cash market indices (such as the Lehman Brothers U.S. Corporate Index) because of composition differences and the cash-CDX basis, as well as the difference in liquidity risk embedded in the index and the individual names. These differences would have only a negligible impact, however, on our conclusions.
7. In fitting the model numerically, we allowed for up to 50 jumps for the first Poisson process, up to 10 jumps for the second Poisson process, and up to 3 jumps for the third Poisson process.
8. In our model, a realization of one of the Poisson processes might trigger the simultaneous default of multiple companies. A more general model, however, might allow for a cluster of defaults in a relatively narrow time interval—although not necessarily simultaneously. We are grateful to the referee for this suggestion.

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