The Value of Financial Flexibility: Equilibrium Liquidation Values and Endogenous Capital Structure Heterogeneity

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Abstract

Firms with lower leverage are not only less likely to experience financial distress but are also better positioned to acquire assets from other distressed firms. With endogenous asset sales and values, each firm's debt choice then depends on the choices of its industry peers. With indivisible assets, otherwise identical firms may adopt different debt policies—some choosing highly levered operations (e.g., to take advantage of tax benefits), others choosing more conservative policies to wait for acquisition opportunities. Moreover, the acquisition channel induces firms to reduce debt when assets become more redeployable. Finally, our paper highlights a simple pitfall: theoretical implications for debt do not always map into empirically measured implications for debt-to-value ratios, because value is also endogenous.

Firms with more leverage are more likely to experience future financial distress. Importantly, their expected costs of bankruptcy are likely to be higher not only when they themselves, but also when their industry peers have taken on more debt. More firms will then want to sell the same types of assets at the same time, and their peer firms—who would otherwise have been the natural asset buyers—become themselves more limited in their capacity to absorb these assets (Shleifer and Vishny (1992)). As a result, the fire-sale discounts relative to fundamental asset values will become steeper. And, therefore, the debt choices of individual firms today, aggregated into industry debt, can themselves influence the asset liquidation values and have anticipative feedback into firms' debt choices in the first place.

Like most earlier literature, in our model, firms choose their capital structures before they learn their profitabilities. Leverage confers direct value benefits, such as tax benefits, signaling benefits, or incentive enhancements. However, leverage can also lead to tough choices for firms that later experience negative shocks. Once in default, the creditors must decide whether to liquidate on the one hand, or to reorganize and continue operations on the other. If they liquidate, firms receive the prevailing market price for their assets. The assets will then be in the hands of buyers who can presumably put them to better use. If they reorganize, firms keep the assets but may still suffer some impairments, such as direct costs and strained relationships with key stakeholders. A distressed firm is not worth as much as it would have been in the absence of default.

Unlike most earlier literature, in our model, debt-laden capital-constrained firms are not only more likely to sell but also less likely to buy. We assume that all firms are competitive and can anticipate but not internalize the effects of their peers. The mechanism in our model that coordinates their debt choices is the endogenous asset price.[1] For example, suppose that some firms adopt more aggressive debt policies. In the future, this will increase the supply and reduce the demand for liquidated assets, resulting in a lower equilibrium price. In turn, the anticipated lower price creates two counterbalancing motivations for the remaining firms today: (1) they will fear running into financial distress more; and (2), if they reduce their own debt, they will be more likely to enjoy future vulture buying opportunities. Thus, their best response to higher debt by their peers is lower debt for themselves. Interestingly, when assets are indivisible, a-priori homogeneous firms may even endogenously split into two coexisting types, with some types levering up and operating more aggressively—even anticipating distress and having to fire-sell—and other types maintaining conservative capital structures ("dry powder") to take advantage of these anticipated future fire sales.

The "opportunistic-acquisition" channel can reverse an important implication of models with only the "financial distress" channel. In Williamson (1988) and Harris and Raviv (1990), when assets are more redeployable, firms take on more debt because their distress costs will be lower (Benmelech, Garmaise, and Moskowitz (2005)). By contrast, in our model, firms may take on less debt to take advantage of more favorable future buying opportunities. Our model can also offer further insights. Depending on

[1] Earlier papers assume the liquidation price is exogenous. The exception is Gale and Gottardi (2015). We will discuss the differences in detail in Section III.A.

parameters, there may be too many or too few asset transfers relative to first-best in our model. Thus, a tax policy designed to improve allocational efficiency must be context sensitive. And, our model can also offer predictions on other observables, such as asset transfer quantities and prices, recovery rates and credit spreads, default and liquidation probabilities, and so on.

Our paper also makes a more general point. Most theories of capital structure are about how parameters influence the optimal debt choice. Most empirical tests use normalized leverage, typically dividing it by firm value. The problem is that not only debt but also firm-value should change with parameters. This matters less when debt and value respond in opposite directions, although what is interpreted as a test on debt could merely be a test on value. It matters more when debt and value respond in the same direction. The empirical metric, debt-to-value, then measures merely whether debt or value changes faster. In our specific model, we illustrate this general point by showing how an increase in the direct benefits of debt always increases debt but not always debt-to-value ratios.

Our paper now proceeds as follows: Section I lays out a basic no-distress model, in which firms with low leverage can later purchase assets from other firms that will turn out to have low productivity. Redeployability favors less leverage, as buyers want the opportunity to purchase poorly performing assets down the line. Debt has an effect only through its influence on this "opportunistic-acquisition" channel. Section II adds the more recognized "financial-distress" channel. Without the acquisition channel, redeployability always favors more leverage, because sellers can rely on the lesser downside. With both the purchase channel and the distress channel, more asset redeployability at first favors higher leverage (lesser distress costs dominate) and then decreases in leverage (greater acquisition opportunities dominate). Section III puts the model in perspective and describes its relation to prior research. Section IV concludes.

I The Opportunistic-Acquisition Channel

[Insert Table 1 here: **Variables**]

In this section, we introduce a model in which lower leverage allows firms to undertake more acquisitions in the future. In the next section, lower leverage will also reduce expected financial-distress reorganization costs. Table 1 summarizes the key variables in our model.

A Model Setup and Assumptions

We consider an industry with risk-neutral competitive firms. Each firm has a manager who maximizes the value of the firm. This is not to discount the real-world importance of intra-firm agency conflicts, but to show that our results can obtain even when they are not present. All information is public upon realization, again to show that asymmetric information concerns are not required for our results, not to discount their real-world importance.

Assets and Types: At time 0, each firm owns one indivisible unit of a productive asset.[2] The productivity of this asset is a random variable, denoted $\tilde{v}_i$, whose realization will be publicly observed at time 1. All firms are ex-ante identical and it is common knowledge that their firm type is distributed uniformly on the interval $\tilde{v}_i \in [0,1]$. After firm productivity is realized at time 1, firms with enough capital (low leverage and high productivity) can acquire assets offered by other firms in the industry at the prevailing endogenous price P. We always assume free disposal, so $P \geq 0$. All assets generate a payoff at time 2, which depends on the holder's realized productivity v_i.

Financing: At time 0, each firm can finance its asset purchase with equity or debt. The face value of debt is constrained to be $D_i \in [0,1]$.[3] All agents are risk-neutral and there is no time discounting, so the expected rate of return on debt is zero. We assume that debt offers immediate net benefits that confer a proportional value $\tau \cdot D_i$. This τ includes the tax benefit of debt (which may or may not be socially valuable), but we have a much broader concept in mind. Tau can reflect the ability of debt to allow financially-constrained firms to take on more productive projects, any positive incentive effects from debt, lower fund-raising costs, and so on; all net of debts' unmodelled costs. This benefit is not dissipated by subsequent events and accrues to the original owners. We show in Appendix D that all our main results hold when the debt benefit is available to pay creditors and fund acquisitions.

Liquidation: At time 1, after each firm has learned its productivity realization v_i, it can decide whether to sell its asset at the prevailing price P or to continue operations. Because managers' objectives are aligned with their firms', their decision to liquidate or continue is efficient, given their earlier time 0 debt choice. The value from continuing operations is v_i. The liquidation price of the asset is determined by perfectly competitive buyers and sellers. Thus, firm i sells iff $v_i < P$. Although the firm's own debt choice has no influence on the asset's price, each firm knows that the asset price is determined by the collective choices of all firms in the industry.

Acquisition: Although firms can acquire liquidated assets, we assume there is some cost associated with redeployment. This could be because assets need to be customized. Repurposing can require, e.g., moving costs, reprogramming, retraining of workers, and coordination with other complementary assets. In our model, we assume that an asset with productivity v_i to its current owner (firm i) has productivity of $\eta \cdot v_j$ to a potential acquirer (firm j), where $\eta < 1$. Higher values of η imply that assets can be redeployed more easily (at lower cost). In this specification, an asset that transfers from a low-productivity seller i to a high-productivity buyer j enjoys upgraded productivity ($v_j > v_i$), but not to the same extent that it would have had if buyer j had owned it all along. Thus, holding productivity fixed across firms, the asset is also worth more to the current owner than to a potential buyer if both have equal productivity. Taking

[2] At time 0, all firms are identical and we can define their preferred investment amount to be one unit. As we describe below, some firms may wish to purchase liquidated assets at time 1, but these buying firms are then aware of their higher productivity.

[3] This convenience assumption is necessary to prevent firms from taking on infinite debt to exploit ex-ante linear benefits on *promised* debt. Firms will optimally choose $D_i \in [0,1]$ if the benefits of debt are linear in the *proceeds* from debt, but this assumption would make the model algebraically intractable and force numerical-only solutions. This upper limit on D_i ensures that the promised debt payment is never greater than the firm's highest possible cash flow (sans direct benefits). Thus, any higher value of D_i would not result in higher proceeds from the debt issuance, because the increment would not be paid.

both firm-specificity and own productivity into account, firms find it worthwhile to buy liquidated assets only if they are sufficiently more productive—acquiring liquidated assets at price P is positive NPV for all firms with $v_j > P/\eta$.

As in Shleifer and Vishny (1992), the natural buyers of liquidated assets are other firms in the industry with appropriate expertise. These firms have limited capital and they are constrained in their ability to acquire the asset at time 1 if they took on too much debt at time 0. Similar limits are also central in other papers, most prominently Duffie (2010). One perspective to justify this assumption is based on debt overhang problems. Potential acquirers may have positive NPV opportunities, but they may not be able to raise new capital if the first claims on the cash flows to any new investments go to existing debtholders. Another perspective is an appeal to *cash-in-the-market* financing (Gale and Gottardi (2015)), where firms are assumed to be unable to raise outside funding on short notice. Our model can go a step further, because it includes one parameter that can help capture at least some cross-sectional or time-series variation in the cash-in-the-market immediacy constraint. Long-term demand curves are more elastic than short-term demand curves. Our parameter eta would be lower when assets are shorter-lived, when they require more informed buyers or due diligence (the crisis time relative to the life time of the cash flows), and when they are more difficult to put to use by outsiders. Of course, eta also has to encapsulate further real-life aspects, such as how quickly transfer activity would have to take place when aggregate economic and financial conditions are worse. In the extreme allowed in our model, eta can approach 1 if industry firms can simply wait out any crisis and search until they can find the nearly perfect buyer.

In our specific model, the only financing available to a firm at time 1 is its internal equity, which is the maximum of zero and $(v_i - D_i)$. We assume that each firm can only acquire one unit of the liquidated asset at time 1.[4] This reflects limited organizational capacity to take on too many new assets at one time.

Timing: Figure 1 illustrates the timing of decisions more precisely.

[Insert Figure 1 here: **Game Tree for the Acquisition Model**]

Objective: At time 0, each firm chooses a debt level D to maximize its ex-ante value,

$$V(P, D_i) = \int_0^P P\,dv + \int_P^1 v\,dv + \int_{P+D_i}^1 \max\{0, \eta \cdot v - P\}\,dv + \tau \cdot D_i\ . \tag{1}$$

The first term represents the payoff P if the asset is eventually liquidated ($v_i \leq P$). The second term represents the payoff if the firm chooses to continue operations ($P < v_i \leq 1$). The third term represents the expected surplus if the firm chooses to acquire liquidated assets. The limits of integration recognize that the firm only has sufficient capital to acquire the asset if $v_i \geq P + D_i$ and the integrand ($\max\{0, \eta V - P\}$)

[4] The indivisibility assumption is important for the existence of a two-type equilibrium. However, our other qualitative results hold if the asset is divisible and firms can acquire as much of the asset as they can afford with their residual equity $(v_i - D_i)$.

recognizes that the firm only acquires the asset if it is positive NPV ($v_i > P/\eta$). The final term represents the immediate benefit of debt.

If the firm's financing constraint is binding (i.e., $P + D_i \geq P/\eta$), the expected surplus associated with acquiring assets at time 1 is

$$\int_{P+D_i}^{1} (\eta \cdot v - P)\, dv \;=\; \frac{\eta \cdot [1-(P+D_i)^2]}{2} \;-\; P \cdot (1-P-D_i)\ ,$$

which is decreasing in the own debt choice D_i. Thus, debt is costly because it reduces future profitable buying opportunities. Moreover, this cost of debt is increasing when future buying opportunities are of higher quality (i.e., assets are more easily redeployed or the price is lower). When the price of the asset is determined endogenously, as in our model, it will depend partly on how easily the asset can be redeployed. Therefore, the net effect of asset redeployability on equilibrium debt choice is not yet clear.

Because there is no aggregate uncertainty in our model, and we have infinitely many industry participants,[5] firms can anticipate the equilibrium price P at time 0. Therefore, each firm can consider its debt choice in one of three regions, outlined by a marginal cost defined by the right-most integral in (1):

1. For low debt, $D_i \leq P/\eta - P$, the marginal cost of debt is zero: increasing debt is not costly because the firm's financing constraint is not binding. Thus, because the marginal benefit of debt is positive (τ), it is always optimal for the firm to increase debt beyond this region.

2. For medium debt, $P/\eta - P < D_i < 1 - P$, the marginal cost is $\eta \cdot (P + D_i) - P$: increasing debt is costly because the firm's financing constraint is now binding, i.e., it may have to forego acquiring positive NPV assets that will be liquidated.

3. For high debt, $D_i \geq 1 - P$, the marginal cost is again zero: the debt is so high that the firm would not be able to finance the acquisition of the asset even if it were to turn out to be the highest productivity, $v_i = 1$. Increasing debt has no additional costs but additional benefits. Therefore, if the optimal debt is at least $1 - P$, given the τ benefit of debt, it is optimal for such a firm to push its debt to the permitted maximum, here $D_i = 1$.

Furthermore, with our indivisible asset, because at $D_i = 1 - P$ the marginal cost of debt jumps from $\eta - P$ to 0, the marginal cost of debt may be equal to the marginal benefit at multiple debt choices. This means that firms may make different debt choices even if they are identical ex-ante. In particular, firms adopting high-debt strategies (to take advantage of the tax debt benefits) will be able to coexist with firms adopting low-debt strategies (to take advantage of future asset buying opportunities at fire-sale prices).

Market Clearing: The equilibrium price for liquidated assets is determined by supply and demand. Because firms may choose different debt strategies, the market clearing price has to be a function of the

[5] An earlier draft considered aggregate uncertainty. The model's implications remained similar, but the algebra became far more involved. To focus on the intuition, we now present only the model without aggregate uncertainty.

frequency distribution of firm debt choices. Let $F(D)$ represent the cumulative distribution function of firms over admissible debt choices $D \in [0,1]$, i.e., the proportion of firms choosing $D_i \leq D$ is given by $F(D)$.

Supply: All firms with values $v_i \leq P$ will liquidate their asset regardless of their debt choice. Therefore, the aggregate supply of the liquidated assets is

$$\int_0^1 \int_0^P 1 \; dv \, dF(D) \;\; (= P) \; . \tag{2}$$

Demand: Acquiring one unit of the liquidated asset is positive NPV iff $v_i > P/\eta$. Firms will have sufficient funding to do so iff $v_i \geq P + D$, and they will have no demand if they have more debt than $1 - P$. Therefore, the aggregate demand for liquidated assets is

$$\int_0^{1-P} \int_{\max\{P+D, P/\eta\}}^1 1 \; dv \, dF(D) \; . \tag{3}$$

B Equilibrium

Definition 1 *An equilibrium is a distribution $F(D)$ over admissible debt choices $D \in [0,1]$ at time 0 and a price $P \in [0,1]$ for the liquidated asset at time 1, such that*

- *Firms act optimally at time 1; and*
- *Given a market clearing price P (and their optimal decisions at time 1), firms choose debt D_i to maximize firm value at time 0, according to the distribution $F(D)$; and*
- *given the distribution of firm debt choices $F(D)$, the price P clears the market for liquidated assets at time 1.*

C Solution

Recall that in the interior region, the value of the firm is

$$V(P, D_L) = \int_0^P P \, dv + \int_P^1 v \, dv + \int_{P+D_L}^1 (\eta \cdot v - P) \, dv + \tau \cdot D_L \; , \tag{1'}$$

with first-order condition for the optimal (interior) debt choice

$$D_L^*(P) = \frac{\tau + (1-\eta) \cdot P}{\eta} \; ,$$

and maximized firm value of

$$V(D_L^*(P)) = \frac{1+P^2}{2} + \frac{\left[\eta^2 + (P+\tau)^2 - 2\cdot\eta\cdot(1+\tau)\right]\cdot P}{2\cdot\eta}\ .$$

The optimal debt choice is higher when the benefits of debt (τ) are greater and when future acquisition opportunities are worse—when assets are more expensive (P) and more difficult to redeploy (η). However, as we explained above, the equilibrium asset price P^* also depends on the exogenous parameters, so the parameter net effects are yet to be determined.

Together, the equilibrium asset price equates supply, as in (2), with demand, as in (3); and each firm, given the asset price and its optimal decisions at time 1, chooses debt at time 0 to maximize its value, as in (1).

Theorem 1 *In the absence of financial-distress reorganization costs:*

- *If $\tau \le \eta^2/(3\cdot\eta+2)$, there is a unique one-type equilibrium with price $P^* = (\eta-\tau)/(1+\eta)$, in which all firms choose $D^* = (2\cdot\tau+1-\eta)/(1+\eta)$.*
- *If $\eta^2/(3\cdot\eta+2) < \tau \le \eta$, there is a unique two-type equilibrium with price*

$$P^* = \eta - \tau + \eta\cdot\tau - \sqrt{\eta^2\cdot\tau^2 + 2\cdot\eta\cdot\tau\cdot(\eta-\tau)}\ ,$$

  in which proportion h^* of firms choose $D_H = 1$, and proportion $1-h^*$ of firms choose D_L, where

$$D_L^*(P^*) = \frac{\tau + (1-\eta)\cdot P^*}{\eta},$$

$$h^* = \frac{1-2\cdot P^* - D_L^*(P^*)}{1-P^* - D_L^*(P^*)}.$$

- *If $\eta < \tau \le 1$, there is a unique one-type equilibrium with price $P^* = 0$, in which all firms choose $D^* = 1$.*

(All proofs are in the appendix.)

If the benefits of debt (τ) are low, all firms choose a low-debt strategy, so that they can maintain financial flexibility to acquire the asset at time 1. For intermediate values of τ, some firms choose a high-debt strategy to take advantage of the immediate benefits of debt, while other firms choose a low-debt strategy to take advantage of future investment opportunities. For high values of τ, the immediate debt benefits outweigh any potential benefit from asset acquisitions, so all firms choose the high-debt strategy. In this case, with no buyers, all assets will end up being discarded rather than being transferred from low-productivity to high-productivity firms.

D Implications

[Insert Figure 2 here: **Comparative Statics for the Acquisition-Only Model ($\phi = 0$)**]

A visual perspective can help the intuition. Figure 2 plots the comparative statics for heterogeneity h and firm-value V^*.

Type Heterogeneity: The left plot shows how heterogeneity in *ex-ante* debt strategies (h^*) arises endogenously. For high redeployability (η) and low debt benefits (τ), all firms choose to operate with very little debt (eager for the opportunity to buy assets from lower productivity firms in the future). For low redeployability and high debt benefits, all firms choose to operate with very high debt (in order to obtain the debt benefits). For intermediate redeployability and debt benefits, ex-ante homogeneous firms naturally divide into two kinds of firms—some pursuing the high-debt operating strategy, others pursuing the lower-debt opportunistic waiting strategy.

This heterogeneity is caused by the indivisibility of the asset. Once a firm has taken on so much debt that it will not be able to purchase the asset, it faces no further marginal cost to taking on more debt. If assets were divisible, our comparative statics below would continue to hold, but all firms would act alike. We can thus speculate that heterogeneity in ex-ante strategies increases in asset indivisibility—for example, in real-world situations in which purchases requires assuming large pieces (like entire divisions or factories), and not just diversifiable and spreadable small bits and pieces (like retail product inventories).

Implication 1 *When assets are indivisible, ex-ante identical firms may specialize: Low-debt firms coexist with high-debt firms.*

Firm Value: The right plot in Figure 2 shows that firm value increases monotonically both in the debt benefits τ and (weakly) in the redeployability η. As obvious as it is—in this surgical context—that *not only debt but also firm-value changes with model parameters*, its consequences are easy to overlook in the adaptation of theories to data.

[Insert Figure 3 here: **Comparative Statics for the Acquisition-Only Model Leverage ($\phi = 0$)**]

Figure 3 plots leverage-related comparative statics in this acquisition-channel-only model.

Leverage: The left and middle plots shows that the face value of debt and the value of debt today *decrease* in asset redeployability.[6] This is because, in equilibrium, future buying opportunities are more attractive when assets are more easily redeployed. Hence, firms choose lower debt upfront to enable more

[6] With one tiny region exception, which can be seen at the bottom left figure, the discussion applies to both the debt of the low firm (D_L^*) and the debt of the industry ($h^* \cdot 1 + (1-h^*) \cdot D_L^*$). Our focus is on industry debt, so the discussion omits some trivial tiny-region caveats.

opportunistic purchasing in the future. It is this opportunistic-acquisition channel that pushes against the more common intuition that firms take on more debt when their assets are more redeployable because distress costs are lower. Naturally, this implication is robust only to the extent that it characterizes an acquisition constraint. If firms in the industry—broadly defined as firms that are suitable buyers—can purchase liquidating assets regardless of their own leverage (e.g., perhaps because they can raise infinite financing instantly, then this implication is unlikely to hold.

Leverage Ratios: Although debt is unambiguously increasing in τ, the right plots in Figure 3 show that this is not true for debt-to-value ratios. The implication of this simple point—that value is also endogenous—is more wide-reaching than just our model. Almost every capital-structure theory has been formulated in terms of debt, while almost every reduced-form empirical capital-structure test has been operationalized in terms of debt-to-value ratios. But with values also always endogenous, debt-to-value ratios measure primarily the relative speed of the change of debt vis-a-vis the speed of change of value. Thus, empirical test coefficients in naive leverage-ratio regressions may not be translatable into support or rejection of underlying theories.

Implication 2 *Because firm value is also endogenous, directional theory implications on debt levels need not be the same as directional theory implications on debt-value ratios.*

[Insert Figure 4 here: **Peer Effects on Debt Choice**]

Peer Effects on Debt Choice: An important aspect of our model is that each firm's debt choice is influenced by its peers via the endogenously determined price of liquidated assets. Recall that the optimal (interior) debt choice is

$$D_L^*(P) = \frac{\tau + (1-\eta)\cdot P}{\eta} \ ,$$

which is increasing in the price P. The intuition is that future vulture buying opportunities are more attractive when the anticipated asset price is low, so firms have more incentives to reduce debt in order to be more likely to have the financing available to make asset acquisitions. When their peer firms take on more debt, the aggregate demand for the asset declines. The resulting lower equilibrium asset price gives other firms the incentives to reduce debt. This is illustrated in Figure 4, which plots the equilibrium price and debt choices as a function of the benefits of debt, τ. (In this example, $\eta = 1/2$.) For high values of τ, a fraction of firms choose a high-debt strategy ($D_H = 1$), resulting in higher industry debt and a lower asset price than would have obtained if all firms had chosen the low-debt strategy (represented by the dashed-lines). Consequently, firms choosing the low-debt strategy—recognizing that more valuable future buying opportunities will become available—shade their leverage below what would have been optimal if industry debt had been lower.

Implication 3 *Holding parameters constant, with endogenous liquidation values, firms' equilibrium debt choices are negatively influenced by those of their peers.*

II The Distress-Reorganization Channel

The main cost of debt in standard trade-off models like Williamson (1988) and Harris and Raviv (1990) is not debt's constraint on future asset purchases, but its financial-distress cost. Firms that have taken on too much debt will suffer not because they can no longer *buy* when there are fire sales, but because they will have to *sell* when they are in trouble.[7] We now extend our model to show how the two channels work in tandem: the opportunistic-acquisition channel means that debt reduces the demand for liquidated assets, while the financial-distress channel means that debt increases the supply of liquidated assets. Each channel plays the dominant role in some parameter region. Moreover, adding the financial-distress channel makes the model more realistic and adds a wealth of implications.

A Setup and Assumptions

The model is similar to the one from the previous section with the following changes:

Impairment: We now assume that there is a dissipative cost from reorganization and continuation in the event of default ($v_i < D_i$). This cost is linear in the shortfall, $\phi \cdot (D_i - v_i)$. The parameter ϕ represents the ϕ
extent of losses to a firm's value associated with being unable to meet its debt obligations. This cost could be due to, e.g., direct distractions; damaged relationships with key stakeholders (suppliers, employees, and customers) when the firm is in distress (Titman (1984));[8] or the residual effects of creditor-manager conflicts (after mitigation by negotiations and side-payments). Our reduced-form specification has the realistic feature that distress costs are lower when the firm is closer to being able to meet its debt obligations. We always assume limited liability, so firm value under continuation is $\max\{0, v_i - \phi \cdot (D_i - v_i)\}$.

Liquidation: At time 1, the manager must decide not only whether to purchase liquidated assets at price P, as in the previous section, but also whether to continue in the event of financial distress or liquidate. Financial distress arises when the firm value is below the face value of debt. The firm value is P if it liquidates, and $\max\{0, v_i - \phi \cdot (D_i - v_i)\}$ if it continues. For lower firm values v_i, liquidation is better; for higher values, impaired operations is better. The firm liquidates for all values v_i below a critical value Λ, Λ

$$\Lambda(D_i) \equiv \frac{P + \phi \cdot D_i}{1 + \phi} . \tag{4}$$

A priori, firms expect to liquidate assets more often when the expected liquidation price (P) is higher and when the relative value from continuing operations in distress is lower (i.e., when debt, D_i, is higher or when the distress impairment, ϕ, is worse). However, ϕ also has an influence on the equilibrium price, so its net effect is yet to be determined.

[7] In debt overhang models, a part of the financial distress cost may arise from the inability to acquire assets.

[8] For example, Opler and Titman (1994) shows that distressed firms lose market share relative to their conservatively financed peers in industry downturns.

Acquisition: As in Section I, the decision whether or not to buy the liquidated asset depends on the transferability of the asset η, the price P, and the firm's capital availability. The asset value to firm i is $\eta \cdot v_i$, so it is positive NPV to acquire the asset iff $v_i > P/\eta$. However, firm i only has sufficient capital to acquire the asset iff $v_i - D_i \geq P$.

[Insert Figure 5 here: **Game Tree for the Full Model ($D > P$)**]

Timing: Figure 5 illustrates the revised model.

Objective: At time 0, the firm chooses its debt, again taking the expected (and fully anticipated) time 1 price P of liquidating assets as given; and anticipating its own optimal time 1 decisions (a) whether to liquidate or continue operating, and (b) whether to purchase or not purchase other firms' liquidating assets. Therefore, the ex-ante (time 0) value of each firm is

$$V(D_i) = \tau \cdot D_i + \begin{cases} \int_0^P P\,dv + \int_P^1 v\,dv + \int_{\min\{P+D_i,1\}}^1 \max\{0, \eta \cdot v - P\}\,dv & \text{if } D_i < P \\ \int_0^{\Lambda(D_i)} P\,dv + \int_{\Lambda(D_i)}^{D_i} [v - \phi \cdot (D_i - v)]\,dv + \int_{D_i}^1 v\,dv + \int_{\min\{P+D_i,1\}}^1 \max\{0, \eta \cdot v - P\}\,dv & \text{if } D_i \geq P \end{cases} \tag{5}$$

If the firm takes on less debt than what the asset will be worth, the first row applies and we are back to the case of the previous model. Each firm would know it would operate without possible impairment by distress. If the firm takes on more debt, the second row applies and there are now five terms in the (always-continuous) value objective. The first term is the τ benefit of debt, which accrues immediately.[9] The second term reflects the payoff, P, if the firm is eventually liquidated ($v_i \leq \Lambda(D_i)$), where $\Lambda(D_i)$ is given by equation (4). The third term represents the payoff to the firm if it is distressed but chooses to continue ($v_i \in [\Lambda(D_i), D_i]$), in which case it receives v_i less the dissipative costs of continuation $\phi \cdot (D_i - v_i)$. The fourth term is the value of the firm if it is not distressed ($v_i \in [D_i, 1]$) and continues unimpaired. The fifth term represents the expected surplus if the firm acquires liquidated assets. The limits of integration recognize that the firm only has sufficient capital to acquire the asset if $v_i \geq P + D_i$, and the integrand ($\max\{0, \eta \cdot v_i - P\}$) recognizes that the firm only acquires the assets if its NPV is positive given its own type ($v_i > P/\eta$).

[9] In this formulation, the debt benefits cannot be used to stave off liquidation or impairment or to finance the purchase of the asset. However, as already noted above, Appendix D shows that a model in which firms can do so is isomorphic to the current one. All our principal conclusions continue to hold.

Market Clearing: The equilibrium price for liquidated assets is determined by supply and demand:

Supply: As explained above, firms choosing $D_i \leq P$ will liquidate when their realized productivity $v_i \leq P$. Firms choosing $D_i > P$ will liquidate when their realized productivity $v_i \leq \Lambda(D_i)$, as described in equation (4). Therefore, the aggregate supply of liquidated assets is

$$\int_0^P \int_0^P 1 \; dv \, dF(D) \; + \; \int_P^1 \int_0^{\Lambda(D)} 1 \; dv \, dF(D) \; . \tag{6}$$

Demand: Acquiring one unit of the liquidated asset is positive NPV iff $v_i > P/\eta$. Moreover, firms will have sufficient funding to do so iff $v_i \geq P + D_i$. Therefore, the aggregate demand is

$$\int_0^{1-P} \int_{\max\{P+D, P/\eta\}}^1 1 \; dv \, dF(D) \; . \tag{7}$$

B Access to Infinite Financing / Eliminating The Acquisition Channel

Before solving the model, it is useful to consider a benchmark in which firms in the industry have infinite access to capital. In this case, the acquisition channel is no longer a constraint. Competition among firms results in an equilibrium with $P^* = \eta$, in which (only) the highest-productivity firms ($v_i = 1$) can purchase *all* assets available for sale. At this high a price, purchasing assets is zero NPV even for the highest-productivity firms and negative NPV for all other firms. Therefore, the acquisition profit terms in both rows in (5) drop out. In the first row ($D_i < P$), there is also no disadvantage to raising debt, so firms would always be better off increasing debt and leaving this region. This leaves only the second row for consideration. Moreover, it can be shown that there is only one firm type in this equilibrium, so we can omit the subscript on D.

Substituting $\Lambda(D) = (P + \phi \cdot D)/(1 + \phi)$ into the objective and taking the derivative with respect to D yields the first-order condition for the (interior) optimal debt choice,

$$D^* = P^* + (1 + 1/\phi) \cdot \tau = \eta + (1 + 1/\phi) \cdot \tau \; .$$

The optimal debt choice is increasing in the benefits of debt (τ) and asset redeployability (η), and decreasing in the costs of financial distress (ϕ). This is the standard result in earlier literature. In particular, debt increases in asset redeployability, because more redeployable assets have higher liquidation values ($P^* = \eta$), thereby reducing distress costs. There is no countervailing cost of debt with unlimited capital—increasing debt never precludes firms from acquiring valuable (high η) assets.[10]

[10] Substituting D^* into Λ yields the equilibrium liquidation threshold $\Lambda(D^*) = P + \tau = \eta + \tau$. The optimized firm value is

$$V^* \equiv V(D^*) = \frac{1 + (\eta + \tau)^2 + \tau^2/\phi}{2} \; .$$

C Equilibrium With Both An Acquisition Channel and A Financial Distress Channel

The trade-offs associated with the firm's debt choice in our general model with both the opportunistic-acquisition channel and the distress-reorganization channel depend again on the level of debt D_i vis-a-vis the predictable asset price P. For example, consider the case where $\eta \geq 1/2$. (All cases are derived in the Appendix.) Each firm takes P as given and considers its possible debt choice in one of four distinct regions:

1. In the first region, $D_i \leq P/\eta - P$, the marginal cost of debt is zero. Firm value is described by the first row of equation (5): increasing debt does not increase reorganization costs (because the firm will always liquidate in distress) and the firm does not forego asset-acquisition opportunities (because the financing constraint is not binding).

2. In the second region, $P/\eta - P < D_i \leq P$, the marginal cost of debt is $\eta \cdot D_i - P \cdot (1-\eta)$. Firm value is still described by the first row of equation (5): increasing debt still does not increase reorganization costs, but the firm now may forego some positive NPV asset-acquisition opportunities.

3. In the third region, $P < D_i < 1-P$, the marginal cost of debt is $\eta \cdot D_i - P \cdot (1-\eta) + (D_i - P) \cdot \phi/(1+\phi)$. Firm value is now described by the second row of equation (5): increasing debt raises the expected reorganization costs *and* results in the firm foregoing some positive NPV buying opportunities.

4. In the fourth region, $1-P \leq D_i \leq 1$, the marginal cost of debt is $(D_i - P) \cdot \phi/(1+\phi)$. Firm value is again described by the second row of equation (5): the firm's debt is now so high that it would *never* be able to buy assets even if it turned out to be the highest productivity type, $v_i = 1$. Therefore, the only remaining marginal cost of debt is the increase in expected reorganization costs.

Importantly, the marginal cost of debt is weakly increasing in D_i over the first three regions, but then jumps down at $D_i = 1-P$ (because the financing constraint is no longer binding), after which it increases again. Consequently, as in our model without reorganization costs, there is again a region with a two-type equilibrium, in which some firms choose low debt and others choose high debt.

Equilibrium requires again that firms make optimal decisions at time 1 (both continuation and asset acquisition); their debt choices at time 0 maximize firm value (5), given the anticipated asset price and optimal decisions at time 1; and the market for liquidated assets clears, i.e., supply in equation (6) is equal to demand in equation (7).

The description of the equilibrium solution for all parameters is very detailed and depends on different parameter regions for the reasons just described. Therefore, for the sake of the exposition, in the following theorem we describe equilibria for a particularly relevant parameter region—when ϕ is modest and η is large, for all values of τ—and leave the full description and proof of the theorem for all parameters for the appendix.

Theorem 2 *Assume (i) $\eta \geq 2/3$ and (ii) $\phi < (3\eta - 2)/(6 - 3\eta)$ and let*

$$\tau_1 = \frac{2(\phi+1)\eta^2 - \phi + \eta \cdot (\phi+1) - \sqrt{(\eta+1)^2 \cdot (\phi+1) \cdot (\eta^2\phi + \eta^2 - 2\phi - \eta\phi)}}{3\eta(\phi+1) + 3\phi + 2},$$

$$\tau_2 = \frac{(2\eta-1) \cdot (2\eta + \phi + 2\eta\phi) - \sqrt{(2\eta-1)^2 \cdot (1+\phi) \cdot [4\eta^2(1+\phi) + \eta\phi - 2(\eta+\phi)]}}{2+3\phi},$$

$$\tau_3 = \frac{2\eta^2(1-\phi^2) + \eta(1 + 9\phi + \phi^2 - 5\phi^3) + 2\phi + 12\phi^2 + 7\phi^3}{2 + 14\phi + 20\phi^2 + 9\phi^3 + 3\eta \cdot (1-\phi) \cdot (1+\phi)^2}$$
$$+ \frac{\sqrt{(1+\phi) \cdot (1 + \eta + 5\phi - \eta\phi)^2 \cdot [\eta^2(1+\phi)(1+3\phi^2) - 3\eta\phi^2(2+\phi) - 2\phi^3]}}{2 + 14\phi + 20\phi^2 + 9\phi^3 + 3\eta \cdot (1-\phi) \cdot (1+\phi)^2},$$

$$\tau_4 = \frac{\eta + \phi + \eta\phi}{1 + 2\phi}.$$

The following is a complete characterization of the equilibrium:

- *If $0 \leq \tau \leq \tau_1$, there exists a unique one-type equilibrium with price*

$$P^* = \frac{\eta - \tau}{1 + \eta},$$

  in which all firms choose

$$D^* = \frac{1 - \eta + 2\tau}{1 + \eta}.$$

- *If $\tau_1 < \tau \leq \tau_2$, there exists a unique two-type equilibrium with price*

$$P^* = \frac{\phi\eta - (1+\phi) \cdot [\tau - \eta(1+\tau)]}{1 + \phi(1+\eta)}$$
$$- \frac{\sqrt{\eta(\phi+1) \cdot \{2\tau \cdot [\eta\phi + (\eta - \tau) \cdot (1+\phi)] + \eta\tau^2(\phi+1) - \phi \cdot (1 + \tau - \eta)^2\}}}{1 + \phi(1+\eta)},$$

  in which fraction h^* of firms choose $D_H^* = 1$, and fraction $1 - h^*$ choose D_L^*, where

$$D_L^* = \frac{\tau}{\eta} + \frac{(1-\eta)}{\eta} \cdot P^*,$$

$$h^* = \frac{(1+\phi) \cdot [\eta - \tau - (1+\eta) \cdot P^*]}{\eta\phi + (1+\phi) \cdot (\eta - \tau) - [1 + \phi(1+\eta)] \cdot P^*}.$$

- *If $\tau_2 < \tau \le \tau_3$, there exists a unique two-type equilibrium with price*

$$P^* = \frac{\phi \cdot [1+2\phi(1-\tau)-3\tau] + \eta(1+\phi)\cdot[1+\tau+(2+\tau)\phi]-\tau}{1+(6-3\eta)\cdot(1+\phi)\cdot\phi}$$
$$- \frac{\sqrt{(1+\phi)\cdot(\eta+\phi+\eta\phi)\cdot\left\{\begin{matrix} 3\eta^2\phi(1+\phi)-2[\phi(\tau-1)+\tau]^2 \\ +\eta[\phi(\tau-1)+\tau]\cdot[2+(\tau-1)\phi+\tau]\end{matrix}\right\}}}{1+(6-3\eta)\cdot(1+\phi)\phi},$$

in which h^* firms choose $D_H^* = 1$, and $1-h^*$ choose D_L^*, where

$$\begin{aligned} D_L^* &= \frac{(1+\phi)\cdot\tau}{\eta+\phi+\eta\phi} + \frac{(1-\eta)\cdot(1+\phi)+\phi}{\eta+\phi+\eta\phi}\cdot P^* \\ h^* &= \frac{(1+\phi)\cdot[\eta+\phi+\eta\phi-(1+2\phi)\tau-(1+\eta+5\phi-\eta\phi)\cdot P^*]}{(1+2\phi)\cdot[\eta+\phi+\eta\phi-(1+\phi)\tau]-(1+5\phi-\eta\phi+5\phi^2-\eta\phi^2)\cdot P^*}. \end{aligned}$$

- *If $\tau_3 < \tau \le \tau_4$, there exists a unique one-type equilibrium with price*

$$P^* = \frac{\eta+\phi+\eta\phi-\tau\cdot(1+2\phi)}{1+\eta+5\phi-\eta\phi},$$

in which all firms choose

$$D^* = \frac{1+2\phi+\tau+(\tau-\eta)\cdot(1+\phi)}{1+\eta+5\phi-\eta\phi}.$$

- *If $\tau_4 < \tau \le 1$, there exists a unique one-type equilibrium with price $P^* = 0$, in which all firms choose*

$$D^* = \min\left\{ 1\ ,\ \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi} \right\}.$$

The debt D^* in the theorem is the face value at time 1. Because the expected return on debt is zero in our model, the value of debt at time 0 is

$$E(D^*) = \begin{cases} D^* & \text{if } D^* < P^* \\ D^* - (1+\phi)\cdot(D^2-\Lambda^2)/2 & \text{otherwise}. \end{cases}$$

For a low face value of debt, there is no possibility of default. For a high face value of debt, the expected payout to creditors is equal to the promised payoff, D_i^*, less the expected loss to creditors.

D Implications

Unlike our model from the previous section, debt is now costly for two reasons: first, it reduces future purchasing opportunities; *and* second, it increases the expected costs of financial distress. The model still has only three parameters—the redeployability of assets (η), which is central to our acquisition channel; the impairment parameter (ϕ), which is central to our financial distress channel; and a compensating direct benefit of debt (τ). Yet, the model can offer many implications. Of course, it remains too stylized to consider its implications to be either quantitative or universal. Instead, our model should be viewed as suggestive of economic forces in contexts in which both the financial-distress and the opportunistic-acquisition channels are important for firms that can become either sellers or buyers of distressed assets in the future.

This subsection discusses the model's comparative statics. They are summarized in Table 2 and illustrated in the graphs that follow. The graphical approach is more intuitive, although the model's implications are also algebraically demonstrable using the closed-form solutions in Theorem 2.

[Insert Figure 6 here: **Debt-Related Comparative Statics for** $\phi = 0.25$]

D.1 Heterogeneity and Firm Value

Figure 6 shows the proportion of firms choosing maximum debt, firm values, and leverage in the case in which $\phi = 0.25$. This parameter means that financial distress would consume one quarter of each dollar's *shortfall*. This seems high for large firms, although it is not unreasonable for midsize and smaller firms (Bris, Welch, and Zhu (2006)).

Type Heterogeneity (h): As in the model of the previous section, heterogeneity in ex-ante leverage strategies can arise endogenously, because our assets are indivisible. The top row in Figure 6 shows the now parabolic convex region that separate the set of homogeneous (one-type) from the set of heterogeneous (two-type) equilibria. These two-type equilibria occur again when otherwise equal firms infer that the corner solution, with maximum permitted debt of $D_H = 1$, is as good for them as the best interior debt choice (D_L^*). Not surprisingly, two-type equilibria can only occur in regions in which firms want to choose fairly high debt to begin with.

Comparing the $\phi = 0$ Figure 2 with this $\phi = 0.25$ Figure 6 shows that distress costs ϕ shrink the heterogeneous region. For sufficiently low values of either debt benefits τ or redeployability η, there are now only homogeneous equilibria. Nevertheless, the set of two-type equilibria remains non-trivially large. In detail:

- When the debt benefits τ are low, all firms choose low debt because the benefits of a high-debt strategy are too small to compensate for the foregone investment opportunities. Similarly, when the redeployability η is low, liquidation values are low and again all firms choose low debt because a high-debt strategy results in excessive distress costs

- At some point, with high enough redeployability and debt benefits, some firms can begin to specialize in waiting for acquisition opportunities. Heterogeneous equilibria appear only for intermediate values of τ and high values of η. Thus, the heterogeneous region becomes smaller than it was in Figure 2.
- Finally, when the debt benefits become overwhelming, all firms end up choosing high debt and no firm finds it worth waiting for opportunities, even though such firms expect large distress costs.

Firm Value (V): Firm value always increases monotonically in the benefit of debt τ and decreases monotonically in distress costs ϕ. Value usually increases in redeployability η, too; but there is a very tiny parameter region (with high redeployability, low distress costs, and high benefits)—too small to be even visible in this graph—in which firm value can decrease.

D.2 Leverage

We are now ready to proceed to the focus of our paper, corporate leverage, when there are both the traditional financial-distress channel and the novel opportunistic acquisition channel.

[Insert Figure 6 here: **Debt-Related Comparative Statics for $\phi = 0.25$**]

The middle and lower plots on the left in Figure 6 show that debt can first increase and then decrease in redeployability η (for low debt benefits τ). For these very low debt benefit values, the financial-distress channel dominates when redeployability is low. At first, when redeployability increases, firms take on more debt. It makes little sense for such firms to speculate on purchasing assets—the assets are simply not valuable enough. Eventually, when redeployability increases further, the potential to buy assets becomes more lucrative, the asset-acquisition channel begins to dominate, and firms again take on less debt. Finally, for higher debt benefits, only the asset-acquisition channel matters again. It dominates for all redeployability parameters η. Firms always find it more important to keep leverage low because of the opportunity to pounce on future opportunities.

Implication 4 *For low debt benefits τ and low asset redeployability η, the financial-distress channel dominates. Firms take on more debt when assets become more redeployable.*

For higher debt benefits τ and higher asset redeployability η, the opportunistic-acquisition channel dominates. Firms take on less debt when assets become more redeployable.

Thus, there are now two reasons why empirical debt-ratios (the middle and lower plots on the right in Figure 6) may not increase in redeployability. The first is the aforementioned endogenous-value effect. The second is the acquisition channel. A simple linear regression explaining leverage ratios with redeployability proxies is not a powerful test. Instead, a better test would posit a U-shape—first an increasing and then a decreasing effect. When redeployability is low, a small increase in redeployability induces firms to fear distress less and they increase leverage. When redeployability is high, a small increase in redeployability induces firms to hold out for better acquisition opportunities and they decrease leverage.

Finally, and as expected, the face value of debt increases with direct benefits τ and decreases in financial distress costs. However, and less expected, this does not translate into unambiguous debt-to-value ratios. Again, the main (but not the only) culprit is the simultaneous endogenous value change.

D.3 Ancillary Implications

[Insert Figure 7 here: **Ancillary Comparative Statics for** $\phi = 0.25$]

Our model can also offer implications on other measures that were not its focus. This section provides a sampling.[11]

Credit Spreads: For what follows, we continue to assume that the benefits of debt (τ) accrue to shareholders. Recall that we normalized the expected rate of return on debt to be zero. The promised debt payment is D_i^*, the expected debt payment is $E(D_i^*)$. Creditors are indifferent between providing funding and not providing funding if the credit spread is

$$r = \frac{D_i^*}{E(D_i^*)} - 1 \ . \tag{8}$$

The top left plot in Figure 7 shows that credit spreads increase when debt benefit are higher. Higher τ encourages firms to take on more debt, which increases the expected loss to creditors. Higher η (redeployability) leads to higher recovery rates and (all else equal) lower credit spreads, but firms may optimally choose higher debt levels which increases the likelihood of default. Although the former effect almost always dominates, resulting in lower spreads when redeployability is greater, there is a very small parameter region in which the credit spread increases when redeployability is greater. Finally (not plotted), just as in Leland (1994), credit spreads may increase or decrease in reorganization costs. Higher ϕ lead to lower recovery rates in the event of default, but also cause firms to choose lower levels of debt, which reduces the likelihood of default.[12]

[11] We could also offer further implications on other outcomes (such as on the average values and discounts of assets in production and transfer) that would be more difficult to measure empirically.

[12] Our qualitative comparative statics results for credit spreads hold even when we allow creditors to have access to the immediate debt benefits (see Appendix D). Of course, quantitative predictions about credit spreads will depend on whether

Asset Liquidation Price (P): All three price-related comparative statics are unambiguous (though they can be quite flat): asset prices increase in redeployability and reorganization costs, and decrease in debt benefits. We already discussed earlier in the context of our model without reorganization costs why the asset price increases with redeployability and decreases with debt benefits. Higher reorganization costs have two competing effects on price: on one hand, they result in greater supply of the asset, because liquidation becomes relatively more desirable than continuing operations in financial distress. On the other hand, they result in greater demand for the asset, because firms take on less debt and therefore have more access to financing. Though not necessarily universal, in our specific model, the latter effect always dominates.

Asset Sales (Q): Asset sales always increase in redeployability and decrease in reorganization costs, but are ambiguous in debt benefits. The dominant effect of greater redeployability is to make the asset more valuable to a potential buyer, resulting in greater demand and higher asset sales. Higher reorganization costs make asset sales more appealing relative to the direct alternative of reorganization. Higher debt benefits increase firm debt. This results both in less demand for the asset (because of tighter financing constraints), and in greater asset supply (because of more firms in trouble). The net effect is ambiguous.

Distress Reorganization Observables: The model also offers secondary predictions for two quantities related to distressed reorganization:

- The liquidation frequency for the low type conditional on being in financial distress is Λ^*/D^* if $D_i \geq P$. If $D_i < P$, firms will always liquidate and never continue. The plot shows that firms liquidate more often in distress when assets are more redeployable. The dominant effect here is that more redeployable assets have higher liquidation values which makes liquidation more desirable. Higher distress costs reduce continuation values *holding debt fixed*, but higher distress costs also result in lower optimal debt which increases continuation values. The first effect dominates and the conditional liquidation frequency increases in ϕ. Increasing the benefits of debt τ leads to higher debt levels and declining liquidation values. This makes liquidation less attractive and therefore less frequent.

- The expected losses associated with reorganizing the firm, $E[\phi \cdot (D^* - \nu)]$—possibly at least a partial transfer to and thus a partial proxy of the size for the legal reorganization industry—increase in τ, decrease in η, and are ambiguous in ϕ. The dominant effect of increasing τ is to increase debt which increases the likelihood of distress and the dissipative cost of continuing in distress. The dominant effect of increasing η is to increase liquidation values which makes impaired continuation less likely and reduces expected reorganization costs. The effect of ϕ is ambiguous because it increases reorganization costs *holding debt fixed* but reduces optimal debt.

creditors have access to the immediate debt benefits—which they may in the real world. It is possible to change the model to entertain different assumptions on the disposition of these benefits.

III Discussion and Literature Context

A Welfare

Our paper has largely deemphasized welfare implications and government policy prescriptions, because we view the model as too stylized to offer policy prescriptions. Our model assumes production, reallocation, incentive and tax[13] effects as parameters; and we are simply not confident enough to take a stance to what extent these aspects are dissipative or redistributive to some parties elsewhere in the economy.

[Insert Figure 8 here: **Allocational Efficiency**]

We are however comfortable to discuss briefly one part of the overall social welfare within the context of our model. This can help to clarify one conceptual aspect of the trade-offs that government should be aware of. How do corporate income taxes—one component of τ—influence re-allocational efficiency?

Figure 8 shows that the answer is ambiguous:

Implication 5 *Increases in the benefits of debt—as can be effectuated by tax code changes—can result in socially less or more efficient redeployment activity.*

This is because there is typically an intermediate level of debt, in which asset transfer activity is socially ideal.[14] Tax policy can then push firms toward or away from this ideal. This is easiest to understand in the context of the total direct debt benefits:

- For low τ, firms choose low leverage, resulting in high demand for liquidated assets. If reorganization costs are high—which makes liquidation more likely in financial distress—this can also result in high supply of assets, and the economy can have too many asset transfers relative to the efficient level. Increasing the tax advantage of debt then pushes firms towards more debt, which helps because it will reduce the expected transfer activity.

[13] To the extent that some of the firm's debt benefits come through the tax shelter (though there are others!), there is a related conceptual puzzle. If, as is widely acknowledged, debt has a potentially negative effect on the stability of firms individually and system-wide, why would the government want to impose them differentially on equity and not on debt? A government could impose taxes on projects instead—for example, in Germany, home ownership is subsidized not through interest deductibility, but through non-recaptured depreciation. We can speculate that default forces more reallocation of resources from less productive to more productive uses; and by increasing the value of debt, the government can calibrate both the equilibrium reallocation frequency and reallocation state dependence. However, debt is a fairly blunt instrument, used by governments that are themselves not great experts about when reallocation is better or worse. The mutual industry-peer externalities discussed in our paper further suggest that it could be a dangerous instrument—if it forces only a few firms to sell, prices are reasonably appropriate, but at some point, feedback effects can reallocate assets less towards the highest-value user of assets and more towards the least-levered users of assets.

[14] Assets are identical and it is always the lowest-use owners who transfer assets to the highest-use owners. Thus, the total quantity transferred is the only metric of relevance.

- For high τ, firms choose high leverage, resulting in low demand for liquidated assets. The economy has too few asset transfers relative to the efficient level. Increasing the tax advantage of debt further would only push firms towards even more debt and thereby worsen the reallocation.[15]

A reasonable interpretation is that government tax policy towards debt should moderate other debt benefits.

For comparison, in Gale and Gottardi (2015), the only other paper with endogenous prices, the thought experiment about the social cost of debt as a tax shelter is different. In their model, in the absence of a corporate debt response (to undo taxes), such taxes would *always* reduce socially beneficial productive operations. Debt, by undoing taxes, tends to increase productive activity and can thereby improve social welfare. Taking the leverage responses of firms into account, the net effect of an increase in taxes on production and thus welfare could be positive or negative. Interestingly, Gale-Gottardi consider a novel mechanism—forcing firms to take on more debt as one policy tool. This can in turn induce firms to increase investment voluntarily.

B Generalizations of the Model

The most important takeaways of our model are that

1. firms' leverage choices are affected by their peers via the equilibrium price of liquidated assets;
2. indivisibility of assets may result in heterogeneity in leverage strategies;
3. leverage level effects are not isomorphic to leverage-ratio effects;
4. the acquisition channel means that increased asset redeployability can also have a negative effect on leverage, especially when these are high to begin with;
5. and tax policy can have ambiguous effects on re-allocational efficiency.

To illustrate them, our model had to employ a set of assumptions for tractability, such as the uniform distribution on values; linearity in eta, phi, and tau; stark integration limits; limited liability and free disposal; limited capital; uncorrelated shocks; no further countervailing important omitted effects (e.g., due to agency or inside information), and so on. None of our takeaways lean especially heavy on specificity in these assumptions, and we would expect the key insights to survive in models in which they are reasonably relaxed. In particular:

- Outside Buyers: Our model is sensitive to the assumption that buying is limited to firms inside the industry. Our qualitative results would continue to hold if there is limited demand from outside the

[15] Similar to this point, when redeployability is low and reorganization costs are high, firms would choose high leverage. This is because maintaining financial flexibility is less valuable when it is unlikely that there will be good buying opportunities later. Again, transfer activity would be too low from a social perspective, and increasing the tax advantage of debt would only hurt more. (The opposite is the case when redeployability is high and reorganization costs are low.)

industry—this would increase liquidation values and mitigate, but not eliminate, the incentive to choose lower debt to take advantage of buying opportunities. It would also have a similar effect as an increase in redeployability, η. But if assets are just as valuable outside the industry and potential buyers have practically unlimited capital, then our acquisition channel vanishes, as discussed in Section II.B. More commonly, neither zero nor infinite capital availability inside the industry, and neither perfect nor useless redeployability outside the industry is likely to be a realistic description; and these forces can help push eta towards higher levels.

- Correlated shocks: It could be that all assets in an industry are simultaneously affected by a recession, or that (e.g., consumer taste) shocks help some firms at the same time they hurt others. For example, if shocks are positively correlated, fire sales will be deeper in bad times (more sellers and fewer buyers) and shallower in good times (fewer sellers and more buyers). This may create an incentive to take on less debt initially to take advantage of the great investment opportunities available in bad times, above and beyond the incentive to avoid financial distress oneself.

- Agency Conflicts: When managers (and equity) have stronger incentives not to declare bankruptcy and even weaker incentives to liquidate (and if creditors cannot renegotiate managers out of collectively inefficient choices, as in Benmelech and Bergman (2008)), then firms would likely be less inclined to liquidate at the same time, given the same amount of debt. However, this would not necessarily be the outcome. In turn, this could have equilibrium repercussions for the optimal level of debt and/or various restrictions written into debt that can enhance the incentives of firms to liquidate. The outcome would likely depend on how extra debt calibrates the relative incentives.

C Related Literature

Our model was built around the fundamental tradeoff between taxes and financial-distress costs, first raised in Robichek and Myers (1966). As this encompasses most of the modern theory of corporate capital structure, we can only highlight some work especially close to the assumptions and results of our own paper. Harris and Raviv (1990), Leland (1994), Leland and Toft (1996), Gryglewicz (2011), and many others, have provided the theoretical formalizations to help understand firm tradeoffs and behavior. Industry debt choices have been proposed by Maksimovic and Zechner (1991), Fries, Miller, and Perraudin (1997), Miao (2005), and others.[16]

The costs of financial distress were further dissected into components, such as debt overhang (Myers (1977)), the damaged relationships with key stakeholders (Titman (1984)), or reduced market share (Opler and Titman (1994)). Like Jensen and Meckling (1976) had for agency issues, the seminal Shleifer and Vishny (1992) paper laid out the research agenda for corporate behavior when asset prices could be

[16]Asset specificity plays a role in Marquez and Yavuz (2013), though assets have exogenous prices. More specific assets can increase productivity (reducing debt) and increase continuation values (increasing debt).

depressed by the simultaneous need to sell and fire sale liquidations.[17] Duffie (2010) went even further, attributing temporarily depressed prices not just to firms and industry assets, but even to financial claims in wide distribution.

Gale and Gottardi (2015) provide, to our knowledge, the only other theory in which fire-sale prices are also endogenous.[18] In their model, frictions and especially taxes lead firms to take on too few projects from a social point of view. Debt can reduce the tax burden and thereby enhance the desire of firms to take projects. An endogenous reduction in price upon resale[19] comes into play, because when many firms have taken on too much debt, the induced price reduction then works against this social advantage of debt. As remedy, they propose forcing firms to take on more debt. This induces them to undertake more projects, which in their model is socially valuable. As noted, our model has a completely different structure, parameters, and focus. It considers social welfare only in passing, because our own model assumes production, reallocation, and tax costs as parameters, and we are less confident about the dissipative/redistributive cost-benefit issues for them.

A number of empirical papers have provided evidence about the existence and nature of these fire sales. Asquith, Gertner, and Scharfstein (1994) showed that financially-distressed firms often liquidate assets at discounts to fundamental value. Pulvino (1998) showed that there are periods in which many airlines were hit by negative shocks at the same time, how this depressed airplane prices, and how financially unconstrained airlines then increased their buying activity, while constrained airlines did not. Acharya, Bharath, and Srinivasan (2007) investigated this effect more generally. Taking this yet a step further, Benmelech, Garmaise, and Moskowitz (2005) showed that firms take on more debt when assets are easier to redeploy. Similarly, Miao (2005) showed that firms with more liquid assets had, on average, higher leverage. Both interpreted their findings as support for an optimal capital theory in which assets that were more redeployable allowed industries to take on more debt.

[17] Our own paper can be viewed as a formalization of some of the more speculative insights in Shleifer and Vishny (1992). Using a specific model, we could explore the argument with more rigor and derive the five major points (and other implications) that were described above.

[18] In Gale and Gottardi (2011), leverage is not a choice that firms consider. (Projects are 100% leverage by assumption.)

[19] Assets are as productive to buyers as they were to sellers. Sales are costly to the firm, but not to the economy.

IV Conclusion

Our paper has sketched a model in which firms could anticipate and participate in industry asset sales, with more levered firms as sellers and less levered firms as buyers. This turns prices into mediators of industry leverage interactions, and ambiguates the role of asset redeployability. When redeployability is low, an increase therein induces firms to take on more debt in order to take advantage of higher fire sales prices as potential sellers. However, then redeployability is high, an increase therein induces firms to take on less debt in order to take advantage of fire sales as potential buyers.

Our paper has further highlighted the mundane but important point that theoretical implication on leverage need not translate one-to-one into empirical implications for leverage-value ratios, because both debt and value are endogenous. The behavior of leverage ratios depends on the *relative* change of debt vis-a-vis the change in firm value. This is particularly problematic for comparative statics that act positively on predicted debt.

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V Tables and Figures

Table 1: Variables

v_i	$v_i \sim U[0,1]$	Unlevered firm/asset type
Exogenous Parameters		
ϕ	$0 \leq \phi \leq 1$	Distress impairment $\phi \cdot (D - v_i)$ for firms continuing in default.
η	$0 \leq \eta \leq 1$	Asset redeployability
τ	$0 \leq \tau \leq 1$	Tax and other (net) benefits of debt
Endogenous Quantities		
D_i	$0 \leq D_i \leq 1$	Face Value of Debt for firm-type i ($\in \{L, 1\}$), promised **for time 1**.
$E(D_i)$	$0 \leq E(D_i) \leq D_i$	Value of Debt **at time 0**, as in (8)
h	$0 \leq h \leq 1$	Proportion of $D^* = 1$ types
Λ	$0 \leq \Lambda \leq 1$	Liquidation/continuation threshold, $(P + \phi \cdot D_i)/(1+\phi)$
$V(D_i, P)$	$V \geq 0$	Firm value at time 0
P	$P \geq 0$	Price of liquidated assets **at time 1**
Q	$Q \geq 0$	Assets transferred **at time 1**
r	$r \geq 0$	Credit Spread, $D_i^*/E(D_i^*) - 1$, as in (8)

Figure 1: Game Tree for the Acquisition Model

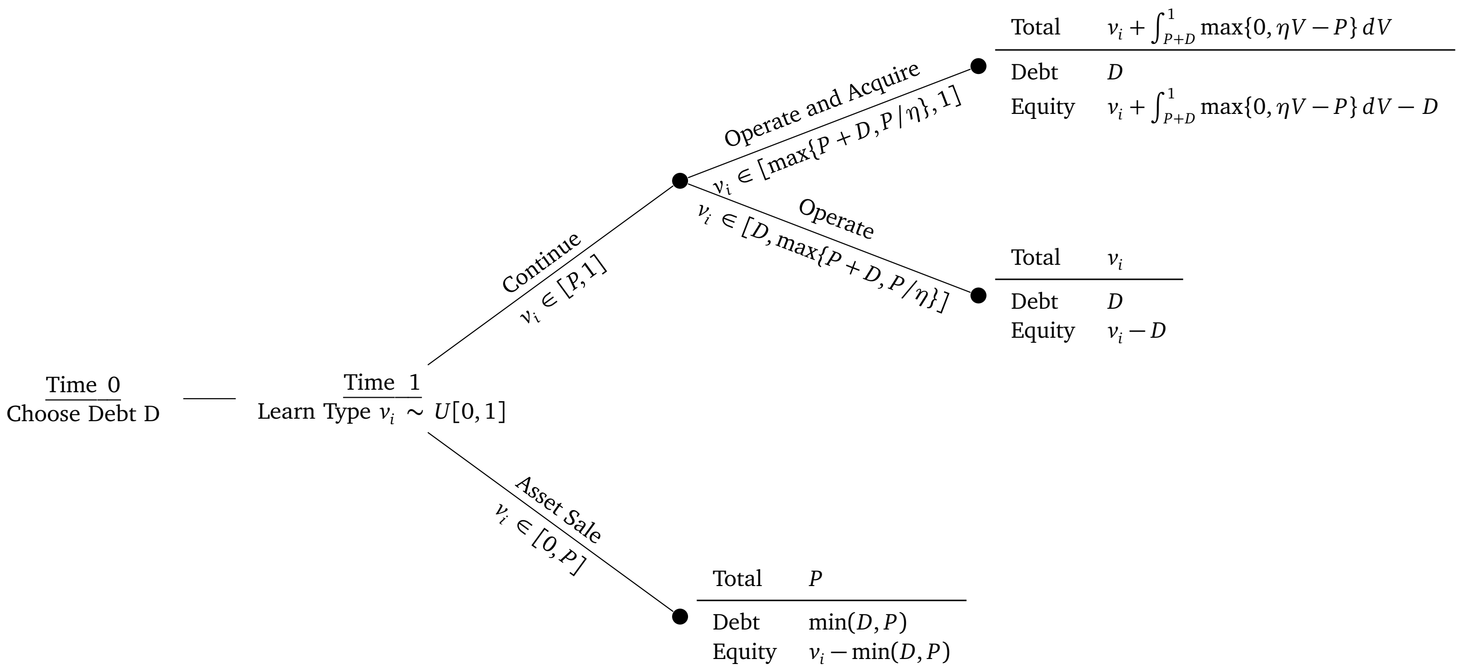
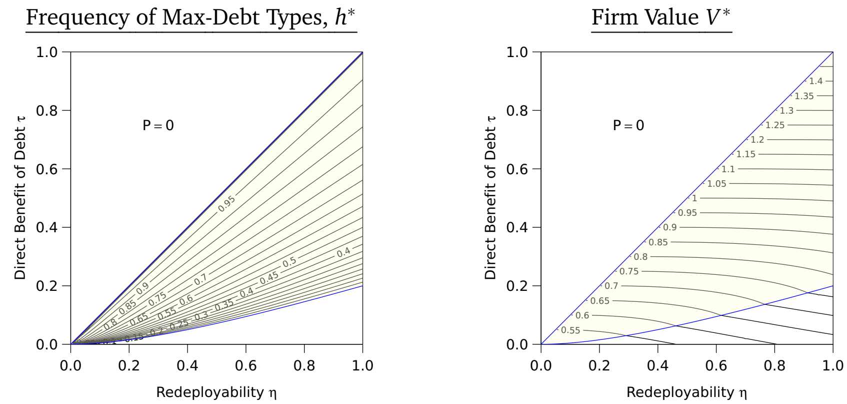


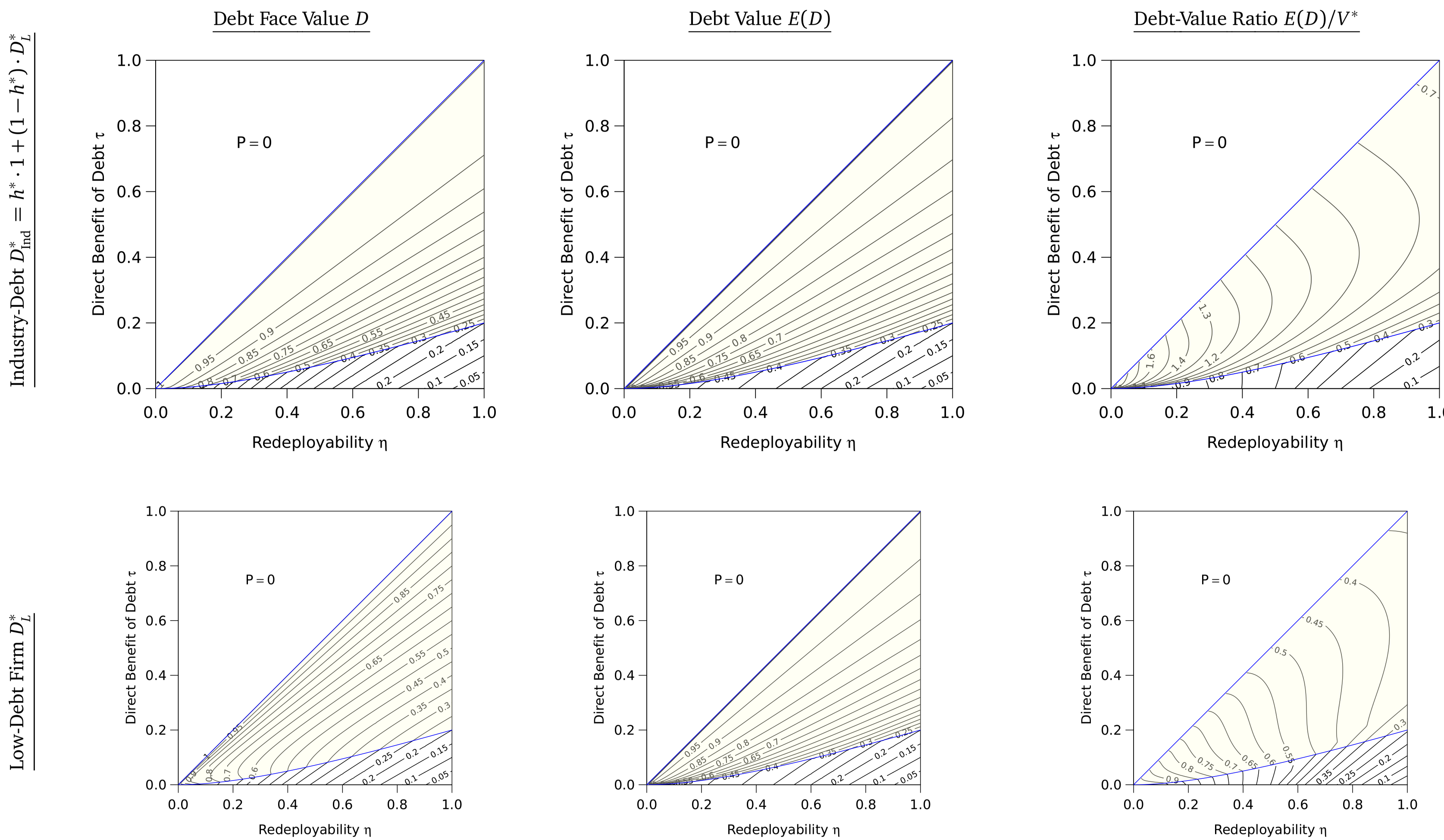
Figure 2: Comparative Statics for the Acquisition-Only Model ($\phi = 0$)



Explanation: These are contourplots. The yellow area contains the two-type equilibria (as defined in Theorem 1 on Page 7).

Interpretation: L: Heterogeneity arises for intermediate values of redeployabilities and direct debt benefits. R: Firm value increases monotonically in redeployability and direct debt benefits.

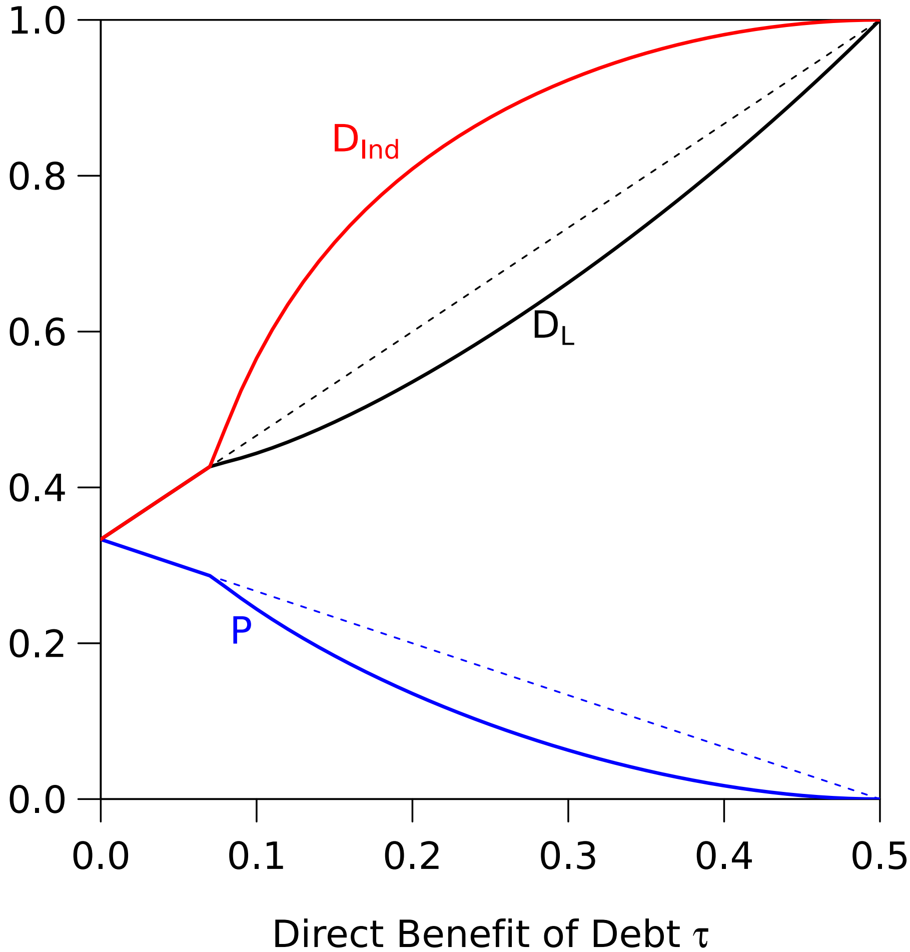
Figure 3: Comparative Statics for the Acquisition-Only Model Leverage ($\phi = 0$)



Explanation: These are contourplots. The yellow area contains the two-type equilibria (as defined in Theorem 1 on Page 7). Patterns that have "∩" or "∪" shapes indicate ambiguous comparative statics in redeployability. "⊂" or "⊃" shapes indicate ambiguous comparative statics in direct debt benefits.

Interpretation: Left and Middle: The face value D^* and current market value of debt ($E(D^*)$) increase [everywhere for the industry, almost everywhere for the low-debt firm] monotonically in debt benefits τ and decrease monotonically in redeployability η. Right: The debt-to-value ratio is ambiguous in debt benefits τ, and very ambiguous in redeployability η.

Figure 4: Peer Effects on Debt Choice



Explanation: In this figure, $\eta = 1/2$.

Interpretation: For high τ, some firms choose a high-debt strategy. Therefore, industry debt (D_{Ind}) is higher and the equilibrium price (P) is lower than what would have occurred if all firms had chosen a low-debt strategy (represented by the dashed lines). Other firms recognize that more valuable buying opportunities will become available and have an incentive to choose debt (D_L) below what is optimal if industry debt was lower.

Figure 5: Game Tree for the Full Model ($D > P$)

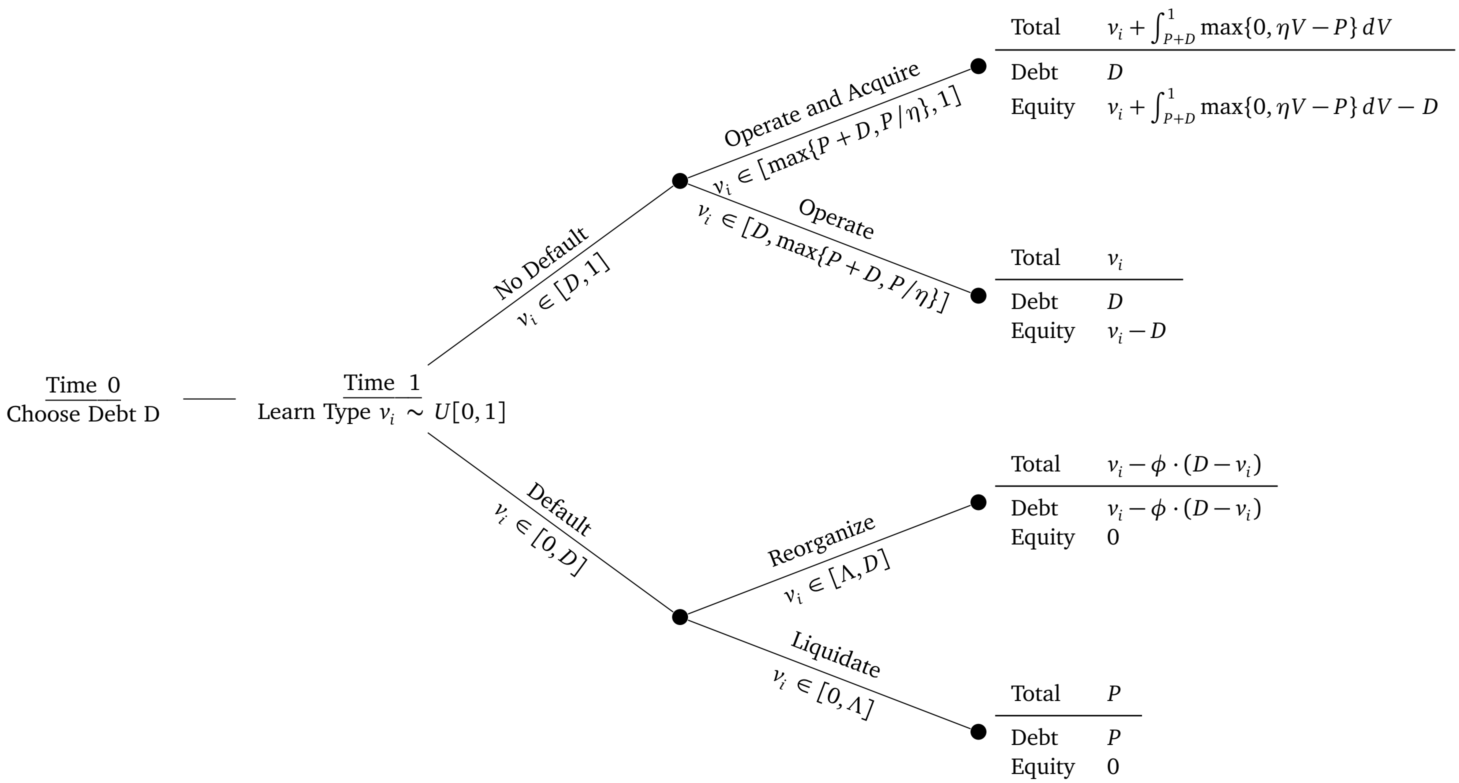


Table 2: Summary of Comparative Statics

Panel A: Key Comparative Statics on Value and Leverage

		Redeploy-ability η	Distress Cost ϕ	Direct Debt Benefits τ
Optimized Firm Value	V^*	0.9,0.2,0.9† 0.9,0.0,0.0	↓	↑
Debt Face Value, Industry	D^*_{Ind}	0.6,0.0,0.1 0.1,0.7,0.0	↓	↑
Low-Debt Firm	D^*_L		0.1,0.2,0.1 0.5,0.0,0.1†	
Debt Value, Industry	$E(D^*_{\text{Ind}})$	0.6,0.0,0.1 0.1,0.1,0.6	0.4,0.0,0.3 0.9,0.5,0.5†	0.3,0.8,0.5 0.1,0.3,0.1
Low-Debt Firm	$E(D^*_L)$			
Debt / Value, Industry	$E(D^*_{\text{Ind}})/V^*$	0.7,0.1,0.1 0.1,0.9,0.1	0.1,0.2,0.1 0.9,0.5,0.5	0.1,0.1,0.1 0.1,0.4,0.1
Low-Debt Firm	$E(D^*_L)/V^*$			

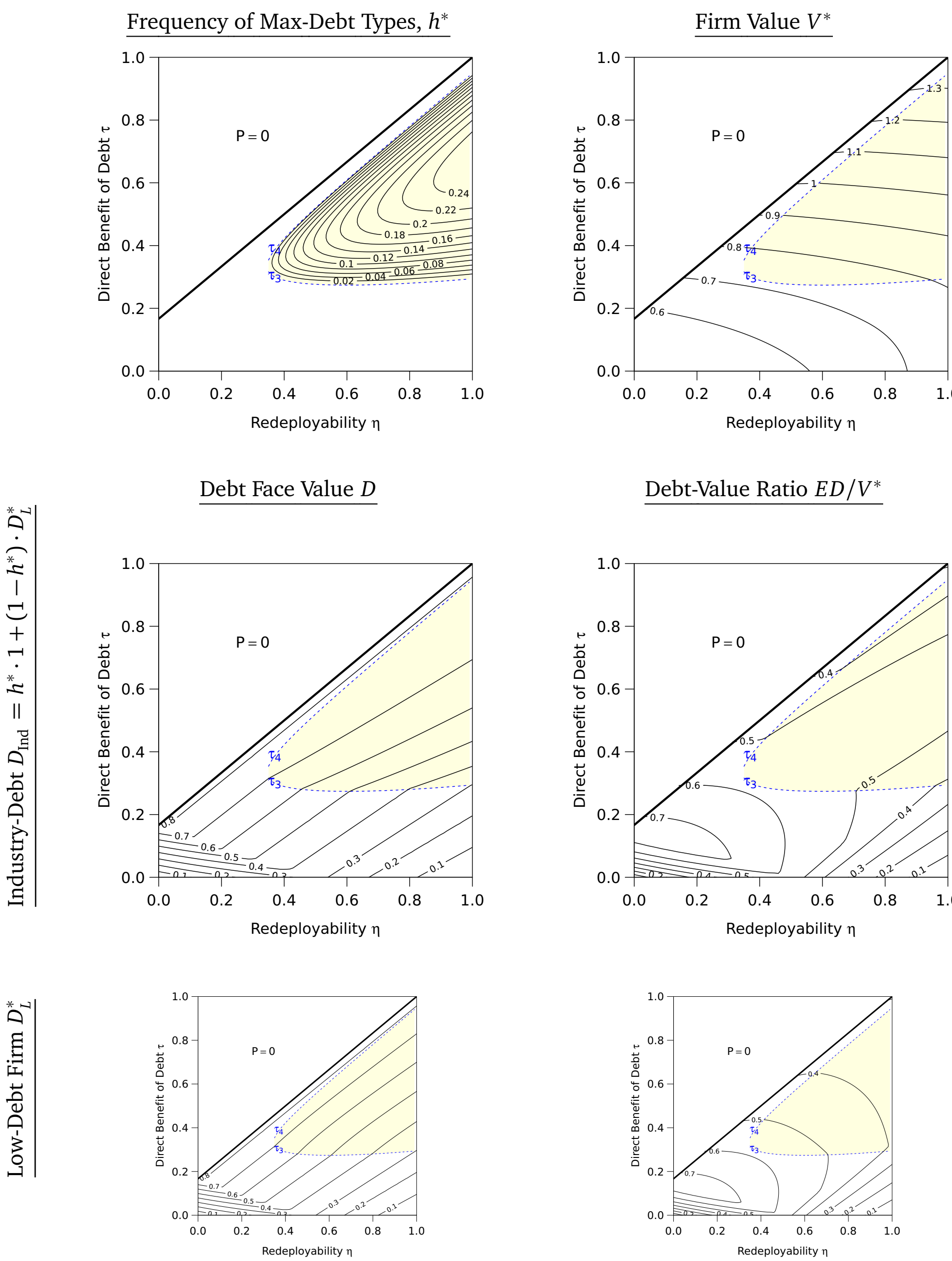
Panel B: Ancillary Comparative Statics

Credit Spread	r	0.3,0.1,0.3 0.1,0.2,0.1†	0.1,0.2,0.1 0.3,0.0,0.1	↑
Asset Price	P^*	↑	↑	↓
Asset Price/Max Value (NPV 0)	P^*/η	0.1,0.5,0.2 0.1,0.2,0.2	↑	↓
Asset Sales #	Q^*	↑	↑	0.6,0.0,0.1 0.1,0.6,0.1
Low Type Liquidation Freq.	Λ^*/D^*	↑	↑	↓
Reorganization Cost	$E[\,\phi\cdot(D^*-V^*)\,]$	↓	0.1,0.2,0.1 0.9,0.0,0.8	↑

† Tiny region.

Explanation: Ambiguous comparative statics are illustrated with two examples (order η, ϕ, τ), in which one derivative is negative (red) and another is positive (blue).

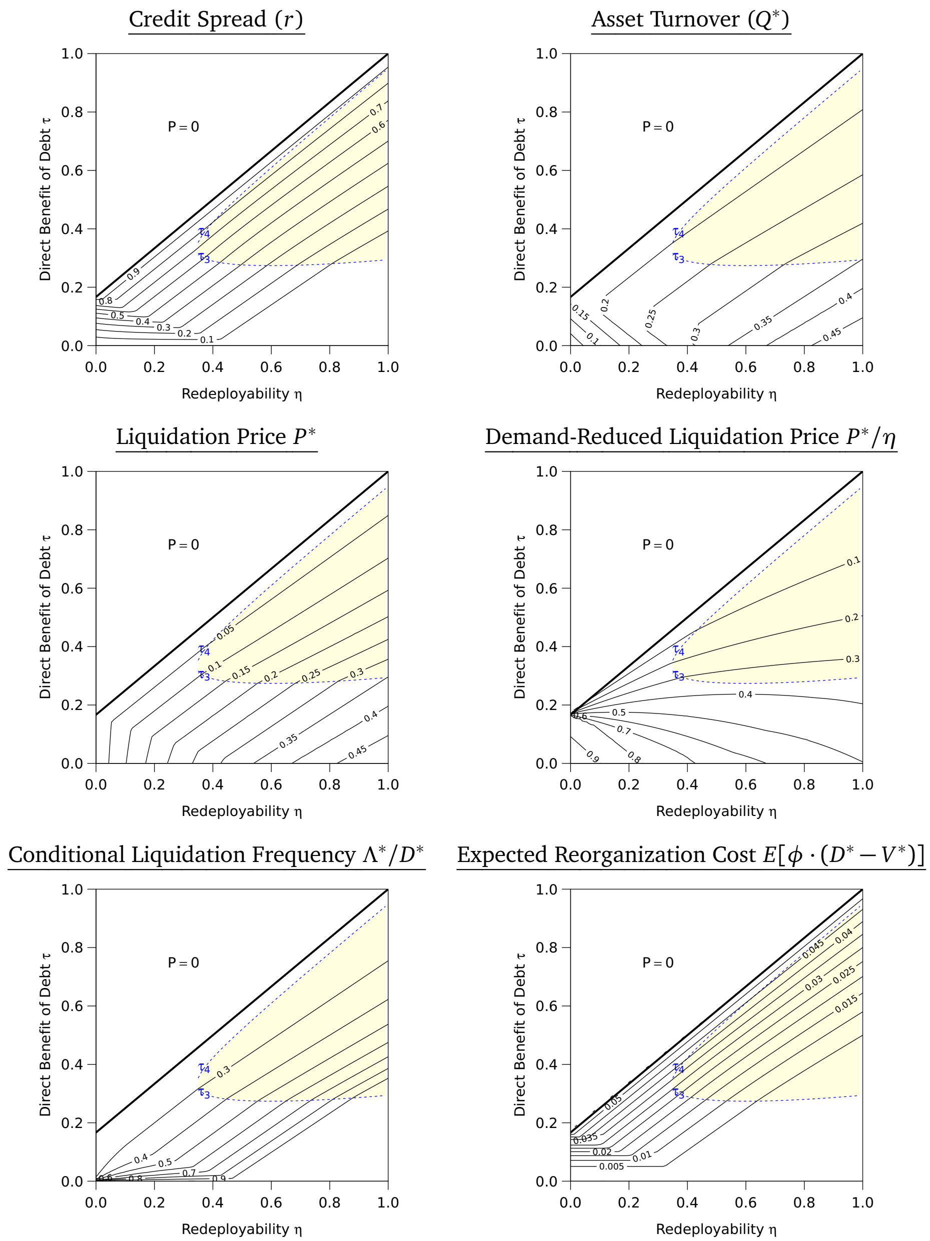
Figure 6: Debt-Related Comparative Statics for $\phi = 0.25$



Explanation: These are contourplots. The yellow area contains the two-type equilibria (as defined in Appendix Section B on Page 38). Patterns that have "∩" or "∪" shapes indicate ambiguous comparative statics in redeployability. "⊂" or "⊃" shapes indicate ambiguous comparative statics in direct debt benefits.

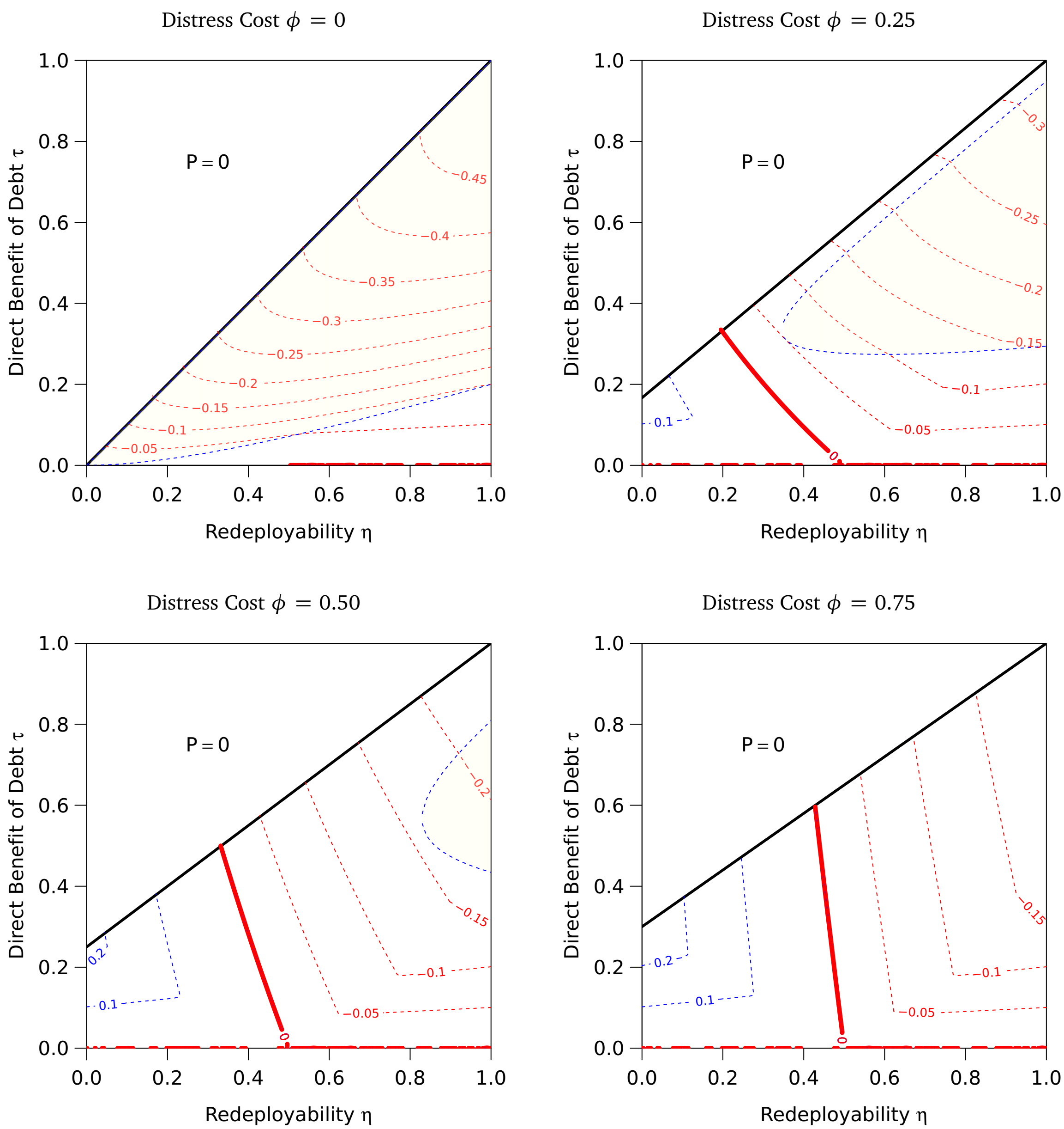
Interpretation: TL: Heterogeneity arises for intermediate values of debt benefits and large values of redeployability. The two-type region is now smaller than it was when $\phi = 0$ in Figure 2. TR: Firm value increases monotonically in redeployability and direct debt benefits. ML/BL/MR/BR: Industry debt increases monotonically with direct debt benefits. However, it can first increases and then decrease in redeployability. MR: Unlike the face value of debt in ML, the industry debt-value ratio can increase and then decrease in direct debt benefits.

Figure 7: Ancillary Comparative Statics for $\phi = 0.25$



Explanation: These are contourplots. The yellow area contains the two-type equilibria (as defined in Appendix Section B on Page 38). Patterns that have "∩" or "∪" shapes indicate ambiguous comparative statics in redeployability. "⊂" or "⊃" shapes indicate ambiguous comparative statics in direct debt benefits.

Figure 8: Allocational Efficiency



Explanation: These are contourplots. The yellow area contains the two-type equilibria, delineated by τ_3 and τ_4 (as in the appendix). The fat line shows the parameters for τ and η where equilibrium results in first-best redeployment.

Interpretation: The area to the left of the fat line has too much transfer activity (q). The area to the right of the fat line has too little transfer activity. If there are no distress costs ($\phi = 0$), there is always too little redeployment.

A Proof of Theorem 1

Each firm is competitive and takes the price, P, as given. The marginal benefit of debt is τ for all values of debt, D. The marginal cost of debt falls into three regions:

1. For $D \in [0, P(1/\eta - 1)]$ the marginal cost is zero
2. For $D \in [P(1/\eta - 1), 1 - P]$ the marginal cost is $\eta(D+P) - P$
3. For $D \in [1-P, 1]$ the marginal cost is zero

In Region 1, increasing debt is not costly to the firm because the marginal project is negative NPV. In Region 2, increasing debt is costly because the firm must forego positive NPV projects. In Region 3, the debt level is so high that the firm cannot finance the acquisition of the asset even if it is the highest productivity type, $V_i = 1$. Therefore, increasing debt further results in no additional costs to the firm.

Since the marginal benefit of debt is positive (equal to τ) it follows that it is never optimal for the firm to choose a debt level in Region 1, i.e., $D \leq P/\eta - P$. Furthermore, since the marginal cost of debt jumps down to zero at $D = 1 - P$ there may be a two-type equilibrium in which some firms choose debt in Region 2 while others choose debt in Region 3. Clearly, in such an equilibrium, firms in Region 3 will choose $D_H = 1$ since the marginal benefit of debt exceeds the marginal cost for all debt choices in Region 3.

The first-order condition for an optimal (interior) debt choice is:

$$D_L(P) = P/\eta - P + \tau/\eta.$$

The second-order condition is clearly satisfied.

One-Type equilibrium

Without reorganization costs the firm liquidates at time 1 if and only if $V_i \leq P$, therefore, the supply of the asset is P. In a one-type equilibrium, the demand is $1 - P - D_L(P)$, therefore, we have:

$$P^* = \frac{\eta - \tau}{1 + \eta}$$

and

$$D^* = \frac{2\tau + 1 - \eta}{1 + \eta}.$$

Equilibrium with two types

We now consider the possibility of a two-type equilibrium. The value of a firm choosing $D_L(P)$ is

$$\begin{aligned} V_L(P) &= 0.5 \cdot (1 + P^2) + \int_{P+D_L}^{1} (\eta V - P) dV + \tau D_L \\ &= 0.5 \cdot (1 + P^2) + \frac{(\eta^2 + P^2 - 2P\eta - \tau^2)}{2\eta} + \tau D_L, \end{aligned}$$

and the value of a firm choosing $D_H = 1$ is

$$V_1(P) = 0.5 \cdot (1 + P^2) + \tau.$$

Setting $V_L(P) = V_1(P)$ and solving for P yields the equilibrium price in a two-type equilibrium:

$$P = \eta - \tau + \eta\tau - \sqrt{\eta^2\tau^2 + 2\eta\tau(\eta - \tau)}.$$

We now find the boundaries of the two-type equilibrium. Since prices are continuous we know $P^*(\tau^c) = P(\tau^c)$ (i.e., prices in the one-type and two-type equilibria are equal at the boundaries). Solving for τ^c gives:

$$\tau^c = \frac{2\eta^2 + \eta \pm \eta(1+\eta)}{3\eta + 2} \Rightarrow \{\tau_{c_1}, \tau_{c_2}\} = \left\{\frac{\eta^2}{3\eta + 2}, \eta\right\}.$$

Therefore, for $\tau \in \left(\frac{\eta^2}{3\eta+2}, \eta\right]$ there is a two-type equilibrium with proportion h of firms choosing $D_H = 1$, proportion $1 - h^*$ of firms choosing $D_L(P^*) = P^*/\eta - P^* + \tau/\eta$, and the price

$$P^* = \eta - \tau + \eta\tau - \sqrt{\eta^2\tau^2 + 2\eta\tau(\eta - \tau)}.$$

The supply of the asset is P^* and in a two-type equilibrium the demand is $(1 - h^*) \cdot (1 - P^* - D_L)$, therefore,

$$h^* = \frac{1 - 2P^* - D_L(P^*)}{1 - P^* - D_L(P^*)}.$$

For $\tau > \eta$ there is a one-type equilibrium in which all firms choose $D^* = 1$ and the equilibrium price is $P^* = 0$.

B Complete Theorem 2

$$
\begin{aligned}
\text{Let } \tau_0 &= \frac{\phi(1-2\eta)}{(1+\eta)(1+\phi)+\phi(1-2\eta)},\\
\tau_1 &= \frac{2\eta^2(\phi+1)-\phi+\eta(\phi+1)-\sqrt{(\eta+1)^2(\phi+1)(\eta^2\phi+\eta^2-2\phi-\eta\phi)}}{3\eta(\phi+1)+3\phi+2},\\
\tau_2 &= \frac{(2\eta-1)(2\eta+\phi+2\eta\phi)}{2+3\phi}\\
&- \frac{\sqrt{(2\eta-1)^2(1+\phi)(4\eta^2(1+\phi)+\eta\phi-2(\eta+\phi))}}{2+3\phi},\\
\tau_3 &= \frac{2\eta^2(1-\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)+2\phi+12\phi^2+7\phi^3}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}\\
&- \frac{\sqrt{(1+\phi)(1+\eta+5\phi-\eta\phi)^2(\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3)}}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2},\\
\tau_4 &= \frac{2\eta^2(1-\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)+2\phi+12\phi^2+7\phi^3}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}\\
&+ \frac{\sqrt{(1+\phi)(1+\eta+5\phi-\eta\phi)^2(\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3)}}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}.
\end{aligned}
$$

Region 1: $\eta \geq 1/2$ **and** $\phi < \frac{3\eta-2}{6-3\eta}$

- If $0 \leq \tau \leq \tau_1$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta-\tau}{1+\eta}$ in which all firms choose $D^* = \frac{1-\eta+2\tau}{1+\eta}$.
- If $\tau_1 < \tau \leq \tau_2$ there exists a unique, two-type equilibrium with price

$$
\begin{aligned}
P^* &= \frac{\phi\eta-(1+\phi)[\tau-\eta(1+\tau)]}{1+\phi(1+\eta)}\\
&- \frac{\sqrt{\eta(\phi+1)(2\tau(\eta\phi+(\eta-\tau)(1+\phi))+\eta\tau^2(\phi+1)-\phi(1+\tau-\eta)^2)}}{1+\phi(1+\eta)}.
\end{aligned}
$$

  in which the proportion $1-f$ of firms choose $D_L = \frac{\tau}{\eta} + \frac{(1-\eta)}{\eta}P^*$ and the proportion f of firms choose $D_H = 1$, where

$$
f = \frac{(1+\phi)\cdot(\eta-\tau-(1+\eta)\cdot P)}{\eta\phi+(1+\phi)(\eta-\tau)-(1+\phi(1+\eta))\cdot P}.
$$

- If $\tau_2 < \tau \leq \tau_4$ there exists a unique, two-type equilibrium with price

$$P^* = \frac{\phi(1+2\phi(1-\tau)-3\tau)+\eta(1+\phi)(1+\tau+\phi(2+\tau))-\tau}{1+(6-3\eta)\phi(1+\phi)} - \frac{\sqrt{(1+\phi)(\eta+\phi+\eta\phi)\begin{pmatrix} 3\eta^2\phi(1+\phi)-2(\phi(\tau-1)+\tau)^2 \\ +\eta(\phi(\tau-1)+\tau)(2+\phi(\tau-1)+\tau) \end{pmatrix}}}{1+(6-3\eta)\phi(1+\phi)}. \tag{9}$$

in which the proportion $1-f$ of firms choose $D_L = \frac{\tau(1+\phi)}{(\eta+\phi+\eta\phi)} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P^*$ and the proportion f of firms choose $D_H = 1$, where

$$f = \frac{(1+\phi)\cdot(\eta+\phi+\eta\phi-\tau(1+2\phi)-P\cdot(1+\eta+5\phi-\eta\phi))}{(1+2\phi)\cdot(\eta+\phi+\eta\phi-\tau(1+\phi))-P\cdot(1+5\phi-\eta\phi+5\phi^2-\eta\phi^2)}. \tag{10}$$

- If $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.

Region 2: $\eta \geq 1/2$, $\phi \geq \frac{3\eta-2}{6-3\eta}$, **and** $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 \geq 0$

- If $0 \leq \tau \leq \frac{2\eta-1}{3}$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta-\tau}{1+\eta}$ in which all firms choose $D^* = \frac{1-\eta+2\tau}{1+\eta}$.
- If $\frac{2\eta-1}{3} < \tau \leq \tau_3$ or if $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.
- If $\tau_3 < \tau \leq \tau_4$ there exists a unique, two-type equilibrium with price (9) in which the proportion $1-f$ of firms choose $D_L = \frac{\tau(1+\phi)}{(\eta+\phi+\eta\phi)} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P^*$ and the proportion f of firms choose $D_H = 1$, where f is (10).

Region 3: $\eta \geq 1/2$, $\phi \geq \frac{3\eta-2}{6-3\eta}$ **and** $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 < 0$

- If $0 \leq \tau \leq \frac{2\eta-1}{3}$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta-\tau}{1+\eta}$ in which all firms choose $D^* = \frac{1-\eta+2\tau}{1+\eta}$.
- If $\frac{2\eta-1}{3} < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.

Region 4: $\eta < 1/2$ **and** $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 \geq 0$

- If $0 < \tau \leq \tau_0$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta(1-\tau)}{1+\eta}$ in which all firms choose $D^* = \frac{\tau(1+\eta+\phi)+\phi\eta}{\phi(1+\eta)}$.
- If $\tau_0 < \tau \leq \tau_3$ or if $\tau_4 \leq \tau \leq \frac{\phi+\eta(1+\phi)}{1+2\phi}$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.
- If $\tau_3 < \tau \leq \tau_4$ there exists a unique, two-type equilibrium with price (9) in which the proportion $1-f$ of firms choose $D_L = \frac{\tau(1+\phi)}{(\eta+\phi+\eta\phi)} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P^*$ and the proportion f of firms choose $D_H = 1$, where f is (10).

Region 5: $\eta < 1/2$ **and** $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 < 0$

- If $0 < \tau \leq \tau_0$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta(1-\tau)}{1+\eta}$ in which all firms choose $D^* = \frac{\tau(1+\eta+\phi)+\phi\eta}{\phi(1+\eta)}$.
- If $\tau_0 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, one-type equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.

Region 6:

- If $\frac{\eta+\phi+\eta\phi}{1+2\phi} < \tau \leq 1$ there exists a unique, one-type equilibrium with price $P^* = 0$ in which all firms choose $D^* = \min\left\{1, \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi}\right\}$.

C Proof

Proof when $\eta \geq 1/2$

We first consider the case $\eta \geq 1/2$. Each firm is competitive and takes the price, P, as given. For now, we assume $P > 0$. We will consider the possibility that $P = 0$ later in the proof. The marginal benefit of debt is τ for all values of debt, D. If $P > 0$ the marginal cost of debt falls into four regions:

1. For $D \in [0, P \cdot (1/\eta - 1)]$ the marginal cost of debt is 0
2. For $D \in (P \cdot (1/\eta - 1), P]$ the marginal cost of debt is $\eta D - P \cdot (1 - \eta)$
3. For $D \in (P, 1 - P)$ the marginal cost of debt is $\eta D - P \cdot (1 - \eta) + (D - P) \cdot \phi/(1 + \phi)$
4. For $D \in [1 - P, 1]$ the marginal cost of debt is $(D - P) \cdot \phi/(1 + \phi)$

Importantly, the marginal cost of debt is weakly increasing in D over the first three regions but then jumps down at $D = 1 - P$ (since the financing constraint is no longer binding) after which it increases again. Consequently, there is the possibility of both symmetric equilibria and asymmetric equilibria in which some firms choose low debt and others choose high debt. We must consider three cases: (i) $0 \leq \tau < (2\eta - 1) \cdot P$, (ii) $(2\eta - 1) \cdot P \leq \tau \leq \eta - P + (1 - 2P)\phi/(1 + \phi)$, and (iii) $\eta - P + (1 - 2P)\phi/(1 + \phi) < \tau \leq 1$.

Case 1: $0 \leq \tau < (2\eta - 1) \cdot P$

If $\tau < (2\eta - 1) \cdot P$ then firms choose either $D_L \in [P \cdot (1/\eta - 1), P)$ or $D_H \in [1 - P, 1]$ where $D_L = \tau/\eta + P \cdot (1/\eta - 1)$ and $D_H = \min\{1, P + \tau \cdot (1 + \phi)/\phi\}$.

Symmetric equilibria

There cannot exist a symmetric equilibrium with $P > 0$ in which all firms choose D_H because the aggregate demand for the risky asset would be zero but the supply is positive $[(P + \phi D)/(1 + \phi)]$. Therefore, if there exists a symmetric equilibrium in the case $\tau < (2\eta - 1) \cdot P$, then all firms choose $D_L = \tau/\eta + P \cdot (1/\eta - 1)$. The demand for the liquidated asset is then $1 - P - D_L$ and the supply of the liquidated asset is P since for $D_L < P$ which implies $\tau < (2\eta - 1) \cdot P$ it is optimal to liquidate the asset for all $V \leq P$. Equating supply and demand gives the equilibrium price $P^* = (\eta - \tau)/(1 + \eta)$.

Importantly, note that if $P^* = (\eta - \tau)/(1 + \eta)$ then we must have $\tau < (2\eta - 1)/3$ to be in a symmetric equilibrium in the case $\tau < (2\eta - 1) \cdot P$.

Asymmetric equilibria with two types

There is also the possibility of an equilibrium with two types (a fraction of firms choosing D_L and the remaining fraction of firms choosing D_H). Firms choosing D_L have ex ante value

$$V_L = \int_0^P P dV + \int_P^1 V dV + \int_{P+D_L}^1 (\eta V - P) dV + \tau \cdot D_L$$

and by substituting $D_L = \tau/\eta + P \cdot (1/\eta - 1)$ yields

$$V_L = 0.5(\eta + (P-1)^2 + (P+\tau)^2/\eta - 2P\tau).$$

Firms choosing D_H have ex ante value

$$V_H = \int_0^\Lambda P dV + \int_\Lambda^{D_H} [V - \phi \cdot (D_H - V)] dV + \int_{D_H}^1 V dV + \tau \cdot D_H.$$

where $\Lambda = (P + \phi D)/(1+\phi)$.

If $D_H = P + \tau \cdot (1+\phi)/\phi$ then ex ante value is

$$V_H = 0.5 \cdot (P+\tau)^2 + 0.5 \cdot \tau^2/\phi + 0.5,$$

but if $D_H = 1$ then ex ante value is

$$V_1 = 0.5 \cdot (P+\phi)^2/(1+\phi) + 0.5 \cdot (1-\phi) + \tau.$$

The following result shows that in a two-type equilibrium the high-type always chooses $D_H = 1$.

Lemma 1: If $\tau < (2\eta - 1)/3$ then in a two-type equilibrium the high type chooses $D_H = 1$.

Proof: Proof by contradiction. Suppose $D_H = P + \tau \cdot (1+\phi)/\phi < 1$. Then we have:

$$G(P) \equiv V_L(P) - V_H(P) = 0.5 \cdot (\eta + (P-1)^2 + (P+\tau)^2 \cdot (1-\eta)/\eta - 2P\tau - \tau^2/\phi - 1).$$

Note, $G'(P) = P/\eta - 1 + \tau \cdot (1/\eta - 2) \leq P/\eta - 1 \leq 0$ where the first inequality follows from our assumption that $1/\eta - 1 \leq 1$ and the second from the fact that $P \leq \eta$ as the price for the asset will never exceed its maximum value. Therefore, G(P) is decreasing in P. Furthermore, in a two-type equilibrium the price is bounded above by the symmetric equilibrium price (i.e. $P \leq (\eta - \tau)/(1+\eta)$) because the introduction of some high-debt firms both reduces the demand and increases the supply of the liquidated asset. Therefore,

$$\begin{aligned}
G(P) &\geq G((\eta-\tau)/(1+\eta)) \\
&= 0.5 \cdot \left[\left(\frac{1+\tau}{1+\eta} \right)^2 (1+\eta-\eta^2) + \eta - 2\tau \left(\frac{\eta-\tau}{\eta+1} \right) - \frac{\tau^2}{\phi} - 1 \right] \\
&> 0.5 \cdot \left[\left(\frac{1+\tau}{1+\eta} \right)^2 (1+\eta-\eta^2) + \eta - 2\tau \left(\frac{\eta-\tau}{\eta+1} \right) - \tau(1-\tau) - 1 \right] \\
&\geq 0 \quad \forall \tau
\end{aligned}$$

where the second inequality follows from the fact that if $D_H^* < 1$ and $P > 0$ then $\tau < \phi/(1+\phi)$ which implies $\phi > \tau/(1-\tau)$; and the third inequality is easily verified numerically. But this contradicts the optimality of D_H^*.

By Lemma 1, we must only compare V_L to V_1 to find a two-type equilibrium.

Conjecture the existence of a symmetric equilibrium in which $D_L = \tau/\eta + P\cdot(1/\eta - 1)$ and $P^* = (\eta-\tau)/(1+\eta)$. Substituting P^* into our expressions for V_L and V_1 implies:

$$F(\tau) \equiv V_L(\tau) - V_1(\tau) = \frac{\eta(\eta-\tau)^2(\phi+1) + \phi(\tau+1)^2 - 2(\eta+1)(\phi+1)\tau(\eta-\tau)}{2(\eta+1)^2(\phi+1)}$$

Therefore,

$$F'(\tau) = \frac{\phi - \eta(1+\phi) - 2\eta^2(1+\phi) + (2+3\phi+3\eta(1+\phi))\tau}{(1+\eta)^2(1+\phi)}$$

and

$$F''(\tau) = \frac{2+3\phi+3\eta(1+\phi)}{(1+\eta)^2(1+\phi)}$$

Note that $F'\left(\frac{2\eta-1}{3}\right) = -\frac{2}{3(1+\eta)(1+\phi)} < 0$ and $F''(\tau) > 0$ for all τ. Therefore, $F'(\tau) < 0$ for all $\tau \leq \frac{2\eta-1}{3}$.

Also, $F\left(\frac{2\eta-1}{3}\right) = \frac{2-3\eta+\phi\cdot(6-3\eta)}{18(1+\phi)} \geq 0$ if and only if $\phi \geq \frac{3\eta-2}{6-3\eta}$. Note that if $\eta < \frac{2}{3}$, then $\frac{3\eta-2}{6-3\eta} < 0$ and $F\left(\frac{2\eta-1}{3}\right) \geq 0$ for any ϕ.

Therefore, if $\phi \geq \frac{3\eta-2}{6-3\eta}$ and $\tau \leq \frac{2\eta-1}{3}$ then $F(\tau) \geq 0$ and all firms optimally choose $D_L = \tau/\eta + P\cdot(1/\eta-1)$ and the conjectured equilibrium price of $P = (\eta-\tau)/(1+\eta)$ is confirmed by the firms' debt decisions.

However, if $\phi < \frac{3\eta-2}{6-3\eta}$, then the conjectured symmetric equilibrium is confirmed only for $\tau \leq \tau_1$ where $F(\tau_1) \equiv 0$. For $\tau > \tau_1$ there is a two-type equilibrium in which some firms choose $D_L = \tau/\eta + P\cdot(1/\eta-1)$ and others choose $D_H = 1$ (by Lemma 1). Solving $F(\tau_1) = 0$ yields

$$\tau_1 = \frac{2\eta^2(\phi+1) - \phi + \eta(\phi+1) - \sqrt{(\eta+1)^2(\phi+1)(\eta^2\phi+\eta^2-2\phi-\eta\phi)}}{3\eta(\phi+1)+3\phi+2}$$

(Note: There is another solution to $F(\tau_1) = 0$ where $F(\tau)$ again becomes positive beyond that point. However, $F'(\tau) < 0$ for all $\tau \leq \frac{2\eta-1}{3}$ so we know $F(\tau) < 0$ for all $\tau_1 < \tau < \frac{2\eta-1}{3}$.)

For $\tau > \tau_1$ there is a unique, two-type equilibrium which is constructed by finding P that equates $V_L = V_1$, which is quadratic in P. There are two solutions, but only one where P is less than the symmetric price (which must be true in equilibrium as argued above) and it is

$$P^* = \frac{\phi\eta - (1+\phi)[\tau - \eta(1+\tau)]}{1+\phi(1+\eta)} - \frac{\sqrt{\eta(\phi+1)(2\tau(\eta\phi+(\eta-\tau)(1+\phi)) + \eta\tau^2(\phi+1) - \phi(1+\tau-\eta)^2)}}{1+\phi(1+\eta)},$$

Let h be the fraction of firms choosing $D = 1$. The demand for the risky asset is then $(1-f)\cdot(1-P-D_L)$ and the supply of the risky asset is $(1-f)\cdot P + f\cdot(P+\phi\cdot 1)/(1+\phi)$, therefore, market clearing requires

$$h^* = \frac{(1+\phi)\cdot(\eta-\tau-(1+\eta)\cdot P)}{\eta\phi+(1+\phi)(\eta-\tau)-(1+\phi(1+\eta))\cdot P},$$

Finally, if there is a two-type equilibrium at the upper boundary we know that $P < (\eta-\tau)/(1+\eta)$ and therefore $\tau < (2\eta-1)/3$ at the boundary. Equating $\tau_2 = (2\eta-1)\cdot P^*(\tau_2)$ yields the upper boundary in this case:

$$\begin{aligned}\tau_2 &= \frac{(2\eta-1)(2\eta+\phi+2\eta\phi)}{2+3\phi} \\ &- \frac{\sqrt{(2\eta-1)^2(1+\phi)(4\eta^2(1+\phi)+\eta\phi-2(\eta+\phi))}}{2+3\phi}\end{aligned}$$

(Note: There is another root but it is greater than $(2\eta-1)/3$ when $\eta \geq 1/2$ so we can ignore it.)

Case 2: $(2\eta-1)\cdot P \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$

If $(2\eta-1)\cdot P \leq \tau \leq \eta-P+(1-2P)\phi/(1+\phi)$ then firms choose either $D_L \in [P, 1-P)$ or $D_H \in [1-P, 1]$ where $D_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P$ and $D_H = \min\{1, P+\tau\cdot(1+\phi)/\phi\}$.

Symmetric equilibria

Again, there cannot exist a symmetric equilibrium with $P > 0$ in which all firms choose D_H because the aggregate demand for the risky asset would be zero but the supply is positive $[(P+\phi D)/(1+\phi)]$. Therefore, in a symmetric equilibrium firms choose

$$D = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P.$$

The demand for the liquidated asset is $1-P-D$ and the supply of the liquidated asset is

$$\Lambda = \frac{P+\phi D}{1+\phi} = \frac{P\cdot((1-\phi)\eta+2\phi)+\phi\tau}{\eta+\phi+\eta\phi}.$$

Equating supply and demand gives the equilibrium price

$$P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}.$$

Substituting into the expression for D yields

$$D^* = \frac{1-\eta-\eta\phi+2\phi+\tau(2+\phi)}{1+\eta+5\phi-\eta\phi}.$$

Asymmetric equilibria with two types

As in Case 1, there is the possibility of an equilibrium with two types. Firms choosing D_L have ex ante value

$$V_L = \int_0^{\Lambda} P\,dV + \int_{\Lambda}^{D_L} [V - \phi(D_L - V)]dV + \int_{D_L}^{1} V\,dV + \int_{P+D_L}^{1} (\eta V - P)dV + \tau \cdot D_L$$

and by substituting $D_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)} P$ yields

$$\begin{aligned} V_L &= \left(\frac{1+\eta+6\phi-3\eta\phi}{2(\eta+\phi+\eta\phi)}\right)P^2 - \left(1 - \frac{\tau(1-\eta+2\phi-\eta\phi)}{\eta+\phi+\eta\phi}\right)P \\ &+ \left(1 + \frac{\eta(\eta-1)+\phi(\eta^2-1)+\tau^2(1+\phi)}{2(\eta+\phi+\eta\phi)}\right). \end{aligned} \tag{11}$$

Firms choosing D_H have ex ante value

$$V_H = \int_0^{\Lambda} P\,dV + \int_{\Lambda}^{D_H} [V - \phi \cdot (D_H - V)]dV + \int_{D_H}^{1} V\,dV + \tau \cdot D_H.$$

If $D_H = P + \tau \cdot (1+\phi)/\phi$ then ex ante value is

$$V_H = 0.5 \cdot (P+\tau)^2 + 0.5 \cdot \tau^2/\phi + 0.5 \tag{12}$$

but if $D_H = 1$ then ex ante value is

$$V_1 = 0.5 \cdot (P+\phi)^2/(1+\phi) + 0.5 \cdot (1-\phi) + \tau. \tag{13}$$

The next result shows that in a two-type equilibrium the high type always chooses $D_H = 1$.

Lemma 2: In a two-type equilibrium the high type chooses $D_H = 1$.

Proof: Proof by contradiction. Suppose $D_H^* = P + \tau \cdot (1+\phi)/\phi < 1$. Since we assume $P > 0$, this implies $\tau < \phi/(1+\phi)$. We have:

$$\begin{aligned} G(P) &\equiv V_L(P) - V_H(P) \\ &= \left(\frac{1+(5-4\eta)\phi}{2(\eta+\phi+\eta\phi)}\right)P^2 - \left(1 - \frac{\tau(1+\phi)(1-2\eta)}{\eta+\phi+\eta\phi}\right)P + \left(\frac{\eta}{2} - \frac{\eta\tau^2(1+\phi)^2}{2\phi(\eta+\phi+\eta\phi)}\right) \end{aligned}$$

Therefore,

$$\begin{aligned} G'(P) &= \left(\frac{1+(5-4\eta)\phi}{\eta+\phi+\eta\phi}\right)P - \left(1 - \frac{\tau(1+\phi)(1-2\eta)}{\eta+\phi+\eta\phi}\right) \\ &\leq \left(\frac{1+(5-4\eta)\phi}{\eta+\phi+\eta\phi}\right)\left(\frac{\eta(1+\phi)+\phi-\tau(1+2\phi)}{(5-\eta)\phi+1+\eta}\right) - \left(1 - \frac{\tau(1+\phi)(1-2\eta)}{\eta+\phi+\eta\phi}\right) \\ &\leq 0 \end{aligned}$$

where the first inequality follows from the fact that in a two-type equilibrium the equilibrium price is less than the symmetric equilibrium price (i.e., $P \leq \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$) and the second inequality holds for all $\eta \geq 1/2$. Therefore,

$$\begin{aligned} G(P) &\geq G\left(\frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}\right) \\ &> 0 \quad \forall \tau < \phi/(1+\phi). \end{aligned}$$

where the last inequality is easily verified numerically. But this contradicts the optimality of D_H^*.

By Lemma 2, we must only compare V_L to V_1 to find a two-type equilibrium. We have

$$\begin{aligned} V_L - V_1 &= \frac{\left(\phi(\phi+1)(2-3\eta)+(2\phi+1)^2\right)}{2(\phi+1)(\eta\phi+\eta+\phi)}P^2 \\ &- \frac{\left((2\phi+1)(\eta+\phi+\eta\phi-\tau(1+\phi))+\eta(\phi+1)^2\tau\right)}{(\phi+1)(\eta\phi+\eta+\phi)}P \\ &+ \frac{((\eta-\tau)(1+\phi)+\phi)^2}{2(\phi+1)(\eta\phi+\eta+\phi)} \end{aligned} \tag{14}$$

Conjecture the existence of a symmetric equilibrium in which case $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$. Substituting P^* into our expressions for V_L and V_1 implies:

$$\begin{aligned} F(\tau) = &\frac{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}{2(1+\phi)(1+\eta+5\phi-\eta\phi)^2}\tau^2 \\ &- \frac{2\eta^2(1-\phi^2)+\phi(2+12\phi+7\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)}{(1+\phi)(1+\eta+5\phi-\eta\phi)^2}\tau \\ &+ \frac{\eta^3(\phi-1)^2(1+\phi)+\phi^2(2+11\phi)+2\eta\phi(1+8\phi+\phi^2)-4\eta^2\phi(-1+2\phi+2\phi^2)}{2(1+\phi)(1+\eta+5\phi-\eta\phi)^2} \end{aligned} \tag{15}$$

We have two possibilities to consider. First, suppose there is a symmetric equilibrium at the upper boundary of Case 1, $\tau = (2\eta-1)\cdot P$. In this case, we know that $P = (\eta-\tau)/(\eta+1)$ which implies $\tau = (2\eta-1)/3$. We also know that $\phi \geq \frac{3\eta-2}{6-3\eta}$. Therefore, at the transition to this case we have $F\left(\frac{2\eta-1}{3}\right) = \frac{2-3\eta+\phi\cdot(6-3\eta)}{18(1+\phi)} \geq 0$ for $\phi \geq \frac{3\eta-2}{6-3\eta}$. Furthermore,

$$F'\left(\frac{2\eta-1}{3}\right) = -\frac{2+(10-6\eta)\phi+6(1-\eta)\phi^2}{3(1+\eta(1-\phi)+5\phi)(1+\phi)} < 0,$$

and

$$F''(\tau) = \frac{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}{(1+\phi)(1+\eta+5\phi-\eta\phi)^2} > 0.$$

We see then that $F(\tau)$ is an upward facing parabola in τ. At the lower boundary of Case 2, where $\tau = \frac{2\eta-1}{3}$, $F(\tau)$ is positive but decreasing.

Solving $F(\tau) = 0$ yields two solutions:

$$\tau_3, \tau_4 = \frac{2\eta^2(1-\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)+2\phi+12\phi^2+7\phi^3}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}$$
$$\pm \frac{\sqrt{(1+\phi)(1+\eta+5\phi-\eta\phi)^2(\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3)}}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}$$

If $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 < 0$, the roots of the solution of $F(\tau) = 0$ are complex so $F(\tau) > 0$ for all τ. Therefore, for $\frac{2\eta-1}{3} < \tau \leq 1$, there is a symmetric equilibrium where all firms choose $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$ and $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$. However, our free disposal assumption implies $P^* \geq 0$ which requires $\tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$. We thus consider symmetric equilibrium in the range $\frac{2\eta-1}{3} < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$.

If $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 \geq 0$, then τ_3 and τ_4 are real. Therefore, for $\tau_3 < \tau \leq \tau_4$, we have $F(\tau) < 0$ and a two-type equilibrium where P^* equates $V_L = V_1$. There are two solutions, but only one where P is less than the symmetric price (which must be true in equilibrium as argued above) and it is

$$P^* = \frac{\phi(1+2\phi(1-\tau)-3\tau)+\eta(1+\phi)(1+\tau+\phi(2+\tau))-\tau}{1+\phi(6-3\eta)(1+\phi)}$$
$$- \frac{\sqrt{(1+\phi)(\eta+\phi+\eta\phi)\begin{pmatrix} 3\eta^2\phi(1+\phi)-2(\phi(\tau-1)+\tau)^2 \\ +\eta(\phi(\tau-1)+\tau)(2+\phi(\tau-1)+\tau)) \end{pmatrix}}}{1+\phi(6-3\eta)(1+\phi)}.$$

The fraction f of firms choosing $D = 1$ supporting the price P is found by equating demand for the risky asset $(1-f)\cdot(1-P-D_L)$ with supply $(1-f)\cdot(P+\phi D_L)/(1+\phi)+f\cdot(P+\phi\cdot 1)/(1+\phi)$ where $D_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P$. Therefore, we have

$$h^* = \frac{(1+\phi)\cdot(\eta+\phi+\eta\phi-\tau(1+2\phi)-P(1+5\phi+\eta-\eta\phi))}{(1+2\phi)(\eta+\phi+\eta\phi-\tau-\phi\tau)-P(1+5\phi(1+\phi)-\eta\phi(1+\phi))}.$$

It has been verified numerically that $\tau_4 \leq 1$ and $P^*(\tau_4) > 0$. This means that for $\tau > \tau_4$, there will be a range of symmetric equilibria with positive prices. Therefore, for $\frac{2\eta-1}{3} < \tau \leq \tau_3$ and $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ we have a symmetric equilibrium where $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$ and $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$

Second, suppose there is an asymmetric (two-type) equilibrium at the upper boundary of Case 1, i.e. $\tau_2 = (2\eta-1)\cdot P^*(\tau_2)$. Then we know that $\phi < \frac{3\eta-2}{6-3\eta}$. It can be verified that $F(\tau_2) < 0$, and, following the arguments above, $\{\tau_3, \tau_4\}$ are the same as described above. However, it can be verified that $\tau_3 < \tau_2$ if $\phi < \frac{3\eta-2}{6-3\eta}$, thus there is an asymmetric equilibrium for all $\tau_2 < \tau \leq \tau_4$. In the asymmetric equilibrium, $D_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P^*$ and $\{P^*, h^*\}$ are characterized above. It is still the case that $\tau_4 \leq 1$ and $P^*(\tau_4) > 0$. Therefore, for $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ we have a symmetric equilibrium where $D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$ and $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$

We must also check that all these equilibria fall under Case 2, which requires $\tau \leq \eta - P + (1-2P)\phi/(1+\phi)$. We know that $P^* \leq \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$, the symmetric equilibrium price, for all equilibria. Therefore, it is sufficient to show that:

$$\tau \leq \eta - \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi} + \left(1 - \frac{2(\eta+\phi+\eta\phi-\tau(1+2\phi))}{1+\eta+5\phi-\eta\phi}\right)\frac{\phi}{1+\phi}$$

which is true if and only if

$$\tau \leq \frac{\eta+2\phi-\eta\phi}{1-\phi}.$$

However, this is satisfied for all $\tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$. Therefore, the equilibria described above all fall within Case 2.

Case 3: $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$

If $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$ then all firms choose $D > 1-P$ which implies that the aggregate demand for the liquidated asset is zero. But, since supply is positive when the price is positive, this case is incompatible with an equilibrium $P^* > 0$.

Equilibrium with $P^* = 0$

We assume free disposal therefore we know $P^* \geq 0$. We now consider the possibility that $P^* = 0$ which can occur in our model because we assume limited liability (i.e., the continuation value of a firm in distress is bounded below by zero). If $P = 0$ the four regions for the marginal cost of debt collapse into one:

- For $D \in [0,1]$ the marginal cost of debt is $\eta \cdot D + D \cdot \phi/(1+\phi)$

Consequently, if $P = 0$ there can only exist a symmetric equilibrium in which all firms choose $D^* = \min\left\{1, \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi}\right\}$. Therefore, the aggregate demand for the asset is $1 - P^* - D^* = 1 - D^* \geq 0$. The aggregate supply of the asset, however, is indeterminate. In particular, because of limited liability the firm will be indifferent between liquidation at $P^* = 0$ and continuation for all $V_i \in \left[0, \frac{\phi D^*}{1+\phi}\right]$. Therefore, the price $P^* = 0$ can be supported in equilibrium if $1 - D^* \leq \frac{\phi D^*}{1+\phi}$ or $\tau \geq \frac{\eta(1+\phi)+\phi}{1+2\phi}$.

Proof when $\eta < 1/2$

We now consider the case $\eta < 1/2$. For now, assume $P > 0$. We will consider the possibility that $P = 0$ at the end of the proof. If $P > 0$ the marginal cost of debt now falls into four regions:

1. For $D \in [0,P]$ the marginal cost of debt is 0
2. For $D \in [P, P \cdot (1/\eta - 1)]$ the marginal cost of debt is $(D-P)\phi/(1+\phi)$
3. For $D \in [P \cdot (1/\eta - 1), 1-P]$ the marginal cost of debt is $\eta D - P(1-\eta) + (D-P)\phi/(1+\phi)$
4. For $D \in [1-P, 1]$ the marginal cost of debt is $(D-P)\phi/(1+\phi)$

We must consider three cases: (i) $0 \leq \tau < (P/\eta - 2P)\phi/(1+\phi)$, (ii) $(P/\eta - 2P)\phi/(1+\phi) \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$, and (iii) $\eta - P + (1-2P)\phi/(1+\phi < \tau \leq 1$.

Case 1: $0 \leq \tau < (P/\eta - 2P)\phi/(1+\phi)$

If $0 \leq \tau < (P/\eta - 2P)\phi/(1+\phi)$ then $D \in [P, P(1/\eta - 1)]$ and all firms equate the marginal cost of debt in this region to the marginal benefit which implies

$$D(P) = P + \frac{\tau(1+\phi)}{\phi}.$$

In this region, all firms for whom the asset is positive NPV ($\eta V \geq P$) will be able to obtain financing to purchase the asset. The demand for the liquidated asset is then $1 - P/\eta$ and the supply of the liquidated asset is $(P + \phi D)/(1+\phi) = P + \tau$. Equating supply and demand gives the equilibrium price

$$P^* = \frac{\eta(1-\tau)}{1+\eta}$$

and, therefore,

$$D^* = \frac{\tau(1+\eta+\phi) + \phi\eta}{\phi(1+\eta)}.$$

To determine the values of τ included in this case, substitute P^* into the expression

$$\tau < (P/\eta - 2P)\phi/(1+\phi) \quad \Rightarrow \quad \tau \leq \frac{\phi(1-2\eta)}{(1+\eta)(1+\phi) + \phi(1-2\eta)} \equiv \tau_0.$$

Case 2: $(P/\eta - 2P)\phi/(1+\phi) \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$

If $(P/\eta - 2P)\phi/(1+\phi) \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$ then firms choose either $D_L \in [P(1/\eta - 1), 1-P)$ or $D_H \in [1-P, 1]$ where $D_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)} P$ and $D_H = \min\{1, P + \tau \cdot (1+\phi)/\phi\}$.

Symmetric equilibria

There cannot exist a symmetric equilibrium with $P > 0$ in which all firms choose D_H because the aggregate demand for the risky asset would be zero but the supply is positive $[(P + \phi D)/(1+\phi)]$. Therefore, in a symmetric equilibrium, firms choose

$$D = \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi} + \frac{(1-\eta)(1+\phi)+\phi}{\eta(1+\phi)+\phi} P.$$

The demand for the liquidated asset is $1 - P - D$ and the supply of the liquidated asset is

$$\Lambda = \frac{P + \phi D}{1+\phi} = \frac{P \cdot [(1-\phi)\eta + 2\phi] + \phi\tau}{\eta(1+\phi)+\phi}.$$

Equating supply and demand gives the equilibrium price

$$P^* = \frac{\eta + \phi + \eta\phi - \tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}.$$

Substituting into the expression for D yields

$$D^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}.$$

Asymmetric equilibria with two types

Again, there is the possibility of an equilibrium with two types. The proof here follows closely the proof in Case 2 when $\eta \geq 1/2$. Firms choosing $D_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P$ have ex ante value V_L as described in equation (11), firms choosing $D_H = P + \tau \cdot (1+\phi)/\phi$ have ex ante value V_H as described in equation (12), and firms choosing $D_H = 1$ have ex ante value V_1 as described in equation (13).

It is straightforward to show that Lemma 2 applies in the case $\eta < 1/2$ when $\tau_0 \leq \tau < \phi/(1+\phi)$. Therefore, in a two-type equilibrium the high type always chooses $D_H = 1$. Therefore, we must only compare V_L to V_1 to find a two-type equilibrium. We also have $V_L - V_1$ as described in equation (14) and $F(\tau)$ as described in equation (15).

We know there is a symmetric equilibrium at the upper boundary of case 1, $\tau = \tau_0$. Therefore, at the transition to this region we have $F(\tau_0) \geq 0$. Furthermore, it can be shown numerically that for all $\phi \in [0,1]$ and all $\eta \in [0,1/2]$ that

$$F'(\tau_0) < 0,$$

and

$$F''(\tau) = \frac{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}{(1+\phi)(1+\eta+5\phi-\eta\phi)^2} > 0.$$

We see then that $F(\tau)$ is an upward facing parabola in τ. At the lower boundary of Case 2, where $\tau = \tau_0$, $F(\tau_0)$ is positive but decreasing.

Solving $F(\tau) = 0$ yields τ_3, τ_4 as before and the remainder of the proof is identical to Case 2 when $\eta \geq 1/2$.

Case 3: $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$

If $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$ then all firms choose $D > 1-P$ which implies that the aggregate demand for the liquidated asset is zero. But, since supply is positive when the price is positive, this case is incompatible with an equilibrium $P^* > 0$.

Equilibrium with $P^* = 0$

Finally, as before, if $P = 0$ the four regions for the marginal cost of debt collapse into one:

- For $D \in [0,1]$ the marginal cost of debt is $\eta \cdot D + D \cdot \phi/(1+\phi)$

Consequently, if $P = 0$ there can only exist a symmetric equilibrium in which all firms choose $D^* = \min\left\{1, \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi}\right\}$. Following the argument in the proof when $\eta \geq 1/2$, the price $P^* = 0$ can be supported in equilibrium if $\tau \geq \frac{\eta(1+\phi)+\phi}{1+2\phi}$.

D Extension: Immediate Use of Debt Benefits

We now show that the solution to our model is isomorphic to one in which the firm can use the immediate benefit τD to pay off debt and purchase liquidated assets.

If $D(1-\tau) < P$ the value of the firm is now

$$\int_0^P P dV + \int_P^1 V dV + \int_{P+D(1-\tau)}^1 \max\{0, \eta V - P\} dV + \tau D.$$

The supply of the risky asset is P and the demand is now $1-P-(1-\tau)D$.

Therefore, if we let $D' = (1-\tau)D$ the value of the firm is

$$\int_0^P P dV + \int_P^1 V dV + \int_{P+D'}^1 \max\{0, \eta V - P\} dV + \frac{\tau}{1-\tau} \cdot D'.$$

The supply of the risky asset is P and the demand is now $1-P-D'$.

If $D(1-\tau) \geq P$ the value of the firm is

$$\int_0^{\Lambda'} P dV + \int_{\Lambda'}^{D(1-\tau)} [V - \phi(D-(V+\tau D))] dV + \int_{(1-\tau)D}^1 V dV + \int_{P+D(1-\tau)}^1 \max\{0, \eta V - P\} dV + \tau D.$$

The supply of the risky asset is $\Lambda' = \frac{P+\phi(1-\tau)D}{1+\phi}$ and the demand is now $1-P-(1-\tau)D$.

Again, if we let $D' = (1-\tau)D$ the value of the firm is

$$\int_0^{\Lambda'} P dV + \int_{\Lambda'}^{D'} [V - \phi(D'-V)] dV + \int_{D'}^1 V dV + \int_{P+D'}^1 \max\{0, \eta V - P\} dV + \frac{\tau}{1-\tau} \cdot D'.$$

The supply of the risky asset is $\Lambda' = \frac{P+\phi D'}{1+\phi}$ and the demand is now $1-P-D'$.

In sum, the solution to our original model yields $\{P(\tau'), D'(\tau')\}$ where $\tau' = \tau/(1-\tau)$. To convert to the equilibrium $\{P(\tau), D(\tau)\}$ note that $\tau = \tau'/(1+\tau')$ and $D = D'/(1-\tau)$. The latter expression for the face value of debt implies that the comparative statics for debt choices with respect to model parameters are not necessarily the same as in the base model. Although we don't report the results here, our main qualitative results continue to hold in this extension; in particular, debt levels and ratios may increase or decrease in η and the comparative statics for debt levels can be different than for debt-to-value ratios.