Evaluating Environmental Investments: A Real Options Approach

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The paper presents a model that determines when (at which output price level) it is optimum for a firm to invest in environmental technologies and which are the main parameters that affect this decision. Our analysis shows that firms require high output price levels to be induced to invest in environmental technologies, because they optimally would not want to commit to a heavy irreversible investment that could turn out to be unprofitable in the event of a price fall. A comparative static analysis predicts that firms in industries with high output price volatility would be more reluctant to invest in environmental protection technologies and would be more willing to operate at low output levels (thus attaining low emission levels). Increases in the interest rate would also reduce optimal environmental investment levels.
(*Real Options; Environmental Economics; Capital Budgeting; Natural Resources*)

1. Introduction

In order to protect the environment, governments impose regulations. These regulations may be enforced in a command-control fashion, but there is a growing consensus that market oriented policies are a more efficient way to reduce contamination levels. Therefore, governments are increasingly setting pollution standards (or imposing taxes depending on pollution levels) and letting firms decide their investment and production policies as long as they abide by the regulations. In order to succeed through these market mechanisms, it is important to understand capital investment evaluation from a private perspective to define appropriate incentives that induce a sound environmental behavior.

There is a multitude of environmental problems associated with production facilities, and even more technologies that try to lessen their effects. Most technologies developed to reduce the environmental impact of production facilities require significant investment levels by the firm, increasing the operating costs and / or reducing the maximum production levels. We treat these environmental investments as irreversible investments under output price uncertainty. This approach is now becoming standard in the finance literature.

The last decade has seen a growing application of financial option theory to real-asset investments, giving rise to what has been called the real options literature. These studies consider a firm that makes some decisions contingent on the particular realizations of one or more relevant random variables. Examples of the analyzed options include the opening and closing of a one-stage firm (Brennan and Schwartz 1985) and of a two-stage firm (Cortazar and Schwartz 1993), the timing of the investment decision (McDonald and Siegel 1986), the investment schedule (Majd and Pindyck 1987, Pindyck 1988), the selection of alternative technologies (He and Pindyck 1989), the output levels under learning (Majd and Pindyck 1989), among others. They all assume sufficiently complete markets that allow hedging the uncertainty (with or without an associated risk premium) by investing in a portfolio of assets.

In this paper we are concerned with the optimal environmental investment decision by a firm that is confronted with an environmental regulation schedule linking maximum production levels and operating costs

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to the level of environmental investment made. We will consider a stochastic output price that induces a nontrivial investment schedule for a firm that is maximizing its economic value in the presence of this regulation.

The paper presents a model that determines when (at which output price level) it is optimum for a firm to invest in environmental technologies and which are the main parameters that affect this decision. Our analysis shows that firms require significantly high output price levels to be induced to invest in environmental technologies, because they optimally would not want to commit to a heavy irreversible investment that could turn out to be unprofitable in the event of a price fall. A comparative static analysis predicts that firms in industries with high output price volatility would be more reluctant to invest in environmental protection technologies and would be more willing to operate at low output levels (thus attaining low emission levels). Also increases in the interest rate would reduce optimal environmental investment levels.

The analytical framework used in this paper is closely related to the one in Cortazar and Schwartz (1993). Even though both papers deal with optimal output levels, Cortazar and Schwartz (1993) look at the optimal levels of production in two different stages of a firm, whereas this paper establishes one optimal level of production and an optimal level of investment. In addition, the problems addressed are quite different. Cortazar and Schwartz (1993) model a firm as a two-stage process with bounded output rates, in which the output of the first stage can be held as an intermediate inventory (work-in-process). The firm can then be thought of as a compound option, in which the exercise of the option to produce in the first stage gives the option to finish the work-in-process and sell the output as its final payoff. The main focus of that paper is to explain how the existence of intermediate inventories may arise as an optimal investment strategy for exploiting possible future price increases, and to analyze the effect of interest rate changes on aggregate inventory levels. This paper is mainly concerned with the factors that affect a firm's optimal investment in environmental technologies.

The paper is organized as follows. In §2 we present a brief discussion on environmental regulations on copper smelters. In §3 we describe a continuous time investment model that allows a firm, at each point in time, to determine its optimal output level and also its optimal environmental investment schedule to expand (or maintain in the presence of regulatory conditions) its capacity. Our model assumes continuous investment decisions and therefore the optimal control variables are the output rate and the investment rate that maximize the value of the firm. This model does not have an analytical solution, but can be solved numerically.

In §4 we assume that a smelter is currently operating with a given capacity constraint set by regulation and must decide when it would be optimal to expand its capacity to a new level by making a fixed and discrete environmental investment. We solve this problem analytically and determine the critical price at which it would be optimal for the smelter to expand capacity. Finally in §5 we apply the discrete investment problem to a real case and discuss its results.

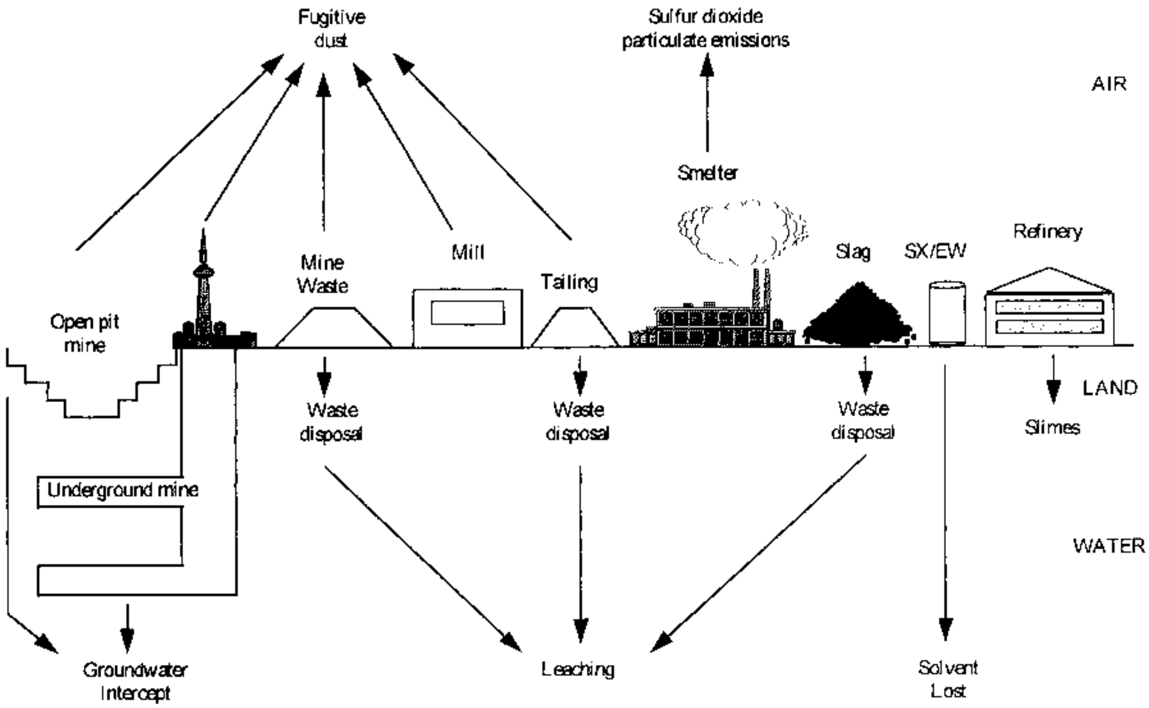
2. Smelter Environmental Regulations in Copper Production

In the following sections we provide a general framework for analyzing investments induced by regulatory requirements, which may affect the optimal operation of a production facility. For concreteness, in this section we describe the main environmental problems related to mining and specifically to air pollution in copper smelters. These problems are generating regulations designed primarily to induce environmental investments that reduce emissions per unit of production. However, private firms may delay these investments, choosing instead to reduce production levels or to pay penalties for regulation violations. This strategy may prove to be optimal from a private perspective and should be considered when setting environmental regulations designed to induce environmental investments.

Figure 1 shows a simplified primary copper production diagram. In order to obtain refined copper the ore has to be extracted from mine pit and go through the Mill, Smelter and Refinery. In each of these processes, there are several environmental impacts on air quality, surface and ground water quality, and the land. One of the most important problems is sulfur dioxide emissions coming from copper smelters which affects the acid rain phenomena. By 1972, in Canada, the mining

Figure 1 Environmental Impacts of Copper Production

Source: Office of Technology Assessment, Congress of United States.



industry was responsible for around 53% of sulfur dioxide emissions (MacDonell 1989).

To curb these environmental impacts most countries have been enacting environmental regulations. For example, Canada has been setting a schedule of restrictions, the latest being the Canadian Sulfur Dioxide Control Strategy which was expected to lower mining sulfur dioxide emission share to around 37% in 1994 (Federal/Provincial Research and Monitoring Coordinating Committee 1990). In the United States, the Clean Air Act of 1970, 1977 and 1990 (General Accounting Office 1986) has established stringent standards on emissions, specially on sulfur dioxide, and has been successful in lowering total emission levels. Other copper producing countries have also enacted or are planning to impose similar environmental regulations.

Even though there is consensus on the effectiveness of these environmental regulations for lowering total sulfur dioxide emissions, it is not so clear that these new levels have been attained through socially optimal economic decisions. While some of the emission lowering has been obtained after heavy investments with major process changes, a significant proportion of the reduction is due to the reduction of production levels. For example, between 1970 and 1984, 44% of emission reduction in the United States was due to environmental investments while 56% was due to production reduction (General Accounting Office 1986).

On the other hand, smelters that achieved regulatory compliance through environmental investments have been affected by significant cost increases both in capital expenditures and in operating costs. For example, the average operating cost increase in 1987 for U.S. smelters to satisfy environmental regulations amounted to US$ 0.032 per pound of copper, or around 26% of the operating costs. Including capital costs, total environmental costs could reach US$ 0.090 (Rothfeld and Towle 1989). Thus, environmental regulations have had a profound effect on operating policies of copper smelters as well as of other mining operations.

We argue that it is likely that some of the production reduction of US smelters may prove to be socially nonoptimal and induced by the regulatory structure. Thus, understanding private investment behavior in the presence of environmental regulation may prove to be helpful in designing regulatory structures that induce socially optimal behavior.

We propose a real options approach to elicit optimal environmental investment behavior. To illustrate, we consider the case of a smelter that has been subject to an emission regulation. The imposition of the emission regulation effectively allows for one of two possible reactions: to invest in environmental technologies that could permit for increases in maximum production levels or, alternatively, not to invest and be restricted to lower production levels. In the next section, we consider a model that allows for continuous environmental investments (and therefore for continuous increases of output levels). This model does not have an analytical solution, but may be solved by resorting to numerical methods. In §4, we solve for the discrete case in which the smelter must decide when it would be optimal to make the environmental investment in order to increase the allowed production level. This model will show how firms may optimally appear to underinvest if copper prices are highly volatile.

3. The Continuous Environmental Investment Schedule

In this section, we analyze how to determine the optimal environmental investment schedule for a smelter that can, at each point in time, make infinitesimal investments to lower emissions, thus being able to expand its production capacity while still meeting environmental regulations. Our economic setting is similar to the one described by Brennan and Schwartz (1985), only that in ours, the manager has two control variables available for maximizing the smelter value: the output rate and the environmental investment schedule. Thus, we are able not only to value a given investment, but also to determine and value the optimal sequence of investments that should be undertaken.

We approach this problem using the continuous time framework of option pricing. In this framework assets are priced consistent with the absence of arbitrage in the economy, i.e., if a portfolio of risky assets has a return that is instantaneously riskless, the return on this portfolio must be the risk free rate of interest. Prices are assumed to follow geometric Brownian motions, which essentially implies that the distribution of futures prices is lognormal. Using Itô calculus to compute the total differential of a function of stochastic variables, the outcome of the analysis is a partial differential equation for the value of the real or financial option. This approach is very closely related to stochastic optimal control for Markov diffusion processes using dynamic programming methods (see Fleming and Rishel 1975).

We start by setting our notation:

- S: spot unit price of smelter output[1];
- X: spot unit price of smelter input (copper concentrate);
- I^a: accumulated environmental investment capital;
- q: rate of production, the first control in the model (constrained to be between 0 and a maximum amount that depends on the accumulated capital, $q^{\max}(I^a)$);
- i: rate of environmental investment, the second control in the model (constrained to be between 0 and a technological maximum, $i^{\max}$);
- $A(q, I^a)$: average unit cost of production[2] if rate of production is q;
- cS: convenience yield on holding one unit of finished good, where c is assumed constant;
- ζX: convenience yield on holding one unit of concentrate, where ζ is assumed constant;
- r: risk free rate of return, assumed constant;
- δ: rate of accumulated environmental investment capital depreciation;
- $F(S, \tau)$: futures price for delivery of one unit of the commodity at time T where $\tau = T - t$;
- $dI^a = (i - \delta I^a)dt$, change in accumulated environmental investment capital;
- $H(S, X, I^a; \phi)$: value of the firm under the operating policy ϕ; and
- ϕ: operating policy which describes q and i for any state variable (S, X, I^a) values.

[1] We will consider that the smelter facility also refines the copper. This is a reasonable approximation given that the refining cost is very low compared to the one at the smelter.

[2] This cost does not include the input cost of buying copper concentrate.

Additions in accumulated environmental investment increases the allowed maximum output rate for a given environmental regulation. So, $q^{max}(I^a)$.

The valuation model considers that the spot prices S, for a unit of output, and X, for a unit of concentrate, are determined competitively and follow Brownian motions. Let

$$\frac{dS}{S} = \mu_1 dt + \sigma_1 dz_1$$

$$\frac{dX}{X} = \mu_2 dt + \sigma_2 dz_2,$$

where μ_1 and μ_2 are the instantaneous trends; σ_1 and σ_2 are the known instantaneous standard deviations; t represents callendar time; dz_1 and dz_2 are increments to standard Gauss-Wiener processes with correlation ρ.

Another critical assumption is the existence of a sufficiently complete market to allow the firm to hedge the output price risk and concentrate price risk. For this matter, we assume the existence of a market for futures contracts on the output (copper). The futures price $F(S, \tau)$ is assumed to be a function of the spot price S and time to maturity τ of the contract. Given that there is no futures market for copper concentrate, to hedge concentrate price risk we must assume that the concentrate price, X, is spanned by the set of existing traded assets in the economy, and can, therefore, be replicated by a dynamic portfolio with price Y and returns perfectly correlated with those of X. So

$$\frac{dY}{Y} = \mu_Y dt + \sigma_Y dz_2.$$

Using Itô's Lemma, we obtain:

$$dF = F_S dS - F_\tau dt + \tfrac{1}{2} F_{SS} S^2 \sigma_1^2 dt. \qquad (1)$$

Assuming that the holder of a unit of output receives a convenience yield (defined as a flow of services unavailable to the holder of a futures contract) equivalent to cS, the instantaneous return on holding one unit of output, while being short on $(F_S)^{-1}$ futures contracts, is

$$\frac{dS + cSdt - (F_S)^{-1} dF}{S},$$

which is riskless and should be equal to rdt. Using Equation (1) and the above equality, we obtain

$$\tfrac{1}{2} F_{SS} S^2 \sigma_1^2 + SF_S(r - c) - F_\tau = 0, \qquad (2)$$

subject to the boundary condition $F(S, 0) = S$.

The next step is to derive the partial differential equation that describes the value of the smelter. Let the value of the smelter H, be represented by an unknown function of the unit prices of copper and concentrate, and also of the accumulated environmental capital. Taking the Itô's differential of the firm value $H(S, X, I^a)$, we have

$$dH = H_S dS + H_X dX + H_{I^a} dI^a + \tfrac{1}{2} H_{SS}(dS)^2 + \tfrac{1}{2} H_{XX}(dX)^2 + H_{SX} dS dX.$$

The cash flows or *dividends* that accrue to the owner of the firm, are $(q(S - X - A(q, I^a)) - i)dt$. Thus, the return on a portfolio consisting in one long position on the firm, a (H_S / F_S) short position on the futures and a $(XH_X\sigma_2 / Y\sigma_Y)$ short position on the Y, is

$$dH + (q(S - X - A(q, I^a)) - i)dt - \frac{H_S}{F_S} dF - \left(\frac{XH_X\sigma_2}{Y\sigma_Y}\right) dY,$$

which is nonstochastic and should be equal to

$$r\left(H - \left(\frac{XH_X\sigma_2}{Y\sigma_Y}\right) Y\right) dt.$$

Rearranging the above equation,[3] we obtain the following differential equation that describes the value of the firm as a function of the state variables, S, I^a, and X:

$$\tfrac{1}{2} H_{SS} S^2 \sigma_1^2 + \tfrac{1}{2} H_{XX} X^2 \sigma_2^2 + H_{SX} XS\rho\sigma_1\sigma_2 + (i - \delta I^a) H_{I^a} + (r - c) SH_S + (r - \zeta) XH_X + (q(S - X - A(q, I^a)) - i) - rH = 0$$

The optimal operating policy and the value of the firm under this optimal policy may be obtained by solving

$$\max_{q,i} [\tfrac{1}{2} H_{SS} S^2 \sigma_1^2 + \tfrac{1}{2} H_{XX} X^2 \sigma_2^2 + H_{SX} XS\rho\sigma_1\sigma_2 + (i - \delta I^a) H_{I^a} + (r - c) SH_S + (r - \zeta) XH_X + (q(S - X - A(q, I^a)) - i) - rH] = 0,$$

[3] See Appendix 1.

subject to the following constraints on the controls:

$$q \le q^{\max}(I^a); \quad (3)$$

$$i \le i^{\max}, \quad (4)$$

$$q \ge 0, \quad (5)$$

$$i \ge 0. \quad (6)$$

The first order optimality conditions for this problem are obtained by taking partial derivatives of the partial differential equation with respect to the two stochastic controls q and i. The first of these conditions may be stated as

$$S - X - A(q, I^a) - q\frac{\partial A(q, I^a)}{\partial q}\begin{cases} \le 0 & \text{if } q = 0, \\ = 0 & \text{if } 0 < q < q^{\max}, \\ \ge 0 & \text{if } q = q^{\max}. \end{cases}$$

The first three terms in the expression in the left hand side represent the marginal benefit (profit) of producing one additional unit, which is given by the unit price of the smelter output minus the sum of the unit price of the smelter input and the average cost of production. The fourth term represents the marginal cost of increasing production by one unit when the level of production is q, and is given by the marginal increase in unit costs times the number of units produced. The expression states the Kuhn-Tucker conditions for a maximum: for an interior solution the left hand side will be equal to zero, for a maximum at the lower constraint ($q = 0$) it will be negative and, for a maximum at the upper constraint ($q = q^{\max}$) it will be positive.

The second optimality condition may be stated as

$$H_{I^a} - 1\begin{cases} \le 0 & \text{if } i = 0, \\ = 0 & \text{if } o < i < i^{\max}, \\ \ge 0 & \text{if } i = i^{\max}. \end{cases}$$

The first expression in the left hand side represents the marginal benefit (increase in the value of the firm) of increasing environmental investment by one dollar and the second term represents the marginal cost of doing so (that is, one dollar). The expression states the Kuhn-Tucker conditions for a maximum with a lower ($i = 0$) and an upper ($i = i^{\max}$) constraint.

This model in its general formulation does not have an analytical solution, but can be solved by resorting to numerical methods. In the next section we impose some additional assumptions that allows us to obtain analytical solutions.

4. The Discrete Environmental Investment Schedule

In this section we assume that existing technological, economical or regulatory considerations effectively restrict production capacity of a smelter to be one of two values: $q^{\max 1}$ or $q^{\max 2}$, with $q^{\max 1} \le q^{\max 2}$. The maximum output level that satisfies environmental regulations with current technology is $q^{\max 1}$, (with unit cost of production A^1). The smelter is considering when it would be optimal to expand to a capacity of $q^{\max 2}$ (with unit cost of production A^2) by investing an additional I^a in more environmentally efficient processes. In order to obtain analytical solutions to our discrete environmental investment problem we make the additional assumption that the concentrate price is a fraction $(1 - \lambda)$ of the copper price, or $X = (1 - \lambda)S$, thus there is only one source of price uncertainty in the model. In Appendix 2 we present an alternative model which does not require perfect correlation between S and X, but restricts the functional form of A and I^a.

Given that we have constant returns to scale, the optimal control solution is bang-bang, which means that the smelter produces either zero or at maximum capacity. We define:

- $W^1(S)$: value of the firm when it is optimal to be closed ($q = 0$) and the smelter has capacity $q^{\max 1}$;
- $W^2(S)$: value of the firm when it is optimal to be closed ($q = 0$) and the smelter has capacity $q^{\max 2}$;
- $V^1(S)$: value of the firm when it is optimal to be open and the smelter has capacity $q^{\max 1}$;
- $V^2(S)$: value of the firm when it is optimal to be open and the smelter has capacity $q^{\max 2}$.

Using a procedure similar to the one described in the past section it can be shown that the value of the smelter under these conditions must satisfy the following system of differential equations:

$$\tfrac{1}{2}W^1_{SS}S^2\sigma^2 + (r - c)SW^1_S - rW^1 = 0,$$

$$\tfrac{1}{2}V^1_{SS}S^2\sigma^2 + q^{\max 1}(\lambda S - A^1) + (r - c)SV^1_S - rV^1 = 0,$$

$$\tfrac{1}{2}W^2_{SS}S^2\sigma^2 + (r - c)SW^2_S - rW^2 = 0,$$

$$\tfrac{1}{2}V^2_{SS}S^2\sigma^2 + q^{\max 2}(\lambda S - A^2) + (r - c)SV^2_S - rV^2 = 0,$$

subject to the following boundary conditions:

$$W^1(0) = 0, \tag{7}$$

$$W^1(S_1^*) = V^1(S_1^*), \tag{8}$$

$$W_S^1(S_1^*) = V_S^1(S_1^*), \tag{9}$$

$$V^2(S_2^*) - I^a = \begin{cases} V^1(S_2^*) & \text{if } S_2^* > S_1^*, \\ W^1(S_2^*) & \text{if } S_2^* < S_1^*, \end{cases} \tag{10}$$

$$V_S^2(S_2^*) = \begin{cases} V_S^1(S_2^*) & \text{if } S_2^* > S_1^*, \\ W_S^1(S_2^*) & \text{if } S_2^* < S_1^*, \end{cases} \tag{11}$$

$$V^2(S_3^*) = W^2(S_3^*), \tag{12}$$

$$V_S^2(S_3^*) = W_S^2(S_3^*), \tag{13}$$

$$W^2(0) = 0, \tag{14}$$

$$\lim_{S\to\infty} \frac{V^2(S)}{S} < \infty, \tag{15}$$

with the following critical output prices:

- S_1^*: Price over which the smelter is opened and under which it is closed if capacity is $q^{\max 1}$;
- S_2^*: Price over which the smelter is expanded from capacity $q^{\max 1}$ to capacity $q^{\max 2}$;
- S_3^*: Price over which the smelter is opened and under which it is closed if capacity is $q^{\max 2}$.

Because there are no opening or closing costs in our model, it is clear that $S_1^* = A^1/\lambda$ and $S_3^* = A^2/\lambda$. However, it is not trivial to determine S_2^* or the critical price at which it is optimal to invest I^a, which will allow for an expansion of capacity by $q^{\max 2} - q^{\max 1}$.

The general solution for the above system of differential equations are

$$W^1(S) = c_1 S^{d_1} + c_2 S^{d_2},$$

$$V^1(S) = c_3 S^{d_1} + c_4 S^{d_2} + q^{\max 1}\left(\frac{\lambda S}{c} - \frac{A^1}{r}\right),$$

$$W^2(S) = c_5 S^{d_1} + c_6 S^{d_2},$$

$$V^2(S) = c_7 S^{d_1} + c_8 S^{d_2} + q^{\max 2}\left(\frac{\lambda S}{c} - \frac{A^2}{r}\right),$$

with

$$d_1 = \alpha_1 + \alpha_2,$$

$$d_2 = \alpha_1 - \alpha_2,$$

$$\alpha_1 = \frac{1}{2} - \frac{(r-c)}{\sigma^2},$$

$$\alpha_2 = \sqrt{\left(\alpha_1^2 + \frac{2r}{\sigma^2}\right)}.$$

Boundary condition (7) implies that $c_2 = 0$, boundary condition (14) implies that $c_6 = 0$ and boundary condition (15) implies that $c_7 = 0$. The other six boundary conditions can be used to solve for the six remaining unknowns, namely c_1, c_3, c_4, c_5, c_8 and S_2^*. Solving the system of equations we obtain

$$c_4 = q^{\max 1} A^1 \lambda^{d_2}\left(\frac{\frac{(d_1-1)}{c} - \frac{d_1}{r}}{(d_2-d_1)A^{1^{d_2}}}\right),$$

$$c_5 = q^{\max 2} A^2 \lambda^{d_1}\left(\frac{\frac{(d_2-1)}{c} - \frac{d_2}{r}}{(d_2-d_1)A^{2^{d_1}}}\right),$$

$$c_8 = q^{\max 2} A^2 \lambda^{d_2}\left(\frac{\frac{(d_1-1)}{c} - \frac{d_1}{r}}{(d_2-d_1)A^{2^{d_2}}}\right).$$

S_2^* can be computed by solving the following equation:

$$(q^{\max 2} A^{2^{1-d_2}} - q^{\max 1} A^{1^{1-d_2}})\left(\frac{\frac{(d_1-1)}{c} - \frac{d_1}{r}}{-d_1}\right)(\lambda S_2^*)^{d_2}$$

$$= \frac{1}{r}(q^{\max 2} A^2 - q^{\max 1} A^1) + I^a$$

$$+ \frac{\lambda S_2^*}{c}(q^{\max 2} - q^{\max 1})\left(\frac{1}{d_1} - 1\right).$$

Finally,

$$c_3 = (c_8 - c_4)\frac{d_2}{d_1} S_2^{*\,d_2-d_1} + \frac{\lambda S_2^{*\,1-d_1}}{d_1 c}(q^{\max 2} - q^{\max 1}),$$

$$c_1 = c_3 + c_4\left(\frac{A^1}{\lambda}\right)^{d_2-d_1} + q^{\max 1}\lambda^{d_1} A^{1^{1-d_1}}\left(\frac{1}{c} - \frac{1}{r}\right).$$

This completes the solution to our environmental investment problem.

5. Discussion and Conclusions

To illustrate our solution, we use approximate figures available for the environmental retrofit performed by the Chino smelter in 1987 (Rothfeld and Towle 1989). By investing \$138 million, the smelter was able to increase production by 70% while satisfying environmental regulations. Approximate parameter values are provided below[4]:

$$q^{\max 1} = 220 \text{ million pounds per year,}$$

$$q^{\max 2} = 374 \text{ million pounds per year,}$$

$$A_1 = 0.148 \text{ dollars,}$$

$$A_2 = 0.18 \text{ dollars,}$$

$$I^a = 138 \text{ million dollars,}$$

$$r = 0.1 \text{ per year,}$$

$$c = 0.04 \text{ per year,}$$

$$\sigma^2 = 0.08 \text{ per year,}$$

$$\lambda = 0.33.$$

Given these parameter values, we are interested in determining at which price level it is optimal for the smelter to expand its capacity from $q^{\max 1} = 220$ to $q^{\max 2} = 374$ million pounds, if the environmental investment required to do so amounts to $I^a = 138$ million dollars. We will determine this critical price, over which it becomes optimal to make the environmental investment, using four different approaches.

First, we use a simple Net Present Value approach and determine the price over which it is profitable to expand production. We define this critical price as $S_2^{*\mathrm{NPV}_1}$. In this case:

$$q^{\max 1}\left(\frac{\lambda S_2^*}{c} - \frac{A_1}{r}\right) = q^{\max 2}\left(\frac{\lambda S_2^*}{c} - \frac{A_2}{r}\right) - I^a,$$

which gives $S_2^{*\mathrm{NPV}_1} = 0.382$.

This approach implicitly assumes that there is no timing option and the smelter is forced to either expand now or never. Notice also that prices are expected to increase on risk adjusted terms, since the interest rate is much larger than the convenience yield. This is the reason why, if forced to make a decision, the smelter accepts to expand now, even for current prices (\$0.382) which make the environmental investment clearly unprofitable.

A second approach considers the timing option by allowing the smelter to postpone environmental investment until the marginal cost of the environmental investment equals the marginal benefit. We define this critical price as $S_2^{*\mathrm{NPV}_2}$. In this case:

$$q^{\max 1}(\lambda S_2^* - A_1) = q^{\max 2}(\lambda S_2^* - A_2) - I^a r,$$

which gives $S_2^{*\mathrm{NPV}_2} = 0.955$.

This approach considers the option to postpone the environmental investment under deterministic prices, disregarding the additional timing option available if prices are assumed stochastic. The critical price using this approach is significantly higher than using the simple Net Present Value approach.

Both of these approaches assume that there is no closing option for the smelter and that it must continue to operate even in the event of unprofitable copper prices.

A third, and more sophisticated approach (even though still incorrect) would be to use the Brennan and Schwartz (1985) option model approach to value the smelter, as a function of the spot price, assuming initially a fixed capacity of $q^{\max 1}$, and to recalculate this value for a smelter with a fixed capacity of $q^{\max 2}$. Given these two value functions, the solution to this problem would be to determine the price level at which the difference of these two functions justifies investing I^a. We define this critical price as $S_2^{*\mathrm{Option}_1}$.

It can be shown that $S_2^{*\mathrm{Option}_1}$ would be equal to \$0.243 in this case. Using this approach amounts to recognizing that prices are volatile and that the smelter always has the option to close down in the event that its operation becomes unprofitable, but disregards the timing option of the environmental investment. Notice that the critical price is even lower than the one obtained in our first approach. The reason is that while neither approach considers the timing option, the inclusion of the closing option, which is more valuable the higher the output capacity, makes it profitable to expand even at lower prices.

[4] Some of the parameter values are assumptions or industry averages. In particular we computed λ, or the fraction of the copper price that can be considered as value added by the smelter operation as the relative cost of the smelter versus the other operations.

Finally, the fourth approach is the one proposed in this paper. It considers both the closing and timing option under stochastic prices available to the smelter. Thus, while the smelter has a capacity of $q^{\max 1}$ it has the option to expand to $q^{\max 2}$ at any point in time, and therefore the value of the smelter without the expansion is higher than the one computed disregarding this option. In addition, it is clear that the value of the expanded smelter is the same as the one we computed before for the smelter with capacity $q^{\max 2} = 374$ million pounds because once the smelter is expanded it has no option to modify its capacity. Thus the critical price at which increasing the capacity justifies the environmental investment should be higher than the one we computed earlier. We define our critical price as $S_2^{*\text{Option}_2}$, which turns out to be \$1.344, the highest critical price of all four approaches.

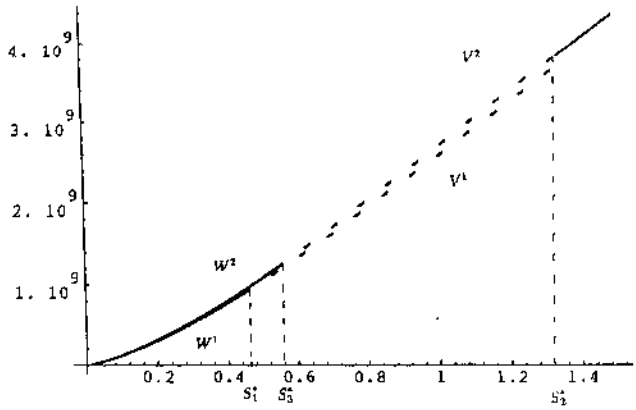
Figure 2 plots the value of the firm for different spot prices and production capacities. W^1 represents the value of the smelter that is currently closed and has a capacity of $q^{\max 1}$. At $S_1^* = 0.448$ it becomes optimal to begin production, and the value of the smelter is V^1. As stated before, the critical price for expanding capacity in our model is $S_2^* = 1.344$. Once the smelter is expanded, whenever the spot price exceeds $S_3^* = 0.545$ the smelter should remain open, and its value is V^2. Spot prices below $S_3^* = 0.545$ induce the closing of the smelter, which has a value of W^2.

Comparative static analysis of our results show that the rise of the critical price increases further with volatility of copper prices and with interest rates. This should be no surprise given that option values increase with both volatility and interest rates, so the opportunity cost of "killing" the option by making the environmental investment becomes higher. Moreover, if we compute our solution with price volatility approaching zero, our results converge to those of the second approach because under deterministic prices, the closing option becomes valueless and the timing option approaches the one computed before.

One of the main conclusions of this paper is that firms under emission restrictions designed to encourage environmental investments may optimally choose to cut back production instead of engaging in heavy environmental investments. The reason is that firms consider both the closing and the timing options available and require very high returns on environmental investment before exercising the option to invest. This effect should be taken into consideration when designing environmental regulations.[5]

Figure 2 **Value of the Smelter for Different Regimes**

W^1: smelter is closed and has capacity $q^{\max 1}$; V^1: smelter is open and has capacity $q^{\max 1}$; W^2: smelter is closed and has capacity $q^{\max 2}$; V^2: smelter is closed and has capacity $q^{\max 2}$.



[5] This is a revised version of our earlier paper entitled "Natural Resource Investments: Determining the Total Cost of Environment Protection Technologies." Cortazar acknowledges the financial support from FONDECYT, FONDEF and Fundación Dictuc. This paper was presented at the meetings of the Institute of Management Science (Alaska, 1994), the Econometric Society (Tokyo, 1995), the European Finance Association (Milan, 1995), the French Finance Association (Geneva, 1996), and the European Economic Association (Istambul, 1996). We thank participants in those conferences for helpful comments.

Appendix 1

In this appendix, we derive the differential equation of the firm value.

Taking the Itô's differential of the firm value $H(S, X, I^a)$, we have

$$dH = H_S dS + H_X dX + H_{I^a} dI^a + \tfrac{1}{2} H_{SS}(dS)^2 + \tfrac{1}{2} H_{XX}(dX)^2 + H_{SX} dS dX.$$

The cash flows or *dividends* that accrue to the owner of the firm, are: $(q(S - X - A(q, I^a)) - i)dt$. Thus, the return on a portfolio consisting of one long position on the firm, a (H_S / F_S) short position on the futures and a $(X H_X \sigma_2 / Y \sigma_Y)$ short position on the Y, is

$$dH + (q(S - X - A(q, I^a)) - i)dt - \frac{H_S}{F_S} dF - \left(\frac{X H_X \sigma_2}{Y \sigma_Y}\right) dY,$$

which is nonstochastic and should be equal to

$$r\left(H - \left(\frac{X H_X \sigma_2}{Y \sigma_Y}\right) Y\right) dt.$$

Rearranging the above equation, we obtain the following differential equation that describes the value of the firm as a function of the state variables, S, I^a, and X.

$$\frac{1}{2}H_{SS}S^2\sigma_1^2 + \frac{1}{2}H_{XX}X^2\sigma_2^2 + H_{SX}XS\rho\sigma_1\sigma_2 + (i - \delta I^a)H_{I^a}$$
$$+ (r - c)SH_S + \left(\mu_2 - \frac{\sigma_2}{\sigma_Y}(r - \mu_Y)\right)XH_X$$
$$+ (q(S - X - A(q, I^a)) - i) - rH = 0$$

By the Capital Asset Pricing Model, the risk-adjusted expected return on Y is $r_Y = \mu_Y = r + \theta\rho_{YM}\sigma_Y$ and the risk-adjusted expected return on X is $r_2 = \mu_2 + \zeta = r + \theta\rho_{XM}\sigma_2$, where $\theta = (r_M - r)/\sigma_M$ is the market price of risk and ρ_{YM} and ρ_{XM} are the instantaneous correlation of Y and X return with the market portfolio return, rearranging the above equation,

$$\frac{1}{2}H_{SS}S^2\sigma_1^2 + \frac{1}{2}H_{XX}X^2\sigma_2^2 + H_{SX}XS\rho\sigma_1\sigma_2$$
$$+ (i - \delta I^a)H_{I^a} + (r - c)SH_S + (r - \zeta)XH_X$$
$$+ \frac{\sigma_2}{\sigma_M}(r_M - r)(\rho_{XM} - \rho_{YM}))XH_X$$
$$+ (q(S - X - A(q, I^a)) - i) - rH = 0$$

with $\rho_{XM} = \rho_{YM} = dz_1dz_2$, then:

$$\frac{1}{2}H_{SS}S^2\sigma_1^2 + \frac{1}{2}H_{XX}X^2\sigma_2^2 + H_{SX}XS\rho\sigma_1\sigma_2 + (i - \delta I^a)H_{I^a}$$
$$+ (r - c)SH_S + (r - \zeta)XH_X$$
$$+ (q(S - X - A(q, I^a)) - i) - rH = 0.$$

Appendix 2. An Alternative Model for the Discrete Environmental Investment Schedule

In this section we assume that existing technological, economical or regulatory considerations effectively restrict production capacity of a smelter to be one of two values: $q^{\max 1}$ or $q^{\max 2}$, with $q^{\max 1} \leq q^{\max 2}$. In order to obtain analytical solutions to our discrete environmental investment problem we make the additional assumption that the unit cost of production A^i is a fraction a^i of the concentrate price. Moreover, we assume that the environmental investment is a function of the concentrate price, kX.

In this model there are two sources of risk, S and X, which follow Brownian motions

$$\frac{dS}{S} = \mu_1 dt + \sigma_1 dz_1,$$

$$\frac{dX}{X} = \mu_2 dt + \sigma_2 dz_2,$$

where μ_1 and μ_2 are the instantaneous trends; σ_1 and σ_2 are the known instantaneous standard deviations; t represents time; dz_1 and dz_2 are increments to standard Gauss-Wiener processes with correlation ρ.

The maximum output level that satisfies environmental regulations with current technology is $q^{\max 1}$, (with unit cost of production a^1X). The smelter is considering when would it be optimal to expand to a capacity of $q^{\max 2}$ (with unit cost of production a^2X) by investing an additional I^a in a more environmentally efficient process.

We define:

- $W^1(S, X)$: value of the firm when it is optimal to be closed ($q = 0$) and the smelter has capacity $q^{\max 1}$;
- $W^2(S, X)$: value of the firm when it is optimal to be closed ($q = 0$) and the smelter has capacity $q^{\max 2}$;
- $V^1(S, X)$: value of the firm when it is optimal to be open and the smelter has capacity $q^{\max 1}$;
- $V^2(S, X)$: value of the firm when it is optimal to be open and the smelter has capacity $q^{\max 2}$.

Using a procedure similar to the one described earlier, it can be shown that the value of the firm under these conditions must satisfy the following equations:

$$\frac{1}{2}V_{SS}^1S^2\sigma_1^2 + \frac{1}{2}V_{XX}^1X^2\sigma_2^2 + V_{SX}^1XS\rho\sigma_1\sigma_2 + (r - c)SV_S^1$$
$$+ (r - \zeta)XV_X^1 + q^{\max 1}(S - X(1 + a^1)) - rV^1 = 0,$$
$$\frac{1}{2}W_{SS}^1S^2\sigma_1^2 + \frac{1}{2}W_{XX}^1X^2\sigma_2^2 + W_{SX}^1XS\rho\sigma_1\sigma_2$$
$$+ (r - c)SW_S^1 + (r - \zeta)XW_X^1 - rW^1 = 0,$$
$$\frac{1}{2}V_{SS}^2S^2\sigma_1^2 + \frac{1}{2}V_{XX}^2X^2\sigma_2^2 + V_{SX}^2XS\rho\sigma_1\sigma_2 + (r - c)SV_S^2$$
$$+ (r - \zeta)XV_X^2 + q^{\max 1}(S - X(1 + a^2)) - rV^2 = 0,$$
$$\frac{1}{2}W_{SS}^2S^2\sigma_1^2 + \frac{1}{2}W_{XX}^2X^2\sigma_2^2 + W_{SX}^2XS\rho\sigma_1\sigma_2$$
$$+ (r - c)SW_S^2 + (r - \zeta)XW_X^2 - rW^2 = 0,$$

subject to the following boundary conditions:

$$W^1(0) = 0, \tag{16}$$

$$W^1(S_1^*(X), X) = V^1(S_1^*(X), X), \tag{17}$$

$$W_S^1(S_1^*(X), X) = V_S^1(S_1^*(X), X), \tag{18}$$

$$V^2(S_2^*(X), X) - I^a = \begin{cases} V^1(S_2^*(X), X) & \text{if } S_2^*(X) > S_1^*(X), \\ W^1(S_2^*(X), X) & \text{if } S_2^*(X) < S_1^*(X), \end{cases} \tag{19}$$

$$V_S^2(S_2^*(X), X) = \begin{cases} V_S^1(S_2^*(X), X) & \text{if } S_2^*(X) > S_1^*(X), \\ W_S^1(S_2^*(X), X) & \text{if } S_2^*(X) < S_1^*(X), \end{cases} \tag{20}$$

$$V^2(S_3^*(X), X) = W^2(S_3^*(X), X), \tag{21}$$

$$V_S^2(S_3^*(X), X) = W_S^2(S_3^*(X), X), \tag{22}$$

$$W^2(0) = 0, \tag{23}$$

$$\lim_{S \to \infty} \frac{V^2(S, X)}{S} < \infty. \tag{24}$$

With the following critical boundary prices, that must be found at part of the solution:

- $S_1^*(X)$: Price over which the smelter is opened and under which it is closed if capacity is q^{max1} and concentrate price is X.
- $S_2^*(X)$: Price over which the smelter is expanded from capacity q^{max1} to capacity q^{max2} and concentrate price is X.
- $S_3^*(X)$: Price over which the smelter is opened and under which it is closed if capacity is q^{max2} and concentrate price is X.

We define $1 + a^1 = \bar{A}^1$ and $1 + a^2 = \bar{A}^2$.

Because there are no opening or closing costs in our case, it is clear that

$$S_1^*(X) = X\bar{A}^1,$$

$$S_3^*(X) = X\bar{A}^2.$$

However it is not trivial to determine $S_2^*(X)$.

We can always define a new function h^x such that $Xh^x(S/X) = H^x(S, X)$, and substitute this expression into the above differential equations, obtaining:

$$\tfrac{1}{2}w_{ZZ}^{x1}Z^2\sigma_Z^2 + (\zeta - c)Zw_Z^{x1} - \zeta w^{x1} = 0,$$

$$\tfrac{1}{2}v_{ZZ}^{x1}Z^2\sigma_Z^2 + (q(Z - \bar{A}^1)) + (\zeta - c)Zv_Z^{x1} - \zeta v^{x1} = 0,$$

$$\tfrac{1}{2}w_{ZZ}^{x2}Z^2\sigma_Z^2 + (\zeta - c)Zw_Z^{x2} - \zeta w^{x2} = 0,$$

$$\tfrac{1}{2}v_{ZZ}^{x2}Z^2\sigma_Z^2 + (q(Z - \bar{A}^2)) + (\zeta - c)Zv_Z^{x2} - \zeta v^{x2} = 0,$$

in which $Z = S/X$ and, $\sigma_Z^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$ is the volatility of Z.

The general solution for the above differential equations is:

$$w^{x1}(X, Z) = (c_1Z^{d_1} + c_2Z^{d_2}),$$

$$v^{x1}(X, Z) = \left(c_3Z^{d_1} + c_4Z^{d_2} + q^{max1}\left(\frac{Z}{c} - \frac{\bar{A}^1}{\zeta}\right)\right),$$

$$w^{x2}(X, Z) = (c_5Z^{d_1} + c_6Z^{d_2}),$$

$$v^{x2}(X, Z) = \left(c_7Z^{d_1} + c_8Z^{d_2} + q^{max2}\left(\frac{Z}{c} - \frac{\bar{A}^2}{\zeta}\right)\right),$$

substituting the value of w and v

$$W^1(X, Z) = X(c_1Z^{d_1} + c_2Z^{d_2}),$$

$$V^1(X, Z) = X\left(c_3Z^{d_1} + c_4Z^{d_2} + q^{max1}\left(\frac{Z}{c} - \frac{\bar{A}^1}{\zeta}\right)\right),$$

$$W^2(X, Z) = X(c_5Z^{d_1} + c_6Z^{d_2}),$$

$$V^2(X, Z) = X\left(c_7Z^{d_1} + c_8Z^{d_2} + q^{max2}\left(\frac{Z}{c} - \frac{\bar{A}^2}{\zeta}\right)\right),$$

with

$$d_1 = \alpha_1 + \alpha_2,$$

$$d_2 = \alpha_1 - \alpha_2,$$

$$\alpha_1 = \frac{1}{2} - \frac{(\zeta - c)}{\sigma_Z^2},$$

$$\alpha_2 = \sqrt{\left(\alpha_1^2 + \frac{2\zeta}{\sigma_Z^2}\right)}.$$

Boundary condition (16) implies that $c_2 = 0$, boundary condition (23) implies that $c_6 = 0$ and boundary condition (24) implies that $c_7 = 0$. The other six boundary conditions can be used to solve for the remaining unknowns:

$$c_4 = q^{max1}\bar{A}^1\left(\frac{\frac{(d_1 - 1)}{c} - \frac{d_1}{\zeta}}{(d_2 - d_1)\bar{A}^{1^{d_2}}}\right),$$

$$c_5 = q^{max2}\bar{A}^2\left(\frac{\frac{(d_2 - 1)}{c} - \frac{d_2}{\zeta}}{(d_2 - d_1)\bar{A}^{2^{d_1}}}\right),$$

$$c_8 = q^{max2}\bar{A}^2\left(\frac{\frac{(d_1 - 1)}{c} - \frac{d_1}{\zeta}}{(d_2 - d_1)\bar{A}^{2^{d_2}}}\right).$$

Substituting the value of I, kX: $S_2^*(X)$ or Z_2^* can be computed by solving the following equation:

$$(q^{max2}\bar{A}^{2^{1-d_2}} - q^{max1}\bar{A}^{1^{1-d_2}})\left(\frac{\frac{(d_1 - 1)}{c} - \frac{d_1}{\zeta}}{-d_1}\right)Z_2^{*d_2}$$

$$= \frac{1}{\zeta}(q^{max2}\bar{A}^2 - q^{max1}\bar{A}^1) + k + \frac{Z_2^*}{c}(q^{max2} - q^{max1})\left(\frac{1}{d_1} - 1\right).$$

Finally,

$$c_3 = (c_8 - c_4)\frac{d_2}{d_1}Z_2^{*d_2-d_1} + \frac{Z_2^{*1-d_1}}{d_1c}(q^{max2} - q^{max1}),$$

$$c_1 = c_3 + c_4\bar{A}^{1^{d_2-d_1}} + q^{max1}\bar{A}^{1^{1-d_1}}\left(\frac{1}{c} - \frac{1}{\zeta}\right).$$

This completes the solution to our alternative environmental investment model.

References

Brennan, M. J. and E. S. Schwartz 1985. Evaluating Natural Resource Investments. *Journal of Business*. **58** **2**.

Cortazar, G. and E. S. Schwartz 1993. A Compound Option Model of Production and Intermediate Inventories. *Journal of Business*. **66** **4**.

Federal / Provincial Research and Monitoring Coordinating Committee 1990. The 1990 Canadian Long-Range Transport of Air Pollutants and Acid Deposition Assessment Report. Part 2. *Emissions and controls*. Environment Canada, Atmospheric Environment Service, Downsview, Ontario, Canada.

Fleming, W. H. and R. W. Rishel 1975. *Deterministic and Stochastic Optimal Control*. Springer-Verlag, New York.

General Accounting Office 1986. Sulphur Dioxide Emissions From Non-Ferrous Smelters Have Been Reduced. Report to the Chairman, Subcommittee on Oversight and Investigations, Committee on Energy and Commerce, House of Representatives, GAO/RCED-86-91.

He, H. and R. S. Pindyck 1989. Investments in Flexible Production Capacity. MIT Working Paper WP-2102-89, MIT, Cambridge, MA.

MacDonell, L. J. 1989. Government Mandated Costs: The Regulatory Burden of Environmental, Health and Safety Standards. *Resources Policy*. March.

Majd, S. and R. S. Pindyck 1987. Time to Build, Option Value, and Investment Decisions. *Journal of Financial Economics*. **18** 7–27.

—— and ——. 1989. The Learning Curve and Optimal Production under Uncertainty. *Rand Journal of Economics*. **20** 3.

McDonald, R. and D. Siegel 1986. The Value of Waiting to Invest. *The Quarterly Journal of Economics*. Nov.

Pindyck, R. S. 1988. Irreversible Investment, Capacity Choice, and the Value of the Firm. *The American Economic Review*. **78** 5.

——. 1991. Irreversibility, Uncertainty, and Investment. *Journal of Economic Literature*. **XXIX** 3.

Rothfeld, L. and S. Towle 1989. Copper Regulatory Study. Environmental, Health and Safety Regulation Impacts on the Southwestern US Smelters and a Comparison with Chile. Unpublished. US Bureau of Mines, Minerals Availability Field Office, Washington, DC.

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