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Implementing a Real Option Model for Valuing an Undeveloped Oil Field

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We present a no arbitrage model for evaluating an undeveloped oil field and its numerical solution and implementation. The model assumes stochastic, but mean reverting, risk adjusted oil spot prices, and includes a timing investment option. The results of using this real options model for evaluating a case study of an undeveloped oil field show that a significant fraction of the oil field value may be provided by the flexibility of delaying development investment, and that this flexibility value decreases as oil price increases. Also the critical price for developing decreases with the available time to develop. We illustrate the use that practitioners could make of the real option methodology to value oil contingent reserves by presenting a user friendly computer program with graphical interface. This implementation could help petroleum companies to use this sophisticated valuation approach that could otherwise be of little practical relevance.

Key words: Real options, contingent claims, oil, software, numerical solution, timing option, undeveloped oil field, evaluation, finance.

INTRODUCTION

Undeveloped oil fields are valuable because someday they will be developed and their oil production sold. Traditional finance teachings state that to value oil fields firms should use the net present value approach which amounts to calculating the expected cash flows and use the market price of risk obtained by the use of some equilibrium model like the Capital Asset Pricing Model to determine a risk premium in the discount rate. Even though this approach appears simple to use, high historic volatility of oil prices induces some fundamental uncertainties difficult to solve using these discount cash flow methods, like when it is optimal to commit heavy investment resources to develop a somewhat marginal oil field and which oil price to use in valuing output. Moreover, oil price uncertainty translates into different risk premiums depending on the specific project operating leverage and its optimal investment and operating strategies.

An alternative approach to the net present value which improves on its weaknesses is the use of Option Theory. The last decade has seen an expanding work in this area, primarily analyzing optimal responses from the firm to different uncertainties and their impact on firm value and giving rise to what has been called Real Options Literature.

One of the seminal papers in the real options literature is Brennan and Schwartz (1985) in which an investment project to extract a finite natural resource is valued using arbitrage arguments. The source of uncertainty is output price and firms can respond by delaying production temporally or permanently depending on price levels and volatility. Given that futures prices are used, the project value is determined without requiring predictions on spot prices, one of the main sources of error on natural resource investment valuations. The optimal response by the firm (when to delay or resume production) is obtained jointly with firm value. In this setting the value of the firm is a function of the available resources to be extracted (output to be produced in the future) and the commodity spot price. In order to maximize its value, the firm manager can respond by modifying the production level.

Several other models have been developed to take into account the particular characteristics of investment projects. For example Majd and Pindyck (1989) include the effect of the learning curve by considering that accumulated production reduces unit costs. Others like McDonald and Siegel (1986) and Majd and Pindyck (1987) consider that the control variable is the investment rate, instead of the

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production level. Some include two production level controls like He and Pindyck (1992) with output for two different products or Cortázar and Schwartz (1993) with a two-stage production system. The source of uncertainty can be commodity price (Brennan and Schwartz, 1985; Cortazar *et al.*, 1996, or Paddock *et al.*, 1988), exchange rates (Dixit, 1989; Cortazar, 1992), or costs (Pindyck, 1993). Finally, models are tailored to different kinds of investment projects like copper mines (Brennan and Schwartz, 1985), oil reserves (Ekern, 1985; Paddock *et al.*, 1988), research and development (Schwartz and Moon, 1994), environmental technologies (Cortazar *et al.*, 1996), or flexible production (He and Pindyck, 1992).

Even though there is some consensus on the merits of the real option approach to valuing commodity linked investments, adoption of this new valuation technology has been rather slow, mostly due to the complexities of its theory and the difficulties of implementation. Most firms that use this new approach limit themselves to approximation formulas like Black and Scholes (1973), developed for financial claims, without properly adapting option theory to the particular economic setting found in commodity investments.

In this paper we present the results of implementing a real option model for valuing an undeveloped oil field. In the next section the model is presented which is followed by a description of its implementation and results. This paper reports some of the results of an industry–government–university funded multi-year project in Chile oriented to the development of methodologies and computer programs that could help in bridging the gap between theory and practice in natural resource investment valuation. In addition to this undeveloped oil field valuation model we have developed and implemented other types of investment project computer models (i.e. exploration, capacity expansion) and analyzed other natural resources in addition to petroleum, such as copper.

THE MODEL

In this section we present a model for valuing an undeveloped oil field contingent on oil spot price. We assume the project manager can decide optimally when to incur in a development investment which, once it is completed, initiates production. Like in most real oil field development projects, the option to delay investment expires and the firm must decide if it commits investment or gives up the concession. The oil field faces a competitive market for oil in which spot prices are uncertain, while firm characteristics like reserves, development investment requirements and production unit costs are considered deterministic. The model considers the oil field as a contingent claim on oil price and uses no arbitrage arguments to determine the functional relationship that must exist between both assets such that an investor is not able to profit without assuming risk.

First we define the stochastic process for the risk-adjusted spot price of oil. Following the continuous-time risk-neutral valuation approach standard in no arbitrage finance models, we define a risk-adjusted brownian motion for oil spot prices. We are interested in keeping the real option model simple, thus we define a one factor model for oil prices, instead of two-factor (Gibson and Schwartz, 1990) or three-factor (Cortazar and Schwartz, 1994) alternatives. However, we accommodate existing evidence of mean reversion in prices (Ross, 1995; Bessembinder *et al.*, 1995), by using a variable convenience yield that depends on the deviation of the spot price to a long term average price $\bar{S}$.

$$\frac{\mathrm{d}S}{S} = (r - c + \beta(\bar{S} - S))\mathrm{d}t + \sigma \mathrm{d}W \tag{1}$$

with the following notation:

S	Spot unit price of oil
$\bar{S}$	Long term average unit price of oil
β	Mean reversion parameter
r	Risk free rate of interest, assumed constant
c	Mean convenience yield on holding one unit of oil
σ	Instantaneous volatility of returns on holding one unit of oil
$\mathrm{d}W$	Increments to standard Gauss-Wiener process

The long term average unit price of oil, $\bar{S}$, is estimated from historical spot prices of oil. To estimate the parameters of the model that cannot be obtained by other sources (like β), we compare historical prices for a futures contract for delivery of oil in τ years, $F(S,\tau)$, with those from a theoretical pricing equation. (Cortazar *et al.* 1997) present a discussion on alternative mean reverting processes and estimation procedures.

The value of an oil future

Applying Itô's Lemma to the value of a future we obtain:

$$\mathrm{d}F(S,\tau) = F_s\mathrm{d}S - F_\tau\mathrm{d}t + \frac{1}{2}F_{ss}S^2\sigma^2\mathrm{d}t$$

It can be noted that returns on the futures contract are perfectly correlated with those on the spot. Thus, if we assume there are no arbitrage opportunities in the oil market, then an investor with a portfolio long in one unit of oil and short in $(F_s)^{-1}$ units of the futures contract has hedged his price risk and should earn the risk free interest rate:

$$\mathrm{d}S + (c - \beta(\bar{S} - S))S\mathrm{d}t - \frac{\mathrm{d}F}{F_s} = rS\mathrm{d}t \tag{2}$$

Replacing (1) in (2) we obtain the following differential equation for the value of a futures contract, if prices have mean reversion as modeled in (1).

$$\frac{1}{2}F_{ss}S^2\sigma^2\mathrm{d}t + SF_s(r - c + \beta(\bar{S} - S)) - F_\tau = 0 \tag{3}$$

Subject to the boundary condition:

$$F(S,0) = S$$

which means that at maturity the futures contract is equal to the spot price.

To solve numerically we impose the requirement that the futures price is linear on the spot price for low and high prices:

$$F_{SS}(0,\tau) = 0$$
$$F_{SS}(\infty,\tau) = 0$$

The value of an oil field

The oil field can be modeled as a real asset which could be in any of three different stages. Stage 1 is before committing to the development, stage 2 is during development, and stage 3 is during production. The value of the oil field at each of these stages can be computed contingent on oil price and other relevant variables as depicted in the Fig. 1.

The remaining notation for the model is:

$H(S,T^{\mathrm{i}})$	Oil field value for stage 1
$U(S,T^{\mathrm{d}})$	Oil field value for stage 2
$V(S,Q)$	Oil field value for stage 3
T^{i}	Remaining time until expiration of development option
T^{d}	Remaining time until investment is completed in stage 2
Q	Remaining reserves of oil
I	Present value of development investment in stage 2
q_{p}	Annual product rate in stage 3
a	Unit production cost in stage 3
cc	Annual capital costs in stage 3
dep	Annual depreciation in stage 3
t	Time
S^*_{ci}	Critical oil spot price, above which it is optimal to develop and under which it is optimal to wait

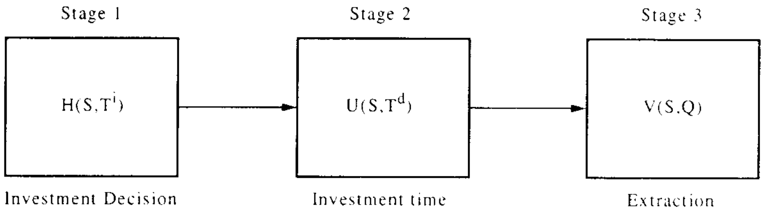


Fig. 1. Oil field model.

R	Taxes: royalty rate
Corp	Taxes: corporate tax rate
λ	Country risk premium

In order to solve the model we start by valuing the oil field in stage 3 and then work our way backwards (in a dynamic programming fashion) to solve for the value of the oil field in stages 2 and 1.

The value of an oil field in stage 3

Applying Itô's Lemma to the value of an oil field in stage 3, $V(S,Q)$, we obtain:

$$\mathrm{d}V(S,Q) = V_s \mathrm{d}S - q_p V_Q \mathrm{d}t + \frac{1}{2} V_{ss} S^2 \sigma^2 \mathrm{d}t$$

As opposed to the investor that holds a futures contract, the one who owns the oil field in stage 3 receives a cash flow which amounts to M:

$$\begin{aligned} M &= (q_p(S - a) - \mathrm{cc} - t1 - t2) \\ t1 &= R q_p S \\ t2 &= \mathrm{Corp}(q_p(S - a) - \mathrm{cc} - \mathrm{dep} - t1) \end{aligned}$$

If an investor takes a long position in the oil field and H_S/F_S short position in futures contracts he hedges his risk and should earn a return equal to the risk free interest plus a risk premium associated with the country where the oil field is located,* so:

$$\mathrm{d}V + M - \frac{V_S}{F_S}\mathrm{d}F = (r + \lambda)V\mathrm{d}t$$

Replacing we obtain the following expression for the value of an oil field in stage 3.

$$\frac{1}{2} V_{SS} S^2 \sigma^2 - q_p V_Q + M + (r - c + \beta(\bar{S} - S))S V_S - (r + \lambda)V = 0 \tag{4}$$

Subject to the boundary condition:

$$V(S,0) = 0$$

which means that when reserves are exhausted, the value of the oil field is zero.

To solve numerically we impose the requirement that the value of the oil field is linear on the spot price for low and high prices:

$$\begin{aligned} V_{SS}(0,Q) &= 0 \\ V_{SS}(\infty,Q) &= 0 \end{aligned}$$

*This country risk premium can be interpreted as the probability of expropriation (without compensation) per unit of time.

The value of an oil field in stage 2

Using similar arguments, it can be shown that the value of an oil field in stage 2 must satisfy the following differential equation and conditions:

$$\frac{1}{2}U_{SS}S^2\sigma^2 - U_{T^d} + (r - c + \beta(\bar{S} - S))SU_S - (r + \lambda)U = 0 \tag{5}$$

Subject to the boundary condition:

$$U(S,0) = V(S,Q)$$

which means that when development is completed, the value of the oil field must equal that of a stage 3 oil field, with Q reserves.

To solve numerically we impose the requirement that the value of the oil field is linear on the spot price for low and high prices:

$$U_{SS}(0,T^d) = 0$$
$$U_{SS}(\infty,T^d) = 0$$

The value of an oil field in stage 1

Finally, the value of an undeveloped oil field in an economy without arbitrage opportunities must satisfy the following equations and conditions:

$$\frac{1}{2}H_{SS}S^2\sigma^2 - H_{T^i} + (r - c + \beta(\bar{S} - S))SH_S - (r + \lambda)H = 0 \tag{6}$$

Subject to the boundary condition:

$$H(S,0) = \mathrm{MAX}(U(S,T^d) - I;0)$$

which means that when the available time to initiate development is exhausted, the value of the undeveloped oil field will depend on whether it is optimal for the owner to make the development investment and obtain a stage 2 oil field, or to give up the concession.

To solve numerically we impose the requirement that the value of the oil field is linear on the spot price for low and high prices:

$$H_{SS}(0,T^i) = 0$$
$$H_{SS}(\infty,T^i) = 0$$

To obtain the market value of the undeveloped oil field we assume that an optimal (value maximizing) investment policy is defined by a critical spot price (S^*_{ci}), below which it is optimal to wait and above which it is optimal to develop:

$$H(S,T^i) = U(S,T^d) - I \text{ if } S \geqslant S^*_{ci}$$

By maximizing the left-hand side of equation (6), subject to the above conditions, the critical spot price (S^*_{ci}), can be determined. The value of the development option when it is optimal to wait ($S < S^*_{ci}$) as a function of the remaining time until expiration, T^i, amounts to the difference between keeping the option alive ($H(S,T^i)$) and killing it ($U(S,T^d) - I$). This development (or timing) option may be very valuable, depending on the characteristics of the oil field.

NUMERICAL SOLUTION

The model presented in the last section does not have an analytical solution, thus it must be solved by utilizing numerical methods. We use finite difference methods to solve a discretized version of the differential equations. For example, for solving the value of the oil field in stage 1, $H(S,T^i)$, the discretization is shown in Fig. 2.

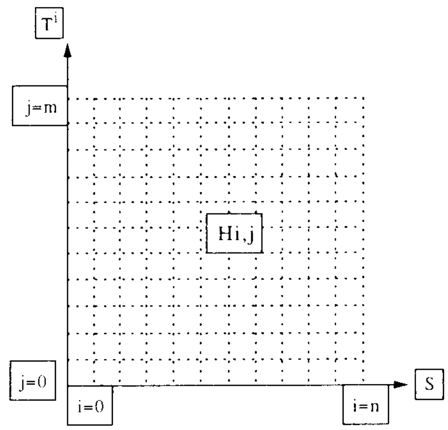


Fig. 2. Discretization of the model.

The differential equation (6), once partial derivatives are replaced by discrete approximations, becomes:

$$a_i H_{i-1,j} + b_i H_{i,j} + c_i H_{i+1,j} = d_i H_{i,j-1} \quad i = 0, \ldots, n \; j = 0, \ldots, m$$

$$a_i = \frac{1}{2} i^2 \sigma^2 - \frac{(r - c + \beta(\bar{S} - i\Delta S))i}{2}$$

$$b_i = -i^2\sigma^2 - \frac{1}{\Delta T^i} - (r + \lambda)$$

$$c_i = \frac{1}{2} i^2 \sigma^2 + \frac{(r - c + \beta(\bar{S} - i\Delta S))i}{2}$$

$$d_i = -\frac{1}{\Delta T^i}$$

To obtain the critical spot price S^*_{ci}, which in our discrete version becomes i^*, we search for the value of **i** which maximizes $H(S,T^i)$, while satisfying the following equation:

$$U_{i^*,m} - I = H_{i^*,j} \quad j = 0, \ldots, m$$

Once determined i^*, the PDE is assumed to hold only for $i < i^*$.

This model gives rise to the following tridiagonal system of equations which must be solved in order to obtain the value of the oil field:

$$\begin{bmatrix} b_0 & c_0 & & & & & \\ a_1 & b_1 & c_1 & & & & \\ & a_2 & b_2 & c_2 & & & \\ & & & \cdots & & & \\ & & & a_{n-2} & b_{n-2} & c_{n-2} & \\ & & & & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & & & a_n & b_n \end{bmatrix} * \begin{bmatrix} H_{0,j} \\ H_{1,j} \\ H_{2,j} \\ \cdots \\ H_{n-2,j} \\ H_{n-1,j} \\ H_{n,j} \end{bmatrix} = \begin{bmatrix} d_0 H_{0,j-1} \\ d_1 H_{1,j-1} \\ d_2 H_{2,j-1} \\ \cdots \\ d_{n-2} H_{n-2,j-1} \\ d_{n-1} H_{n-1,j-1} \\ d_n H_{n,j-1} \end{bmatrix}$$

From the boundary conditions, we have:

$$\begin{aligned} b_0 &= b_0 + 2a_0 \\ c_0 &= c_0 - a_0 \\ a_0 &= 0 \\ a_n &= a_n - c_n \\ b_n &= b_n + 2c_n \\ c_n &= 0 \end{aligned}$$

This completes the numerical solution to the stage 1 oil field.

A CASE STUDY: COMPUTER IMPLEMENTATION AND RESULTS

As was stated in the introduction of this paper, there is a growing literature of real option models for valuing investments, but relatively few practical implementations of this theory. In what follows we apply a computer implementation of this model to a case study illustrating its use through a user-friendly graphical interface which could help to promote acceptance of this rather sophisticated valuation model by the industry. The model runs in an IBM-compatible PC and uses Windows, FORTRAN and Visual Basic.

We present a case study of an undeveloped oil field concession which, if developed, would produce for 7 years a decaying annual amount of oil ranging from 1.7 million barrels during the first year to 0.07 million barrels during the 7th. Operating annual costs also would vary from 3.04 million dollars during the first year to 0.47 million dollars during the 7th. The owner of the concession may exercise the option to develop the oil field at any moment during a 5 year time-span, after which if no investment is made the concession expires. The development investment amounts to 29.66 million dollars and requires 1 year to be concluded after which production begins.

We show the computer implementation of the model and its results by printing several screens of the *Reserves Evaluator* Software of the case study.

Figures 3–6 show computer screens designed to input data to the model. Figure 3 defines the undeveloped oil field reserve characteristics, including the available time to exercise the development option, the investment cash flows during stage 2 and the extraction cash flows of stage 3. Figure 4 shows the input screen for market data, including interest rate, and country risk premiums, and the parameter values for specifying the stochastic process for risk adjusted oil spot prices. Figure 5 defines the tax data, allowing for real estate, royalty or corporate taxes. We evaluate the oil field assuming a 35% corporate tax rate. Finally, Fig. 6 is designed to input the parameter values required for the numerical solution of the oil field model.

Figure 7 shows the optimal investment policy for developing the oil field. It can be noted that the critical price over which it is optimal to invest is lower the less available time we have to delay investment without loosing the concession. Throughout the software critical prices are expressed in terms of the 1 year future equivalent, instead of the spot price, because spot prices tend to be

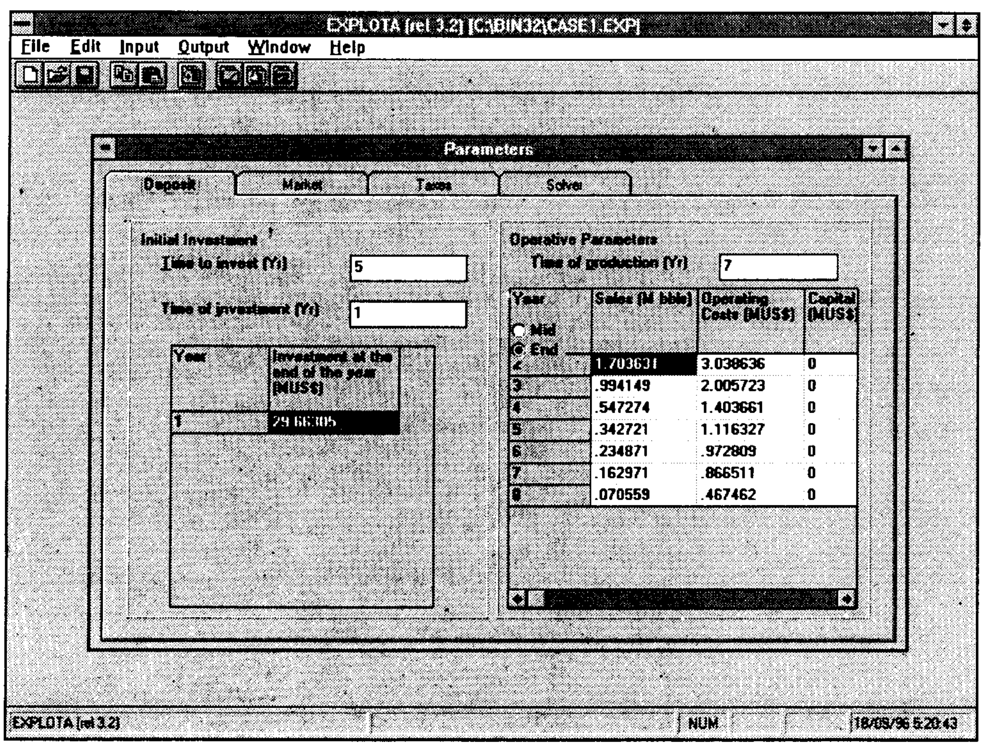


Fig. 3. Oil field characteristics.

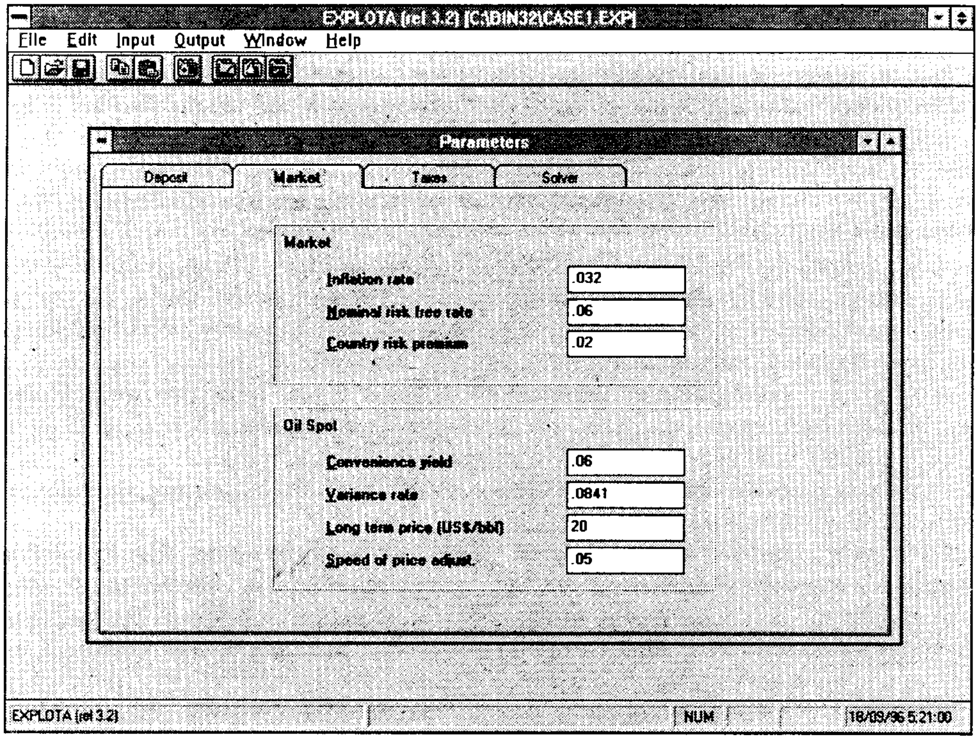


Fig. 4. Market data.

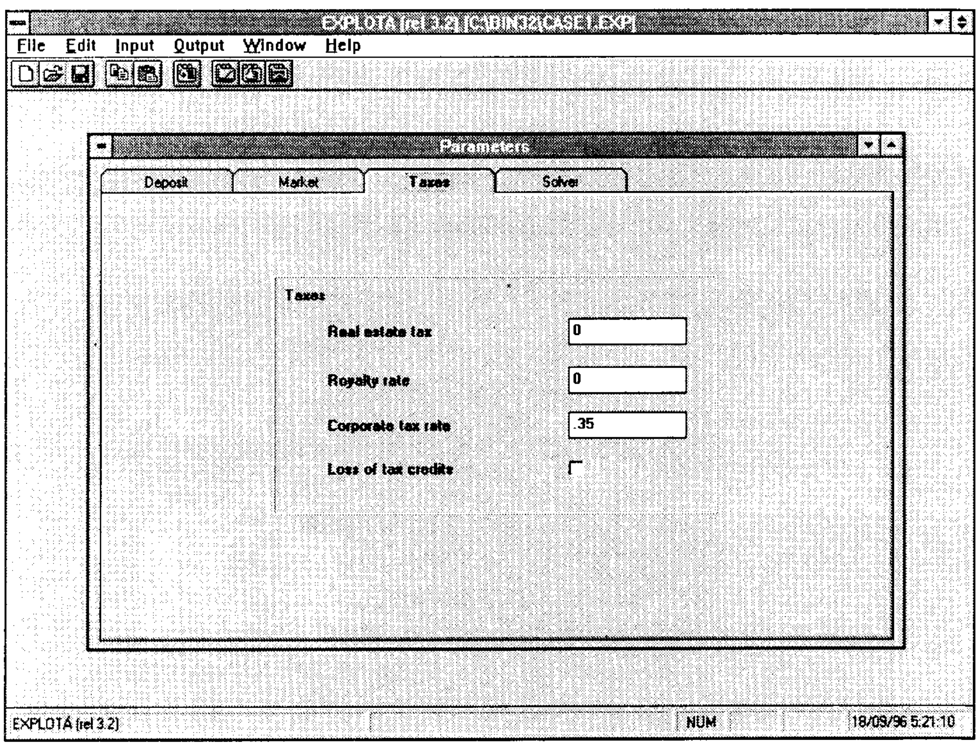


Fig. 5. Tax data.

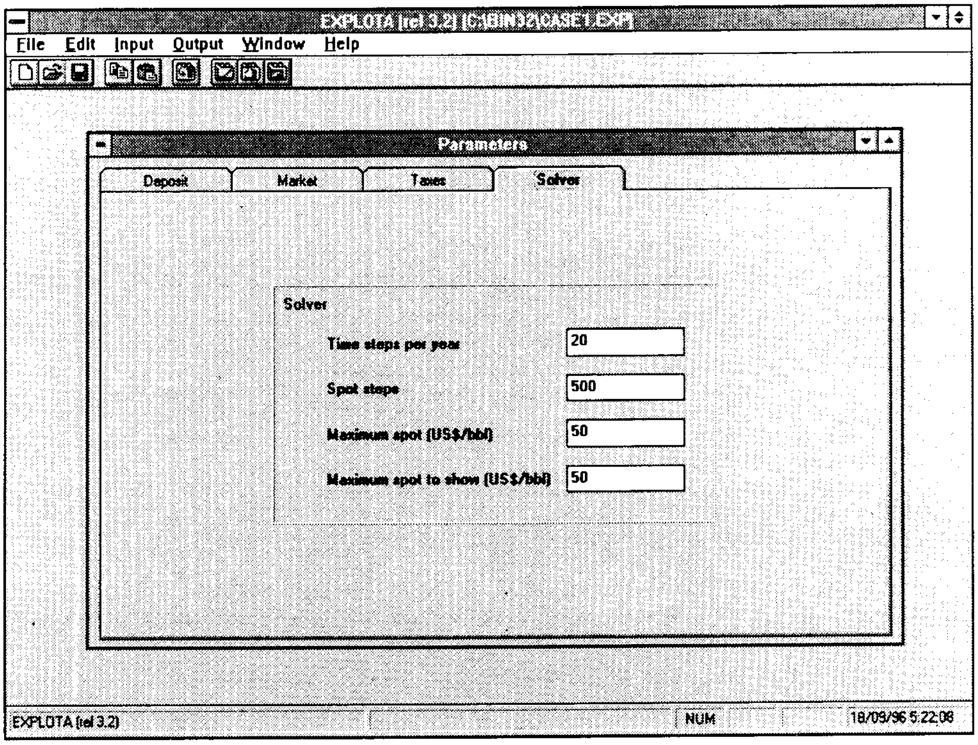


Fig. 6. Numerical solution parameters.

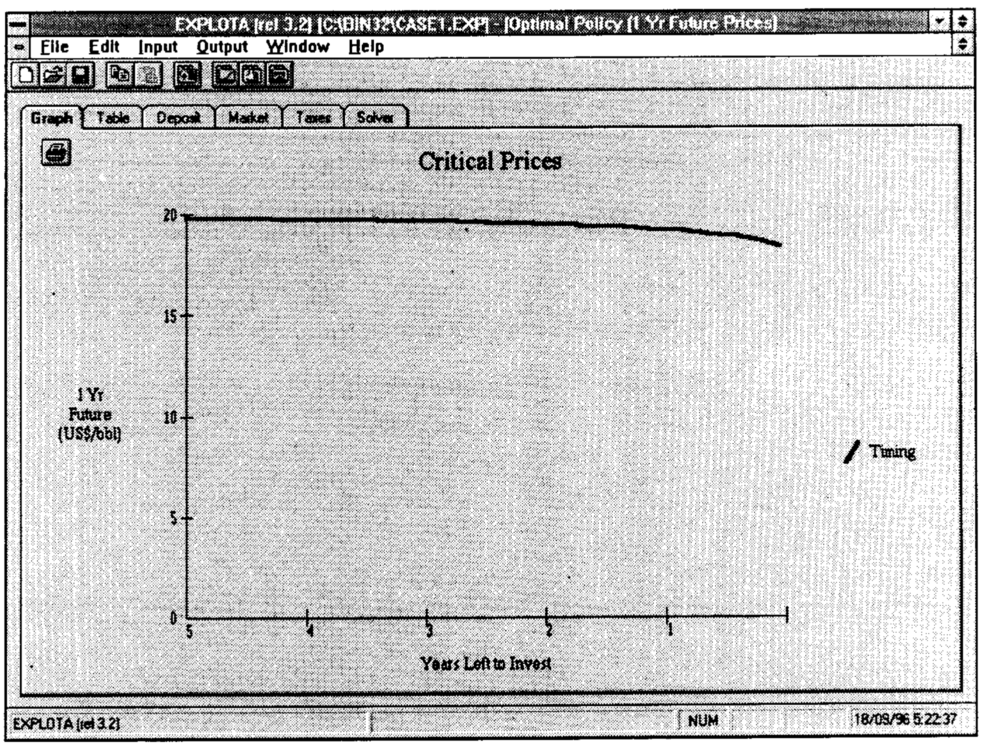


Fig. 7. Optimal investment policy.

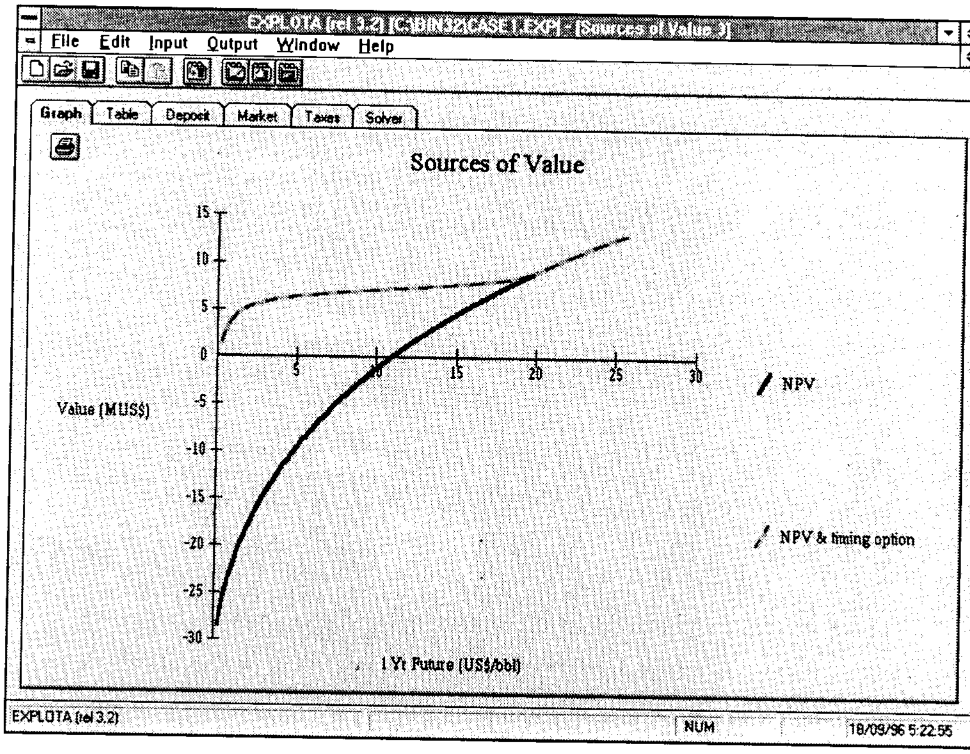
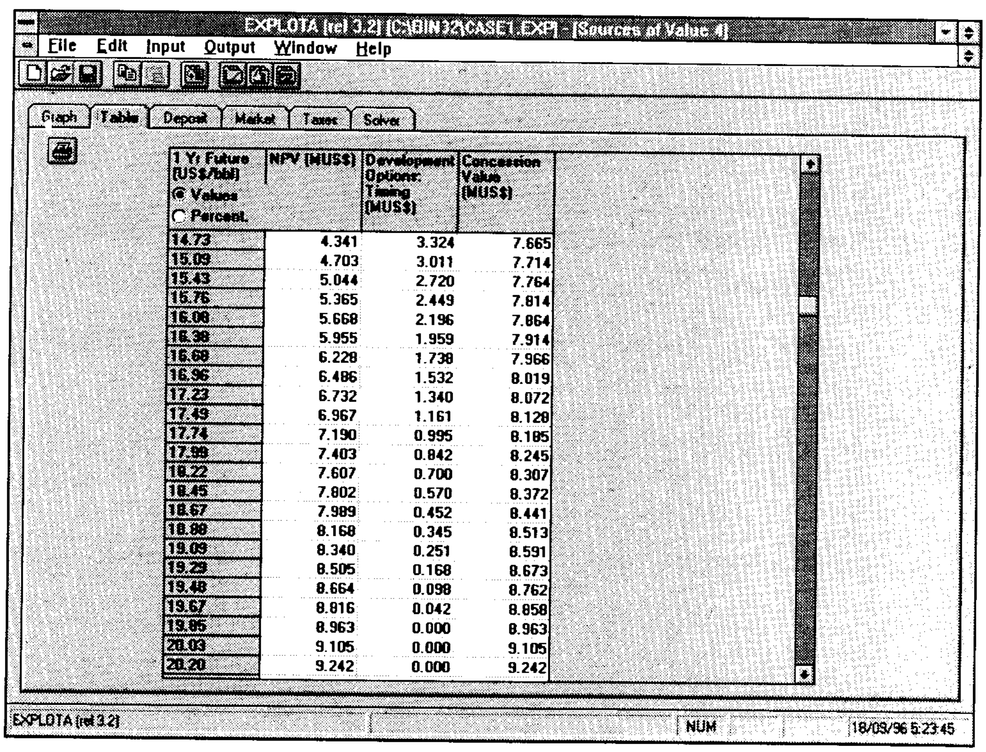
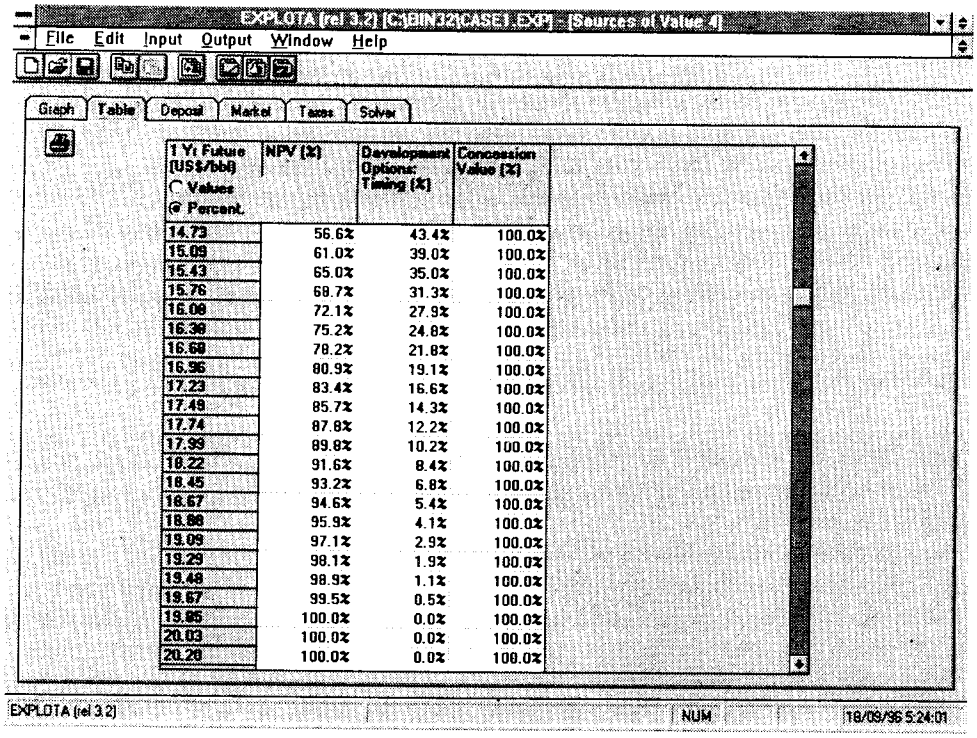


Fig. 8. Source of oil field value.



1 Yr Future (US$/bbl)	NPV (MUS$)	Development Options: Timing (MUS$)	Concession Value (MUS$)
14.73	4.341	3.324	7.665
15.09	4.703	3.011	7.714
15.43	5.044	2.720	7.764
15.76	5.365	2.449	7.814
16.08	5.668	2.196	7.864
16.38	5.955	1.959	7.914
16.68	6.228	1.738	7.966
16.96	6.486	1.532	8.019
17.23	6.732	1.340	8.072
17.49	6.967	1.161	8.128
17.74	7.190	0.995	8.185
17.99	7.403	0.842	8.245
18.22	7.607	0.700	8.307
18.45	7.802	0.570	8.372
18.67	7.989	0.452	8.441
18.88	8.168	0.345	8.513
19.09	8.340	0.251	8.591
19.29	8.505	0.168	8.673
19.48	8.664	0.098	8.762
19.67	8.816	0.042	8.858
19.85	8.963	0.000	8.963
20.03	9.105	0.000	9.105
20.20	9.242	0.000	9.242

Fig. 9. Sources of value in dollars.



1 Yr Future [US$/bbl]	NPV (%)	Development Options: Timing (%)	Concession Value (%)
14.73	56.6%	43.4%	100.0%
15.09	61.0%	39.0%	100.0%
15.43	65.0%	35.0%	100.0%
15.76	68.7%	31.3%	100.0%
16.08	72.1%	27.9%	100.0%
16.38	75.2%	24.8%	100.0%
16.68	78.2%	21.8%	100.0%
16.96	80.9%	19.1%	100.0%
17.23	83.4%	16.6%	100.0%
17.49	85.7%	14.3%	100.0%
17.74	87.8%	12.2%	100.0%
17.99	89.8%	10.2%	100.0%
18.22	91.6%	8.4%	100.0%
18.45	93.2%	6.8%	100.0%
18.67	94.6%	5.4%	100.0%
18.88	95.9%	4.1%	100.0%
19.09	97.1%	2.9%	100.0%
19.29	98.1%	1.9%	100.0%
19.48	98.9%	1.1%	100.0%
19.67	99.5%	0.5%	100.0%
19.85	100.0%	0.0%	100.0%
20.03	100.0%	0.0%	100.0%
20.20	100.0%	0.0%	100.0%

Fig. 10. Sources of value in percentage.

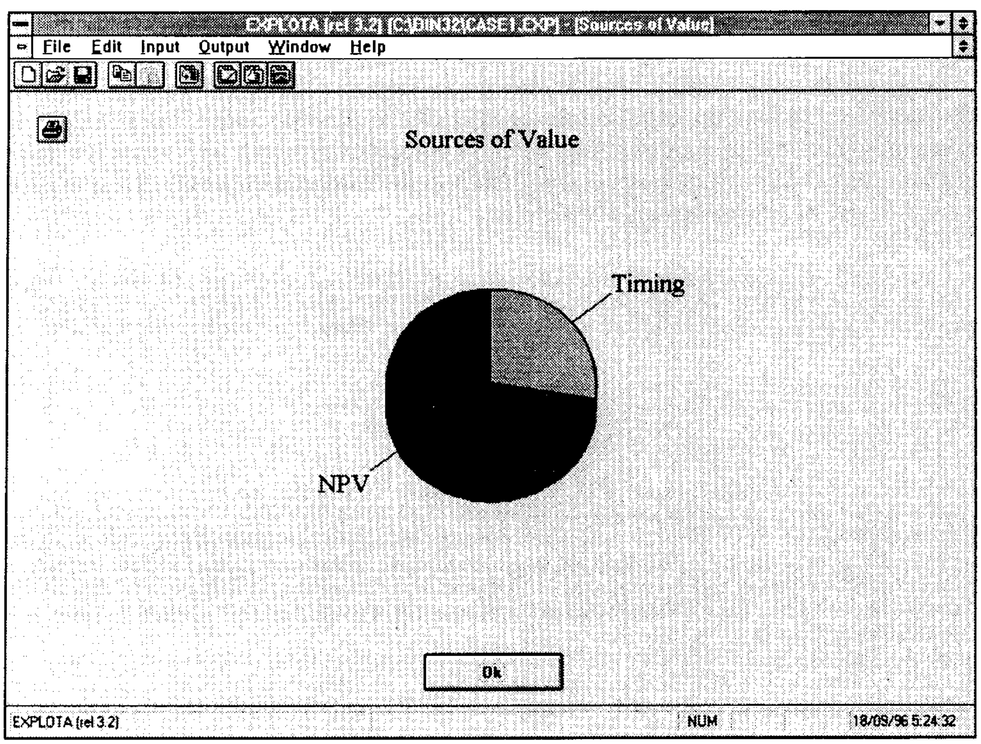


Fig. 11. Sources of value for a 1 year futures price of US$ 16.

extremely volatile and with an extremely high mean reverting behavior, while 1 year futures seem to behave better.

Figures 8–11 show the valuation output of the model. Figure 8 defines the sources of the oil field value, decomposing the concession value in the net present value and in the timing option. It can be seen that for oil prices above the critical price there is no option value, while for lower prices the option value is a sizable fraction of total concession value. This can also be seen in Figs 9 and 10 which show the dollar and percentage value of the timing option with respect to total concession value for a range of oil prices. It can be seen that for a 1 year futures price of US\$18, the timing option represents around 10% of total value, while for a futures price of 15, it represents around 40%. A similar situation is illustrated in the pie chart of Fig. 11, for a futures price of US\$16.

CONCLUSIONS

In this paper we present a no arbitrage model for evaluating an undeveloped oil field and its numerical solution and implementation. The model assumes stochastic, but mean reverting, risk adjusted oil spot prices.

The real options model developed has several advantages over using traditional net present value methodologies. First, it efficiently uses market price information in the oil futures market, which would normally be neglected. Second, it does not require estimates for spot prices over the life of the project, which has great uncertainty given its high historic volatility. Third it does not require estimates of the risk premiums, but it uses risk free interest rates, which are subject to little estimation error. Fourth, the model values the operational flexibility of being able to optimally delay development investments. Finally, the model determines the critical price over which it is optimal to develop the oil field.

The results of using this real options model for evaluating a specific undeveloped oil field show that a significant fraction of the oil field value may be provided by the flexibility of delaying development investment, and that this option value decreases as oil price increases. Also the critical price for developing decreases with the available time to develop.

We illustrate the use that practitioners could make of the real option methodology to value oil contingent reserves by presenting a user friendly computer program with graphical interface. This implementation could help petroleum companies to use this sophisticated valuation approach that could otherwise be of little practical relevance.

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