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Public disclosure, private information collection, and short-term trading*

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This paper examines how public disclosure affects private information acquisition activity in a market economy. We analyze a setting where traders with short-term investment horizons are allowed to trade on their private information prior to a public disclosure. We demonstrate in this setting that public disclosure stimulates investment in private information acquisition. This result is shown to have implications for the magnitude of the pre-announcement and announcement price reactions to the disclosure.

1. Introduction

This paper examines how public disclosure affects private information acquisition activity in a market setting, an issue of importance to both regulators and academics. Regulators often justify required public disclosure as a way to 'level the playing field' by providing equal access to information across investors.[1] To fully understand how mandatory public disclosures affect the extent to which there is equal access to information, however, it is necessary to examine how these disclosures affect investment in pre-announcement private information collection. If mandatory disclosures actually stimulate such investment by one set of investors relative to another, then they decrease the extent to which

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[1] For further discussion of this point, see Beaver (1977) and Hakansson (1977).

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the 'playing field' is level prior to the disclosures. For academics interested in empirical investigation of the impact of public disclosures on security prices, this research provides an understanding of how such disclosures affect prior information acquisition activity and pre-announcement security prices.

Prior studies of this issue, such as Diamond (1985), Bushman (1991), Lundholm (1991), and Alles and Lundholm (1990), have assumed that traders observe their private information and the public disclosure at the same time. As a result, the public signal serves as a substitute for private information and generally reduces traders' incentives to invest in information acquisition. In contrast, in the economy we consider, traders are allowed to acquire and trade on private information prior to the public signal's disclosure. Consequently, a forthcoming public disclosure can stimulate private information collection in the pre-announcement period.

A second distinction between this paper and prior research concerns the assumption made about the investment horizon of privately informed traders. While in prior research it has been assumed that informed traders hold their positions in a firm's shares until its liquidating value is revealed, we assume that informed traders reverse their positions prior to the liquidation of the firm.[2] That is, informed traders are either unable or unwilling to hold security investments for an indefinite period of time. A preference for short-term profits may be due to either capital or labor market imperfections that make long-term positions more costly to hold than those of shorter term. Professional portfolio managers, for example, are thought to prefer short-term profits because their performance is evaluated over short intervals.[3]

When a trader has a finite investment horizon, positive returns to private information collection depend on his information being at least partially reflected in the firm's market price by the time that he closes out his position. This is in the spirit of the conventional Wall Street wisdom that 'a bargain that remains a bargain is no bargain'. By advancing the time when information about the firm's liquidating value is incorporated in the firm's market price, public disclosure is of value to the informed trader. In contrast, public disclosure need not have any value to a trader with a long horizon, who can hold his shares until the firm's liquidating value is revealed.

Given both the ability of the informed trader to trade on his private information before the public signal is announced and his short-term trading horizon, it

[2]This assumption is plausible on empirical grounds, given the long lives of many firms, and is reinforced by the observation that institutions hold over 50% of the shares of U.S. equities [Jacobs (1991)] and have average turnover in excess of 50% of their portfolios each year [Lang and McNichols (1992)].

[3]It is also possible to think of short-term traders as traders with longer-term horizons but alternative profitable uses for their funds. Such investors find it optimal to invest in private information just prior to a public disclosure, as that is the point in time at which they expect to realize the profits from their investment.

is shown here that public disclosure stimulates investment in private information acquisition. More specifically, it is demonstrated that an increase in either the probability or the precision of a public disclosure increases the expected trading profits of the informed trader and gives him an incentive to increase the precision of his private information. This result stands in contrast to Diamond's (1985) conclusion that the introduction of a public disclosure reduces the incentive to collect private information.[4]

It is also shown here that, holding constant the precision of the informed trader's private information, his expected trading profits are an increasing function of the covariance between the errors in the public and private signals. This relation arises because the greater the covariance, the greater is the informed trader's ability to predict the price change that will occur when the public signal is announced. Further, when the covariance is a choice variable for the informed trader, it is demonstrated that as either the probability or precision of the public disclosure increases, the optimal covariance also rises.

Finally, we examine the effect that increases in the probability and precision of public disclosure have on pre-announcement and announcement date price changes. Allowing for the informed trader to choose the precision of his private information, we show that the absolute pre-announcement price change is increasing in both the probability and precision of the public disclosure. As a consequence, the magnitude of the announcement date price change is decreasing in the probability of disclosure, and in some cases, is also decreasing in its precision. This last result implies that it is not always possible to use the magnitude of the announcement date price reaction to assess the informativeness of a public disclosure.

Recent models by Demski and Feltham (1994) (hereafter DF) and Kim and Verrecchia (1991) (hereafter KV) also examine the impact of public disclosure on private information collection in multiperiod rational expectations economies. Our setting contrasts with those of DF and KV along several dimensions. First, DF work in a setting similar to that of Grossman and Stiglitz (1980), where traders who choose to become informed acquire the same private signal. KV follow a setting similar to that of Hellwig (1980), where traders acquire different private signals. This paper uses the Kyle (1985) framework, in which there is a single informed trader, who chooses the precision of his private signal, and a competitive market maker, who establishes prices. The focus on a single informed trader simplifies our analysis, by abstracting from the effect the

[4]Other research on public disclosure in single-period settings examine somewhat different issues. Bushman (1991) allows for a monopolistic seller of private information and shows that the more precise the public information, the more precise are the signals that the monopolist sells. Lundholm (1991) considers the effect of a public signal on the concentration of private information among investors. He finds that disclosure of public information results in both fewer informed traders and a lower dispersion of private information. Alles and Lundholm (1990) focus on the welfare effects of public disclosure.

presence of other traders has on the relation between public disclosure and the trader's private information acquisition.[5]

Our model also differs from DF and KV in that we assume the informed trader is risk-neutral and takes into account how his demand for a firm's shares affects the firm's market price. In contrast, both DF and KV assume that informed traders are risk-averse and set their demand for the firm's shares believing that it does not affect the firm's market price. By assuming risk neutrality, we are able to abstract from the risk-sharing motivations for trade and private information collection and focus on the incentives to collect information that derive from the impact of private information on the informed trader's expected profits. In this setting, we are able to characterize the effect of the public signal on the informed investor's trading and information acquisition decisions in an intuitive way.

Another difference between our model and those of DF and KV is that our informed trader has a finite horizon, while DF and KV allow traders to hold their positions until liquidation of the firm. A final difference among the models relates to the information structure assumed. The simplifying assumptions described above permit us to analyze a very general information structure, in which the relation between the private and public signals can take many forms. In contrast, DF assume that the public disclosure is sufficient for the private signal,[6] while KV assume that the errors in the public and private signals are independent. Having a general information structure allows us, in particular, to treat the covariance between the two signals as a choice variable of the informed trader. Our setting also allows for a public report to occur with a probability less than 1, in contrast to DF and KV, who assume the public report occurs with certainty. Consequently, our analysis applies to both mandatory and discretionary disclosures.

The layout of the paper is as follows. Section 2 describes the setting and equilibrium of Kyle's (1985) model, which is the framework we use for our analysis. Section 3 introduces a public signal and short-term trading horizons, and characterizes the equilibrium. The relation between the informed trader's expected profits and the characteristics of both his private information and the public signal are analyzed in section 4. In section 5, we allow characteristics of the informed trader's private information to be endogeneously determined as

[5]If there were multiple informed traders, information acquisition would be influenced by the extent to which price reflects the private information acquired by other traders and the extent to which that other information is a substitute for the information each investor could directly acquire. (This effect arises in both KV and DF.) However, the nature of our results would remain unchanged as long as some uninformed traders still exist in the market. In particular, an increase in either the probability or the precision of a public disclosure would still stimulate each informed trader to increase the precision of his private information.

[6]This reflects a setting in which informed traders are attempting to forecast the forthcoming public disclosure.

a function of the characteristics of the public disclosure. Section 6 analyzes the price reactions to public disclosures in this setting, and section 7 relaxes the assumption that traders have short-term horizons. Section 8 concludes the paper.

2. Equilibrium in the basic setting

The basic framework for our analysis follows along the lines of Kyle (1985). We begin by briefly describing the equilibrium in his setting as a point of departure for characterizing the equilibrium in a setting that incorporates public disclosure and short-term trading. Kyle assumes a one-period economy in which two assets are traded, a riskless security and shares in a risky firm. The end-of-period liquidating value of the firm, denoted by v, is assumed to be normally distributed with mean $\bar{v}$ and variance σ^2. Without loss of generality, the riskless rate of interest is set equal to zero.

There are three types of risk-neutral traders in this economy, each of whom may trade each security at the beginning of the period. One trader, referred to as the informed trader, privately receives imperfect information about the firm's end-of-period liquidating value before he makes his trading decision. The informed trader receives a signal θ that is of the form

$$\theta = v + \varepsilon_1, \tag{1}$$

where ε_1 is a normally distributed random variable that is uncorrelated with v, and has a mean of zero and variance of σ_1^2 (or, alternatively, a precision of $1/\sigma_1^2$). Kyle actually assumes that the informed trader observes the liquidating value of the firm without error. In describing his setting here, we introduce imperfect information since this is an important feature of our model, as will be described below. His purchases of shares of the risky asset at the beginning of the period, denoted by x, are chosen to maximize his expected profits conditional on his private information, taking into account the effect which his demand has on the firm's market price. Liquidity traders are the second group of traders. Their demand for shares at the beginning of the period, denoted by u, is unrelated to any information in the market and is assumed to be normally distributed with a mean of 0 and variance of σ_u^2. The third type of trader is the market-maker, whose task is to set the firm's beginning-of-period price, P, and offer shares for sale so as to clear the market at that time. Under the assumption that he acts as a perfect competitor, the market-maker sets P equal to his expectation of the liquidating value, v, given his priors and his observation of the beginning-of-period order flow, $y = x + u$; that is, $P = \mathrm{E}(v|y)$. Given the market-maker's pricing strategy, the informed trader chooses x to maximize

$$\max_x \mathrm{E}[(v - \mathrm{E}(v \mid y))x \mid \theta].$$

Kyle shows that an equilibrium in this setting can be characterized by a pricing relation $P(y)$, for which the market-maker earns zero profits, and a trading strategy $x(\theta)$, which maximizes the informed trader's conditional expected profits. The demand for shares by the informed trader is given by

$$x = \beta(\theta - \bar{v}), \tag{2}$$

where

$$\beta = \left(\frac{\sigma_u^2}{\sigma^2 + \sigma_1^2}\right)^{1/2}. \tag{3}$$

The equilibrium price is

$$P = \mathrm{E}(v \mid y) = \bar{v} + \lambda y, \tag{4}$$

where

$$\lambda = \frac{\sigma^2}{2\sigma_u(\sigma^2 + \sigma_1^2)^{1/2}}. \tag{5}$$

Eqs. (2) and (3) reflect the intuition that the informed trader trades more aggressively on his private information as its precision increases and as the variance of liquidity trade increases. Note however that even though the informed trader takes a more aggressive position as the precision of his information increases, the variance of the order flow is unaffected. This occurs because increased precision has the offsetting effect of reducing the variability of the informed trader's private information. This can be seen mathematically by noting that $\sigma_y^2 = \beta^2(\sigma^2 + \sigma_1^2) + \sigma_u^2 = 2\sigma_u^2$. Finally, note from (4) and (5) that as σ_1^2 decreases, the market-maker places greater weight on the order flow in setting price. This occurs because the covariance between v and x, or alternatively between v and y, increases with the precision of the informed trader's information.

3. Equilibrium with short-term traders and a public signal

We next modify Kyle's basic setting in two ways. First, we assume that the informed trader has a short-term horizon, so that he closes out any position he takes in the firm before the firm is liquidated.[7] Second, we allow for the

[7] As we note in section 7, our analysis is unaffected if we make the weaker assumption that there is a positive probability (less than 1) that the informed trader will trade before the liquidation of the firm. Other papers examining the implications of investment horizons that end prior to the firm's liquidation include Shleifer and Vishny (1990) and Froot, Scharfstein, and Stein (1992).

possibility that a public signal of firm value is released before the informed trader closes out his position. Specifically, in this one-period economy, there are four dates. At date 0, the informed trader acquires private information about the liquidating value of the firm. Subsequently, at date 1, trading takes place in the two assets. At date 2, the firm releases some information about its liquidating value with probability p. This release can be thought of as a voluntary disclosure by the firm or as a disclosure mandated by the Financial Accounting Standards Board or the Securities and Exchange Commission (in which case $p = 1$). Also at date 2, the informed trader and liquidity traders close out their positions in the firm. Finally, at date 3, the firm is liquidated and the payoff on the riskless security is realized.

Having the informed trader liquidate his position before the end of the period captures the assumption that he has a short-term trading horizon and does not hold his shares until the firm's liquidating value is realized. Consequently, he can profit on his private information only if a public disclosure is made at date 2.[8] While the probability p most obviously can be interpreted as the probability that the firm makes a public disclosure during the period, it also can be thought of as the probability that the informed trader is able to keep his shareholding position until after the firm releases information. The requirement that the liquidity traders also liquidate at date 2 is imposed solely to simplify the analysis, as it implies that the market-maker cannot make additional inferences about the liquidating value of the firm from the date 2 order flow.[9] While relaxing this assumption would complicate the analysis, it would not change the nature of the results.

If released, the date 2 public signal of the firm, denoted by z, is assumed to take the form

$$z = v + \varepsilon_2, \tag{6}$$

where ε_2 is a normally distributed random variable that is uncorrelated with v and has a mean of 0 and a variance of σ_2^2 (or, alternatively, a precision of $1/\sigma_2^2$). The covariance between ε_1, the error in the informed trader's private signal, and ε_2 is denoted by σ_{12}, and is assumed to be nonnegative.

An equilibrium is characterized by the informed trader's optimal date 1 holdings in the risky firm and by the market prices set for the firm at dates 1 and 2.

[8]The analysis is not affected if a party other than the firm makes the disclosure, but since our interest is in the relation between corporate disclosure policy and private information collection, we will think of the firm as the information provider.

[9]This assumption is also made by Froot, Scharfstein, and Stein (1992) in studying short-horizon traders.

The date 1 holdings arise as the solution to the informed trader's optimization problem

$$\max_x \mathrm{E}[(P_2 - P_1)x \mid \theta], \tag{7}$$

where P_2 (P_1) is the price of the firm as set by the market-maker at date 2 (date 1). As discussed in the previous section, the market-maker sets P_1 equal to his expectation of the liquidating value, v, given his priors and his observation of the date 1 order flow, $y = x + u$. In setting P_2, the market-maker also conditions his expectation on the public signal z, if released, so that $P_2 = \mathrm{E}(v \mid y, z)$. The following proposition provides a characterization of the equilibrium.

Proposition 1. For the case where a public signal is released at date 2, an equilibrium exists in which x, the informed trader's demand for shares, is given by

$$x = \beta(\theta - \bar{v}), \tag{8}$$

where

$$\beta = \operatorname{sign}(b_2)\left(\frac{\sigma_u^2}{\sigma^2 + \sigma_1^2}\right)^{1/2}. \tag{9}$$

Further, the date 1 equilibrium price is

$$P_1 = \mathrm{E}(v \mid y) = \bar{v} + \lambda y, \tag{10}$$

where

$$\lambda = \frac{\operatorname{sign}(b_2)\sigma^2}{2\sigma_u(\sigma^2 + \sigma_1^2)^{1/2}}, \tag{11}$$

and the date 2 equilibrium price is

$$P_2 = \mathrm{E}(v \mid y, z) = \bar{v} + b_1 y + b_2(z - \bar{v}), \tag{12}$$

where

$$b_1 = \frac{\operatorname{sign}(b_2)\sigma^2(\sigma^2 + \sigma_1^2)^{1/2}(\sigma_2^2 - \sigma_{12})}{\sigma_u[2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2) - (\sigma^2 + \sigma_{12})^2]}, \tag{13}$$

$$b_2 = \frac{\sigma^2(2\sigma_1^2 + \sigma^2 - \sigma_{12})}{2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2) - (\sigma^2 + \sigma_{12})^2} \tag{14}$$

If z is not announced, $P_2 = P_1$.

Proof. See the appendix. ▌

Three observations follow. First, in this setting, the informed trader's optimal demand and the date 1 market price are similar in form to those in Kyle's basic model, with one important difference. When the parameters of the economy are such that a higher realization of the public signal, conditional on the order flow, results in a lower date 2 price (that is, when b_2 is negative), both β and λ are the negative of their values in the simpler setting. In other words, the informed trader takes a long (short) position in the firm's shares when he receives unfavorable (favorable) private information. We believe that this equilibrium cannot be justified on empirical grounds and therefore restrict the remainder of our analysis to settings where the parameters of the economy are such that b_2 is positive, that is, where $2\sigma_1^2 + \sigma^2 > \sigma_{12}$.[10] A second observation from Proposition 1 is that the effect of an increase in the value of the public signal on the market price of the firm at date 2 (as reflected in the coefficient b_2) is decreasing in σ_2^2. The noisier the public signal, the less its revelation causes a revision in the market-maker's expectation of v. Finally, it is straightforward to show that a unit increase in the date 1 order flow has a greater effect on the date 1 price than on the date 2 price; that is, λ is greater than b_1. This is consistent with recent empirical findings of Daley, Hughes, and Rayburn (1991). They document for their sample of block trades that permanent price effects before earnings announcements (captured by λ in our model) are significantly greater than those after earnings are announced (captured by b_1).

4. Properties of the informed trader's expected trading profits

The informed trader's expected trading profits, $\mathrm{E}(\pi)$, are derived by substituting expressions (8)–(14) into (7) and integrating over all possible values of θ,

$$\mathrm{E}(\pi) = p\left[\frac{\sigma^2}{2(\sigma^2 + \sigma_1^2)^{1/2}} - \frac{\sigma^2(\sigma_2^2 - \sigma_{12})(\sigma^2 + \sigma_1^2)^{1/2}}{2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2) - (\sigma^2 + \sigma_{12})^2}\right]\sigma_u . \tag{15}$$

Eq. (15) can be rewritten to provide additional intuition about the source of the informed trader's expected profits. First,

$$\mathrm{E}(\pi) = p[\mathrm{cov}(P_2 - P_1, \theta - \bar{v})\beta] . \tag{16}$$

[10]Even in an equilibrium in which b_2 is positive, b_1 can be negative; that is, 'good news' in the sense of an increase in the order flow can have a negative effect on the date 2 price of the firm. See Lundholm (1988) for a similar result.

This expression shows that the informed trader's expected profits depend on the covariance between his private signal and the change in prices over his trading horizon. Intuitively, the better that his information allows him to predict price changes, the greater his expected profits.

Second, eq. (15) can be written as

$$\mathrm{E}(\pi) = p[\lambda - b_1]\,\sigma_u^2. \tag{17}$$

The quantity $\lambda - b_1$ equals the liquidity traders' expected loss from purchasing a dollar of the firm's shares at date 1 and selling them at date 2. This immediately follows from the observation that λ (b_1) measures the impact of a one-dollar increase in liquidity demand on the firm's date 1 (date 2) share price. Since the informed trader's expected trading profits are equal to the expected loss of the liquidity traders (given that the market-maker earns zero expected profits), his expected profits must be directly related to $\lambda - b_1$.

Finally, eq. (15) can be written as

$$\mathrm{E}(\pi) = pb_2\beta(\sigma^2 + \sigma_{12})/2. \tag{18}$$

This expression shows that expected profits are related to three factors: the aggressiveness with which the informed trader trades on his private information, β, the covariance of the errors in the private and public information, σ_{12}, and the extent to which public information affects the market price at date 2, b_2.

The remainder of this section examines several interesting properties of the informed trader's expected profits for each of three different scenarios. In the first scenario, which we refer to as the independent variance/covariance case, it is assumed that σ_1^2, σ_2^2, and σ_{12} can all be varied independently of each other, subject to the constraint that $\sigma_{12} \leq \sigma_1^2\sigma_2^2$. Further, no specific relation is posited between the errors in the private and public signals, ε_1 and ε_2. In this scenario, an increase in σ_{12}, holding σ_1^2 and σ_2^2 constant, can be interpreted as an increase in the variance of the information common to the private signal and public disclosure and a decrease in the variance of the signal-specific and disclosure-specific information. Similarly, an increase in σ_1^2 (σ_2^2) can be interpreted as an increase in the variance of the information that is specific to the private signal (public disclosure), holding constant the variance of the information common to the two signals.

In the second scenario, referred to as the independent errors case, it is assumed that the errors in the public and private signals are independent, so that σ_{12} is equal to zero. This is the case analyzed by KV. In the third scenario, referred to as the sufficiency case, the public disclosure is assumed to be a sufficient statistic for the private signal. Sufficiency is captured by the specification that $\varepsilon_1 = \varepsilon_2 + \varepsilon_3$, where ε_3 is a normally distributed random variable that is independent of both v and ε_2 and has a mean of 0 and a variance of σ_3^2. This is the case analyzed by DF. Given this specification, $\sigma_1^2 \geq \sigma_2^2 = \sigma_{12}$, so that a decrease in the variance of the error in the public signal implies an equal reduction in the covariance between the errors in the two signals. It is also assumed to imply an equal reduction in the variance of the error in the private signal (or, in other words, that σ_3^2 is unaffected by a reduction in σ_2^2).[11] Note, however, that the variance of the error in the private signal can still be varied independently of both the variance of the error in the public signal and the covariance between the errors in the two signals. Analyzing this specific setting is interesting because it reflects the type of relation that often exists between public and private information. It is the appropriate setting, for example, in the case where the private signal provides information about forthcoming earnings and the public signal is the actual earnings announcement.

The first property of the informed trader's expected profits is captured by the following observation.

Observation 1. In all three scenarios, the informed trader's expected profits are increasing in both p and $1/\sigma_2^2$.

Proof. The observation follows immediately from differentiation of (15), noting for the case of sufficiency that a change in σ_2^2 causes an equal change in σ_{12} and σ_1^2, holding σ_3^2 fixed.[12] ∎

Since the informed trader can gain from his private information only if the firm makes a public disclosure during the period, an increase in p results in higher expected trading profits. As $1/\sigma_2^2$ increases, there is a greater covariance between the price change at date 2 and the informed trader's private information, which again results in higher expected trading profits.[13]

The next observation considers the effect that the precision of the private signal has on the informed trader's expected profits.

Observation 2. In the case of sufficiency, expected trading profits are increasing in the precision of the private signal. In the independent variance/covariance case

[11]An alternative assumption would be that a reduction in σ_2^2 is accompanied by an equal increase in σ_3^2, leaving the variance of the private signal unaffected. This alternative is among the possibilities considered by DF in their analysis.

[12]If it is alternatively assumed in the case of sufficiency that a reduction in σ_2^2 implies an equal increase in σ_3^2, so that σ_1^2 remains unchanged, then expected profits are unaffected by an increase in $1/\sigma_2^2$. This is because, under these assumptions, the covariance between the price change and the informed trader's private signal is not a function of σ_2^2.

[13]Using (16) and (18), $\mathrm{cov}(P_2 - P_1, \theta - \bar{v}) = b_2((\sigma^2 + \sigma_{12})/2)$, which, in the independent variance/covariance case, is increasing in $1/\sigma_2^2$ since b_2 is increasing in $1/\sigma_2^2$. In the case of sufficiency, σ_{12} decreases with $1/\sigma_2^2$. However, it is straightforward to show that the quantity $b_2((\sigma^2 + \sigma_{12})/2)$ still increases in $1/\sigma_2^2$.

as well as in the independent errors case ($\sigma_{12} = 0$), expected trading profits are increasing in the precision of the private signal if b_2 is at least weakly increasing in $1/\sigma_1^2$. If b_2 is decreasing in $1/\sigma_1^2$, expected trading profits are still increasing in $1/\sigma_1^2$ as long as $\sigma_1^2 \geq \sigma_2^2$. However, if *$\sigma_2^2$ is sufficiently large and σ_1^2 is sufficiently small, the informed trader's expected profits are* decreasing *in $1/\sigma_1^2$.*

Proof. From (18), $\partial E(\pi)/\partial\sigma_1^2 = p(\sigma^2 + \sigma_{12})/2 \cdot \partial(b_2\beta)/\partial\sigma_1^2$. Straightforward differentiation of (9) reveals that $\partial\beta/\partial\sigma_1^2$ is negative. Therefore, $\partial E(\pi)/\partial\sigma_1^2$ is also negative (so that expected profits are increasing with $1/\sigma_1^2$) if $\partial b_2/\partial\sigma_1^2$ is less than or equal to 0. Differentiation of (14) shows this to be the case whenever $\sigma_{12} \geq \sigma_2^2$, as is true when the public signal is sufficient for the private signal. Examination of $\partial(b_2\beta)/\partial\sigma_1^2$ confirms the remainder of the observation. ▮

The finding that expected profits can decrease with the precision of the informed trader's private information in both the independent variance/covariance case and in the independent errors case is, at first glance, surprising. To better understand why this may occur, note from (18) that expected profits increase with $1/\sigma_1^2$ if, and only if, $b_2\beta$ increases with $1/\sigma_1^2$. Recall that b_2 measures the price impact of the public disclosure, while β represents the size of the position that the informed trader takes in the firm. As discussed previously, β increases with $1/\sigma_1^2$. But, with the informed trader taking a more aggressive position and, consequently, the date 1 price impounding more of his information, the impact which the public disclosure itself has on price may decrease as $1/\sigma_1^2$ increases. If this happens, the informed trader's expected profit per unit of investment will decrease. This does not occur in the sufficiency case, where b_2 is independent of $1/\sigma_1^2$. Therefore, in this scenario, $b_2\beta$ and expected profits increase with the precision of the private information. However, in the independent variance/covariance and independent errors cases, there are parameter values for which the price impact of the public disclosure decreases sufficiently with $1/\sigma_1^2$ so that $b_2\beta$, and hence expected profits, fall as the precision of the private information rises.

The effect of an increase in σ_{12}, the covariance between ε_1 and ε_2, on the informed trader's expected profits is given in the following observation.

Observation 3. In the independent variance/covariance case, the informed trader's expected profits are increasing in σ_{12}, while in the case of sufficiency they are decreasing in σ_{12}.[14]

Proof. See the appendix. ▮

[14]Analyzing the effect of an increase in σ_{12} on expected profits is not meaningful in the independent errors case.

In the independent variance/covariance case, an increase in σ_{12} increases expected profits because it results in a greater covariance between the informed trader's private information and the price change between dates 1 and 2. In the case of sufficiency, an increase in σ_{12} can only come about through an increase in σ_2^2 which, by Observation 1, decreases the informed trader's expected profits.

A related issue is whether, given the choice, the informed trader would rather collect information about the forthcoming public disclosure or information about the liquidating value of the firm. Insight into this question is provided by the following observation.

Observation 4. Holding constant the precision of the public signal, $1/\sigma_2^2$, *the informed trader's expected profits are* (*weakly*) *greater when his signal is identical to the public disclosure than when it is equal to the firm's liquidating value.*

Proof. When θ is identical to the public signal, the parameters σ_1^2, σ_2^2, and σ_{12} are equal and positive. In constrast, when θ is identical to the firm's liquidating value, the parameters σ_1^2 and σ_{12} both equal 0. Imposing these parameter restrictions separately on (15), the observation follows from straightforward algebra. ∎

Perfect foreknowledge of the forthcoming public disclosure is more valuable to the informed trader than perfect information about the firm's liquidating value as a consequence of the trader's short-term horizon, in that the ability to predict the price change over his horizon is more valuable to him than the ability to predict it over a longer time period. Observation 4 thus formalizes the notion of Keynes (1936, pp. 154–157), Hirshleifer (1971), and Hakansson (1977), that it is more profitable to forecast a forthcoming public disclosure than to forecast the firm's liquidating value.

The following observation will prove useful when the informed trader's optimal investment in information is analyzed.

Observation 5. (a) *In all three scenarios, the increase in the informed trader's expected profits resulting from an increase in the precision of his private information,* $1/\sigma_1^2$ (*for those parameter values in which expected trading profits increase with precision*), *is increasing in* p *and* $1/\sigma_2^2$. (b) *In the independent variance/covariance case, the increase in expected profits resulting from an increase in the covariance between the errors in the public and private signals,* σ_{12}, *is increasing in* p *and* $1/\sigma_2^2$.

Proof. See the appendix. ∎

This observation captures the notion that incremental investment in private information collection (in terms of either precision or covariance with the public signal) becomes more valuable the more likely the information is to be reflected in the subsequent market price through a public disclosure.

It should be noted that part (b) of Observation 5 refers only to the independent variance/covariance case. This is because it is not meaningful for the informed trader to choose σ_{12} in the other two scenarios. In the case of sufficiency, where σ_{12} is equal to σ_2^2, if the informed trader was allowed to choose σ_{12}, he would also be given discretion over the nature of the public signal. In the independent errors case, σ_{12} is constrained to equal zero.

5. Endogenous private information collection

In the previous section, the characteristics of the informed trader's private information were exogenously given. It is reasonable to expect, however, that the trader can affect the variance of his signal or its covariance with the public information (or both) by varying the magnitude of his investment in information acquisition activities. The first part of this section considers the informed trader's optimal choice of σ_1^2, denoted by σ_1^{2*}, when this is the parameter under his control. The second part focuses on the optimal level of σ_{12}, denoted σ_{12}^*, when this is the informed trader's choice variable.

Consider first the optimal choice of σ_1^2 in the independent variance/covariance setting and in the independent errors case. Denote by $C_1(\sigma_1^2)$ the investment in information acquisition activities that the informed trader must make to obtain a private signal with precision $1/\sigma_1^2$. $C_1(\sigma_1^2)$ is assumed to have the properties that $C_1'(\sigma_1^2) < 0$, $C_1''(\sigma_1^2) > 0$, and $C_1'(\sigma_1^2)$ approaches minus infinity (zero) as σ_1^2 approaches zero (infinity). These conditions imply that a signal of higher variance is less costly than one of lower variance and that the cost decreases with the variance at a decreasing rate. They also ensure that there is an interior optimum for σ_1^2. The informed trader's optimization problem is then to choose the σ_1^2 that maximizes his expected trading profits, less the cost of information collection,[15]

$$\sigma_1^{2*} = \arg\max \left\{ p \left[\frac{\sigma^2}{2(\sigma^2 + \sigma_1^2)^{1/2}} - \frac{\sigma^2(\sigma_2^2 - \sigma_{12})(\sigma^2 + \sigma_1^2)^{1/2}}{2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2) - (\sigma^2 + \sigma_{12})^2} \right] \sigma_u - C_1(\sigma_1^2) \right\}. \tag{19}$$

[15]There may be more than one local maximum, depending on the shape of $C_1(\sigma_1^2)$. If so, the value of σ_1^2 chosen by the informed trader must not only satisfy the first- and second-order conditions from (19) but must also be the global maximum.

For the remainder of this analysis we will refer to expected trading profits net of the cost of information collection as expected *net* profits to distinguish them from the expected profits due to trading activity alone.

The effect of the parameters p and σ_2^2 on the optimal σ_1^2 is reflected in the following proposition.

Proposition 2. In the independent variance/covariance setting as well as in the independent errors case, $1/\sigma_1^{2*}$ *is increasing in both* p *and* $1/\sigma_2^2$.

This proposition is a direct result of the observation made in the previous section (Observation 5), that the effect of an increase in the precision of private information on the informed trader's expected profits is an increasing function of both p and $1/\sigma_2^2$.[16]

Determination of the informed trader's optimal level of precision when the public signal is a sufficient statistic for the private information proceeds along lines fundamentally the same as in the other scenarios. The one difference is that in the case of sufficiency, the informed trader does not have complete freedom in his choice of precision, $1/\sigma_1^2$. Specifically, the precision of his information cannot be increased above $1/\sigma_2^2$, the precision of the public disclosure. It is convenient in this case to consider that the informed trader's object of choice is the precision, $1/\sigma_3^2$, of the error, ε_3, with which the private signal reflects the forthcoming public disclosure. Focusing on $1/\sigma_3^2$ and proceeding in a manner similar to that for the cases discussed above, it is straightforward to show the following corollary.

Corollary. In the case of sufficiency, the optimal level of precision, $1/\sigma_3^{2*}$ *(as well as the optimal level of* $1/\sigma_1^2$*), is increasing in both* p *and* $1/\sigma_2^2$.

Consider now the case where the informed trader's choice variable is σ_{12}, the covariance between the errors in the public and private signals, rather than the precision of the private signal. Recall that such a choice is only possible in the independent variance/covariance case, where σ_{12} can be varied independently of the variance of the public signal, σ_2^2.[17] Let $C_2(\sigma_{12})$ denote the informed trader's

[16]While this proposition has been formally proven for a short-term trader, it is reasonable to expect that it would extend to a trader who has a long-term horizon with alternative profitable uses for his funds. An increase in the probability and precision of the public disclosure gives such a trader an incentive to expend more resources on information collection just before the public disclosure, since his immediate expected return on that investment is higher.

[17]One example in which σ_{12} can be chosen by the informed trader independently of both σ_1^2 and σ_2^2 is where the public disclosure is a managerial earnings forecast and the private signal is also a forecast of earnings. By focusing his efforts on the collection of information that more closely resembles that employed by management to estimate the firm's sales and its various expenses (for example, firm-specific rather than industry- or economy-wide information), the trader increases the covariance between the errors in the managerial and private forecasts without affecting their variances.

cost to obtain a covariance of σ_{12}. $C_2(\sigma_{12})$ is assumed to have the properties that $C_2'(\sigma_{12}) > 0$, $C_2''(\sigma_{12}) > 0$, and $C_2'(\sigma_{12})$ approaches infinity (zero) as σ_{12} approaches its maximum value of $\sigma_1\sigma_2$ (minimum value of zero).[18] These properties ensure an interior optimum for σ_{12}. The informed trader's optimization problem is to choose the σ_{12} that maximizes his expected net profits:

$$\sigma_{12}^* = \arg\max\left\{p\left[\frac{\sigma^2}{2(\sigma^2+\sigma_1^2)^{1/2}} - \frac{\sigma^2(\sigma_2^2-\sigma_{12})(\sigma^2+\sigma_1^2)^{1/2}}{2(\sigma^2+\sigma_1^2)(\sigma^2+\sigma_2^2)-(\sigma^2+\sigma_{12})^2}\right]\sigma_u - C_2(\sigma_{12})\right\}. \tag{20}$$

The following proposition is analogous to Proposition 2 and again follows directly from Observation 5.

Proposition 3. In the independent variance/covariance case, σ_{12}^* is increasing in both p and $1/\sigma_2^2$.

6. Price reactions to public announcements

That public disclosure affects private information collection has important implications for both the amount of information reflected in the firm's pre-announcement price and for the price reaction to the disclosure.[19] The expected absolute pre-announcement price change, conditional on the private information θ, $\mathrm{E}(|P_1-\bar{v}|\,|\theta)$, is given by

$$\mathrm{E}(|P_1-\bar{v}|\,|\theta) = \frac{\sigma^2|\theta-\bar{v}|}{2(\sigma^2+\sigma_1^2)}. \tag{21}$$

Eq. (21) follows directly from eqs. (8) and (10), noting that the expectation of y over all possible values of liquidity demand u is just x. Taking the expectation of (21) over all θ gives

$$\mathrm{E}(|P_1-\bar{v}|) = \frac{\sigma^2}{[2\pi(\sigma^2+\sigma_1^2)]^{1/2}}. \tag{22}$$

[18]Continuing the example of the preceding footnote, the assumption that cost is an increasing function of σ_{12} captures the notion that it is easier for the informed trader to forecast earnings using his own methods than to infer and implement those used by management.

[19]Predictions about pre-announcement price changes are also made by DF, while predictions about announcement price reactions are made by both DF and KV.

As seen from (22), the magnitude of the expected pre-announcement price change is an increasing function of the precision of the informed trader's information, $1/\sigma_1^2$. This follows because the more precise the informed trader's information, the more aggressively he acts on it, and the greater its impact on the pre-announcement price. Given the positive relation between $1/\sigma_1^2$ and both the probability and the precision of the forthcoming public signal, the following proposition results.

Proposition 4. In all three scenarios, allowing σ_1^2 to be a choice variable for the informed trader, the magnitude of the expected pre-announcement price change is positively related to both p and $1/\sigma_2^2$.[20]

Proof. The proposition follows directly from Proposition 2 and its Corollary. ▮

Pre-announcement price changes are commonly thought to be caused by news leakages from the firm's insiders as well as by their pre-announcement trading activity. While these factors may certainly contribute to any pre-announcement price movement, the analysis here reveals an additional factor, the anticipation of a forthcoming public signal and its effect on private information collection by outsiders that causes the pre-announcement price change to anticipate the public signal's information content. Further, as shown in Proposition 4, the probability and the precision of the public disclosure determine the extent to which this factor impacts the firm's pre-announcement price.

Also of interest is the expected price reaction to the release of the public signal z (where the expectation is again taken over all realizations of the liquidity demand). From (10) and (12), the expected price change conditional on z, $\mathrm{E}(|P_2 - P_1|\,|z)$, is given by

$$\mathrm{E}(|P_2 - P_1|\,|z) = \frac{\sigma^2(2\sigma_1^2 + \sigma^2 - \sigma_{12})|z - \bar{v}|}{2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2)}. \tag{23}$$

Taking the expectation of (23) over all z gives

$$\mathrm{E}(|P_2 - P_1|) = \frac{\sigma^2(2\sigma_1^2 + \sigma^2 - \sigma_{12})}{(\sigma^2 + \sigma_1^2)[2\pi(\sigma^2 + \sigma_2^2)]^{1/2}}. \tag{24}$$

[20] DF examine the effect of varying the precision of the public and private signals on the informativeness of the date 1 price, which they define as the squared correlation between the price and the private signal. In contrast to the result obtained here, they find in one of their scenarios that an increase in the precision of the public disclosure reduces the informativeness of price. In that scenario, DF hold constant the variance of the private signal. This, combined with the reduction in the covariance between the public and private signals (which arises from the increased precision of the public disclosure), leads to a decrease in the share demand of the informed traders. Consequently, there is a reduction in the extent to which the date 1 price reflects their private information.

Examination of eq. (24) yields the following proposition:

Proposition 5. In all three scenarios, allowing σ_1^2 to be a choice variable for the informed trader, the magnitude of the expected price reaction to the public announcement decreases as p increases.

Proof. The proposition follows directly from Proposition 2 and its Corollary. ▮

This result follows from the positive relation which exists between the disclosure probability and the precision of the informed trader's information. Since more information is incorporated into the pre-announcement price, there is less of a price reaction at the time of the announcement.

It is clear from (24) that the magnitude of the expected announcement price reaction is a decreasing function of the variance, σ_2^2, of that announcement, *holding all other parameters in the economy constant.*[21] This is a standard result. [See, for example, Holthausen and Verrecchia (1988).] However, if the precision of the private information is endogenously determined by the informed trader, such a conclusion need not hold. Taking the positive relation between σ_1^{2*} and σ_2^2 into account, the total effect of an increase in σ_2^2 on the expected announcement price reaction is

$$\frac{\mathrm{dE}(|P_2 - P_1|)}{\mathrm{d}\sigma_2^2} = \frac{\partial \mathrm{E}(|P_2 - P_1|)}{\partial \sigma_2^2} + \frac{\partial \mathrm{E}(|P_2 - P_1|)}{\partial \sigma_1^{2*}} \frac{\partial \sigma_1^{2*}}{\partial \sigma_2^2}. \tag{25}$$

Given that $\partial \mathrm{E}(|P_2 - P_1|)/\partial \sigma_1^2$ is positive, for $\partial \sigma_1^{2*}/\partial \sigma_2^2$ sufficiently large, (25) can also be positive. The magnitude of the expected price reaction to a public disclosure may then actually *increase* in the variance of that disclosure.

Similarly, if the covariance between the public and private signals is instead the informed trader's choice variable (which is possible only in the independent variance/covariance setting), then the effect of an increase in the variance of the public information on the expected price reaction to the announcement is still of ambiguous sign. It is given by

$$\frac{\mathrm{dE}(|P_2 - P_1|)}{\mathrm{d}\sigma_2^2} = \frac{\partial \mathrm{E}(|P_2 - P_1|)}{\partial \sigma_2^2} + \frac{\partial \mathrm{E}(|P_2 - P_1|)}{\partial \sigma_{12}^*} \frac{\partial \sigma_{12}^*}{\partial \sigma_2^2}. \tag{26}$$

Given that $\partial \mathrm{E}(|P_2 - P_1|)/\partial \sigma_{12}$ is less than 0, for negative enough $\partial \sigma_{12}^*/\partial \sigma_2^2$, eq. (26) can be positive. An increase in the variance of the public signal may again result in an increase in the price reaction to the disclosure.

[21]This statement is only meaningful in the independent variance/covariance and independent errors cases, where σ_2^2 can be varied independently of the other parameters.

7. An extension to long-term traders

In this section, the effect of a public disclosure on the value of private information to a trader with a long-term horizon is explored. As the preceding analysis has demonstrated, a public disclosure of information is necessary for investment in private information acquisition to be of any value to a short-term trader. In contrast, public disclosure is not necessary for private information to be valuable to a trader who can hold his share position in the firm until its liquidating value is revealed. In fact, the value of his information may not be enhanced at all. For example, in the case where the public signal measures the firm's liquidating value without error (so that $z = v$) or where the public and private signals are equal (so that $\theta = z$), it is straightforward to show that the expected profits of an informed trader with a long-term horizon are the same whether he trades at date 2, when the public disclosure is made, or he holds his position until date 3, when the firm is liquidated. However, there are cases where the expected profits of the long-term trader, and so the value of private information, are increased by the presence of a public disclosure. To see this, consider a slightly modified setting in which the informed trader establishes his position at date 1 as before but now has the choice of closing out his position at either date 2 or date 3. Further, in line with the preceding analysis, assume that if the trader closes out his position at date 2, the trade does not reveal anything about his private information.[22] In this setting, the following proposition can be shown.

Proposition 6. The expected profits of a long-term trader are greater when he trades at the date of the public disclosure if, and only if, $\sigma_{12} > \sigma_2^2$ *or, equivalently,* $b_1 < 0$.

Proof. In the case where the long-term trader closes out his position at date 2, his expected profits are identical to those of a short-term trader and given by $p[\lambda - b_1]\sigma_u^2$. [See eq. (17).] In the case where he holds his position until date 3, his expected profits are identical to what he would earn if $\sigma_2^2 = \sigma_{12} = 0$ and he liquidated at date 2. When σ_2^2 equals 0, it is straightforward to see that b_1 also equals 0 and that the informed trader's expected profits are given by $p\lambda\sigma_u^2$. $p\lambda\sigma_u^2$ is lower than $p[\lambda - b_1]\sigma_u^2$ if, and only if, b_1 is negative or, equivalently, $\sigma_{12} > \sigma_2^2$. ∎

As stated in the proposition, a public disclosure enhances the long-term trader's expected profits if, and only if, the private and public signals are

[22] In a more general setting, trading at date 2 would reveal to the market-maker some of the informed trader's private information, and hence would affect the price at which the informed trader closes out his position.

sufficiently highly correlated and the error in the public signal is sufficiently small (and smaller than the error in the private signal). If this is the case, the private information is more correlated with the price change at the time of the public signal than at the time of liquidation, so that the informed trader's expected profits are higher if he closes out his position at date 2, just after the public disclosure.

In the case of sufficiency, where $\sigma_{12} = \sigma_2^2$, the long-term trader's expected profits are the same whether he trades at date 2 or date 3. Consequently, the public disclosure is of no value to him. In the case where $\sigma_{12} = 0$, the long-term trader's expected profits are strictly greater if he trades at date 3 and so, again, the public disclosure does not have any value. Only in the independent variance/covariance case could the public disclosure be valuable to the long-term trader, so that he would prefer to close out his position just after the public disclosure at date 2. However, as stated in the proposition, this can only arise if b_1 is negative. While it is possible for this condition to hold, so that the more favorable news embodied in a higher order flow would cause a decrease in price, it seems unlikely to occur. It appears more likely that a greater order flow would result in a higher market price, in which case the long-term trader's expected trading profits again would not be enhanced by the public disclosure at date 2.

That the long-term trader has a (weak) preference not to trade at date 2 in both the independent errors and sufficiency cases stands in contrast to the findings of KV and DF in those settings. The difference arises because of their assumption that the informed traders are risk-averse; given their speculative positions at date 1, risk aversion induces them to trade to a more diversified position after the beliefs of all traders become more homogeneous.

Finally, it is important to recognize that the conclusion that the long-term trader's expected trading profits are not enhanced by a public disclosure at date 2 (when $b_1 > 0$) holds only if there is a zero probability that he will need to close out his position before the liquidating value of the firm is revealed. If he will be required to liquidate at date 2 with positive probability, then his expected trading profits would be enhanced by a public disclosure. The analysis of sections 4 and 5 would be applicable in this case.

8. Summary and conclusions

The central result of this paper is that a forthcoming public disclosure by a firm has the effect of stimulating investment in private information acquisition in anticipation of that disclosure. As noted, this result stands in contrast to that of Diamond (1985) and is a consequence of allowing the informed trader to trade on his private information before the public disclosure and recognizing that his investment horizon is of finite duration. Specifically, it was demonstrated in this setting that the greater the probability of a public disclosure or its precision, the more precise the information collected by the informed trader in advance of the disclosure. It was further shown that in the independent variance/covariance setting the informed trader's expected profits are increasing in the covariance between the private signal and the forthcoming public disclosure. Consistent with this, the informed trader prefers to receive perfect information about the public disclosure rather than perfect information about the firm's liquidating value.

The results of this model yield predictions concerning security analysts' information collection activities, to the extent that they are geared toward providing private information to clients. That this is an important factor in their information collection decisions is supported by the fact that security analysts typically do not publicly disclose their forecasts before privately revealing them to their firm's favored clients. Our model predicts that security analysts would have a preference for collecting information that is more highly correlated with a firm's forthcoming disclosures. This is consistent with the observation that security analysts tend to forecast earnings to a greater extent than other financial variables, such as cash flow, that are also released on a quarterly basis. The value of forecasting any of these variables stems from the covariance of the forecast with the price change at the time of the subsequent quarterly report, as can be seen in eq. (16). There is considerable empirical evidence in support of a greater association between earnings forecast errors and contemporaneous stock price changes than for any other financial statement variable, suggesting that private information about forthcoming earnings is more valuable to investors than private information about forthcoming cash flows.

Finally, it was shown that an increase in the precision of a public disclosure results in an increase in the magnitude of the pre-announcement price change and, potentially, a decrease in the price reaction to the announcement itself. This result had not previously been recognized in either the empirical or theoretical accounting literature and provides an avenue for direct empirical testing of the model developed here.

Appendix

Proof of Proposition 1

To prove the proposition it must be shown that (a) the pricing rules are optimal given the market maker's conjecture of the informed trader's demand, and (b) given the pricing rules, the conjectured demand function is optimal.

(a) Conjecture that $x = \beta(\theta - \bar{v})$, where

$$\beta = \operatorname{sign}(b_2)\left(\frac{\sigma_u^2}{\sigma^2 + \sigma_1^2}\right)^{1/2}.$$

Given a competitive market-maker, the price at each date is set equal to his current expectation of the firm's end of period value. Using Bayes' rule, this means that

$$P_1 = \mathrm{E}(v|y) = \bar{v} + \lambda y,$$

where

$$\lambda = \frac{\mathrm{cov}(x+u, v)}{\mathrm{var}(x+u)} = \frac{\beta\sigma^2}{\beta^2(\sigma^2 + \sigma_1^2) + \sigma_u^2} = \frac{\mathrm{sign}(b_2)\sigma^2}{2\sigma_u(\sigma^2 + \sigma_1^2)^{1/2}}. \tag{A.1}$$

Similarly,

$$P_2 = \mathrm{E}(v|y,z) = \bar{v} + b_1 y + b_2(z - \bar{v}).$$

To solve for b_1 and b_2, note that

$$\mathrm{cov}(v,y) = b_1 \,\mathrm{var}(y) + b_2 \,\mathrm{cov}(y,z), \tag{A.2}$$

$$\mathrm{cov}(v,z) = b_1 \,\mathrm{cov}(y,z) + b_2 \,\mathrm{var}(z).$$

Solving for b_1 and b_2 and substituting yields

$$b_1 = \frac{\mathrm{sign}(b_2)\sigma^2(\sigma^2 + \sigma_1^2)^{1/2}(\sigma_2^2 - \sigma_{12})}{\sigma_u[2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2) - (\sigma^2 + \sigma_{12})^2]}, \tag{A.3}$$

$$b_2 = \frac{\sigma^2(2\sigma_1^2 + \sigma^2 - \sigma_{12})}{2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2) - (\sigma^2 + \sigma_{12})^2}.$$

This proves part (a).

(b) The optimal x is chosen to maximize

$$\begin{aligned}\max_x \mathrm{E}(\pi|\theta) &= \mathrm{E}((P_2 - P_1)x(\theta)) \\ &= p\mathrm{E}[(b_1 - \lambda)(x+u) + b_2(z - \bar{v})|\theta]x \\ &= p[(b_1 - \lambda)x + b_2(\mathrm{E}(z|\theta) - \bar{v})]x.\end{aligned}$$

The first-order condition is given by

$$\frac{\partial \mathrm{E}(\pi|\theta)}{\partial x} = p[2(b_1 - \lambda)x + b_2(\mathrm{E}(z|\theta) - \bar{v})]. \tag{A.4}$$

Rearranging gives

$$x = \frac{b_2(\mathrm{E}(z|\theta) - \bar{v})}{2(\lambda - b_1)}. \tag{A.5}$$

We can write

$$\mathrm{E}(z|\theta) = \bar{v} + \gamma(\theta - \bar{v}),$$

where

$$\gamma = \frac{\sigma^2 + \sigma_{12}}{\sigma^2 + \sigma_1^2}.$$

Thus, $x = \beta(\theta - \bar{v})$, where

$$\beta = \frac{b_2\gamma}{2(\lambda - b_1)} = \frac{b_2(\sigma^2 + \sigma_{12})}{2(\lambda - b_1)(\sigma^2 + \sigma_1^2)}.$$

Using the definitions of λ, b_1, and b_2 and simplifying yields

$$\beta^2 = \left(\frac{\sigma_u^2}{\sigma^2 + \sigma_1^2}\right).$$

To determine whether β equals the positive or negative root of $(\sigma_u^2/(\sigma^2 + \sigma_1^2))$, note that the second-order condition of the informed trader's maximization problem requires that $\lambda > b_1$. Rewriting (A.2),

$$b_1 = \frac{\operatorname{cov}(v, y) - b_2 \operatorname{cov}(y, z)}{\operatorname{var} y}.$$

Subtracting this expression from (A.1) gives

$$\lambda - b_1 = \frac{b_2 \operatorname{cov}(y, z)}{\operatorname{var}(y)}. \tag{A.6}$$

The variance of y is always positive, and the covariance between y and z is positive (negative) when β is positive (negative). Thus, $\lambda - b_1$ is positive if β is positive (negative) when b_2 is positive (negative). [The denominator of b_2 in (A.3) is always positive so b_2 is positive if, and only if, $2\sigma_1^2 + \sigma^2 > \sigma_{12}$.] This means that

$$\beta = \mathrm{sign}(b_2)\left(\frac{\sigma_u^2}{\sigma^2 + \sigma_1^2}\right)^{1/2}.$$

This proves part (b).

Proof of Observation 3

To find the sign of $\partial \mathrm{E}(\pi)/\partial\sigma_{12}$, note that

$$\frac{\partial(\lambda - b_1)}{\partial\sigma_{12}} = \frac{-\partial b_1}{\partial\sigma_{12}},$$

$$\frac{\partial b_1}{\partial\sigma_{12}} = \frac{-b_1}{\sigma_2^2 - \sigma_{12}} + \frac{2b_1(\sigma^2 + \sigma_{12})}{2(\sigma^2 + \sigma_1^2)(\sigma^2 + \sigma_2^2) - (\sigma^2 + \sigma_{12})^2}. \tag{A.7}$$

If $b_1 < 0$, then (A.7) is easily seen to be negative, so that $\partial(\lambda - b_1)/\partial\sigma_{12} > 0$. To show that this is also true for $b_1 > 0$, note that $\partial b_1/\partial\sigma_{12} < 0$ at both $\sigma_{12} = \sigma_2^2$ and $\sigma_{12} = 0$. This means the $\partial b_1/\partial\sigma_{12}$ can be positive for some σ_{12} between 0 and σ_2^2 only if (A.7) equals zero at two (or more) such values of σ_{12}. Setting (A.7) equal to zero and solving for σ_{12},

$$\sigma_{12} = \sigma_2^2 \pm \sqrt{\sigma_2^4 - 2\sigma_1^2\sigma_2^2 - 2\sigma^2\sigma_1^2 - \sigma^4}.$$

The positive root implies that $\sigma_{12} > \sigma_2^2$, so there is at most one value of σ_{12} between 0 and σ_2^2 for which (A.6) equals zero. Therefore $\partial b_1/\partial\sigma_{12}$ can never be positive.

Proof of Observation 5

It is trivial to show that the increase in the informed trader's expected profits resulting from an increase in either the precision of the private information (for those parameter values in which expected profits increase with precision) or its covariance with the public signal is increasing in p. To prove the remainder of the observation, it must be shown that $\partial^2 \mathrm{E}(\pi)/\partial\sigma_2^2\partial\sigma_1^2 > 0$ in all three scenarios and that $\partial^2 \mathrm{E}(\pi)/\partial\sigma_2^2\partial\sigma_{12} < 0$ in the independent variance/covariance setting. To find the sign of $\partial^2(\mathrm{E}(\pi))/\partial\sigma_2^2\partial\sigma_1^2$ in both the independent variance/covariance case as well as when $\sigma_{12}=0$, note first that

$$\mathrm{E}(\pi)=p(\lambda-b_1)\sigma_u^2=p\left(\frac{b_2(\sigma^2+\sigma_{12})}{2\beta(\sigma^2+\sigma_1^2)}\right)\sigma_u^2$$

and that

$$\frac{\partial(\mathrm{E}(\pi))}{\partial\sigma_2^2}=p\left(\frac{-b_2(\sigma^2+\sigma_{12})}{2\beta(\sigma^2+\sigma_1^2)}\cdot\frac{2(\sigma^2+\sigma_1^2)}{2(\sigma^2+\sigma_1^2)(\sigma^2+\sigma_2^2)-(\sigma^2+\sigma_{12})^2}\right)\sigma_u^2.$$

Define

$$z\equiv\frac{2(\sigma^2+\sigma_1^2)}{2(\sigma^2+\sigma_1^2)(\sigma^2+\sigma_2^2)-(\sigma^2+\sigma_{12})^2}.$$

Then

$$\frac{\partial(\mathrm{E}(\pi))}{\partial\sigma_2^2}=-\mathrm{E}(\pi)z. \tag{A.8}$$

Differentiating (A.8) with respect to σ_1^2 gives

$$\frac{\partial^2(\mathrm{E}(\pi))}{\partial\sigma_2^2\partial\sigma_1^2}=-\frac{\partial\mathrm{E}(\pi)}{\partial\sigma_1^2}z-\mathrm{E}(\pi)\frac{\partial z}{\partial\sigma_1^2}.$$

That $\partial^2\mathrm{E}(\pi)/\partial\sigma_2^2\partial\sigma_1^2>0$ follows immediately since

$$z>0,\quad \mathrm{E}(\pi)>0,\quad \partial z/\partial\sigma_1^2<0,$$

given the assumption that $\partial(\mathrm{E}(\pi))/\partial\sigma_1^2<0$.

For the case of sufficiency, $\partial^2\mathrm{E}(\pi)/\partial\sigma_2^2\partial\sigma_1^2$ is equal to $\partial^2\mathrm{E}(\pi)/\partial(\sigma_1^2)^2$, since a change in σ_2^2 elicits an equal change in σ_1^2. Referring to expression (17) and noting that $b_1=0$ in the case of sufficiency (since $\sigma_2^2=\sigma_{12}$), the sign of $\partial^2\mathrm{E}(\pi)/\partial(\sigma_1^2)^2$ is the same as that of $\partial^2\lambda/\partial(\sigma_1^2)^2$. Straightforward differentiation shows the sign of this to be positive.

To find the sign of $\partial^2\mathrm{E}(\pi)/\partial\sigma_2^2\partial\sigma_{12}$ in the independent variance/covariance case, differentiate (A.8) with respect to σ_{12}. This gives

$$\frac{\partial^2\mathrm{E}(\pi)}{\partial\sigma_2^2\partial\sigma_{12}}=-\frac{\partial\mathrm{E}(\pi)}{\partial\sigma_{12}}z-\mathrm{E}(\pi)\frac{\partial z}{\partial\sigma_{12}}.$$

That $\partial^2 E(\pi)/\partial\sigma_2^2 \partial\sigma_{12} < 0$ follows immediately since $z > 0$, $E(\pi) > 0$, $\partial(E(\pi))/\partial\sigma_{12} > 0$, and $\partial z/\partial\sigma_{12} > 0$.

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